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Some General Remarks on Hjorth's Parameters Used in EEG Analysis

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Automatic analysis of the EEG requires the definition of parameters able to describe, as completely as possible, the basic characteristics of the phenomenon under investigation.

Two different approaches to the problem have mainly been followed: one based on the analysis of the EEG in the time domain, the other based on the analysis of the EEG in the frequency domain, the two methods being equivalent insofar as they convey the same information about the characteristics of the time series analyzed (Burch, 1959; Remond, 1973).

In recent years HJORTH (1970, 1973) has proposed a method of analysis based on time properties of the EEG. This method characterizes the time series by three time domain descriptors: activity (A), mobility(M) and complexity (C). The meaning of A, M, C has been widely described by HJORTH (1970, 1973), and from a physical point of view may be summarized as follows:

A: gives the measure of the mean power of the signal (expressed in V^2) over the analyzed epoch;

M: may be considered as the mean frequency of the signal (expressed in Hz);

C: describes the bandwidth of the signal.

From a mathematical point of view, for time functions f (t) with a mean value equal to zero, the three descriptors are defined as follows:

$$A = m_o = \sigma_0^2$$

$$M = \sqrt{m_2/m_o} = \sigma_1/\sigma_o$$

$$C = \left[\sigma_2^2/\sigma_1^2 - \sigma_1^2/\sigma_0^2\right]^{\frac{1}{2}}$$

with:

$$\sigma_n^2 = \frac{1}{T} \int_{t-T}^{t} f^{n^2}(t)\,dt = \int_{-\infty}^{+\infty} \omega^{2n} S(\omega)\,d\omega = m_{2n}$$

where:

$f^n(t)$ is the n^{th}-order derivative of f (t)

σ_n is the variance of $f^n(t)$

$S(\omega)$ is the power density spectrum (PDS)

m_n is the n^{th}-order moment of $S(\omega)$

Figures 1, 2 and 3 show the relationship among A, M and C and the amplitude and frequency characteristics of f (t). In the same figures is also shown the PDS of f (t) at some particular epochs of analysis: t = a, t = b, t = c.

Fig. 1 shows the trend of the three descriptors when

$$f(t) = \sqrt{2}\,(1 - t/T_o) \sin \omega_o t$$

with:

$$\omega_o = 20\pi\ s^{-1}$$

$$T_o = 128\ s$$

This type of function is sketched in Fig. 1 upper-left.

It can be seen that, while M and C are constant (M = 10 Hz, C = 0), A decreases from 1 to zero with a square law.

