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ASSIGNMENT OF CREWS TO TRAIN IN RAILROAD NETWORK

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ASSIGNMENT OF CREWS TO TRAIN IN RAILROAD NETWORK

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Abstract

In this paper we consider the assignment of personnel of a railroad company to trains. As well known, the complexity of the problem makes it pratically impossible to take into account, in a mathematical model, all the real aspects. For this reason, here an approach has been adopted which links a rigorous model of a certain part of the problem to an approximate treatment of other parts. It yields a "good" approach, which takes into ac= count most real aspects and does not require an unreasonable com= puter time, as a good deal of actual applications have shown.

1 - Introduction, notation and definition of problem

A railroad network is generally partitioned into districts; each of them provides the personnel required for a given set of trains. For each district the personnel is located in a "residence"; with each residence there is associated a set of quadruplets: starting and arrival times and stations names. Our problem is that of optimizing the assignment of shift schedules for the train engineers with the preassigned train time table.

The personnel is assumed to work according to shifts belonging to a given set. The assignment of personnel to trains consists in partitioning the set of trains into subsets and in establishing a cyclic order for each subset.

The personnel of every residence works sequentially and repeatidly on all the trains of the shift.

In practice, the set of the trains, to be assigned to the agents of a certain residence, is selected by empiric organizational criteria which take into account several conditions among which, e. g., the specialization of the engines to be used, the nature of the trains, and so on.

Once the subset of the trains has been assigned, the problem consists in choosing, among the (n-1)! cyclic permutations of the n trains, the one which, as far as possible, minimizes the time during which the personnel remains unemployed between one train and the next and consequently minimizes the number of agents used.

In the construction of a shift, a series of limits of a physical nature and of a normative union nature must be respected, so that not all cyclic permutations constitute an admissible shift. For example, if the place of arrival of a train does not coincide with that of departure of the next, the interval of time between the two services must allow for the movement of the agent from one place to the other, and this movement must not conflict with any normative condition.

We shall quote some union rules governing the italian rail service, which will be described in greater detail in section 2:

- An agent cannot work on a night shift (from 0 h. to 5 h.) more than two times consecutively (3 times if one of the night services lasts less than one hour).
- between two rest periods in residence (at home) not more than 30 hours can pass; the limit is of 32 hours if the time extra is used by the agent to return to his residence with a trip off duty.

- The total amount of the driving shift lengths is not to exceed the time of 7 h. 30 m., after a residence-rest, and of 4 h. 30 m., after a rest outside residence.
- The rest outside residence must last between 7 and 14 hours.

In the case of the italian railways, to which the procedure here described has been applied, the characteristics of the normative rules can be grouped into two fundamental classes:

- normative rules relating to the work period carried out in the interval included between two rest periods in residence.
- normative rules relating to the repeated succession of work periods defined above.

We have considered as a distinct entity, or "work period", a group of operations, carried out by a train engineer, according to certains conditions and in the interval between two rest periods. The work which a train engineer carries out in the interval between two periods of residence (at home) rest is defined as "service". Collecting together the daily work in residence in a set of services it is possible to think of the organization of the work, in a given period, as a sequence of services.

Since it is possible to partition the set of union rules in two distinct subsets concerning, the one, the modalities for defining the services and, the other, their successions for each train engineer, the programme in question may be divided into two parts, each related to only one of the above mentioned subsets. In both cases the methods of solution use optimization criteria based on the work efficiency.

Given the fact that a rest period outside residence must always be followed by a rest period in residence, a service Z can be constituted by one or two work periods L, in this last case separated by a rest outside residence R. This can be stated shortly in the following way:

$$Z = L' \left[, R , L'' \right] \qquad (1.1)$$

We use square brackets to indicate that the elements contained in them may also be missing. It is understood that in the formula (1.1), as in the definition which follow, there is a time relation of succession between the "events", from the left to the right: therefore L' precedes R and this, in turn, precedes L".

In what follows, the curly brackets will be used in order to point out that what is contained in them may be repeated more than once.

We shall now define some elements of the work period of a train engineer:

1) The "stop" ("waiting hours"). It is a non-active time interval $S < 24$ h. . To all normative effects, the following intervals are considered work: stop in residence lasting ≤ 1 h.; the day stops outside residence lasting ≤ 2 h.; the night stops outside residence ≤ 3 h... These types of stops are also called "working stops".

