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PROPERTIES OF ASYNCHRONOUS CONTROL NETWORKS

P.Corsini G.Frosini

Italy

I-INTRODUCTION

In this paper digital systems constituted by mutually asynchronous units are examined. In a unit only the portion of the control part that regulates the information exchange with other units (<u>control node</u>) is considered . A control node is seen as a device that correctly realizes an asynchronous sequential function between input states and output states. Some input variables of a control node are regarded as <u>parameters</u>, whose values are determined by some agents, independently from the control information flow among the units.

A set of several interconnected control nodes constitutes a <u>control network</u>. The operation speed of every control node and the size of the delays associated with the interconnection wires are unknown. In order to assure that a control node is correctly driven, synchronizers [1] are supposed to be inserted, if necessary, at its input terminals.

Two important properties of a control network are <u>persistence</u> [2] and <u>activity</u> [3] . A control network is persistent if no information is lost. A control network is active if there are not hang-up situations.

The problem of obtaining a larger persistent and active control network by interconnecting two smaller ones is explicitly considered in this work. For this aim a model for describing control networks is utilized, that uses state diagrams [2] as a means of description of network behaviour. A two-way relation is established between the states of two control networks B^1 and B^2 to be interconnected, and the states of the resulting control network C. Such a relation allows us to determine conditions on

the state diagrams of B^1 and B^2 that assure that C is persistent and/or active.

II-THE MODEL

The portion of a control network beginning at the input terminals of a control node N^h and ending at the input terminals of the control nodes to which N^h is connected, is considered as a whole, and its behaviour is schematized by means of an abstract device called <u>element</u>.

<u>Definition 1</u>. An <u>element</u> is a device with <u>n</u> control input variables $x_1,x_2,\dots,x_n$, <u>p</u> parameters $\varphi_1,\varphi_2,\dots,\varphi_p$, <u>m</u> internal variables $y_1,y_2,\dots,y_m$, $q\leq m$ output variables $z_1,z_2,\dots,z_q$, that realizes i) <u>m</u> logical asynchronous functions

$$Y_i=f_i(x_1,x_2,\dots,x_n,\varphi_1,\varphi_2,\dots,\varphi_p,y_1,y_2,\dots y_m)\ ,i=1,2,\dots,m\ ,$$

possibly excluding some sequences of input variable values, and ii) <u>q</u> transfer operations

$$z_j\leftarrow y_j\ ,\ j=1,2,\dots,q.$$

Each logical function $f_i(\cdot)$ gives the new value of the internal variable y_i. When $Y_i\neq y_i$, the variable y_i is <u>unstable</u>, and the time required for the variable to attain its new value (to <u>switch</u>) is supposed to be equal to zero. Each output variable z_j takes the value of the corresponding variable y_j. When $z_j\neq y_j$, the variable z_j is <u>excited</u>, and the time required for the variable to switch is neither constant nor bounded, but finite. The variable z_j, if excited, actually switches only if the value of y_j does not change while z_j is excited.

<u>Definition 2</u>. An element is <u>normal</u> if i) each unstable internal variable becomes stable as it switches, and ii) each stable internal variable remains stable until the value of at least one control input variable changes.

In the following, we always refer to normal elements. In Fig. 1 some simple elements are described.

<u>Definition 3</u>. A <u>net</u> is an interconnection of elements, where each control input (output) terminal of an element is connected to exactly one output (control input) terminal of another element. No delay is present on the interconnection wires among the elements.

