Trading dependability and energy consumption in critical infrastructures: focus on the rail switch heating system

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***Abstract*—Traditionally, critical infrastructures demand for high dependability, being the services they provide essential to human beings and the society at large. However, more recent attention to cautious usage of energy resources is changing this vision and calls for solutions accounting for appropriate multi-requirements combinations when developing a critical infrastructure. In such a context, analysis supports able to assist the designer in envisioning a satisfactory trade-off among the multi-requirements for the system at hand are highly helpful. In this paper, the focus is on the railway sector and the contribution is a stochastic model-based analysis framework to quantitatively assess trade-offs between dependability indicators and electrical energy consumption incurred by the rail switch heating system.**

Moving from a preliminary study that concentrated on energy consumption only, the analysis framework has been extended to become a solid support to devise appropriate tuning of the heating policy that guarantees satisfactory trade-offs between dependability and energy consumption. An evaluation campaign in a variety of climate scenarios demonstrates the feasibility and utility of the developed framework.

I. Introduction

Critical infrastructures are characterized by stringent dependability requirements, being the services they provide essential to citizens and the society in the large. However, more recent attention to cautious usage of planet resources and environmental safeguarding, is increasingly pushing towards reduction in energy consumption. For example, as part of the 'Clean energy for all Europeans package', the new amending EU Directive on Energy Efficiency (2018/2002) updated the policy framework to 2030 and beyond, by raising the energy efficiency target of at least 32.5% (it was 30% in the previous 2016 Directive)[1]. This changes the traditional vision and calls for solutions accounting for appropriate multi-requirements combinations when developing a critical infrastructure.

In this paper, the focus is on the railway sector, and specifically on the rail switch heating system in charge of avoiding the freezing of rail switches, which are mechanical installations that regulate track junctions and diversions. Reliability of rail switches is critical to ensure safe railway transportation of both passengers and goods, but low temperature and snowfall may block the switch mechanism, with detrimental impact on operating efficiency and, in extreme cases, causing train derailments [1]. To avoid these problems, a switch heating system is employed to keep the temperature of the switch at a safe value. Depending on the climate conditions of the place where the railway system operates, the energy consumed by this heating system can be very relevant. For instance, as reported in [2], a heating system for a single switch that operates continuously during cold months (i.e. from October to April) may require up to 34,000 kWh per year. In Germany, Deustche Bahn (DB) alone has 64,000 switches heated with electrical resistance and gas heaters, a combined power of 900 MW which consume up to 230 GWh/year. As reported in [3], to heat the 6800 switches and crosses in Sweden, the cost can amount to 10-15 MEuro/year.

So, pursuing energy efficiency is undoubtedly significant from the cost point of view. To this purpose, several directions have been and are currently investigated, including: i) enhancing the technology of the heaters and their insulation on one side, and ii) performing a careful adjustment of the energy supply policy, typically an on-off technique based on thresholds [2] decided on the basis of climate conditions as well as other possible constraints (e.g., criticality of the specific rail station, compliance to regulation of a specific country, etc). However, the reduced energy consumption should not put in danger the dependability properties, given the critical role of the railroad switch in the correct operation of the railroad infrastructure. In such a context, supports helping finding satisfactory trade-offs among the multi-requirements for the system at hand are highly helpful. The contribution offered in this paper has this objective.

Specifically, the paper presents a stochastic model-based analysis framework to quantify trade-offs between dependability indicators (in particular, reliability and availability are considered) and electrical energy consumption incurred by the rail switch heating system. This is a useful instrument in the hands of the system designer/railroad operator, to assist in accomplishing efficient tuning of the energy management policy, while respecting the dependability constraints. Previous studies in the literature addressed the analysis of the railroad switch heating system from either more simplistic assumptions and/or restricted focus on indicators under assessment. The analysis framework presented in the following is a more general and solid support. Special attention is devoted to the realistic and general management of climate aspects (dynamics of both temperature and humidity are represented), thus

[1] https://ec.europa.eu/energy/en/topics/energy-efficiency/targets-directive-and-rules/energy-efficiency-directive

allowing to analyze a large variety of weather profiles.