2) The "accessory operations". These are checking and control operations which are carried out by driving personnel before and after a trip. The accessory operations, which precede and follow a trip, are indicated respectively by A' e A".

3) The "driving". This is the engine driving work. This type of work is indicated by C.

4) The "travelling by carriage". A train engineer can be ordered to go from one locality to another to take up service or to come back to residence. This is indicated usually with the symbol W. In particular, let us call W_R the carriage travel, made by the train engineer as passenger, to go back to his residence when he has finished the service, and with W_N every other type of travel by carriage.

To define the working period it is necessary to specify also the succession of the various working activities within the period itself. For this purpose two characteristic types of travel, from the union point of view, may be put in evidence and are called "simple trip" and "compound trip":

1) "Simple trip". It is the set of work operations (driving and accessory operations or travelling by carriage) which are carried out by a train engineer to go from one locality to another. A simple trip is preceded and followed by rests or stops. It is therefore:

$$V_s = \begin{Bmatrix} A' , C , A'' \\ W \end{Bmatrix} ,$$

where V_s is a simple trip.

2) "Compound trip". This is the set of various simple trips separated by working stops. Indicating by V_c the compound trip, it is:

$$V_c = \{ V_s , S_L , V_s \} ,$$

where S_L is a working stop.

The structure of a working period can now be indicated:

$$L = V_c , S_R' , V_c ,$$

where S_R is a non-working stop.

The union rules fixe the minima and the maxima allowed for the different types of productive and non-productive activity, such as, e. g., the stops and the rests. We indicate with $d(a)$ the duration of any activity a.

The maximum duration of service is:

$$d(Z) = d(L') + d(R) + d(L'') \begin{cases} \leq 30 \text{ h.} \\ \leq 32 \text{ h.} \end{cases}$$

The maximum limit of 32 h. also holds for those services in which the reentering in residence of the train engineer takes place by carriage, i. e. Z ends with a W_R.

For the working period it is:

$$d(L) \leq 11^h\, 05^m .$$

From the (2.4) we may note that, from the point of view of union rules, the effective work L_E, in a working period, is given by the sum of the compound trips V_c:

$$d(L_E) = \sum d(V_c) .$$

For the duration of work L_E maximum limits have also been fixed which vary according to the type of service Z:

1) if the service is of the type $Z = L$, it is

$$d(L_E) \leq 9^h 05^m ;$$

2) if it is $Z = L' , R , L''$;

(1.2) $$d(L') \leq 7^h 05^m ;$$

3) and for L", if it ends with a trip by carriage:

$$d(L_E'') \leq 7^h 05^m + d(S_L^*) + d(W_R),$$

since S_L^* is the working stop ($d(S_L^*) \geq 0$) preceding W_R.

4) If, finally, L" does not end with a W_R, (1.2) also holds for L".

Other maxima have been fixed for the duration of the driving. A type of working stop, which is at the same time considered as a time of effective driving, is the one corresponding to an interruption of driving lasting $\leq 30^m$, more precisely, if between two simple trips $V_c(i)$ and $V_c(i+1)$ there is a driving interruption $S_c(i,i+1)$ which lasts:

(1.3) $$d\, S_c(i,i+1) = d\, A''(i) + d\, S(i,i+1) + d\, A'(i+1) \leq 30^m$$

then the driving duration of the compound trip

$$V_c(i,i+1) = V_s(i) \; , \; S(i,i+1) \; , \; \tilde{V}_s(i+1)$$

is

$$d\left[C(i,i+1)\right] = d\left[C(i)\right] + d\left[S_c(i,i+1)\right] + d\left[C(i+1)\right] \; .$$

If, on the other hand, formula (1.3) is not satisfied, then $d\left[S_c(i,i+1)\right] = 0$.

The maxima allowed concerning the driving duration also depend from the type of service; if $Z = L$, then

$$d(C) \leq 7^h 30^m \; .$$

Otherwise both for L' and for L" it is:

$$d(C) \leq 4^h 30^m \; .$$

Since the localities, belonging to a district, are always connected to the residence by many trains, the number of the service sets which can be formed is almost always quite large. It is therefore necessary to look for the one which has the highest work efficiency.