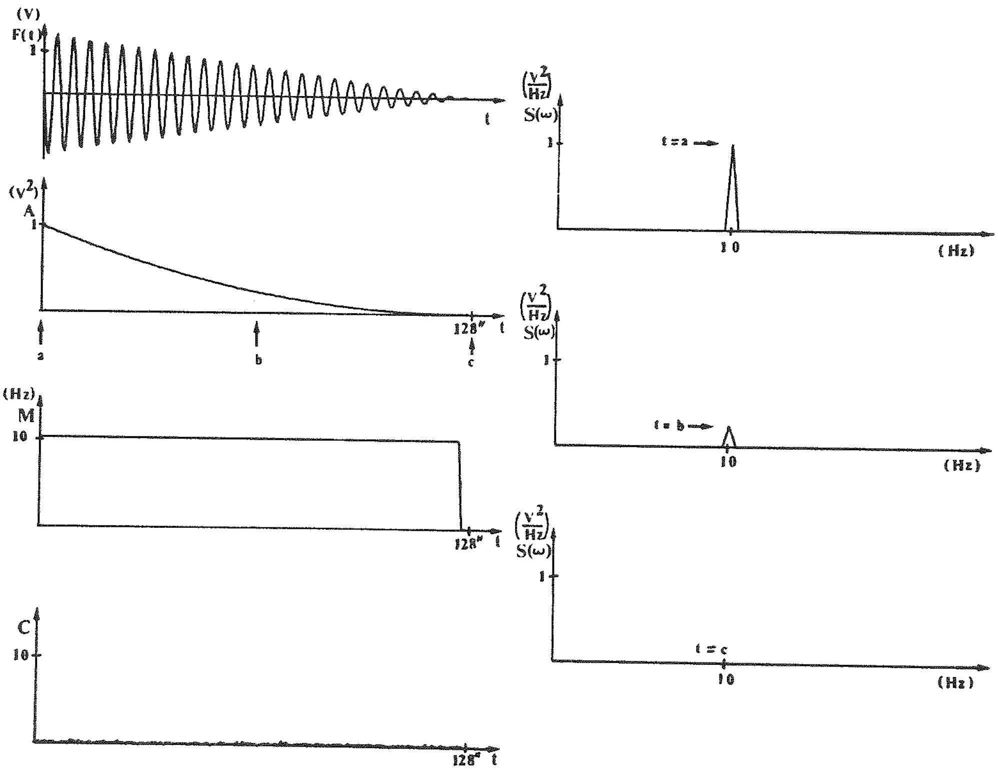

Fig. 1: Dependence of A, M and C on the amplitude of the signal. Upper left: type of function analyzed; the amplitude remains constant over each epoch of analysis, but changes from epoch to epoch.

In Fig. 2 the trend of the three descriptors is shown when

$$f(t) = \sqrt{2} \sin \omega t$$

with:

$$\omega = \frac{50\ X}{128}\ t$$

In this case A and C are constant, while M increases linearly with time.

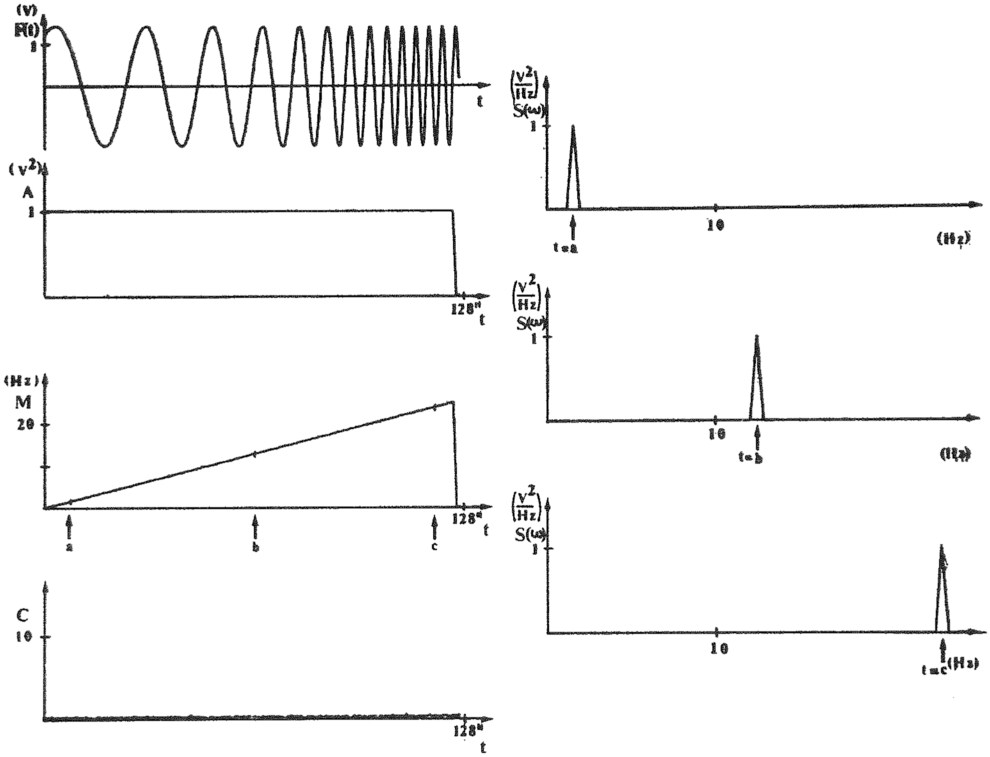

Fig. 2: As in Fig. 1, but with constant amplitude and changing frequency.

In Fig. 3 the trend of the three descriptors is shown when:

$$f(t) = \sqrt{2}\,(\sin \omega_0 t + \sin \omega t)$$

with:

$$\omega_0 = 20\pi$$

$$\omega = (100\pi/128)\,t$$

In this case both M and C change with time. In particular, C reaches its minimum value when the two components have the same frequency, whereas it increases according to the difference of the frequencies of the two components.

The main advantages that can be obtained by characterizing the EEG by A, M and C are:

- the descriptors can be calculated in the time domain, so that the calculation of the PDS is not required;
- the descriptors completely characterize signals generated by second-order systems.

