# Topological Classification of Vittorio Giorgini's Sculptures

Daniela Giorgi[1], Marco Del Francia[2], Massimo Ferri[3] and Paolo Cignoni[1]

[1]ISTI-CNR, Pisa, Italy; daniela.giorgi@isti.cnr.it, paolo.cignoni@isti.cnr.it
[2] B.A.Co - Archivio Vittorio Giorgini, Italy; marco@archiviovittoriogiorgini.it
[3]University of Bologna, Italy; massimo.ferri@unibo.it

## Abstract

The Italian architect Vittorio Giorgini (1926 – 2010) was among the first artists to incorporate the topological concepts of space and transformation into architecture and art. In this paper, we first present Giorgini's beautiful pieces of art, which were largely unrecognized in his lifetime. Then we unveil the true mathematical types of some of his most famous sculptures.

## Introduction

The 19th century saw the development of a new branch of mathematics, *topology*, which would have a great impact on the idea of shape and space, not only for mathematicians, but also for the arts. Topology is commonly referred to as the study of properties of geometrical objects that are invariant under continuous transformations: "*[Topology] has as its object the study of properties of geometrical figures that persist even when the figures are subjected to deformations so drastic that all their metric and projective properties are lost*" [1].

As observed by Michele Emmer [4], this idea of a continuous distortion that still preserves the properties of an object was inevitably sensed by artists and architects in the past decades. Alicia Imperiale noted that "*the scientific discoveries have radically changed the definition of the word 'Space', attributing a topological shape to it. Rather than a static model of consecutive elements, space is perceived as something malleable, mutating, and its organization, its division, its appropriation becomes elastic*" [6]. Di Cristina talked about "*a topological tendency in architects at both theoretical and operative levels . . . The topologisizing of architectonic form according to dynamic and complex configurations leads architectural design to a renewed and often spectacular plasticity, in the wake of the Baroque of Organic Expressionism*" [3].

Among the artists who followed this tendency to incorporate the topological idea of space and dynamics into architecture, we focus this paper on the italian architect Vittorio Giorgini (1926 – 2010). Giorgini pioneered the notion of form as dynamics of transformation, though it was only recently that he received the attention he deserved [2, 9]. Beside his architectural works, Vittorio Giorgini left a few enchanting sculptures representing non-orientable surfaces, in which he explored the connections between shape and topology.

Our aim is twofold: to present Giorgini's beautiful works of art, and to explain the topological ideas behind some of his sculptures, in a formal mathematical language. In particular, we show that, surprisingly enough for non-mathematicians, four of his most famous sculptures – *Modified Klein*, *Giorgini Sphere*, *Giorgini Torus I*, and *Giorgini Torus III* – are topologically equivalent, that is, they are the same object up to continuous deformations, even though their geometric appearance is very different.

We begin with an introduction to Vittorio Giorgini's life and work, then focus on his sculptures and analyse them from a topological perspective.

## Vittorio Giorgini: a morphological architect

Vittorio Giorgini was born in Tuscany in 1926. He came to maturity during the 1950's, when a new current of thought was coming forward which associated design with nature, uniting artists such as Gaudí, Le Corbusier, Fuller among others. Nature started to be investigated not only as a source of solutions to copy, but as a true methodological guide: natural evolution proceeds with dynamic transformations and continuous adaptations, to achieve both optimal functioning and economy. In architecture, this corresponds to achieving static efficiency and minimum amount of material. Vittorio Giorgini, as a *morphological architect*, was a forerunner in the study of natural objects in terms of the formation of their shape and structure, the latter intended as the way elements and parts connect. Giorgini outlined the difference between an object which is built and one which grows and transforms itself [2]. He experimented with the study of natural structures and the way form develops with his first experiments in architectural design, namely *Casa Esagono* and *Casa Saldarini* – also known as *Casa Balena* for its zoomorphic form – at Gulf of Baratti, Livorno (Figure 1(a,b)).