Evaluation results obtained considering a wide range of weather condition profiles demonstrate the feasibility and utility of the developed framework.

The rest of the paper is organized as follows. Section II overviews related work, to properly position the current contribution. The logical architecture of the railroad switch heating system, the weather aspects and related stochastic processes, as well as the definition of the measures of interest, are presented in Section III. Section IV describes the modeling framework to assess the considered energy consumption and dependability related indicators. Section V discusses the results obtained exercising the model in interesting scenarios. A cost function is also introduced, expressing in a compact way the impact of the dependability and energy related aspects. Section VI draws the conclusions and briefly indicate some future research lines.

II. Related work

Stochastic model-based analysis has been adopted since long time as an effective method to perform quantitative assessment targeting trade-offs among system properties showing contrasting effects on each other (typical case is when enhancing one property has a detrimental effect on another one). Just to make general examples based on trade-offs involving power consumption, in [4], the power-performance trade-offs in IaaS cloud is addressed, while in [5] the focus is on the energy-performance trade-off in data centers.

The analysis of energy consumption induced by supply policies in smart cities contexts (such as the railway domain, but also home/buildings) has been the subject of few studies in the literature [6]–[9]. However, as far as the stochastic model-based analysis of rail switch heating system is concerned, to the best of our knowledge previous work contributed an evaluation framework to assess the energy consumption under specific assumptions on the system behavior, fault model and environment conditions. However, such studies: i) either considered trade-offs between reliability and energy efficiency, but under simplistic assumptions on weather representation (characterized by the temperature only) [7], or ii) adopted a more realistic weather representation accounting for both temperature and humidity, but restricted to specific weather profiles and concentrated on the assessment of energy consumption only [10].

This paper presents a significant advancement with respect to the literature by adopting a realistic, general and parametric weather profile characterization, and studying trade-offs such as in [7], but extended to availability, in addition to reliability.

III. Logical view of the system under analysis

The logical architecture of the system under analysis encompasses the same elements as in [10], since the same system is represented, but here the assumption that the rail switch never freezes is relaxed, so correct operation is no more guaranteed. This translates in a different model of weather's aspects because here they can lead to the failure of a switch and to its unavailability for the time needed to recover from the freezing.

In particular, in this paper, a wide and general variety of weather profiles is considered, without any limitation on the observable variability in temperature values between two observation instants (t^w time window), as in [10] where temperature and humidity changes are synchronized with control actions to assure no switch freezing. Consequently, the threshold temperature, adopted by the heating control system to decide on whether to switch on or off the heating, plays a double role on both energy consumption and on the probability that the switch gets frozen.

The logical architecture of the system under analysis is composed of three levels: i) a set of n railroad switch heaters $SH_1, \dots, SH_n$, that heat the switches through electricity to prevent them from freezing, ii) the weather conditions, i.e., the weather data used to decide the control actions, and iii) the heater control system, that decides when to turn on or off the heaters. Each heater SH_i is installed close to a different switch RS_i and is powered through power lines connected to a power system. Different heaters can be connected in series on the same power line. The state of each heater is *on*, when it consumes electrical energy, or *off*, when no electrical energy is consumed.

A. Weather aspects

Stochastic methods are increasingly adopted in comprehensive weather and climate prediction models, where stochastic elements are typically added to deterministic (physical) ones for handling different spatial and time scales [11]. However, when employing a weather stochastic model as a component of a larger model (typically, when the modeling targets a cyber-physical system influenced by meteorological conditions), some specificities characterizing the weather could be irrelevant, while additional aspects related to other components of the overall system under analysis need to be properly harmonized. In this paper, the primary objective is to address the variability of temperature and humidity of a restricted geographical area while keeping the management of weather and dependability-related aspects compatible. Thus, a pure-jump stochastic process has been defined, taking also into account the need of evaluating the measures of interest on the overall model (component failures and weather changes) through discrete event simulation.