Hence the time domain descriptors introduced by Hjorth appear to be powerful method for analysis of the EEG.

However, it is necessary to consider some mathematical and physical aspects in order to establish the actual capabilities of these descriptors in characterizing the EEG and the actual possibility of calculating their value with small digital computers.

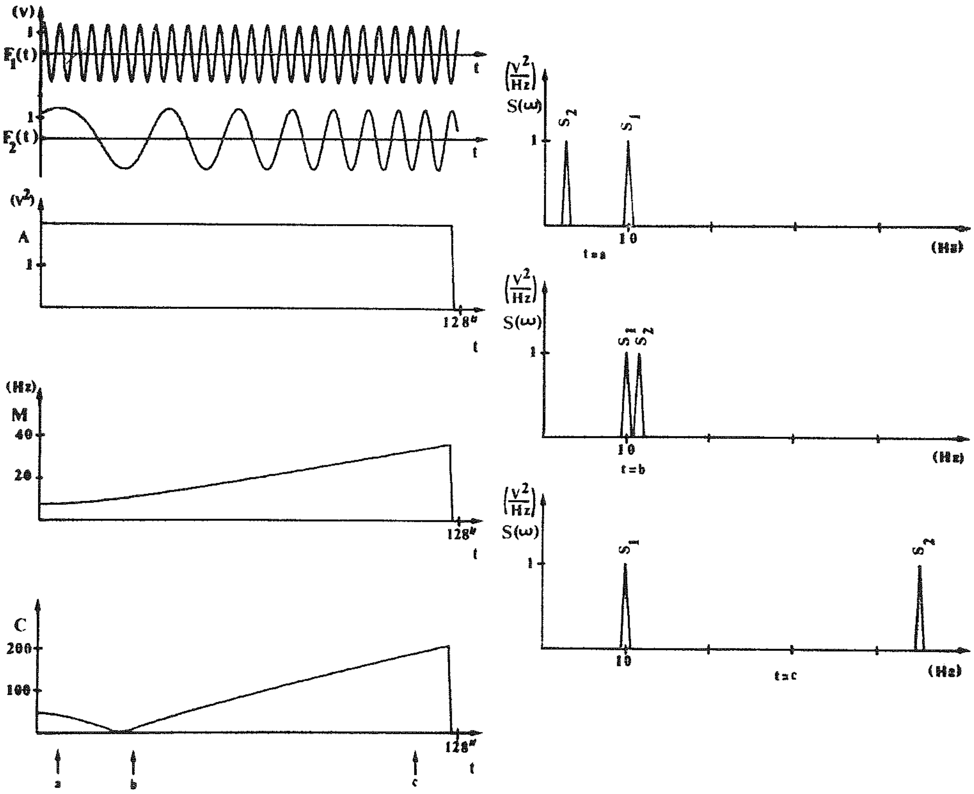

Fig. 3: Trend of A, M and C in the case of f (t) = F1 (t) + F2 (t). F1 and F2 are sketched in the upper-left part of the figure.

It is possible to demonstrate (HJORTH, 1973) that the three descriptors completely identify a system formed by (mutually identical) second order subsystems.

Such systems have a response to an impulse excitation with a PDS described by the function:

$$\left| \frac{\omega_0}{(\alpha - j\,\omega)^2 - \omega^2} \right|^2$$

that is, by a function with only one maximum.

Under the hypothesis that the PDS completely describes the basic characteristics of the EEG, the limitations inherent in HJORTH's descriptors appear evident, since it is commonly the case that the PDS has more than one relative maximum, even after a smoothing process has been applied.

From a physical point of view the validity of the descriptors may be limited to situations in which the signal gives rise to a PDS with only one maximum, as in those sketched in

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Fig. 4: Relationship between single pulse response of a second order system (upper part) and descriptor value (lower part), increasing the damping factor of the system going on from A to C. Other tracings show the effect of random superposition of 500 single pulse responses and the PDS of a single response.

