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THE BOUNDARY GRAPHS: AN APPROACH TO THE DIAGNOSABILITY WITH REPAIR OF DIGITAL SYSTEMS

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WITH REPAIR OF DIGITAL SYSTEMS. *

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ABSTRACT

Determination of diagnosability with repair of a digital system, composed by n subunits, is considered. The class of systems considered is that in which each subunit alone is capable of testing another. Some particular graphs (boundary graphs) are defined, and on their basis results are presented that improve the sufficient conditions about the t-fault diagnosability with repair, which are in the literature. Moreover, necessary and sufficient conditions for particular values of t and n are stated.

INTRODUCTION

Several papers [1-6] have been concerned with the problem of system level fault diagnosis, with the aim of determining the conditions for the diagnosability of systems, when multiple hardware failure occur.

Interest in this topic is motivated by the need of highly available digital systems, which can continue their operation, even though at reduced capacity, when faults are present.

Any approach to this problem requires the detection and the location of the malfuntioning elements, or, in other words, the system must be diagnosable.

In this paper attention is focused on the diagnostic model which was introduced by Preparata et alt. [1].

A large system S is partitioned into n subunits.

The sets $F = \{f_1, f_2, \dots, f_n\}$ of the n faults which can occur, and $T = \{p_1, p_2, \dots, p_m\}$ of the m tests which are applied, are defined, relatively to the system S. Any subunit v_i is either faulty or fault free. Any subunit v_i is able to test all the subunits to which it is connected, and to monitor the outcome of those subunits on the application of the tests. The outcome is binary (faulty or fault free) and is the judgement of the testing subunit on the tested one. All the test outcomes represent the condition of the system at the same point in time. The outcome of the subunit v_i is reliable only if v_i is fault free. No subunit tests itself. The test which v_i applies to the other subunits is complete relatively to the class of faults, which are considered for those subunits, in the sense of [4]. It is possible to represent this model as a directed graph $G = G(V,E)$: the diagnostic graph. The node $v_i \in V$ corresponds to the subunit v_i of the system S; a directed edge (v_i, v_j) from the node v_i to the node v_j, represents a diagnostic connection from v_i to v_j. (v_i, v_j) is labelled by p_k, the complete test which the subunit v_i apply to the subunit v_j.

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In [3]- [4] two types of diagnosability are defined; the diagnosability without repair and the diagnosability with repair.

A system S is t-fault diagnosable without repair if and only if one application of the test set T is sufficient to identify precisely which faults are present in S, provided the number of faults present does not exceed t.

A system S is t-fault diagnosable with repair if and only if there exists a sequence of applications of the test set T and repairs of faults that allows all the faults originally present to be identified, provided the number of faults originally present does not exceed t.

In the following, t_M means the maximal value of t for which the definitions given are valid in a system S.

The problem of the t-fault diagnosability without repair is completely solved by Russell and Kime [3]- [4].

The problem of the t-fault diagnosability with repair is not yet solved.

About this topic in [3]- [4] the necessary and sufficient conditions are given in the case of t = 1,2 and 3, and some sufficient conditions for $t \geq 4$.

In our paper the sufficient conditions given in [3] - [4], are improved, and the necessary and sufficient conditions for particular values of t and n are given.

1. T-FAULT DIAGNOSABILITY WITH REPAIR OF CONNECTED GRAPHS

Let us consider a system S of n subunits and its diagnostic graph G = G(V,E). Let G be a connected graph.

In [7] is shown that G can be outlined, pointing out all the Maximal Strongly Connected (M.S.C.)* subgraphs present in G. The M.S.C. subgraphs are mutually disjoint and are connected by link subgraphs.