The connection of an output terminal with more than one con-

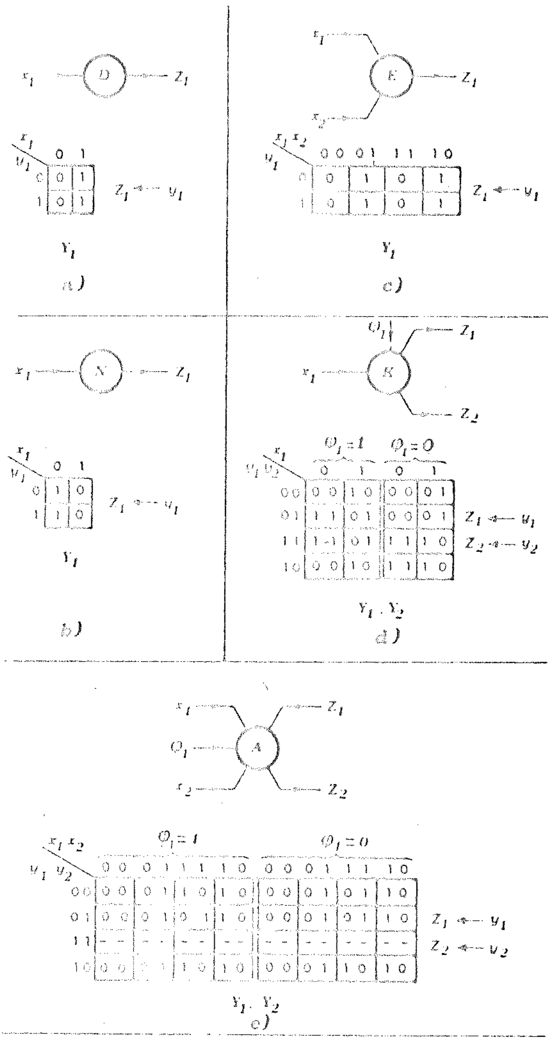

Fig. 1. a) A Delay element. b) A Not element. c) An Exclusive-or element. d) An Arbiter element.

trol input terminal can be accomplished by regarding a fork of wires as an element. A set of elements connected to an external world can be seen as a net, by regarding the external world as one or more elements.

Definition 4. In a net, each value combination of the internal variables and of the output variables of every element constitutes a state of the net. Each pair of variables (y_j^h, z_j^h) of an element E^h is called state pair. The four values of a state pair, i.e., 00, 01, 11, 10, are respectively denoted by $0, 1^*, 1, 0^*$.

Note that the state pair values 0^* and 1^* represent situations in which the output variables are in the process of taking on the values of the internal variables.

Definition 5. Let S_a and S_b be two states of a net. S_b is consecutive to S_a if it is immediately reached from S_a when some output variables of some elements, that are excited in S_a, switch. The set constituted by a state S_a and by all its consecutive states is denoted by $cs(S_a)$. If in a state S_a no output variable of any element is excited, there is no consecutive state to S_a, and S_a is stable.

Definition 6. A sequence of states of a net $S_a, S_{a+1}, \ldots, S_{a+n}, S_{a+n+1}, \ldots, S_b$, such that S_{a+n+1} is consecutive to S_{a+n}, is an allowed sequence starting from S_a. A state S_d is successive to a state S_c if S_d follows S_c in a allowed sequence. The set constituted by a state S_c and by all its successive states is denoted by $ss(S_c)$.

The behaviour of a net, placed initially in a state S_a, is represented by a state diagram with respect to S_a. Such a diagram is a directed graph in which the nodes are associated with the states of the allowed sequences starting from S_a, and a directed arc connects the node associated with a state S_c to the node associated with a consecutive state to S_c. If a state S_d is consecutive to a state S_c for one or many, but not all, combinations of the parameter values, the directed arc connecting the nodes associated with S_c and S_b, respectively, is marked by such combinations.

Definition 7. In a net, an output variable of an element is persistent with respect to a state S_a if in each allowed sequence starting from S_a, the variable, when it is excited, remains excited until it switches. A net is persistent with respect to a state S_a if each output variable of every element is persistent with respect to S_a.

Note that in a persistent net no information is lost, as it is not possible for a state pair value to change from 0^* to 0 or from 1^* to 1. The persistence property is an extension to our nets of the "semimodularity" introduced in [2].

Definition 8. A net is active with respect to a state S_a if no state $S_b \in ss(S_a)$ is stable.

Note that in a active net a part of the net may hang-up, but the net as a whole may not. The concept of active net is similar to the concept of "well-formed control network" used in [3].

An example of a persistent and active net with respect to the state S_1 is given in Fig. 2. In each state of the diagram in Fig. 2b, the i-th symbol represents the value of the i-th state pair (y_i, Z_i), i=1,2,3,4. Another example of a persistent an active net with respect to its state S_1 is given in Fig. 3.

III-MODULAR NETS

In the following, we consider those nets (modular nets) constituted by several parts (modules) interconnected by means of pairs of wires that are obtained by merging Delay elements and Not elements of two proper smaller nets (blocks).

Definition 9. A net constituted by a set M of elements, by $\underline{r}$ Delay elements and by $\underline{s}$ Not elements ($\underline{r}$ or $\underline{s}$ can be equal to 0) is called block. The set M is called module.