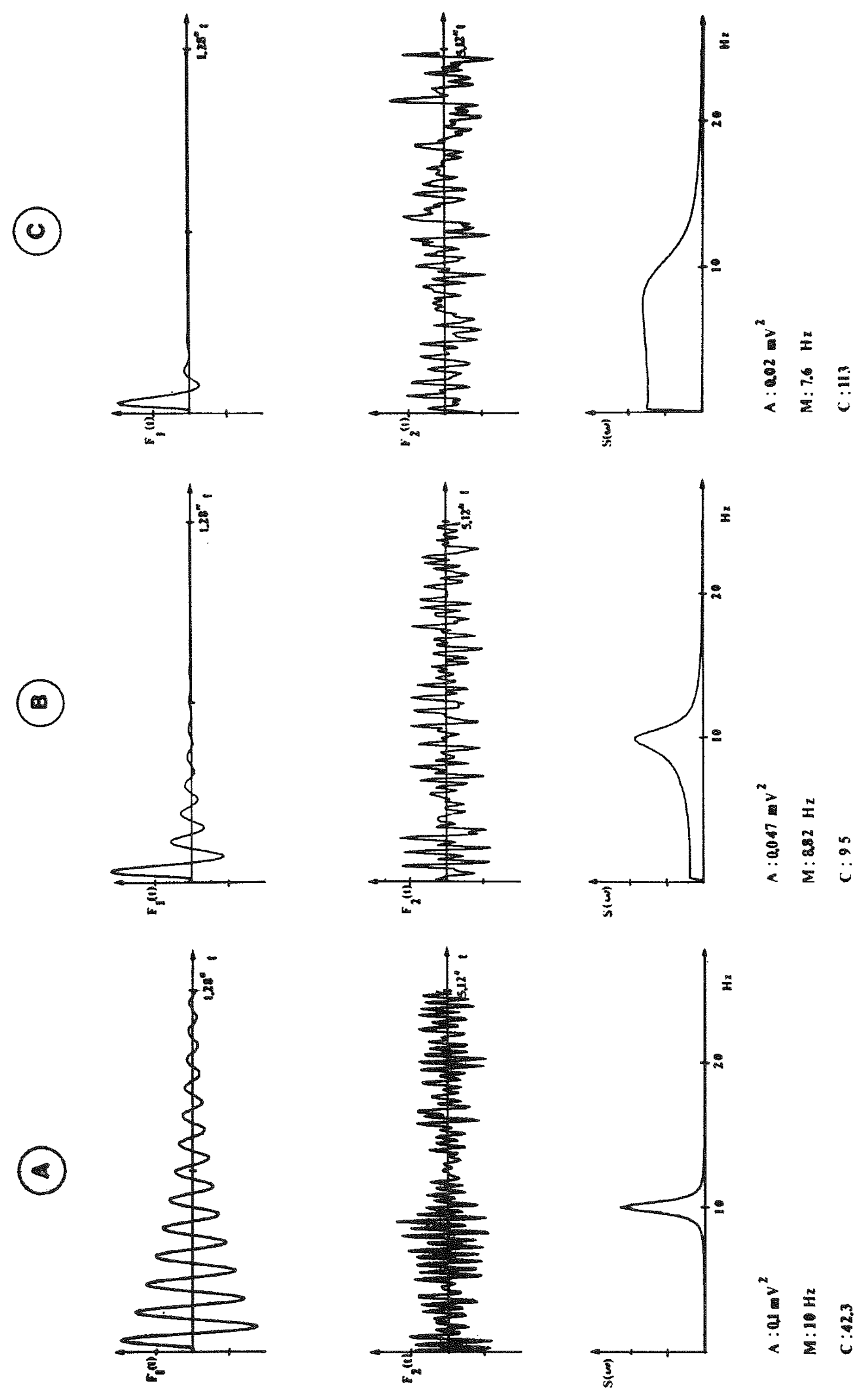
A
B
C
A : 0,1 mV²
M : 10 Hz
C : 42,3
A : 0,047 mV²
M : 8,82 Hz
C : 95
A : 0,02 mV²
M : 7,6 Hz
C : 113
1,28" t
5,12" t
S(ω)
10
20
Hz

Fig. 4 A, B, C. In this figure the upper tracings show the single pulse response of a second order system with a damping factor that increases from A to C. The middle tracings show the result of a superposition of 500 single responses, randomly distributed over a time interval of 5.12 sec. As shown by HJORTH (1973), the random superposition of single pulse responses gives a curve very similar to an actual EEG, with a synchronized activity decreasing as the damping factor increases. The lower tracings of Fig. 4 show the PDS and the value of A, M and C. It can be noted that the damping factor affects C much more than M.