Let us now consider the subset of all the M.S.C. subgraphs, so that no link subgraph is in-connected with them. Let $G_1, G_2, \ldots, G_k$ be such subgraphs, and let $t_{M1}, t_{M2}, \ldots, t_{Mk}$ be the value t_M of their t-fault diagnosability with repair. Let:

$$t_{Mi} = \min \left\{ t_{M1}, t_{M2}, \ldots, t_{Mk} \right\}$$

<u>Theorem 1.1</u>: Let S be a system and let G = G(V,E) be its n-node connected diagnostic graph; a) if $\exists\, v_i \in V$ exists such that $(v_j, v_i) \notin E$ for j = 1,2,...,n, then S is not diagnosable; b) if no such v_i exists, then S is t_M-fault diagnosable with repair, where $t_M = t_{Mi}$.

<u>Proof</u>:

a) If v_i is faulty it cannot be diagnosed by any other subunit.

b) The system S cannot be t_j-fault diagnosable with repair if $t_j > t_{Mi}$. If this is the case, let us suppose that the $t_j > t_{Mi}$ faulty subunits are present in the M.S.C. subgraph G_i. As G_i is M.S.C. and no edge of G is directed toward G_i, it is not possible to identify and locate the $t_j > t_{Mi}$ faulty subunits. So $t_M \leq t_{Mi}$.

Let us now show that $t_M = t_{Mi}$. If the t_{Mi} faulty subunits are shared in $G_1, G_2, \ldots, G_k$, these M.S.C. subgraphs are completely diagnosed, that is after some repair it is possible to make $G_1, G_2, \ldots, G_k$ fault free.

Moreover the set of the paths from every node $v_i \in G_1, G_2, \ldots, G_k$ covers all the nodes

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* A maximal strongly connected subgraph is a strongly connected subgraph that includes all the possible nodes which are strongly connected with each other.

of G. If this is not the case, let V_1 be the set of nodes which are not covered by the set of paths from every node $v_i \in G_1, G_2, \ldots, G_k$, and V_2 be the set of nodes covered by those paths. The connection between V_2 and V_1 can be only from V_1 to V_2, so V_1 must contain at least one M.S.C. subgraph $G_m \notin \{G_1, G_2, \ldots, G_k\}$ with no edge of G directed toward G_m. But as $\{G_1, G_2, \ldots, G_k\}$ contains all the M.S.C. subgraphs of that type, this is not possible. Then every node of G is covered by at least one path which has as initial node a fault free subunit and it can be diagnosed.

Q.E.D.

Therefore, in the following, the t-fault diagnosability with repair of strongly connected graphs will be considered.

2. T-FAULT DIAGNOSABILITY WITH REPAIR OF STRONGLY CONNECTED GRAPHS

A direct consequence of the results given in [1] and [4] is the following: any n-node strongly connected graph is t_M-fault diagnosable with repair, with $t_m \leq t_M \leq t_{Max}$. where $t_{Max} = \left\lfloor \frac{n-1}{2} \right\rfloor$ and $n = \left\lfloor \left(\frac{t_m+2}{2}\right)^2 \right\rfloor + 1$.
$\lfloor x \rfloor$ means the greatest integer not more than x.
$\lceil x \rceil$ means the smallest integer not less than x.
Let $G = G(V,E)$ be a strongly connected graph that is t_M-fault diagnosable with repair. The following theorem makes clear why G is not (t_M+1)-fault diagnosable with repair.

A fault pattern $F^i = \{f_1, f_2, \ldots, f_j\}$ is a subset of F.
In G, F^i corresponds to the subset $V_{fi} = \{v_1, v_2, \ldots, v_j\}$ of faulty nodes of the set V. $|F^i|$ is the cardinality of the fault pattern F^i. F^i and F^j are indistinguishable if the system outcome, on the application of the test T, can be the same when F^i or F^j are present.

Theorem 2.1: Let S be a system and let $G = G(V,E)$ be its n-node strongly connected diagnostic graph.