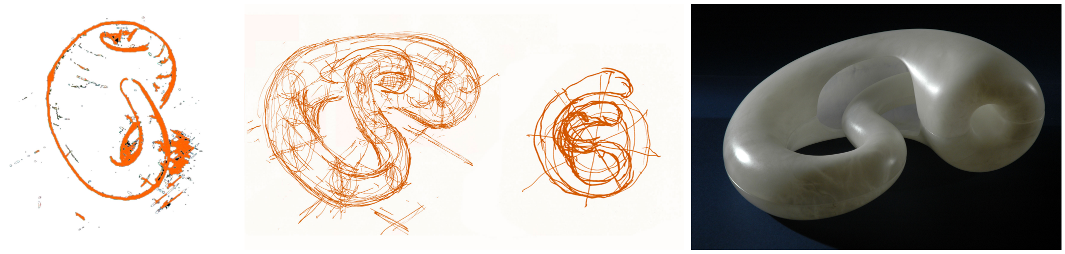
Giorgini coined the term *Spatiology* to define its studies in morphology, during the first Triennial Itinerant Exhibition of Italian Contemporary Architecture in Florence in 1965: spatiology means "*the study of geometry as a mathematical discipline and the backbone of statics, systematic taxonomy and technology*" [5]. Giorgini was highly influenced by the English naturalist, mathematician and orator D'Arcy Wentworth Thompson, who first expressed the importance of investigating natural form in a fully quantitative manner. In its famous book *On growth and form* [8], in the context of biological taxonomy, Thompson wrote that "*the study of form may be descriptive merely, or it may become analytical. We begin by describing the shape of an object in the simple words of common speech: we end by defining it in the precise language of mathematics; and the one method tends to follow the other in strict scientific order and historical continuity."*

At the end of the 1960's, still largely unrecognized by his colleagues, Giorgini left Italy and migrated to the United States, where he remained until the mid 1990's. In the US, in-between 1976 and 1979, he organized an educational workshop with students with the aim of designing and building the *Liberty Rural Community Centre*, to educate and help youths (Figure 1(c)). The scale models and drawings Giorgini made for Liberty are preserved at the Frac Centre-Val de Loire at Orléans.

**(a)** **(b)** **(c)**

**Figure 1:** *(a) Casa Esagono and (b) Casa Saldarini, Gulf of Baratti, Livorno, Italy. (c) The Liberty project, New York, USA.*

Unfortunately, the Liberty project was suspended for lack of funds. Yet, thanks to this project, Giorgini further tested and refined his building techniques based on geometrical and topological research, and developed new new three-dimensional figures: the *Modified Klein* (Figure 2 and 3), the *Giorgini Sphere* (Figure 4), and variations I-IV on the *Giorgini Torus* (Figure 5, 6, 7 and 8). Giorgini studied Leonardo da Vinci's drawings, and inherited the Leonardesque idea of *variations* in nature. The term *variations* refers to the process by which nature transform existing forms into new creations by using the same basic elements (e.g., limbs and body parts in animals, and trunk, branches and leaves in trees). The basic elements in Giorgini's variations are topological surfaces, and Giorgini shows how removing parts of the surfaces and connecting the so-formed boundaries give rise to different form variations.



**(a)** **(b)** **(c)**

**Figure 2:** *Giorgini's drawings of (a) the Klein bottle, and (b) the Klein bottle with a disk removed around the self-intersection of the surface; (c) Giorgini's sculpture of the Klein bottle minus a disk: Modified Klein; width: 20 inches; height: 6 inches; material: alabaster.*

**(a)** **(b)**

**Figure 3:** *(a) Giorgini's drawing of the Möbius strip, which can be seen as half of the Klein bottle, like (b) one of the two pieces that together make the Modified Klein sculpture.*