Specifically, weather conditions are represented at each instant of time t by the temperature and dew point provided by the reference meteorological station and by the temperature and humidity close to each switch heater RS_i, i.e., formally, by a stochastic process composed by a $(2+2n)$-tuple of random variables:

$$\big(T_0(t_k), T_d(t_k), T_1(t_k), \dots, T_n(t_k), H_1(t_k), \dots, H_n(t_k)\big),$$

where $k \in \mathbb{N}, T_i(t_k) \in [-50^oC, 50^oC], H_i(t_k) \in [0.01, 1]$, $t_k = t_{k-1} + X^w$ for $k \geq 1$, and X^w is a random variable uniformly distributed over $[a^w, b^w]$. $T_0(t_k)$ and $T_d(t_k)$ are the temperature and the dew point, respectively, of the geographical area where the railroad switches are installed, and are provided by the reference meteorological station. The random variables representing the weather conditions are piece-wise constant over time and change value every X^w units of time. Thus, $T_0(t) = T_0(t_k)$, for each t such that $t_k \leq t < t_{k+1}$, and analogously for the other random variables.

The dew point is assumed to be the same in the geographical area where the railroad switches are installed. $T_i(t_k)$ and $H_i(t_k)$ with $i = 1, \cdots, n$ are the temperature and the air relative humidity, respectively, close to RS_i. The values of $T_i(t_k)$ are provided by sensors local to the switches and can be different from those of $T_0(t_k)$, respectively, due, e.g., to exposition to sun or the presence of shadow. The values of $H_i(t_k)$ are obtained as a function of $T_i(t_k)$ and $T_d(t_k)$, according to the formulation (from [12], [13]):

$$H_i(t_k) = e^{\left(\frac{aT_d(t_k)}{b+T_d(t_k)} - \frac{aT_i(t_k)}{b+T_i(t_k)}\right)}, \tag{1}$$

where $a = 17.27$ and $b = 237.7\,°\mathrm{C}$, as suggested in [12].

The measurement of the dew point is related to humidity. A higher dew point means that there is more moisture in the air [12], [13]. When $T_d(t_k) \leq 0^oC$, the dew point is called the *frost point*, as frost is formed. In this case, i.e., when $T_i(t_k) \leq T_d(t_k) \leq 0^oC$, the moisture on RS_i turns into ice that can prevent the switch from working correctly.

Four key parameters are introduced and used to define the stochastic processes $\hat{T}_0(t_k)$, $T_0(t_k)$, $T_d(t_k)$ and $T_i(t_k)$:

- $\hat{T}_0$ Is the temperature assigned at time 0 to $T_0(t_k)$.
- T_0^a Represents how much $T_0(t_k)$ varies from $\hat{T}_0$.
- $\bar{T}_d$ Is the difference between the temperatures assigned at time 0 to $T_0(t_k)$ and $T_d(t_k)$, such that $T_d(0) = \hat{T}_0 - \bar{T}_d$.
- T_d^a Represents how much $T_d(t_k)$ varies from $T_d(0)$.

Different weather profiles can be represented as different combinations of the parameter values of the 4-tuple $(\hat{T}_0, T_0^a, \bar{T}_d, T_d^a)$ (having fixed all the other parameters values): medium, low and very low temperatures ($\hat{T}_0$), low and high temperature variations from $\hat{T}_0$ (T_0^a), dew point values close to or far from the temperature ($\bar{T}_d$) and low and high variation of dew point values (T_d^a).

Specifically, the stochastic processes $T_0(t_k)$, $T_d(t_k)$ and $T_i(t_k)$ are defined by the stochastic process $T(t_k)$ representing a generic temperature:

$$T(t_k) = \begin{cases} \frac{1}{1-\sigma_f} T^a f'(t_k)(t_k - t_{k-1}) + T(t_{k-1}) & \\ \qquad\qquad \text{if } k \geq 1 \text{ and } Y = 0, \\ (2 - \frac{1}{1-\sigma_f}) T^a f'(t_k)(t_k - t_{k-1}) + T(t_{k-1}) & \\ \qquad\qquad \text{if } k \geq 1 \text{ and } Y = 1, \\ \hat{T} \qquad\qquad \text{if } k = 0. \end{cases} \tag{2}$$