The problem of the search for the optimum service set is not, in general, such as to be solved in an absolute manner. It is nevertheless possible to find a satisfactory solution by subdividing the problem into two parts: the search for the optimum set of the round trips, with or without rest outside residence, and the search for the optimum set of services consisting of round trips. In practice the large majority of services constists of a single round trip and so the solution found is very near the optimal one.

2 - A linear programming formulation of a sub-problem

The problem of the search of the service set which ensures a high work efficiency of the train engineers may be brought back to a set of problems of linear programming, the solution of which approximizes noticeably the absolute optimum.

As has already been said, every train, assigned to a given shed residence, is characterized by a departure locality and time, and by an arrival locality and time. The shed residence is not always a departure locality or an arrival locality. There are in fact localities which, although depending on a residence, are not connected to it by trips programmed in the time-table. In these cases it is necessary to make travel off duty the train engineers, in order to make them arrive at the departure locality of the train or to make them reenter in the residence. Since sometimes the number of the trains which leave a locality is not equal to that of the trains which come back (and this because a seasonal accumulation of trains in some areas of the railway network is foreseen), it is therefore necessary, in this case too, to make the personnel travel by carriage off duty.

In the solution method presented, in order to obviate these anomalous cases, there should be taken into account, besides the time schedule of the residence, also the general time table of the railways in order to have at our disposal all the elements necessary to be able to evaluate the most economical connections between on duty and off duty trips. This is not part of the aim of the present work and therefore we shall limit the problem by introducing the following limitations:

1) in the set of trips, belonging to each shed residence, the residence itself is always either the departure of the arrival locality.
2) In every locality, a certain number of trains leave and arrive daily.

In order to make the composition of the round trips of the personnel easier, for the movements off duty we shall use, besides a certain number of passenger trains, the trains of the problem itself, if they have the requisites required.

For the sake of simplicity, we shall, from now on, consider the times of departure and of arrival of the trains, comprehensive of the accessory times. We shall therefore consider the time-table of the train engineers instead of the one of the passengers.

For the composition of a service, the trips which originate in the residence, directed to a certain locality may be connected only by trips which return from that locality to the residence. However all the trips which converge on the residence, from whatever locality they come, may be connected among themselves. Therefore the problem of the composition of the round trips, from the residence to the terminal localities of the district, may be decomposed in as many distinct problems as there are terminal localities.

Let us indicate with m_1 and n_1 the number of trips, respectively of out-going and back-going trips, which connect the shed residence with the k-th locality the carrying out of which is assigned to the shed personnel. With m_2 and n_2 we shall indicate the number of trips, respectively of out-going and back-going trips, which can, in the case, be used to make the train engineer travel off duty. For the sake of simplicity of calculation, let us include among the latter, as many fictitious trips, with the same time-table, as are the trips of the deposit that have the required requisites to carry out such a service.

Let us indicate with u_i the i-th out-going trip, from the residence to the k-th locality of the district, and with v_j the j-th back-going trip to the residence from the same locality. With $s(u_i)$, $s(v_j)$ and $a(u_i)$, $a(v_j)$ we shall indicate the starting and arrival times of both types of trips. The duration of the stop S(i, j) called "waiting hours", subsequent on the round trip (u_i, v_j) is

$$S(i,j) = \begin{cases} s(v_j)-a(u_i) & \text{if both of them belong to the same day} \\ 24^h-a(u_i)+k.24^h+s(v_j) , & k = 0,1,2, \ldots , \text{otherwise.} \end{cases}$$

For $S(i,j) < 24^h$, as we shall assume in what follows, then

$$S(i,j) \equiv s(v_j) - a(u_i) \quad (\text{ mod. } 24^h) \quad .$$

For the sake of simplicity in writing, we shall indicate with $|a|_g = b$; the relation a congruent b, module g.

We shall indicate by x_{ij} the number of agents travelling, in service or in carriage, on the i-th trip u_i and returning to the residence, in service or in carriage, on the j-th trip v_j.