Giorgini designed his three-dimensional figures in-between the late 1960's (*Giorgini Sphere*) and the years 2004–2006 (*Modified Klein* and *Giorgini Torus IV*). The figures only existed as bi-dimensional drawings for many years, apart from a hand-carved wax model of the *Giorgini Sphere*, which is now lost. Then, Giorgini asked David Dainelli and Alessandro Marzetti to carve alabaster sculptures from his drawings, in their artistic workshop in Volterra (Pisa, Italy), under his direct supervision (Figure 9). Dainelli and Marzetti had to design custom tools to carve the particular cavities of Giorgini's models from alabaster. The supervision of the work by Dainelli and Marzetti gave Giorgini inspiration for the design of the *Modified Klein*, and of the *Giorgini Torus IV* while the artists were realizing the *Giorgini Torus III*. The individual names were given to the scupltures by Giorgini himself. With those names, the sculptures were shown at the exhibition *Vittorio Giorgini Architetto. MorfoTopoSpazio-Logia*, in Volterra, Tuscany, in 2006.

In what follows, using concepts from algebraic topology [7], we analyse the sculptures by Vittorio Giorgini, reporting on their topological classification.

## Topological classification of Vittorio Giorgini's sculptures

Giorgini's sculptures offer interesting topological insights we believe are worth being discussed. The figures represent *non-orientable surfaces*. A well-known example of non-orientable surface is the Möbius strip, which can be obtained by cutting a long strip of paper, and gluing the ends of the strip together after making

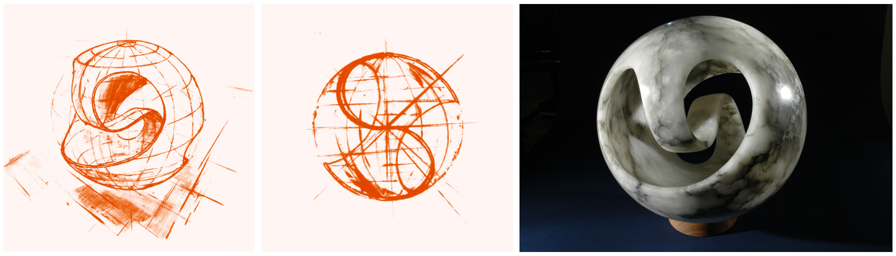


**Figure 4:** *Drawings for the Giorgini Sphere, and the actual sculpture: diameter: 16 inches; material: alabaster. The surface is a Möbius short, topologically equivalent to the Klein bottle minus a disk.*

**Figure 5:** *Drawings for the Giorgini Torus I, and the actual sculpture: diameter: 16 inches; height: 5 inches; material: alabaster. The surface is a Möbius short, topologically equivalent to the Klein bottle minus a disk.*

a half twist. Intuitively, non-orientable surfaces are one-sided: one could paint in colour the whole surface without crossing its boundary and without detaching the brush. That would not be possible for orientable surfaces such as the cylinder and the sphere, which are two-sided.

Another well-known example of non-orientable surface is the Klein bottle. The Klein bottle was discovered by Felix Klein in 1882. It is even more fascinating than the Möbius strip, as not only it is one-sided, but also closed (it has no boundaries) while it does not enclose an interior. Therefore, it has been both visualized with computer graphics techniques and physically reproduced many times in its three-dimensional projection, where it shows a self-intersection (absent in dimension four, where the surface actually sits).