In (2), $f'(t)$ is the derivative of the differentiable function $f(t)$ and Y is a (discrete) random variable that can take the values 0 and 1 with probabilities $p_Y(1)$ and $1 - p_Y(1)$, respectively and expected value equal to 0.5. $f(t)$ represents the qualitative trend of the temperature over a day. The distance of $T(t_k)$ from $f(t_k)$ depends on the value of σ_f. In particular, the expressions $1/(1 - \sigma_f)$ and $(2 - 1/(1 - \sigma_f))$ are instrumental in (2) to obtain the mean of $T(t_k)$ independent of σ_f. The variance of $T(t_k)$ is governed by $\sigma_f^2/(1 - \sigma_f)^2$, so for $\sigma_f = 0$ it is small, wherease for $\sigma_f \in [0.5, 1)$ it grows fast. Y represents the event that $T(t_k)$ changes direction ($Y = 1$), flipping the slope, with respect to $f(t_k)$, or not ($Y = 0$). Being the expected value of Y equal to 0.5, $T(t_k)$ is supposed to follow the increase/decrease pattern dictated by $f(t_k)$. For a fixed value of σ_f, T^a represents how much $T(t_k)$ varies from $\hat{T}$. Increasing T^a, also the maximum and minimum values that $T(t_k)$ can assume increase.

The stochastic process $T_0(t_k)$ is defined by $T_0(t_k) = T(t_k)$ assigning in (2) $\hat{T} = \hat{T}_0$ and $T^a = T_0^a$.

The stochastic process $T_d(t_k)$ is defined by:

$$T_d(t_k) = \begin{cases} T(t_k) & \text{if } T(t_k) \leq T_0(t_k), \\ T_0(t_k) & \text{if } T(t_k) > T_0(t_k), \end{cases}$$

assigning in (2) $\hat{T} = \hat{T}_0 - \bar{T}_d$ and $T^a = T_d^a$ (being the dew point always lesser than or equal the temperature of the air).

Also the definition of $T_i(t_k)$, $i = 1, \ldots, n$ is obtained from (2), by $T_i(t_k) = T(t_k)$, assigning $\hat{T} = \hat{T}_0 - \bar{T}_i$ and $T^a = T_i^a$, and replacing the expression $f'(t_k)(t_k - t_{k-1})$ with $T_0(t_k) - T_0(t_{k-1})$. In this way, each sample of $T_i(t_k)$ follows the same trend as the corresponding sample of $T_0(t_k)$, although the sample of $T_i(t_k)$ can assume values different from the corresponding sample of $T_0(t_k)$. Assigning small values to $\bar{T}_i$ and T_i^a (e.g., 3 and 1, respectively), small variations of $T_i(t_k)$ respect to $T_0(t_k)$ can be represented.

A couple of examples of $T_0(t)$, $T_d(t)$ and $T_i(t)$ are depicted in Figure 2. Notice that for small values of $\bar{T}_i$ and large values of T_i^a the curves for $T_0(t)$ and $T_i(t)$ could cross each other, when the temperature $T_i(t)$ rises above and decreases below the reference temperature $T_0(t)$.

B. The heater control system

The heater control system has a hierarchical, distributed organization, with a number of coordinating components (which depends on the size of the railway station), each controlling a set of heaters located close to the switch through a rather simple local logic. Traditionally, heaters are operated on an on/off basis [2], i.e. they are turned on as soon as the temperature falls below a defined threshold and work at full power until the temperature raises again above the threshold.

In this paper, we refer to an heater control system composed of the following logical components: i) m coordinators C_h, with $h = 1, \ldots, m$, ii) n controllers, $L_1, \ldots, L_n$, partitioned in m different subsets, a subset for each different coordinator, iii) a Power-Line Communication (PLC), composed by n logical communication channels $CH_1, \ldots, CH_n$, each CH_i connecting L_i with its coordinator C_h. Thus, for each switch heater SH_i, there is a controller L_i, coordinated by a coordinator C_h. C_h and L_i decide every t^w units of time to turn on or off the heaters, in accordance to the heating policy described later in this section. More specifically, whenever the communication channel CH_i works properly, the controller L_i implements the command issued by the coordinator C_h. Using the temperature and dew point received from the meteorological station, and the local temperatures received from the associated controllers, each coordinator C_h decides which heaters should be turned on or off and transmittes the computed action to the corresponding controllers L_i. Instead, for the period in which the controller L_i is disconnected from the coordinator C_h, due to the failure of CH_i, L_i implements an autonomous policy.