Let us suppose, for each variable, a cost c_{ij}, function of u_i and v_j. The search for the most economical composition of round trips, from the residence to the terminal locality k, is reducible to a problem of linear programming whose objective function is

$$\min z = \sum\sum c_{ij} \; x_{ij}$$

and the linear constraints are:

$$(1) \qquad \sum_{j=1}^{n_1+n_2} x_{ij} = 1 \qquad i = 1, 2, \ldots , m_1 \quad ;$$

which expresse the fact that the agent who was on duty on u_i returns to the residence by one of the n_1+n_2 trips;

$$(2) \qquad \sum_{j=1}^{n_1} x_{ij} \leqslant n_1 \qquad i = m_1+1, m_2+2, \ldots , m_1+m_2 \quad ;$$

these constraints fix at n_1 the maximum number of agents who travel on the trip u_j, off duty.

In the same way:

$$(3) \qquad \sum_{i=1}^{m_1+m_2} x_{ij} = 1 \qquad j = 1, 2, \ldots , n_1 .$$

(4) $$\sum_{i=1}^{m_1} x_{ij} \leq m_1 \qquad j=n_1+1, n_1+2, \ldots, n_1+n_2 \quad .$$

Having set $g=24^h$ and $d_{ij}= |a(v_j)-s(u_i)|_g$, the values which c_{ij} may assume depend on the following cases:

- if the agent was on duty during u_i and v_j, then

$$c_{ij} = \frac{|s(v_j) - a(u_i)|_g}{d_{ij}}$$

- if the agent was on duty on u_i and has returned by carriage on v_j, then:

$$c_{ij} = \frac{|a(v_j) - a(u_i)|_g}{d_{ij}}$$

- if the agent has travelled by carriage on u_i and has returned driving on v_j:

$$c_{ij} = \frac{|s(v_j) - s(u_i)|_g}{d_{ij}}$$

- if the combination, between the i-th train for the out-going trip and the j-th train for the return, is not compatible:

$$c_{ij} = M \quad ,$$

where M is a very large positive integer.

Our aim is now to bring back the previous problem to the clas_ sical formulation of the linear transportation problem of Hitchcock-K opmans. For this purpose we shall introduce the artificial variables x_{ij} , $i= m_1+1, m_1+2, \ldots , m_1+m_2$, $j = n_1+1, n_1+2, \ldots , n_1+n_2$, to which we assign, in the object function, a weight $+\infty$, in order to ensure that, in the optimal solution, all of them are equal to zero.

Let us then introduce the variables $x_{m_1+m_2+1,j}$, $j = 1, 2, \ldots$ $\ldots , n_1+n_2+1$, and assign to them, in the object function, a weight $+\infty$ to the first n_1, which consequently turn out to be artificial variables, and a weight zero to the remaining n_2+1 variables, whose meaning is that of slack variables. This introduction allows to trans form relations (4) into equations. In the same manner let us intro_ duce the variables x_{i,n_1+n_2+1} , $i = 1, 2, \ldots , m_1+m_2+1$, giving to the first m_1 variables (artificial variables) a weight $+\infty$, and to the remaining m_2+1 variables (slack variables) a weight zero.

By introducing such variables, we now can look at the problem as a classical transportation problem where the sum of the constant terms of the row equations may be not equal to the sum of the constant terms of the column equations. These sums amount to

$$m_1 + m_2 . n_1 \quad \text{and} \quad n_1 + n_2 . m_1$$

respectively. If these two numbers are equal, the problem is formulated correctly, otherwise if, e. g., the first is larger than the second one, a row equation is added, or a column equation in the other case, assigning to the corresponding slack variables a weight zero and setting, in the first case:

$$m = m_1 + m_2 + 2 \quad , \quad n = n_1 + n_2 + 1$$

and in the second one:

$$m = m_1 + m_2 + 1 \quad , \quad n = n_1 + n_2 + 2$$

where m and n are the dimensions of the transportation array.

Finally, by defining c_{ij} , i = 1, 2, ... , m , j = 1, 2, ... , n , as indicated above, the problem (1), (2), (3) and (4) is in the canonical form.

3 - A method of composition of round trips into services

Let us indicate by $\{V_i\}$, i = 1, 2, ... , b , the set of round trips obtained with the method presented in the previous section. Given the limits fixed for the duration of the working period and for the duration of the work, a service, consisting of a single working period, cannot be formed by more than a couple of round trips; whereas, a service of the type Z = L', R, L" can be formed even by three round trips, the second of which must necessarily imply a rest outside residence.