The limitation mentioned above has been pointed out by HJORTH himself (HJORTH, 1973), who suggested to introduce into the model a short of mutual interaction between the sub-systems; that is, assuming that the probability of response of one sub-system depends on the phase of a preceding response. Such a hypothesis improves the capability of the model of describing the observed EEG, but not in a satisfactory way.

However, it raises the problem of defining a new parameter describing the probablity of synchronization. In practice, without this parameter, the PDS, such as the ones shown in the lower trace of Fig. 5, are characterized by the same set of descriptor values. Fig. 5 indicates that curves with a very different structure may be described by constant values of A, M and C. In particular, a curve with a well organized activity (Fig. 5, example A) may have the same values of A, M and C as one with poorly organized activity (Fig. 5, example C). All of the differences are well indicated by the PDS (Fig. 5, lower trace).

From a mathematical point of view it must be noted that the calculation of the descriptors requires very simple operations, but deals with very large quantities. In fact operations involved are limited to time derivative calculations and zero crossing counts of time functions (SALTZBERG and BURCH, 1971). Nevertheless, the order of magnitude of the numbers involved in the calculation of C is such that great errors are often introduced. In fact, the calculation of C requires the calculation of the second order derivative; under the hypothesis that the signal has significant components at $\omega = 100\ s^{-1}$ ($\simeq 16$ Hz), a factor of the order of 10^8 is obtained by squaring the second derivative; moreover, C is obtained from the difference of numbers of the order of 10^4. Hence it is necessary to adopt particular computing procedures in order to handle numbers with the described order of magnitude with sufficient precision on a small digital computer.

Even if errors attributable to the performance of the computer may be kept as low as desired by using of special algorithms, there is another source of error: the analog to digital (A/D) conversion process.

To get an idea of the errors that affect the calculation of m_2 and m_4 (linked to the calculation of M and C), when starting from a sampled signal, the percentage error, $\left[\frac{\text{calculated value}}{\text{true value}} - 1\right] \cdot 100$, is plotted in Fig. 6 as a function of the A/D converter resolution (10 bit or 16 bit) and as a function of the ratio: sampling frequency/signal frequency (F_S/F_O).

From the analysis of Fig. 6 it follows that, in order to limit the maximum error of m_4 (linked to the calculation of C) at $\pm$ 10%, the bandwidth of the signal must be limited to a range of 1:7 (1:55/8) when a 10 bit A/D converter is used, and to a range of 1:20 (1:180/8) when a 16 bit A/D converter is used.

This means that high performance A/D converters must be used in order to get a good estimate of the complexity.

The errors due to the A/D conversion process may be drastically reduced by sampling the signal at very high speed and applying some averaging techniques. This means that a