The state of SH_i is modeled through the random variable Z_i: at a given time instant t, if SH_i is on then $Z_i(t) = 1$, otherwise $Z_i(t) = 0$. Each L_i is installed close to SH_i, includes a sensor to measure the temperature of RS_i, and is powered by the same power line as SH_i. Each C_h is powered by the power line connected to all the Ls controlled by C_h. The power lines powering the Ls and the Cs are the physical layers of the communication channels.

The failure of CH_i, due to a fault at level of the power line, interrupts the communication between L_i and its coordinator C_h. The time to the physical fault of CH_i at time t is represented by a random variable exponentially distributed with rate $\lambda_i(t)$, which depends on the relative humidity $H_i(t)$, and other influencing aspects that are constant over time. The recovery time of CH_i is a random variable exponentially distributed with constant rate μ_i. A physical fault that interrupts CH_i can also affect other communication channels, depending on the power line topology. For example, if a subset of controllers are connected in series to their coordinator, the failure of CH_i may affect also all the channels CH_j, with $i < j \leq n$, relative to controllers in subsequent positions with respect to L_i.

The protocol constituting the switch heating policy takes into account the failure of communication channels, shifting the decision on keeping the heater on/off from the coordinator to the individual local controllers affected by the communication failure. Moreover, it considers the temperature threshold ΔT that, for values of ΔT greater than 0, aims to prevent switches from freezing due to random variations of the weather conditions between two consecutive control actions. The protocol is specified as follows. At time instant $t_z = z \cdot t^w$, for $z = 0, 1, \ldots$, each coordinator C_h receives from a weather forecasting service the value of $T_0(t_z)$ and $T_d(t_z)$, together with measured value of $T_i(t_z)$ received instantaneously from L_i under its control. Then C_h sends to L_i one of the following commands:

- turn on SH_i, if $T_i(t_z) \leq T_d(t_z) + \Delta T$ and $T_i(t_z) \leq \Delta T$,
- turn off SH_i, if $T_i(t_z) > T_d(t_z) + \Delta T$ or $T_i(t_z) > \Delta T$.

Otherwise, when CH_i does not work, at each instant of time t_k, L_i turns on SH_i as soon as $T_i(t_z) \leq \Delta T$ and turns off SH_i as soon as $T_i(t_z) > \Delta T$. Through the paper, ΔT is considered the most relevant parameter.

Notice that, C_h issues control actions at constant time intervals (t_z is an integer multiple of t^w), while variations of the weather profile are determined by the random variable t_k. This is the reason why temperature can lead to switch freezing at a time when the control is not ready. Depending on the value of t^w this phenomenon can have different degrees of impact. The presence/absence of ice on SH_i is modeled through the random variable U_i: at a given time instant t, $U_i(t) = 1$ if SH_i is frozen, i.e., if $T_i(t) \leq T_d(t)$ and $T_i(t) \leq 0$, $U_i(t) = 0$ otherwise. For $t = 0$ the switch heaters are considered not frozen, then $U_i(0) = 0$.

When a switch gets frozen, its recovery from the freezing does not occur instantaneously, but requires an interval of time.

C. *Measures of interest*

Three measures are of interest in this paper: one is representative of the energy consumption, while the other two are dependability-related indicators.