In the case of a service consisting of two round trips, the stop between the first and the second is certainly shorter than the duration of the rest in residence, whose minimum duration is 16 h.. Since the stops successive to the round trips, whatever the connections between them, have a constant incidence on the global balance of the waiting time, it follows that, whenever possible, it is useful to carry out the highest number of connections between round trips.

Let us order the V_i according to the increasing values of the starting times and let us recall the indices; $\beta = [V_i]$, i = 1, 2, ..

.. , b , is then the ordered set of the round trips in a shed residence. Let $\gamma = \{1, 2, \dots, b\}$ be the set of the indices of the elements of β and $\gamma' \in \gamma$ the set of indices of the V_i which imply a rest outside residence. Let us now form all the possible couples among the elements of β and let $\Psi = \beta \times \beta$. From Ψ we shall discard all the couples which are not compatible with the union constraints, and let Ψ' be the new set:

$$(3.1) \qquad \Psi' = \{(V_i , V_j)\} .$$

Certainly $i \neq j$ since $d(L) \leq 11^h 05^m$. From what has been said above, a service cannot be formed by more than three round trips and, therefore, the combination of a pair of couples of round trips:

$$(3.2) \qquad [(V_i , V_j) , (V_h , V_k)]$$

would have a duration not compatible with the maxima fixed by the union norms. If however in the (3.1) i, j, h and k are not all distinct, and if it is:

$$(3.2) \qquad (j = h \in \gamma') \wedge (i, k \in \gamma - \gamma')$$

then the service is of the type Z = L',R,L", in which the working periods L' and L" are both formed by three round trips.

We may therefore conclude that it is possible to find services formed by triplets of round trips looking for them among the pairs of couples of round trips which satisfy the condition (3.2).

The search for sets of services, formed respectively by couples and triplets of round trips, whose working efficiency is maximal, may be brought back to two distinct problems of linear programming.

Let us consider all the possible couples (V_i, V_j) which satisfy the condition foreseen in the union regulations, discarding however, because of (3.2), all those which for a given $j \in \gamma'$ (or $i \in \gamma'$) do not have at least one corresponding couple (V_j, V_k) (or (V_k, V_i)).

Let us then form all the possible pairs of couples, as in (3.1) satisfying the condition (3.2). If we call x_{ijk} the number of services corresponding to the triplet (V_i, V_j, V_k); then the following constraints characterize a set of feasible services:

$$(3.3) \qquad \sum_{i \in \bar{\gamma}'} \sum_{k \in \bar{\gamma}'} x_{ijk} \leq 1 \qquad \text{for every } j \in \gamma'$$

where $\bar{\gamma}' = \gamma - \gamma'$. Therefore the (3.3) does not consider the possibility of forming more than a triplet with the round trips V_j.

In the same manner it is, for V_i and V_k:

$$(3.4) \qquad \sum_{j \in \gamma} \sum_{k \in \bar{\gamma}'} x_{ijk} \leq 1 \qquad \text{for every } i \in \bar{\gamma}'$$

(3.5) $\sum_{i \in \beta'} \sum_{j \in \beta'} x_{ijk} \leq 1$ for every k

since it is evidently:

(3.6) $x_{ijk} \in \{0, 1\}$.

Let $\bar{S}(i,j)$ be the stop in residence, as a consequence of the connection (V_i, V_j) (difference between the starting time of V_j and the arrival time of V_i), to each variable x_{ijk} there corresponds a waiting time:

$$\bar{S}(i,j,k) = \bar{S}(i,j) + \bar{S}(j,k) ,$$

since the aim is that of obtaining the maximum number of connections with the minimum waiting time, to the x_{ijk} is assigned a cost

$$c_{ijk} = \left[\bar{S}(i,j,k)\right]^{-1} + c^o$$

where

$$c^o = \sum \left[\bar{S}(i,j,k)\right]^{-1} ;$$

that is to say the coefficients c_{ijk} are increased by a very large constant c^o, so that the number of formed triplets be, in any case, a maximum. In this case too, $\bar{S}(i,j,k) = \infty$ if the coupling of two round trips is a service not compatible with union rules.

The objective function is therefore

$$\max \quad z = \sum c_{ijk} \cdot x_{ijk} .$$

Let $\beta' \subseteq \beta$ be the set of the round trips which form the above mentioned services; among the elements $\beta' = \beta - \beta'$ one has to look for the most numerous set of services formed by couples of round trips with the maximum efficiency.