The first sculpture we examine is *Modified Klein*, a Klein bottle with a disk removed (Figure 2). Indeed, as it often happens with physical reproductions of the Klein bottle, Giorgini cut a disk around the surface self-intersection. Moreover, for both allowing simpler manufacturing and exposing the interior of the bottle, he produced the sculpture in two separate halves; separating a Klein bottle into two halves by cutting along

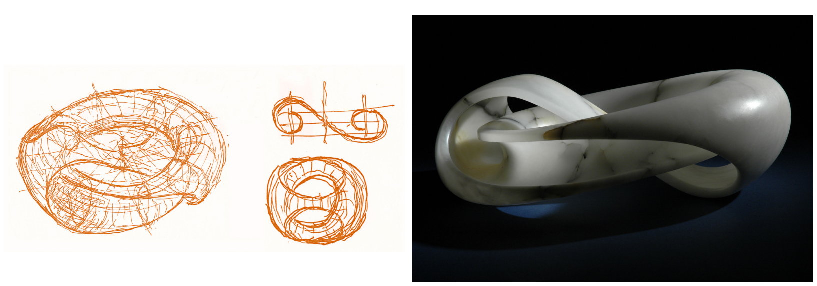


**Figure 6:** *Drawings for the Giorgini Torus III, and the actual sculpture: diameter: 16 inches; height: 5 inches; material: alabaster. The surface is a Möbius short, topologically equivalent to the Klein bottle minus a disk.*

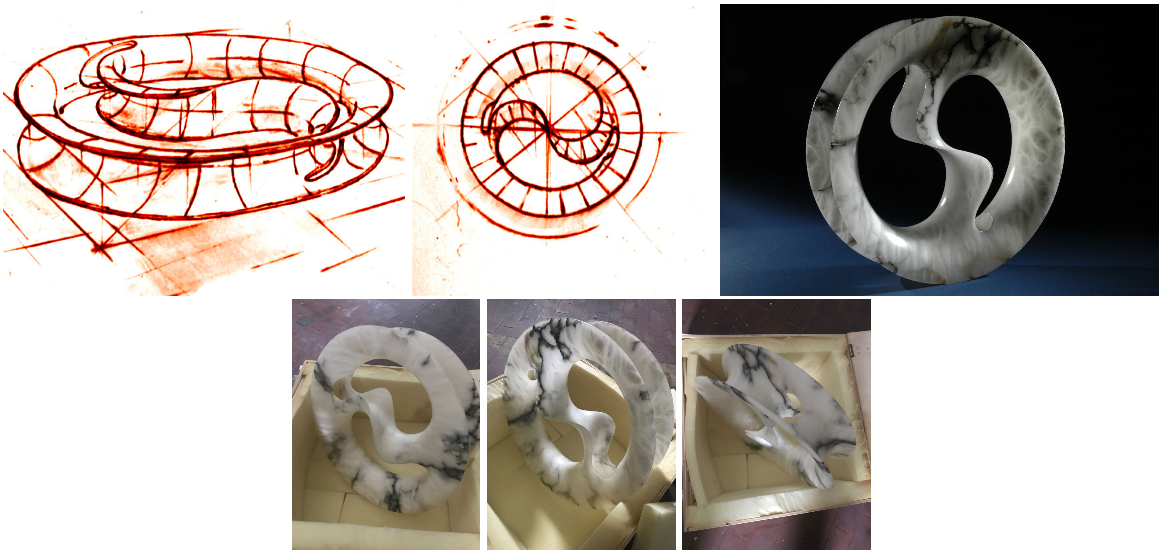


**Figure 7:** *Drawings for the Giorgini Torus II, and the actual sculpture: diameter: 16 inches; height: 5 inches; material: alabaster. The surface is topologically equivalent to the a Möbius strip minus two disks.*

**Figure 8:** *Drawings for the Giorgini Torus IV, and the actual sculpture: diameter: 16 inches; height: 5 inches; material: alabaster. The surface is topologically equivalent to the connected sum of three projective planes minus two disks.*

**Figure 9:** *A photograph taken during the realization of the sculptures: Vittorio Giorgini (middle), Alessandro Marzetti (left), and David Dainelli (right).*

its plane of symmetry results in two Möbius strips, one right-sided and one left-sided (Figure 3).