Let P^{SH} be the electrical power required by each switched-on heater SH_i, i.e., when $Z_i(t) = 1$. The electrical energy consumed in the interval of time $[0, t]$ by all the heaters is

$$\mu_E(t) = \mathbb{E}\Big[P^{SH} \sum_{i=1}^{n} \int_0^t Z_i(t)\, dt\Big]. \tag{3}$$

The first dependability-related indicator is representative of the unreliability of the railroad switches, and is defined as the probability $P_F(t)$ that at least one switch experiences weather conditions that lead to its freezing. Its formulation is as follows:

$$P_F(t) = \mathbb{P}\{\text{there exists } i \text{ and } x \in (0, t] \text{ s.t. } U_i(x) = 1\}. \tag{4}$$

The second dependability-related indicator is representative of the unavailability $\mu_U(t)$ of the switches due to the freezing condition, and is determined as the expected cumulative time between the freezing and the end of recovery, for each switch belonging to the system configuration that experiences a freezing in the time interval $[0, t]$. Its formulation is:

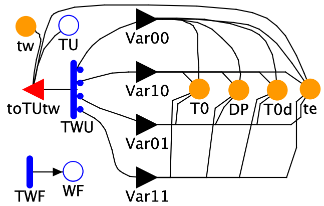
$$\mu_U(t) = \mathbb{E}\Big[\sum_{i=1}^{n} \int_0^t U_i(t)\, dt\Big]. \tag{5}$$

IV. SAN Model

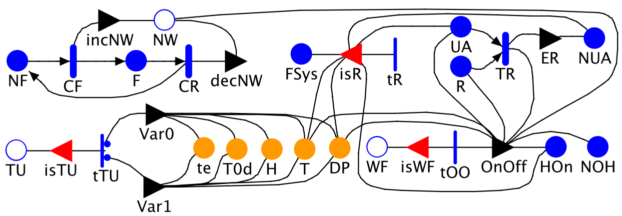
The logical architecture of the system under analysis is modeled and evaluated in the Möbius [14] modeling environment. Two atomic template models *TM_W* and *TM_L*, as in Figure 1, are defined using the Stochastic Activity Network (SAN) formalism [15], a stochastic extension of Petri nets. The model is open sourced[2].

Both the template models are a significant update and extension of the corresponding models proposed in [10]. In fact, the structure and all the activities of the SAN, excluded *CF* and *CR* and the related primitives, (i.e., 6 activities over 8 and 12 gates over 14), are new or considerably modified, and 13 new places over 19 are defined (shared places are counted one time only), with respect to the model proposed in [10].

[2]The link to download the source code is https://github.com/106ohm/STINGRAY_Availability.



(a) TM_W



(b) TM_L

Figure 1. SAN template model for weather conditions (a) and SAN template model for CH_i and *TM_L*$_i$ (b); white places are dependency-aware SVs.

The overall system model is obtained generating and composing automatically through the $\mathcal{D}$ operator (supported by the *DARep* approach [16]) one instance *TM_W*$_1$ of the template model *TM_W* and n instances *TM_L*$_1, \ldots,$ *TM_L*$_n$ of the template model *TM_L*. Dependency-aware SVs are defined in the template models to share each instance of the dependency-aware SV among different instances of the template model, following the topology of dependencies associated to the dependency-aware SV. They are represented in Figure 1 by white places. *TM_W*$_1$ represents the changes of the weather conditions at each instant of time t_k, through the timed activity *TWU* (and the linked primitives: places and input and output gates), and the instants of time $zt^w, z \in \mathbb{N}$, when the weather forecast service provides to the coordinators the current weather conditions, through the timed activity *TWF*. Each instance *TM_L*$_i$ represents: i) the failure and repair of CH_i, through the timed activities *CF* and *CR*, and the failure propagation based on the dependency-aware SV *NW* ii) the updating of $T_i(t_k)$ and $H_i(t_k)$ through the instantaneous activity *tTU*, iii) the ice formation event and the recovery time, through respectively the instantaneous and timed activities *tR* and *TR*, iv) the actions of L_i turning on and off SH_i at each instant of time kt^w, through the instantaneous activity *tOO*.