If S is t_M-fault diagnosable with repair, p indistinguishable fault patterns $F^1, F^2, \ldots, F^p$ must exist (or p subsets of nodes $V_{f1}, V_{f2}, \ldots, V_{fp}$ in G) such that:

a) $\exists V_{fi} : |V_{fi}| = t_M + 1$;

b) $(\forall j \mid j \neq i) : |V_{fj}| \leq t_M + 1$;

c) $\bigcap_{i=1,\ldots,p} V_{fi} = \emptyset$

d) $\bigcup_{i=1,\ldots,p} V_{fi} = V$

Proof:

The points a),b),c) are necessary directly from the definition of t_M-fault diagnosability with repair.

If the point d) is not valid, it exists a not empty subset $V' \subset V$ such that $\exists v_j : v_j \in V'$, $v_j \notin \bigcup_{i=1,\ldots,p} V_{fi}$.

If this is the case, $(v_j, v_k) \notin E$, where $v_k \in \bigcup_{i=1,\ldots,p} V_{fi}$, as the outcome relative to these testing edges would distinguish the fault patterns to which v_k belongs from the others. As this holds for $\forall v_j \in V'$, then from every v_j it is not reachable v_k, contradicting the hypothesis that G is strongly connected.

Q.E.D.

The following corollaries are direct consequences of the theorem 2.1. The proofs are given in [8].

Corollary 2.1: If $t_M < \left\lfloor \frac{n-1}{2} \right\rfloor$, then $p \geq 3$.

Corollary 2.2: Let $V_{f1}, V_{f2}, \ldots, V_{fp}$ be the indistinguishable subsets of nodes of the strongly connected graph $G = G(V,E)$. If $t_M < \left\lfloor \frac{n-1}{2} \right\rfloor$, for $i,j = 1,\ldots,p$ must be:

$$V_{fi} \cap V_{fj} \neq \emptyset$$

Corollary 2.3: If $v_j \in V_{f1}, V_{f2}, \ldots, V_{fm}$ and $v_j \notin \bigcup_{i=m+1,\ldots,p} V_{fi}$, then
$\bigcap_{i=m+1,m+2,\ldots,p} V_{fi} \neq \emptyset$

Corollary 2.4: Let $V_{f1}, V_{f2}, \ldots, V_{fp}$, be the subsets of nodes of $G = G(V,E)$, which fulfill the conditions of the theorem 2.1, and let $\{V_{f1}, V_{f2}, \ldots, V_{fk}\}$ be the minimal set of $k \leq p$ subsets of nodes, such that:

$$\bigcap_{i=1,2,\ldots,k} V_{fi} = \emptyset$$

Then:

a) $(\exists V_{fi} \mid 1 \leq i \leq k) : |V_{fi}| = t_M + 1$

b) $(\forall j \mid j = 1,\ldots,k) : \bigcap_{i=1,\ldots,j-1,j+1,\ldots,k} V_{fi} \neq \emptyset$

c) $\bigcup_{i=1,\ldots,k} V_{fi} = V$

It is now possible to collect all the conditions given, in a theorem, which is a necessary condition for the t_M-fault diagnosability with repair of a system S.

Theorem 2.2: Let S be a system and let $G = G(V,E)$ be its n-node strongly connected diagnostic graph.

If S is t_M-fault diagnosable with repair, where $t_M < \left\lfloor \frac{n-1}{2} \right\rfloor$, k indistinguishable subsets of nodes $V_{f1}, V_{f2}, \ldots, V_{fk}$ exist such that:

a) $k \geq 3$

b) $\exists V_{fi} : |V_{fi}| = t_M + 1$

c) $(\forall j \mid j \neq i) : |V_{fj}| \leq t_M + 1$

d) $\bigcap_{i=1,2,\ldots,k} V_{fi} = \emptyset$

e) $\bigcup_{i=1,2,\ldots,k} V_{fi} = V$

f) $(\forall j \mid j = 1,2,\ldots,k) : \bigcap_{i=1,\ldots,j-1,j+1,\ldots,k} V_{fi} \neq \emptyset$

A direct consequence of the theorem 2.2, is a relation between the number n of nodes of any graph G and the value t_m, getting the same result given in the theorem 4.4.1 in [4].