As for *the Giorgini Sphere*, the *Giorgini Torus I*, and the *Giorgini Torus III* in Figures 4, 5, and 6, respectively, what would sound surprising to a non-topologist is that, beside being neither spheres nor toruses, they are topologically equivalent to *Modified Klein*. Topological equivalence is defined as the existence of a continuous map with continuous inverse between two objects. If the two objects live inside a space like ours and it is possible to continuously deform one object to the other, then equivalence is granted. This is the case of our four sculptures: they can be continuously deformed into one another, even though their geometric appearances are so different from each other. The deformation process is illustrated in Figure 10.

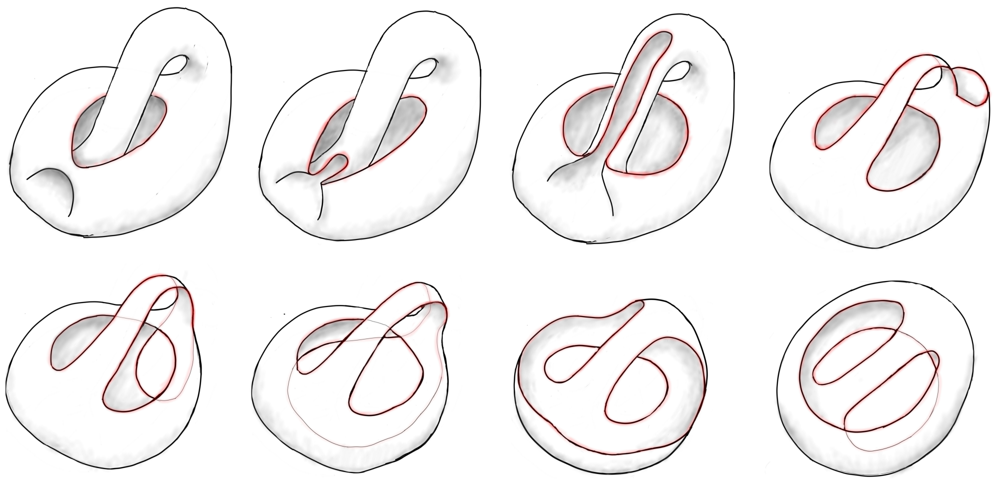
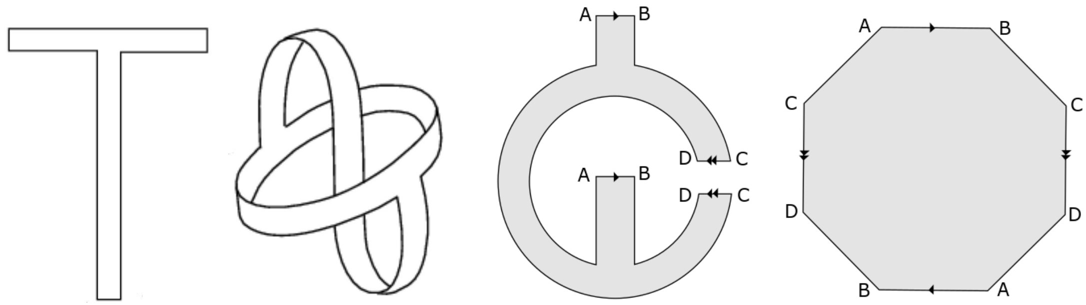


**Figure 10:** *Steps in the continuous deformation taking* Modified Klein *to* Giorgini Torus I. *The red line marks the disk around the surface self-intersection, along the deformation process.*

The topological equivalence stems from the fact that all three sculptures are Möbius shorts, which are exactly Klein bottles with one hole [10]. To get an intuition, with reference to Figure 11(a), the Möbius shorts can be constructed by starting with a $T$-shape, gluing the top part sides to form a ring (a cylinder), passing the bottom side upward to the ring, then turning it down, and finally gluing it without twisting to the border of the ring. Examining the *Giorgini Sphere*, one can easily see it can be obtained with the same construction: starting with a a spherical surface, one cuts away two disks, which makes a cylinder; then, one has to glue a strip between the two boundaries as described above. For the *Giorgini Torus I*, the cylinder to start with is obtained by removing the internal half of the torus (the part with negative curvature), while the rest of the procedure stays the same. Similar reasonings apply to the *Giorgini Torus III*. The result, in all three cases, is a non-orientable surface with one boundary.