Definition 10. Let B^1 and B^2 be two blocks, and let $S_a^1 1$ and $S_a^2 2$ be, respectively, their initial states. Let B^1 be constituted by a module M^1, by $\underline{r}'$ Delay elements D_i^1, i=1,2,...,r', and by $\underline{s}'$ Not elements N_j^1, j=1,2,...,$\underline{s}'$. Let B^2 be constituted by a module M^2, by $\underline{s}'$ Delay elements D_j^2 and by $\underline{r}'$ Not elements N_i^2. If there are two states $S_d^1 1 \in ss(S_a^1 1)$ and $S_d^2 2 \in ss(S_a^2 2)$, of B^1 and B^2 respectively, such that, for all i and for all j:

i) the value of the control input (output) variable $z\hat{a}d_i^1$ $(z\hat{b}d_i^1)$ of D_i^1 in $S_d^1 1$ is equal to the value of the output (control input) variable $z\hat{b}n_i^2$ $(z\hat{a}n_i^2)$ of N_i^2 in $S_d^2 2$,

ii) the value of the control input (output) variable $z\hat{a}n_j^1$ $(z\hat{b}n_j^1)$ of N_j^1 in $S_d^1 1$ is equal to the value of the output (control input) variable $z\hat{b}d_j^2$ $(z\hat{a}d_j^2)$ of D_j^2 in $S_d^2 2$,