The minimal number n_{min} of nodes which cannot be covered by the k indistinguishable subsets of nodes $V_{f1}, V_{f2}, \ldots, V_{fk}$, where at least $|V_{fi}| = t_m$, is given by the following relation:

$$n_{min} = k\,t_m - k\,(k-2) + 1 \qquad (1)$$

In (1) $V_{fi} = t_m$ for $i = 1,\ldots,k$, and from the value $k\,t_m$ is subtracted the minimal number of nodes, that is k, which was considered (k-2) times more.

Any graph for which (1) is valid, is at least t_m-fault diagnosable with repair. Maximizing (1), we get:

$$n_{min} = \left\lfloor \left(\frac{t_m+2}{2} \right)^2 \right\rfloor + 1$$

The maximal number n_{max} of nodes, which can be covered by the k indistinguishable subsets of nodes $V_{f1}, V_{f2}, \ldots, V_{fk}$, where at least $|V_{fi}| = t_m + 1$, is given by the following relation:

$$n_{max} = k(t_m + 1) - k(k-2) \qquad (2)$$

Maximizing (2), as before, we get:

$$n_{max} = \left\lfloor \left(\frac{t_m+3}{2} \right)^2 \right\rfloor$$

Therefore any n-node strongly connected graph is at least t_m-fault diagnosable with repair if:

$$\left\lfloor \left(\frac{t_m+2}{2} \right)^2 \right\rfloor + 1 \leq n \leq \left\lfloor \left(\frac{t_m+3}{2} \right)^2 \right\rfloor \qquad (3)$$

2.1 Boundary graphs

Reversing the arguments which have brought to the theorem 2.2, given n nodes and the value t_M, it is possible to cover the n nodes with k subsets $V_{f1}, V_{f2}, \ldots, V_{fk}$ so that the conditions a) to f) of the theorem 2.2 are verified.

If the n nodes are connected with edges, accordingly to one of the following procedures, we make the k subsets indistinguishable. Every node v_j partitions the set $\{V_{f1}, V_{f2}, \ldots, V_{fk}\}$ in two blocks V_f' and V_f''

such that:

$(\forall i \mid i=1,\dots,k)$ if $v_j \in V_{fi}$ then $V_{fi} \in V'_f$

$(\forall i \mid i=1,\dots,k)$ if $v_j \in V_{fi}$ then $V_{fi} \in V''_f$

without loss of generality let us consider that:

$$V'_f = \{V_{f1}, V_{f2}, \dots, V_{fh}\}$$

and

$$V''_f = \{V_{f(h+1)}, V_{f(h+2)}, \dots, V_{fk}\}$$

Procedure 1 (outdegree of a node): The edge (v_j, v_p) is drawn, if:

$a_1)$ $v_p \in \bigcap_{i=h+1,\dots,k} V_{fi}$

$b_1)$ $v_p \in V - \bigcup_{i=h+1,\dots,k} V_{fi}$

Procedure 2 (indegree of a node): The edge (v_p, v_j) is drawn if:

$a_2)$ $v_p \in \bigcap_{i=h+1,\dots,k} V_{fi}$

$b_2)$ $v_p \in \bigcap_{i=1,\dots,h} V_{fi}$

In [8] is shown that these two procedures are equivalent, or in other words, the same graph is drawn, using one or the other.

Theorem 2.1.2: The graph G, drawn with one of the two procedures, is not (t_M+1)-fault diagnosable with repair.

Proof:

As the k subsets $V_{f1}, V_{f2}, \dots, V_{fk}$ verify the conditions a) to f) of the theorem 2.2, it is shown that the edges, drawn with the procedure 1 or the procedure 2, make $V_{f1}, V_{f2}, \dots, V_{fk}$ indistinguishable. Let us consider the procedure 1.