The demonstration of the topological equivalence between the Möbius shorts (hence the *Giorgini Sphere* and the *Giorgini Torus I* and *III*) and the Klein bottle minus a disk (hence the *Modified Klein*) relies on the notions of *non-orientable genus* and *Euler characteristic*, and on the classification theorem for surfaces. In what follows, the term *sphere* refers to the spherical surface. An orientable surface has *genus* $g$, $g \geq 0$, if it is topologically equivalent to a sphere with $g$ handles sewn on; for example, the sphere has genus 0, and the torus has genus 1. A non-orientable surface has *non-orientable genus* $k$, $k \geq 1$, if it is topologically equivalent to a sphere with $k$ Möbius strips sewn on; for example, the projective plane has non-orientable genus 1, and the Klein bottle has non-orientable genus 2. The *Euler characteristic* $\chi$ of a surface is defined as the alternate sum of the number of $0-$, $1-$, and $2-$cells into which the surface can be decomposed. It is a topological invariant, and it is independent of the decomposition. For example, the sphere has Euler characteristic equal to 2. Indeed, a sphere can be obtained by starting with a 0-cell (i.e. a point); attaching a 1-cell (i.e. a segment) by glueing its boundary (i.e. its endpoints) to the 0-cell, thus getting a circle; then




**(a)** **(b)**

**Figure 11:** *(a) The construction of the Möbius shorts, from Ralph P. Boas Jr. [10]. (b) The fundamental polygon representation of the Möbius shorts. The closed surface obtained by closing the boundary with a disk has four* 0*-cells (*$A,B,C,D$*), six* 1*-cells (up to pairwise identifications of sides* $AB$ *and* $CD$*), and two* 2*-cells (including the disk to close the boundary); therefore, its Euler characteristic is* $\chi = 4 - 6 + 2 = 0$*, and its non-orientable genus is* $k = 2 - \chi = 2$*. It follows from the classification theorem that the closed surface is homeomorphic to the Klein bottle, and the surface with boundary is homeomorphic to the Klein bottle minus a disk.*

glueing two 2-cells (i.e. disks) to the circle. Alternatively, one can build a sphere by attaching a disk to a point, by glueing the disk border to the point. In these examples, the Euler characteristic of the sphere can be computed as $\chi = 1 - 1 + 2 = 1 - 0 + 1 = 2$. The Euler characteristic is related to the surface genus. For orientable closed surfaces, it holds that $\chi = 2 - 2g$, with $g$ the surface genus. For non-orientable closed surfaces, it holds that $\chi = 2 - k$, with $k$ the surface non-orientable genus.

The *classification theorem* gives a complete catalogue for closed (i.e. without boundary) surfaces. It states that the topological type of a closed surface is completely determined by two pieces of information: its Euler characteristic (and therefore its genus), and whether it is orientable or not. To classify surfaces with boundary, one counts the number of connected components (i.e. separate pieces) of the boundary. Since all connected components are topologically circles, if one caps each circle with a disk, one gets a closed surface, for which we can compute the Euler characteristic (or the genus), and follow the classification rules for closed surfaces.