In *TM_W*, the completion time of the always enabled activity *TWU* represents the random variable X^w. At each completion, *TWU* selects one of the output gates *Var00*, *Var01*, *Var10*, *Var11* with the probability associated to one of these events ($Y^0 = 0, Y^d = 0$), ($Y^0 = 0, Y^d = 1$), ($Y^0 = 1, Y^d = 0$), ($Y^0 = 1, Y^d = 1$), respectively, where Y^0 and Y^d are the Y variables associated to $T_0(t_k)$ and $T_d(t_k)$, respectively. Next, *TWU* performs the code of the input gate *toTUtw* that: i) assigns to the extended place *te* (shared among all models) the value of t_{k-1}, saved in the the local extended place *tw*, and to *tw* the current time t_k, and ii) triggers the immediate activity *tTU* of each *TM_L*$_i$, by assigning one token to each i-th instance TU_i of the dependency-aware SV *TU*. Next, *TWU* performs the code of the selected output gate that updates the local extended places *T0*, and the extended places *DP* and *T0d* (shared among all models), representing $T_0(t_k)$, $T_d(t_k)$, and $T_0(t_k) - T_0(t_{k-1})$ respectively, as defined in (2). The timed activity *TWF* completes every t^w instant of times, when one token is added to each i-th instance WF_i of the dependency-aware SV *WF* shared among all the models, to trigger the immediate activity *tOO* of each *TM_L*$_i$.

At each completion in the instance *TM_L*$_i$, when $TU_i = 1$, *tTU* performs the code of the selected output gate that updates the values of the local places T and H, representing $T_i(t_k)$ and $H_i(t_k)$, respectively, as defined in (2). The immediate transition *tR* can complete, setting the places *Fsys* and *UA* to 1, when the input gate *isR* detects the ice formation event, based on the values of the places *T*, *DP* and *HOn*. The local place *HOn* represents the status of the heater, such that if *HOn* $= 0$ the heater is turned off. *TR* is enabled after the ice formation on RS_i at completion of *tOO* (*UA*=1), when the control action defined in Section III-B is performed by the code in the output gate *OnOff*, if needed (setting *R* to 1 when the heater is turned on). *tOO* is enabled and completes in the instance *TM_L*$_i$ when $WF_i = 1$. At completion of *TR*, the place *UA* is set to 0 and *NUA* (the place shared among all *TM_L*$_i$ that represents the number of switches that are not frozen) is decreased by 1.

The measures of Section III-C are evaluated through the definition of performance variables [14] on the SAN model *TM_W*$_1$. In particular: (4) corresponds to an instantaneous measure, the reward is equal to 1 if there is one token in place *Fsys*; (3) is a cumulative measure, characterized by the reward equal to the number of tokens in place *NOH*; and (5) is a cumulative measure, characterized by the reward equal to 1 if there is a token in place *UA*.

The behavior of the developed model has been extensively tested. Specifically, it has been exercised adopting parameter settings corresponding to a number of simplified scenarios for which the same outcome of the model can be manually computed. Among the others, extreme scenarios where the communication channel never fails or is always failed have been analyzed. All the tests were successful, thus reinforcing the correctness of the modeling framework implementation.

V. Evaluation Results

To demonstrate the applicability of the framework, the kind of analyses that it allows to perform and the usefulness of the instrument, an evaluation campaign is presented in the following. In particular, the impact of variations of the threshold value ΔT is assessed in different scenarios. A first set of experiments relates to the effect of ΔT variations on the individual selected metrics $\mu_E(t)$, $P_F(t)$ and $\mu_U(t)$. This analysis is useful to gain insights on the dynamics, at varying weather profiles and temperature threshold values, and to understand appropriate trade-offs by observing the obtained trends (see Section V-C). Then, the impact of ΔT variations on a cost function, that accounts for the contribution of P_F, μ_E and μ_U in a comprehensive way, is studied (see

Section V-D). All the analyses are performed on a reference case study, described in Section V-A, under the parameters setting, provided in Section V-B.