Let $v_j \in \bigcap_{i=1,\dots,h} V_{fi}$ and $v_p \in \bigcap_{i=h+1,\dots,k} V_{fi}$. The edge (v_j, v_p) is labelled with 1 (that is v_j is fault free and diagnoses v_p faulty) when $h+1 \leq i \leq k$. (v_j, v_p) is labelled with X (the outcome of v_j is not meaningful, as v_j is faulty) when $1 \leq i \leq h$. Let $v_j \in \bigcap_{i=1,\dots,h} V_{fi}$ and $v_p \in V - \bigcup_{i=h+1,\dots,k} V_{fi}$. The edge (v_j, v_p) is labelled with 0 (v_j is fault free and diaggnoses v_p fault free) when $h+1 \leq i \leq k$. (v_j, v_p) is labelled with X when $1 \leq i \leq h$.

Therefore the test outcomes are compatible and $V_{f1}, V_{f2}, \dots, V_{fk}$ are indistinguishable.

Q.E.D.

In [8] it is shown that the graph drawn with one of the given procedures are the maximal graphs which make indistinguishable the k subsets on which they are drawn.

Let now $G' = G'(V,E')$ and $G = G(V,E)$ be two graphs drawn respectively on the p subsets of nodes $V_{f1}, V_{f2}, \dots, V_{fp}$ and on the k subsets of nodes $V_{f1}, V_{f2}, \dots, V_{fk}$, where $k < p$.

In [8] it is shown that the following theorem holds:

Theorem 2.1.3: The graphs $G' = G'(V,E')$ and $G = G(V,E)$ are such that:

$$E' \subset E.$$

We define boundary graphs those n-node graphs, drawn on all the possible minimal subsets of k elements $V_{f1}, V_{f2}, \dots, V_{fk}$, which verify the conditions a) to f) of the theorem 2.2.

In the following a boundary graph will be indicated with $B_i = B_i(V, E_i)$.
We can now conclude:

Theorem 2.1.4: Let S be a system and let $G_1 = G_1(V, E_1)$ be its n-node strongly connected diagnostic graph. If S is t_M-fault diagnosable with repair it must exist at least one boundary graph $B_i = B_i(V, E_i)$ relative to those values of n and t_M, such that:

$$E_1 \subseteq E_i .$$

It is now given a procedure to determine the value t_M of diagnosability with repair of a system S, whose diagnostic graph G_1 is strongly connected.

Procedure 3:

Step 1) Set $t = t_m$, go to step 2).

Step 2) Draw all the boundary graphs relative to t and n. Let $B = B_1, B_2, \dots, B_s$ be the set of boundary graphs. Compare $G_1 = G_1(V, E_1)$ with every $B_i = B_i(V, E_i)$. If $\exists B_i : E_1 \subseteq E_i$ stop ($t_M = t$); otherwise go to step 3).

Step 3) Set $t = t+1$, go to step 2).

Generally, several $B_i = B_i(V, E_i)$ can be removed when, drawing them, it is found that it exists $B_j = B_j(V, E_j)$ such that:

$E_i \subseteq E_j$.

A direct consequence of what has been said till now, is that every system S of n subunits, with $3 \leq n \leq 8$ or $n = 10$, and whose diagnostic graph is strongly connected, can have only one value t_M, as for those values of n, $t_m = t_{Max}$.

Example 1: Let S be a system composed by $n = 9$ subunits; S can be either 3 or 4-fault diagnosable with repair. Let us draw the boundary graphs.

The only arrangement of k subsets of nodes, for $n = 9$ and $t = 3$, which verify the conditions a) to f) of the theorem 2.2 is shown in Fig. 1.