Let us recall the indices of the elements of $\beta' = \{V_i$, $i = 1, 2, \ldots, g'$ $(g' \leq g)$. Let $\Phi = \{1, 2, \ldots, g'\}$ be the set of the indices of the elements of β' , and x_{ij}, the number of connections which can be carried out between the i-th and the j-th round trip; we may deduce the following relations:

$$\sum_{\substack{i \in \Phi \\ i \neq j}} x_{ij} \leq 1 \qquad \text{for every } j \in \Phi$$

(3.7)

$$\sum_{\substack{j \in \Phi \\ j \neq i}} x_{ij} \leq 1 \qquad \text{for every } i \in \Phi$$

where the variables x_{ij} are non negative integers: $x_{ij} \geq 0$.

Relation (3.7) gives the condition that, for every $i \in \Phi$, the sum of the services of the type $(V_i, ..)$ be at most 1. The same relation is valid for services of the types $(.., V_j)$, $j \in \Phi$. However, since i and j are elements of the same set Φ, not necessarily distinct, the relation (3.4) does not discard the case where the same round trip V_i follows and precedes two different round trips. Therefore the following constraints are also set:

(3.8) $x_{hi} + x_{ij} \leq 1$ for every $h, i, j \in \Phi$.

In this problem too, there is assigned to every x_{ij} a non-active time $\bar{S}(i,j)$, as a consequence of the connection between V_i and V_j, and a cost coefficient

$$c_{ij} = \left[\bar{S}(i,j)\right]^{-1} + c^{+}$$

where

$$c^{+} = \sum \left[\bar{S}(i,j)\right]^{-1} .$$

Even in this case, $\bar{S}(i,j) = \infty$ if the coupling of two round trips is not compatible. The objective function is, therefore:

(3.9) $\max \quad z = \sum c_{ij} \cdot x_{ij}$.

The solution of this problem of linear programming gives the set of services formed by couples of round trips with, or without, rest outside residence. Finally, the round trips, which do not belong to the services formed by triplets or by couples of round trips, are themselves services. Let us indicate by $\sigma = \{Z_j\}$, $j = 1, 2, \ldots, M$ this set.

In the problem (3.3), (3.4), (3.5) and (3.6), the number of possible triplets is generally very rare and so it seems right and adequate to use an exhaustive method for the search for compatible combinations of round trips, instead of making use of the general computing algorithm. The same applies to the problem (3.7), (3.8) and (3.9) which would require a computing procedure too difficult for and out of proportion with the type of problem. In fact, at least for what concerns the italian railways, the possibility of combining into a single service a couple of round trips is such that the cost matrix (c_{ij}) contains a large number of values equal to 0. Because of this, it has been considered more adequate to make use of a solution procedure of the heuristic type. The compatible combination (V_i, V_j) has been represented by an edge connecting the nodes V_i and V_j. Deriving a graph from (c_{ij}), we tried to extract from this graph the maximum number of arcs which, as a consequence, imply a minimum global cost. The procedure is articulated in two parts according to whether in fathoming the graph one meets a final edge of the tree or a cycle.

4 - Some properties of the set of services

In the union regulations a set of limitations are foreseen which concern the recurrence of certain overtime work activities as, e.g., night work, the rests outside residence, etc.. These limitations have a noticeable influence on the way of defining the temporal succession of the services, since they concern essentially the cumulation of these work activities in given time intervals.

The modalities of these limitations may all be brought back to clauses of the type:

(4.1) $P(n,d,x)$ = " not more than n times in d days is the working activity x allowed " .

For example, the limitation set concerning night services: " night services cannot be more than three between two weekly rests and not more than two consecutive ones. Three night services can be consecutive if one of them does not exceed the duration of 1 hour. Finally, in a time interval of 28 days, they cannot exceed the number of 12",
can be decomposed into three clauses of the type (4.1). If we consider the following extra activities:

x_1 = night service,

x_2 = night service of duration greater than 1 hour,

the above mentioned limitation set may be worded, alternatively, in the following way:

$P(3,7,x_1)$ ="not more than 3 times in 7 days is the x_1 allowed ".

$P(2,3,x_2)$ = " not more than 2 times in 3 days is the x_2 allowed".

$P(12,28,x_1)$ = " not more than 12 times in 28 days is the x_1 allowed".