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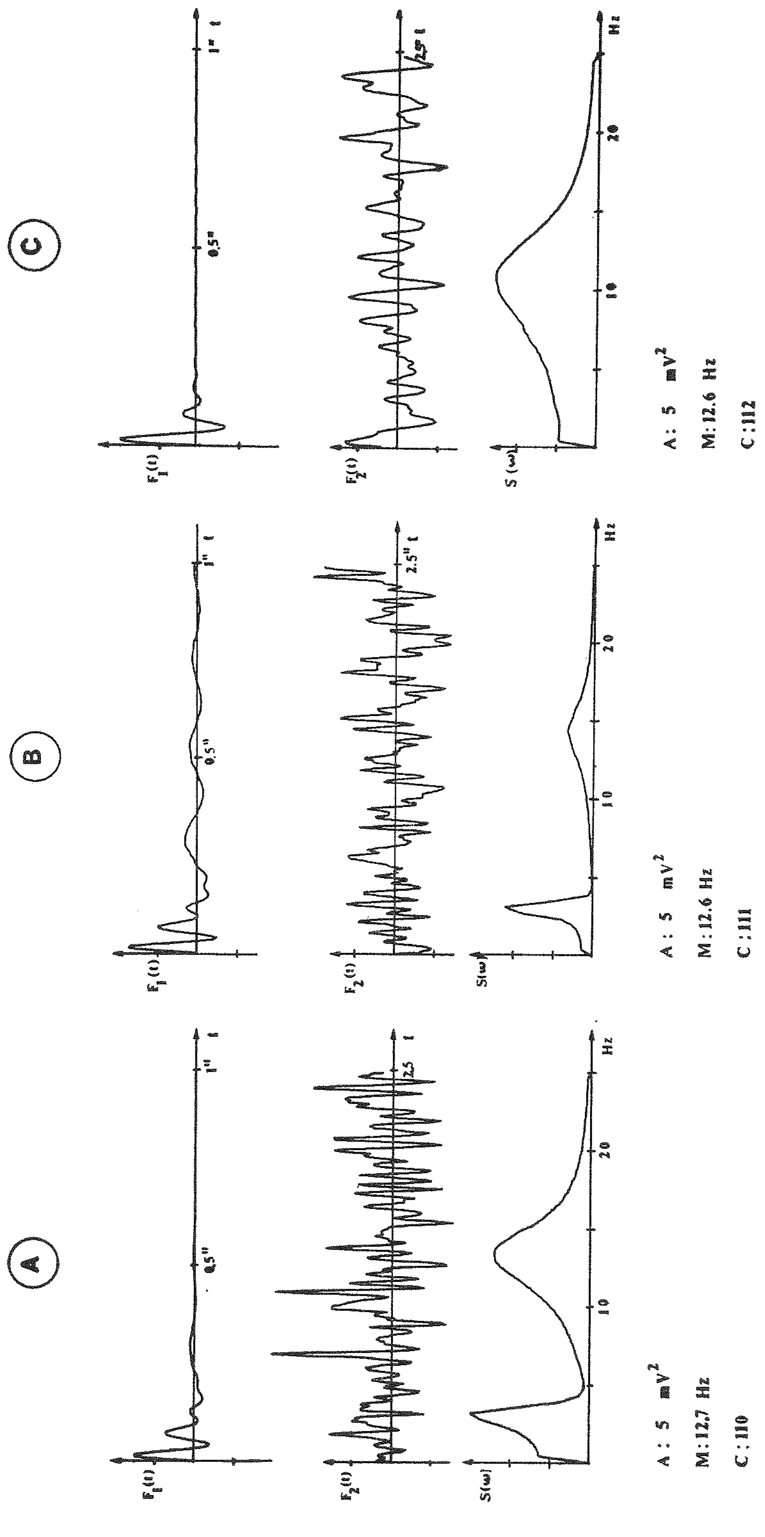
Fig. 5: Some examples of signals with different PDS and equal set of descriptors.


A
B
C
A : 5 mV²
M: 12.7 Hz
C: 110
A : 5 mV²
M: 12.6 Hz
C: 111
A : 5 mV²
M: 12.6 Hz
C: 112