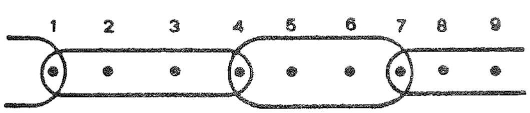

FIG. 1

The boundary graph, drawn with one of the given procedures, is outlined in Fig. 2. Each block in Fig. 2 represents a 3-node complete subgraph, and the arrow outcoming from one node of the block represents the fact that that node has outdegree toward all the other nodes.

The particular structure of the boundary graph, shown in example 1 in the case of $n = 9$, can be generalized when $n = \left\lfloor \left(\frac{t_m+3}{2} \right)^2 \right\rfloor$.

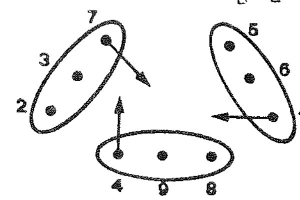

FIG. 2

In [8] it is shown that when $n = \left\lfloor \left(\frac{t_m+3}{2} \right)^2 \right\rfloor$ and t_m is odd, only one boundary graph exists. This graph can be outlined, as in Fig. 3, in k blocks $P_1, P_2, \dots, P_k$, where $|P_i| = k$.

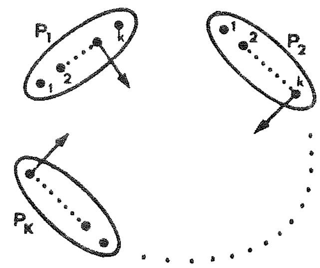

FIG. 3

If $n = \left\lfloor \left(\frac{t_m+3}{2} \right)^2 \right\rfloor$ and t_m is even, only two boundary graphs exist. They can be outlined as in Fig. 4; the former has k blocks $P_1, P_2, \dots, P_k$ with $|P_i| = k+1$, the latter has $k+1$ blocks $P_1, P_2, \dots, P_{k+1}$ with $|P_i| = k$.

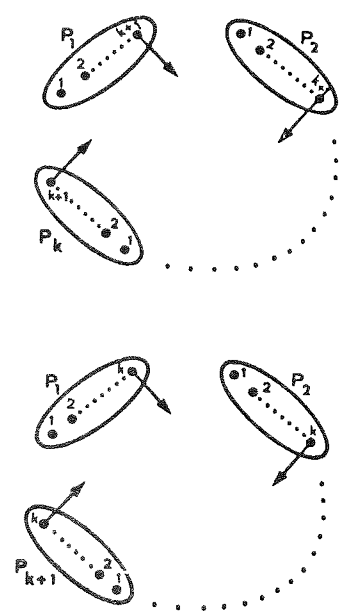

FIG. 4

It follows directly:

Theorem 2.1.5: Let S be a system and let $G_1 = G_1(V, E_1)$ be its n-node strongly connected diagnostic graph, where $n = \left\lfloor \left(\frac{t_m+3}{2}\right)^2 \right\rfloor$;

S is t_M-fault diagnosable with repair, and it holds $t_M = t_m$, if and only if there exists at least one partition of the n nodes in k blocks $P_1, P_2, \dots, P_k$ with $|P_i| = k$, such that each block has outdegree to the others from only one node of it, when t_m is odd; or there exists at least one partition of the n nodes in k blocks $P_1, P_2, \dots, P_k$ with $|P_i| = k+1$, or one partition in k+1 blocks $P_1, P_2, \dots, P_{k+1}$ with $|P_i| = k$, such that each block has outdegree to the others from only one node of it, when t_m is even.

When the number of boundary graphs is high it is difficult to apply the procedure 3; however the approach based on the boundary graphs allows to state some sufficient conditions about the t-fault diagnosability with repair which improve those given in [4].