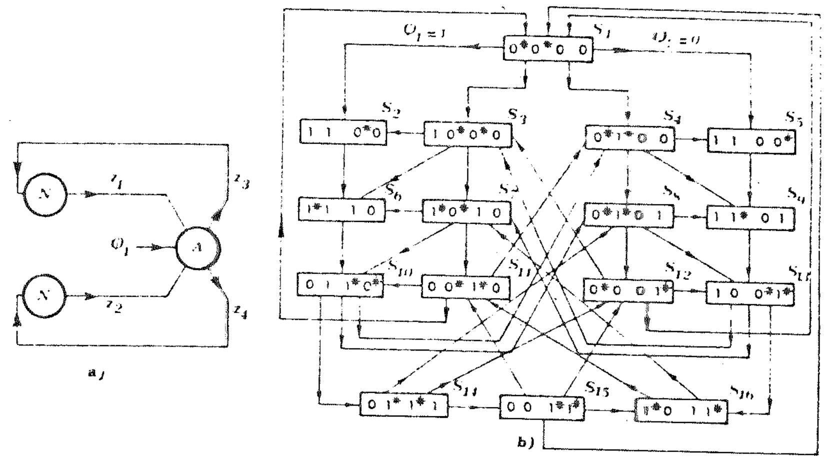

Fig. 2

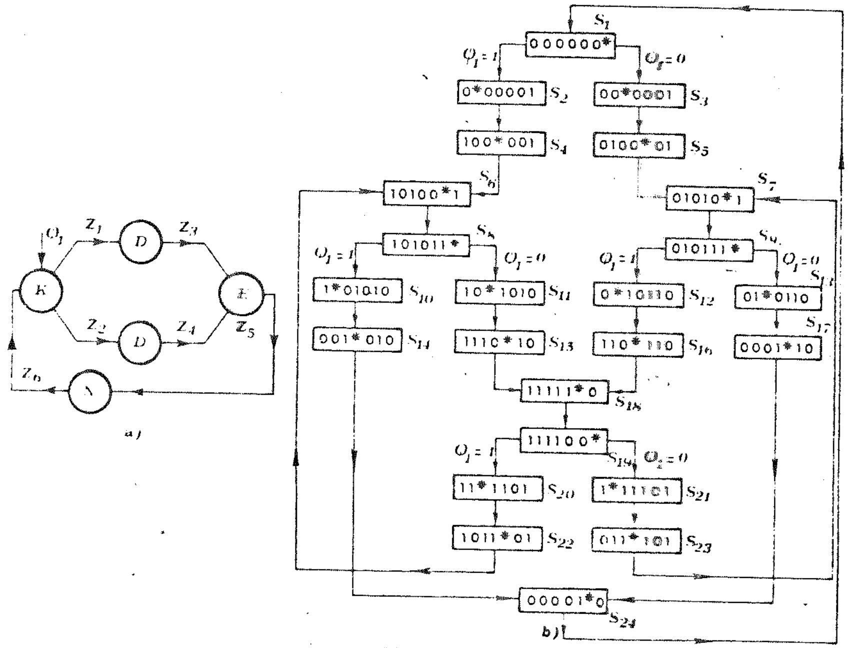

Fig. 3

then B^1 and B^2 are two connectible blocks with respect to the pair of states (S^1_d1, S^2_d2). The net obtained by connecting the control input (output) wire of D^1_i with the output (control input) wire of N^2_i, the control input (output) wire of N^1_j with the output (control input) wire of D^2_j, and by removing D^1_i, N^1_j, D^2_j, N^2_i, for all i and for all j, is a modular net, and it is denoted by $C = M^1 \cup M^2$. The initial state of C (connection state) is denoted by $S^c_dc = S^1_d1 \cup S^2_d2$. The variables of C obtained by merging z^1ad_i with z^2bn_i, z^1bd_i with z^2an_i, z^1an_j with z^2bd_j, and z^1bn_j with z^2ad_j are denoted, respectively, by $z^{12}d_i$, $z^{21}n_i$, $z^{12}n_j$, $z^{21}d_j$.

Two connectible blocks with respect to the pair of states (S^1_1=000000*, S^2_1=0*0*00) are shown in Fig. 4a (this is the net shown in Fig. 3) and in Fig. 4b (this is the net shown in Fig.2). The resulting modular net is shown in Fig. 4c.

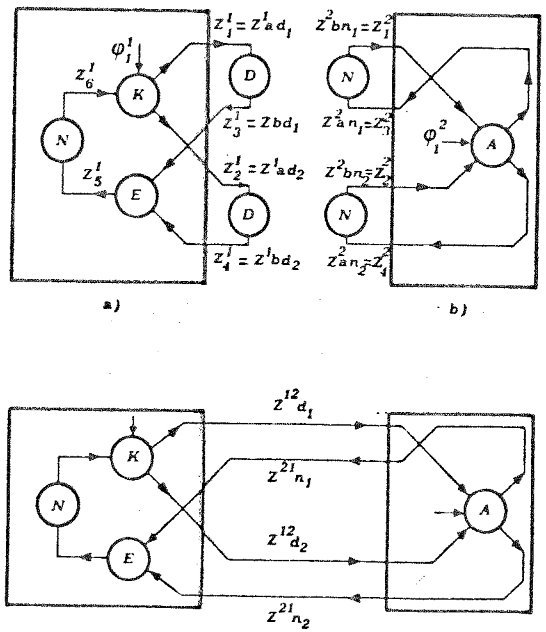

Fig. 4

In the following, we always refer to the notations used in the previous Definition. Moreover, S^1, S^2 and S^c, with subscript symbols, denote a state of B^1, of B^2 and of C, respectively.

Definition 11. Two states $S^1_m1 \in ss(S^1_d1)$ and $S^2_m2 \in ss(S^2_d2)$ are said to be correspondent, and are denoted by $S^1_m1 \leftrightarrow S^2_m2$, if, for all i and for all j, i) the values of z^1ad_i, z^1bd_i, z^1an_j, z^1bn_j in S^1_m1 are equal, respectively, to the values of z^2bn_i, z^2an_i, z^2bd_j, z^2ad_j in S^2_m2, and ii) $S^1_m1 = S^1_d1$, $S^2_m2 = S^2_d2$, or $S^1_m1 \in cs(S^1_p1)$, $S^2_m2 \in cs(S^2_p2)$, where $S^1_p1 \leftrightarrow S^2_p2$.

In Table I are listed the pairs of correspondent states for the blocks shown in Fig.4, a and b . Superscripts 1 and 2 refer to the states of blocks in Fig. 4a and 4b, respectively.

Table I

$S^1_1 \leftrightarrow S^2_1$, $S^1_2 \leftrightarrow S^2_1$, $S^1_3 \leftrightarrow S^2_1$, $S^1_{24} \leftrightarrow S^2_1$, $S^1_4 \leftrightarrow S^2_3$, $S^1_5 \leftrightarrow S^2_4$, $S^1_{15} \leftrightarrow S^2_6$,
$S^1_6 \leftrightarrow S^2_7$, $S^1_8 \leftrightarrow S^2_7$, $S^1_{10} \leftrightarrow S^2_7$, $S^1_{11} \leftrightarrow S^2_7$, $S^1_7 \leftrightarrow S^2_8$, $S^1_9 \leftrightarrow S^2_8$, $S^1_{12} \leftrightarrow S^2_8$,
$S^1_{13} \leftrightarrow S^2_8$, $S^1_{16} \leftrightarrow S^2_9$, $S^1_{14} \leftrightarrow S^2_{11}$, $S^1_{17} \leftrightarrow S^2_{12}$.