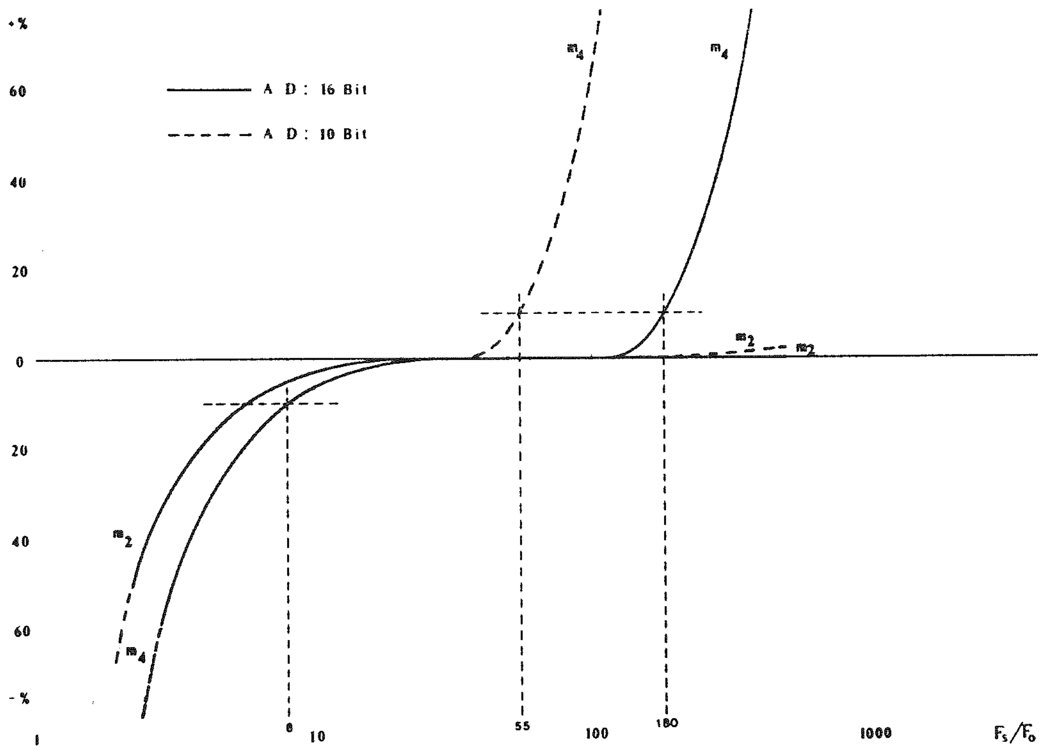


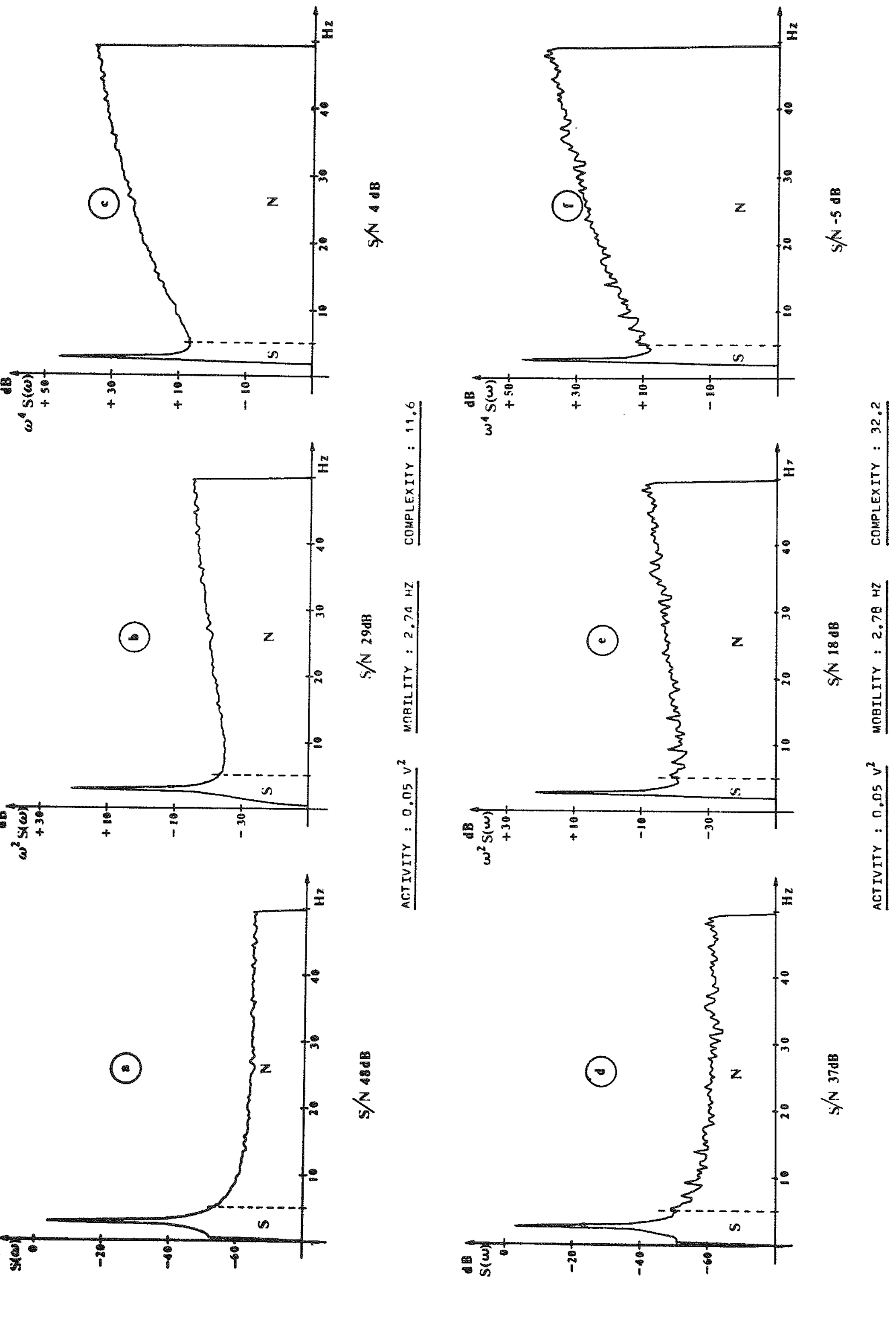
Fig. 6: Percentage error in the calculation of m2 and m4 as a function of the ratio: sampling frequency/signal frequency (F_s/F_a), for two different resolution of the A/D converter.