In the same manner the non-working activities, such as the rests in residence or the weekly rests, of the type: " an agent has a right to a weekly rest of 2 days every 7 days", can be brought back to the (4.1): " not more than 5 times in 7 days is the x_q allowed", where x_q is the working day.

This new formulation of the above mentioned constraints and of others, puts better in evidence the concentration limits, in given time intervals, of certain working and non-working activities of personnel. These relations allow us to compute, for every depot residence and for a given set of services, a theoretical minimum number of train engineers, necessary for carrying out the work of the personnel depot. This indication is very useful, as we shall see, in the procedure of service composition for roster definition.

Let $P(n_i, d_i, x_i)$ be the i-th limitation on x_j and, e.g., let m_i be the number of services, of the type $Z = L$, subject to the x_i; from all this, it follows that one, and the same, train engineer cannot carry out x_j more than once in d_i/n_i days. Therefore, in the residence, there should be available, every day, at least N engineer, where:

(4.2) $$N \geq \max\ (d_i \cdot m_i\ /\ n_i,\ M)$$

where M is the total number of services. After considering that the services of the type $Z = L', R, L''$, because of their duration (32 hours at most) imply, at the most, only one activity x_i; the relation (4.2), extended to all services, becomes:

(4.3) $$N \geq \max\ (d_i \cdot m_i\ /\ n_i) \qquad i = 1, 2, \ldots, q$$

where q-1 is the number of extra activities and $m_q = M.7/5$.

5 - The roster composition

In the previous paragraph we have seen that a set of constraints controls the temporal distribution of the extra working activities. In the procedure of service composition for roster definition (a sequence of assignment which periodically repeats itself), one essentially tries to order the services in such a way that all the constraints are satisfied.

Therefore, from different initial situations, there are generated, one at time, N rosters (where N is the theorically minimum number of engineers) by use of the progressive concatenation of the services of σ, alternating them adequately with daily and weekly rests.

In order for the distribution of the extra activities to be as uniform as possible. let us assign, before beginning the procedure of the shift composition, to everyone of them, i.e. to the hypothetical train engineer who has to carry it out, a fictitious set of activities already carried out, which are called "histories", so as to simulate an initial average load of work. In this way, the assignment of the services is conditioned by these histories and so it is possible to avoid that, at the beginning of the shift, there should be a concentration of extra work, higher than average and that there should be also anomalies in the distribution of the work and of the rests between the end and the beginning of the roster.

The histories, which are initially assigned to the N hypothetical engineers, depend on the number and on the type of services which have to be made into a roster. If, e.g., the activity x_1 has to be carried out n_1 times daily, then only to n_1 train engineers, among the N ones, is the x_1 assigned, at the time t_0-f, where t_0 is the initial time at which the actual roster originates and f are the days during which the assignements are simulated. The same thing is done for all the other activities. At the date $t_0-(f-k)$, where $k = 0, 1, \ldots, f$, the random assignment to the extra activities are repeated, and the daily and the weekly rests are naturally included. In the end, the composition of the roster will begin. The assignment of the service Z_j to the train engineer $M(h)$, $h = 1, 2, \ldots, N$, shall obey the following rules:

1) The starting time of Z_j and the time of reentering on duty are such that their difference is a minimum.

2) The limitations concerning the extra activities, relative to Z_j, must be compatible with the past activity of the engineer.

3) The theorically minimum number of engineers, computed on the remaining services, after Z_j has been tentatively subtracted, must be less than the one computed at the previous step.

If such conditions are satisfied, then Z_j is linked together with the previous service, otherwise another one has to be chosen and the control of the above mentioned conditions is carried out once more. If it is not possible to link together any service, a supplementary rest of 24 hours is assigned to the engineer.

At every assignment, be it concerning a service, a weekly rest or an interval, the history of the train engineer is brought up to date. When all the services are assigned, one continues with the control of conditions 1) and 2), for all the services of the roster, in order to check that the union constraints are satisfied even when it is carried out more than once.

In the end, among the N generated rosters, the shortest one will be chosen, namely to one which requires the minimum number of engineers for its execution.

Conclusion

A Fortran IV code has been written and tested on a good deal of actual applications and on real problems by the State Railroads. The result has been a reduction of about 7% in the required number of train engineers. Moreover, a code has been written also for as-