Definition 12. Two states $S^1_m1 \in ss(S^1_d1)$ $(S^2_m2 \in ss(S^2_d2))$ and $S^c_mc \in ss(S^c_dc)$ are said to be congruent, and are denoted by $S^1_m1 \sim S^c_mc$ $(S^2_m2 \sim S^c_mc)$, if each control input variable, each internal variable and each output variable of every element of M^1 (M^2) has the same value in S^1_m1 (S^2_m2) and in S^c_mc.

IV-PROPERTIES OF MODULAR NETS

For two blocks B^1 and B^2, persistent with respect to S^1_a1 and S^2_a2, respectively, and connectible with respect to (S^1_d1, S^2_d2), the Theorems of this section hold. Theorem proofs are omitted and they can be found in [4].

Theorem 1. For every state $S^c_pc \in ss(S^c_dc)$, where S^c_dc is the connection state, there are exactly two correspondent states S^1_p1 and S^2_p2 such that $S^1_p1 \sim S^c_pc$ and $S^2_p2 \sim S^c_pc$.

Theorem 2. For every pair of correspondent states S^1_p1 and S^2_p2

there is exactly one state S^c_pc such that $S^c_pc \sim S^1_p1$ and $S^c_pc \sim S^2_p2$.

Theorems 1 and 2 establish a two-way relation between the pairs of correspondent states of two connectible blocks on one side, and the states of the resulting modular net on the other side. Such a relation is the starting-point for proving the following Theorems.

Theorem 3. In a modular net $C = M^1 \cup M^2$, the modules M^1 and M^2 communicate according to a ready-reply rule, i.e., if a variable zd^{12}_i (zn^{12}_j) switches, it does not become excited until zn^{21}_i (zd^{21}_j) has switched, and viceversa.

Theorem 4. The modular net C is always persistent with respect to S^c_dc.

Theorem 5. The modular net C is active with respect to S^c_dc if and only if, for each pair of correspondent states S^1_p1 and S^2_p2, there is at least one output variable of an element of M^1 that is excited in S^1_p1, or an output variable of an element of M^2 that is excited in S^2_p2.

The modular net shown in Fig. 4c is not active with respect to $S^1_1 \cup S^2_1$ as the pair of correspondent states S^1_{15} and S^2_6 does not satisfy the hypothesis of the previous Theorem.

Note that a modular net constituted by several modules can be obtained iteratively, by considering two blocks at every step, by constructing the corresponding modular net and by regarding it as a block. At every step the state diagram of the obtained modular net must be constructed. The following rule allows us to construct the state diagram of $C = M^1 \cup M^2$ by starting from the state diagrams of B^1 and of B^2.

Construction Rule. Let D^1, D^2 and D^c be, respectively, the state diagrams of B^1 with respect to S^1_a1, of B^2 with respect to S^2_a2 and of C with respect to S^c_dc. D^c can be obtained by performing the following steps.

1) Construct the states of C by considering all the pairs of correspondent states. The state of C associated by means of the two-way relation with a pair of correspondent states S^1_p1 and S^2_p2 is obtained by taking from S^1_p1 (S^2_p2) the value of each internal variable and the value of each output variable of every element of M^1 (M^2). This state is denoted by $S^c_pc = S^1_p1 \cup S^2_p2$.
2) Connect, by means of a directed arc, the node associated with a state S^c_pc to the node associated with a state S^c_qc, if $S^c_pc = S^1_p1 \cup S^2_p2$, $S^c_qc = S^1_q1 \cup S^2_q2$, and $S^1_q1 \in cs(S^1_p1)$, $S^2_q2 \in cs(S^2_p2)$.
3) Mark the directed arc connecting the node associated with S^c_pc to the node associated with S^c_qc by the union of the markings that, possibly, appear in D^1 (on the directed arc connecting the nodes associated with S^1_p1 and S^1_q1) and in D^2 (on the directed arc connecting the nodes associated with S^2_p2 and S^2_q2).

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Paolo Corsini, Dipartimento Sperimentale di Elettrotecnica ed Elettronica, Università di Pisa, via Diotisalvi, 2, Pisa, Italy.

Graziano Frosini, Istituto di Elaborazione della Informazione, Consiglio Nazionale delle Ricerche, via S. Maria, 46, Pisa, Italy.