large quantity of data must be processed by the computer, thereby reducing the main advantages of very simple and fast calculation of the descriptors. Moreover, the descriptors must be calculated over an f (t) with zero mean value. Highpass filtering of the analog signal does not satisfy this requirement over each epoch to be analyzed, hence a pre-processing of the signal by means of the digital computer is required.

Another source of error in the computation of the time descriptors is due to the noise introduced by a possible record/reproduce process of the event on a FM magnetic tape unit. It is well known that a good FM recorder has a signal to noise ratio (S/N) of about 45 dB, referring to the maximum recorder signal range. To avoid saturation during the peak amplitude of the signal, the actual S/N ratio obtainable in EEG records is about 30 db. This is quite satisfactory in calculating A and M but gives rise to large errors in computing C.

The noise introduced by the record/reproduce process may be considered «white noise» over the whole EEG frequency band ($\simeq 0.5 = 50$ Hz); on the other hand, in some particular epochs of analysis, the main EEG activity may be concentrated in the low frequencies (i.e. δ activity). In this case ω^4 has an order of magnitude of 10^6 in the signal band and an order of magnitude of 10^{10} at the highest frequency of the noise, so that the S/N ratio is drastically reduced when computing C.

Fig. 7: Noise introduced by a record/reproduce process. Upper: actual signal processing; Lower: reproduced signal processing.


a
b
c
d
e
f
dB S(ω)
dB ω² S(ω)
dB ω⁴ S(ω)
Hz
N
S
S/N 48dB
S/N 29dB
S/N 4 dB
S/N 37dB
S/N 18 dB
S/N -5 dB
ACTIVITY : 0.05 V²
MOBILITY : 2.74 HZ
COMPLEXITY : 11.6
ACTIVITY : 0.05 V²
MOBILITY : 2.78 HZ
COMPLEXITY : 32.2

This reduction may be estimated by considering the ratio:

$$\frac{S}{N} = \frac{\int_{\omega_m}^{\omega_a} F(\omega)\, d\omega}{\int_{\omega_a}^{\omega_M} F(\omega)\, d\omega}$$

where:

$\omega_a - \omega_m$ = signal bandwidth

$\omega_M - \omega_a$ = noise bandwidth

$F(\omega) = S(\omega), \omega^2 S(\omega), \omega^4 S(\omega)$

The effect of the record/reproduce noise is shown in Fig. 7 for $f(t) = 0.31 \sin 5.4 \pi t$.

In this figure the PDS and its product by ω^2 and ω^4 are shown both for the actual signal (Fig. 7a, b, c) and for the recorded/reproduced signal (Fig. 7d, e, f). In this test a high performance FM recorder has been used (S/N $\simeq$ 50dB). Nevertheless, the complexity calculated on the actual signal is $C = 11.6$, while the complexity calculated on the basis of the reproduced signal is $C = 32.2$, that is, increased by a factor 3.

It may be pointed out that the A/D conversion error ($F_S = 100$, $F_S/F_O = 37$, A/D converter on 10 bit) and the computing errors increase the value of C from O (theoretical value) to 11.6 (calculated value on the basis of the actual signal).

In what has been described above, we have pointed out some limitations in computing Hjorth's time domain descriptors by means of a small digital computer and also some limitations of their capability to characterize a general EEG. In spite of such limitations, it is our opinion that these time descriptors may find some application in particular cases where the structure of the EEG eliminates any misunderstanding. Moreover, the approach to the problem suggested by Hjorth is a very efficient attempt to attain a quantitative description of the EEG on the basis of a physical model, acceptable from a neurophysiological point of view. For this reason the approach has a great promotional value.

It is our opinion that further development of the method will permit wide application in the field of clinical neurophysiology.

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