2.2 Sufficient conditions for the t-fault diagnosability with repair or strongly connected graphs

Let us consider all the boundary graphs, for n and t. Let $v_j \in \bigcap_{i=1,2,\dots,p} V_{fi}$ and $v_j \notin \bigcup_{i=p+1,p+2,\dots,k} V_{fi}$. The indegree of v_j, ind v_j, from procedure 2 is:

$$\text{ind } v_j = \left|\bigcap_{i=1,2,\dots,p} V_{fi}\right| + \left|\bigcap_{i=p+1,p+2,\dots,k} V_{fi}\right| - 1 \quad (4)$$

Let us outline the n nodes in Fig. 5.

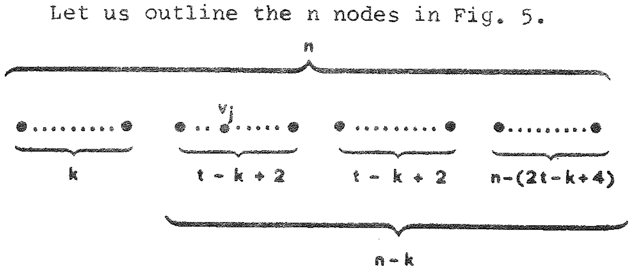

FIG. 5

In Fig. 5, the k nodes, which are common to the (k−1)-uples of subsets, are pointed out. $\bigcap_{i=1,2,\dots,p} V_{fi}$ and $\bigcap_{i=p+1,p+2,\dots,k} V_{fi}$ could each cover $t-k+2$ nodes of the remaining $n-k$.

If this is the case, the k subsets could not cover all the n nodes. Infact they would cover $2t-k+4$ nodes and $2t-k+4 < n$, as $t < \left\lfloor \frac{n-1}{2} \right\rfloor$.

Therefore:

$$\text{ind } v_j < 2t - k + 3 \quad (5)$$

In order to find the real maximal value of ind v_j the following theorem is useful.

Theorem 2.2.1: The maximal indegree of a node v_j, considering all the boundary graphs for n and t, is that one of a node v_j which is covered by one and only one subset V_{fi}.

The proof is given in [8].

Theorem 2.2.2: Any node v_j of any boundary graph, for n and t, is such that:

ind $v_j \leq \text{ind}_{max}\, v_j$, where

$$\text{ind}_{max}\, v_j = 2(t+1) - 2 - \left\lceil \sqrt{n-2(t+1)} \right\rceil - \left\lceil \frac{n-2(t+1)}{\left\lceil \sqrt{n-2(t+1)} \right\rceil} \right\rceil$$

Proof:

Let $v_j \in V_{fj}$. It follows from the procedure 2:

$$\text{ind } v_j = |V_{fj}| + \left|\bigcap_{i=1,\dots,j-1,j+1,\dots,k} V_{fi}\right| - 1 \quad (6)$$

The maximal value of the (6) is when $|V_{fj}| = t+1$ and $\left|\bigcap_{i=1,\dots,j-1,j+1,\dots,k} V_{fi}\right| = h_{max} - (k-1)$, where h_{max} is the maximal number of nodes common to the (k−1)-uple $V_{f1}, \dots, V_{f(j-1)}, V_{f(j+1)}, \dots, V_{fk}$, and $k-1$ is the minimal number of nodes, covered by the (k−1)-uples, which are covered by V_{fj}. In [8] is shown:

$$h_{max} = \left\lfloor \frac{k(t+1)-n}{k-2} \right\rfloor$$

Therefore:

$$\text{ind } v_j = t+1-k+\left\lfloor \frac{k(t+1)-n}{k-2} \right\rfloor \qquad (7)$$

Maximizing (7), with respect to k, as shown in [8], we get:

$$\text{ind}_{max}\, v_j = 2(t+1)-2-\left\lceil \sqrt{n-2(t+1)} \right\rceil - \left\lceil \frac{n-2(t+1)}{\left\lceil \sqrt{n-2(t+1)} \right\rceil} \right\rceil$$

Q.E.D.