Concerning our sculptures, if we close (univocally) the boundary of the *Giorgini Sphere* (the *Giorgini Totus I*, the *Giorgini Torus III*) with a disk, we get a closed surface. For closed surfaces, the Euler characteristic cam be computed using *fundamental polygons*, polygons with oriented sides, which can be identified in pairs. For example, the fundamental polygon of the torus is a square, whose opposite sides are identified, and have the same orientation; if the top and bottom sides have opposite orientation, one gets the Klein bottle. Figure 11(b) shows the fundamental polygon for our three sculptures. By counting the number of 0−, 1− and 2−cells of the polygon, once performed the identifications, we find that the Euler characteristic $\chi$ is 0. Since for non-orientable, closed surfaces it holds that the Euler characteristic equals 2 minus the surface non-orientable genus, we get that the non-orientable genus of the surface is $k = 2 - \chi = 2 - 0 = 2$. Therefore, according to the classification theorem of closed surfaces, it is topologically equivalent to the Klein bottle. If we cut a disk, to go back to the original surface, we end up with a surface with boundary, which is topologically equivalent to the Klein bottle minus a disk. Therefore, all three sculptures have the same topological type: they are Klein bottles minus a disk, as the *Modified Klein*.

Similarly, we can classify the sculpted surface in Figure 7, the *Giorgini Torus II*, which is a non-orientable surface with three boundaries. In this case, the Euler characteristic is 0 and therefore the non-orientable genus is 1, which means that the sculpted surface is topologically equivalent to the projective plane minus three disks. Since the projective plane minus a disk is topologically equivalent to the Möbius strip, we get that the sculpture has the topology of the Möbius strip minus two disks.

Finally, the *Giorgini Torus IV* (Figure 8) is a non-orientable surface with two boundaries. From its fundamental polygon we get that its Euler characteristic is $-1$, and its genus is 3. From the classification theorem for surfaces, it turns out that the sculpted figure is equivalent to the connected sum of three projective planes minus two disks, or equivalently, a sphere with five holes, three of which are capped by Möbius strips.

## Conclusions

Henri Poincaré, the father of topology, referred to his studies as *Analysis situs*, which literally means the *analysis of place* (1895). Retrospectively, the name itself suggests how topology, the science of transformations and invariants, would have influenced a lot the way in which artists understood, perceived, and represented space. We have analysed the work of Vittorio Giorgini, an Italian architect who pioneered the idea of elastic surfaces, and of form as a constructive process. Giorgini said that "*the structure of a system is the order by which the elements are organized*" – which rings a bell in the ears of mathematicians. In this context, we analysed the structure – the topological properties – of some of Giorgini's sculptures. We disclosed the type of six sculptures, and showed that four of them are topologically equivalent. But Giorgini also observed that "*all systems have one or more structures relative to the criteria of observation, as is well-known in the natural sciences*" – therefore, he reminds us that shapes are essentially an individual experience.

## References


[1] R. Courant and H. Robbins. *What is Mathematics? An Elementary Approach to Ideas and Methods.* Oxford University Press, 1941.

[2] M. Del Francia. *Vittorio Giorgini Architetto. Morfologia, Topologia, Spaziologia.* Generazioni in Arte, 2006.

[3] G. Di Cristina (Editor). *Architecture and Science.* Wiley Academy, 2001.

[4] M. Emmer. "The Idea of Space in Art, Technology, and Mathematics." *Applications of Mathematics in Models, Artificial Neural Networks and Arts*, Springer, 2007.

[5] V. Giorgini. *Spatiology: the Morphology of the Natural Sciences in Architecture and Design.* L'Arca Edizioni, 1995.

[6] A. Imperiale. *New Bidimensionality.* Birkhauser, 2001.

[7] W. S. Massey. *A Basic Course in Algebraic Topology. Graduate Texts in Mathematics*, Springer-Verlag, New York, vol. 127, 1991.

[8] D'A. W. Thompson. *On Growth and Form.* 2nd Edition, Cambridge University Press, 1942.

[9] D. Ulivieri, L. Giorgetti and B. Tognetti. "Vittorio Giorgini Spatiology-Morphology Architect." *Nexus Network Journal*, vol. 22, 2020, pp. 191-210.

[10] Ralph P. Boas Jr. "Möbius Shorts." *Mathematics Magazine*, vol. 68, no. 2, 1995, p. 127, 1995.