It is possible now to state:

<u>Theorem 2.2.3</u>: Let S be a system and let $G_1 = G_1(V,E_1)$ be its n-node strongly connected diagnostic graph;

S is t_M-fault diagnosable with repair, where $t_M \geq t$, if a node $v_j \in V$ exists such that:

$$\text{ind } v_j \geq 2t-1-\left\lceil \sqrt{n-2t} \right\rceil - \left\lceil \frac{n-2t}{\left\lceil \sqrt{n-2t} \right\rceil} \right\rceil \qquad (8)$$

The proof follows obviously from the theorem 2.2.2, replacing $t+1$ with t.

In a similar way it is possible to state some other sufficient conditions about the number of distinct predecessor nodes of a couple of nodes $v_j, v_l \in V$ in the diagnostic graph $G_1 = G_1(V,E_1)$ of a system S.

The proof of the following theorem is given in [8].

<u>Theorem 2.2.4</u>: Let S be a system and let $G_1 = G_1(V,E_1)$ be its n-node strongly connected diagnostic graph;

S is t_M-fault diagnosable with repair, with $t_M \geq t$, if two nodes $v_j, v_l \in V$ exist such that the number ind $|v_j, v_l|$ of distinct predecessor nodes is:

$$\text{ind } |v_j, v_l| \geq 2t - k_{min} + 1 \qquad (9)$$

where k_{min} is the smallest value of the number k of subsets of nodes $V_{f1}, V_{f2}, \ldots, V_{fk}$, given n and t.

The sufficient conditions given in the theorem 4.4.4 of [4] are:
A system S is t-fault diagnosable with repair if:

a) ind $v_j \geq 2t-1$; the worst case of (8) is when $t = \left\lfloor \frac{n-1}{2} \right\rfloor$. If n is odd the (8) has the value ind $v_j \geq 2t-3$. If n is even the (8) has the value ind $v_j \geq 2t-4$.

b) ind $|v_j, v_l| \geq 2t$ for any couple v_j, v_l;

c) ind $|v_j, v_l| \geq 2t-1$ if either $(v_j, v_l) \notin E_1$ or $(v_l, v_j) \notin E_1$;

d) ind $|v_j, v_l| \geq 2t-2$ if $(v_j, v_l) \notin E_1$ and $(v_l, v_j) \notin E_1$.

As $k_{min} \geq 3$, in the worst case the (9) is equal to d), but improves points b) and c). When $k_{min} > 3$ all b), c) and d) are improved.

<u>Example 2</u>: Let $G_1 = G_1(V,E_1)$ be the graph with $n=16$, shown in Fig. 6.

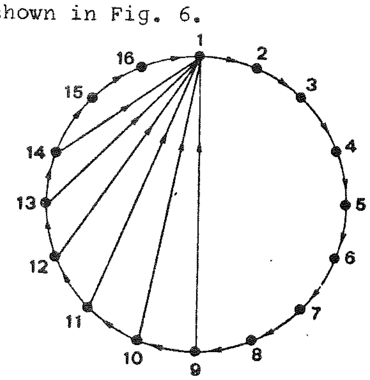

FIG. 6

Any strongly connected graph with $n=16$ has $t_m = 5$.

As ind $(1) = 7$, from (8) we can conclude that $t_M \geq 6$, while using point d) of the theorem 4.4.4 of [4], as ind $|1,8| = 8$, it is only possible to say that $t_M \geq 5$.

3. CONCLUDING REMARKS

In this paper the boundary graph approach is introduced to study the t-fault diagnosability with repair of digital systems.

Many problems are not yet solved and the study is in progress.

First of all a formal characterization of the elimination rules among the boundary graphs would allow the drawing up of a catalog, so that the procedure 3 could be more efficiently used. Moreover, it is opinion of the authors, by a first draft, that the boundary graph approach is very interesting for what concerns the optimal design of systems, that is those designs which use the minimal number of connections among the subunits, given n and t_M. Some results for particular values of n and t_M have been just found, but they are not yet completely formalized and generalized to any value of n and t_M.

These topics will be the object of a next report.

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