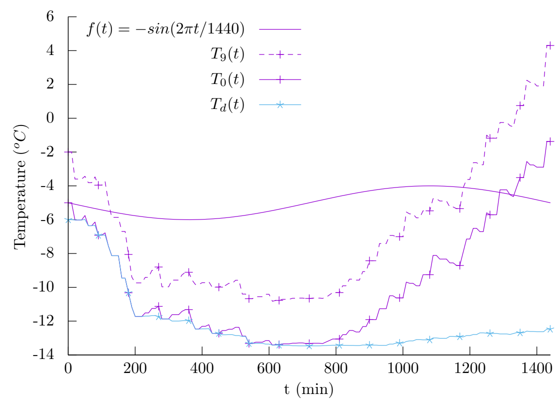
A. Case study

To reuse case studies already proposed in the literature, here the same switch heating system configuration as in [10] is adopted, since it is well representative of a small/medium size railway station and is fully adequate to show how the framework works and its utility. Specifically, the rail switch heating system consists of 2 coordinators (C_{North} and C_{South}), with 9 and 10 L_i controllers, respectively. In each of the two groups, local controllers are connected in series through a single communication network, which means that the failure of a communication channel between a coordinator and one of its associated local controllers interrupts also the communication with all the subsequent local controllers. As studied in [10], this is a critical dependency topology when compared to the alternative one where all the communications happen through independent power lines, so certainly more interesting to analyze from the reliability and availability perspective.

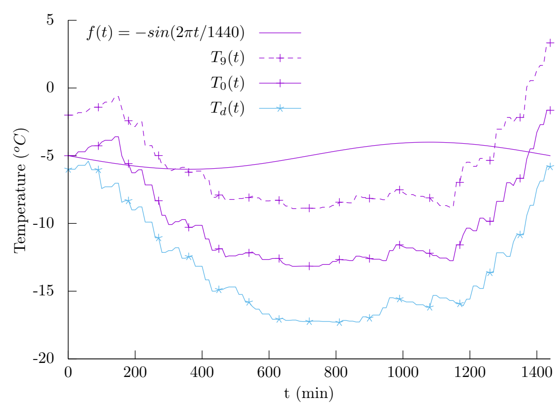
B. Parameters setting and scenarios

The introduction of reliability and availability indicators as defined in Section III, shifts the focus of the analysis on the conditions primarily impacting on them, which are the temperature and humidity, responsible for freezing the railroad switches. Therefore, the following analysis considers:

- fixed values for parameters related to both the failure and repair rate of the communication channel, selected among those adopted in the studies in [10]. Specifically, the channel failure rate is $\lambda_i(t) = 5^{H_i(t)} \cdot 3.472 \cdot 10^{-4}$ minutes^{-1}, meaning that, without considering humidity, a channel failure is expected once every two days, and the relative humidity - at worst - can cause an increase of half order of magnitude of the failure rate. The communication channel recovery rate is $\mu_i = 1.66 \cdot 10^{-2}$ minutes^{-1}, i.e., on average the recovery takes one hour;
- varying weather profiles, by assigning a set of values to related parameters. The chosen profiles are representative of rather cold weather conditions, since they are the most interesting both from the energy consumption perspective (the heaters are on for longer time) and from the dependability perspective (adverse temperature values are more prone to block the switches). Specifically, $\hat{T}_0$, which represents the value of the temperature of the considered geographical area at analysis starting time, can assume values in the set $\{0, -15\}$°C. T_0^a, which represents the maximum variability of the temperature with respect to its initial value (both as an increase or a decrease of $\hat{T}_0$), can assume values in the set $\{1, 10\}$°C. Specularly, the other two parameters related to the dew point, $\bar{T}_d$ and T_d^a, can assume values in the set $\{1, 10\}$°C;
- the weather change time interval is defined by $a^w = 5$ and $b^w = 15$, and the variability is set to $\sigma_f = 0.6$ for $T_0(t)$ and $\sigma_f = 0.3$ for $T_i(t)$ with $i = 1, \ldots, n$;
- to differently characterize the weather between the groups of heaters placed in the north and in the south of the railway station, the combination $T_i^a = 0.9$ and $\bar{T}_i = -3$ has been selected for L_i under C_{North}, while the values for those L_i under C_{South} are $T_i^a = 1.1$ and $\bar{T}_i = 3$;
- varying values of the temperature threshold ΔT to activate/deactivate the switch heating (in the range $[0.2, 3]$°C);
- the control is activated every $t^w = 10$ minutes, as already assumed in the studies in [10];
- the time needed to defrost a frozen switch is set to 15 minutes.



(a) $T_d^a = 1$°C

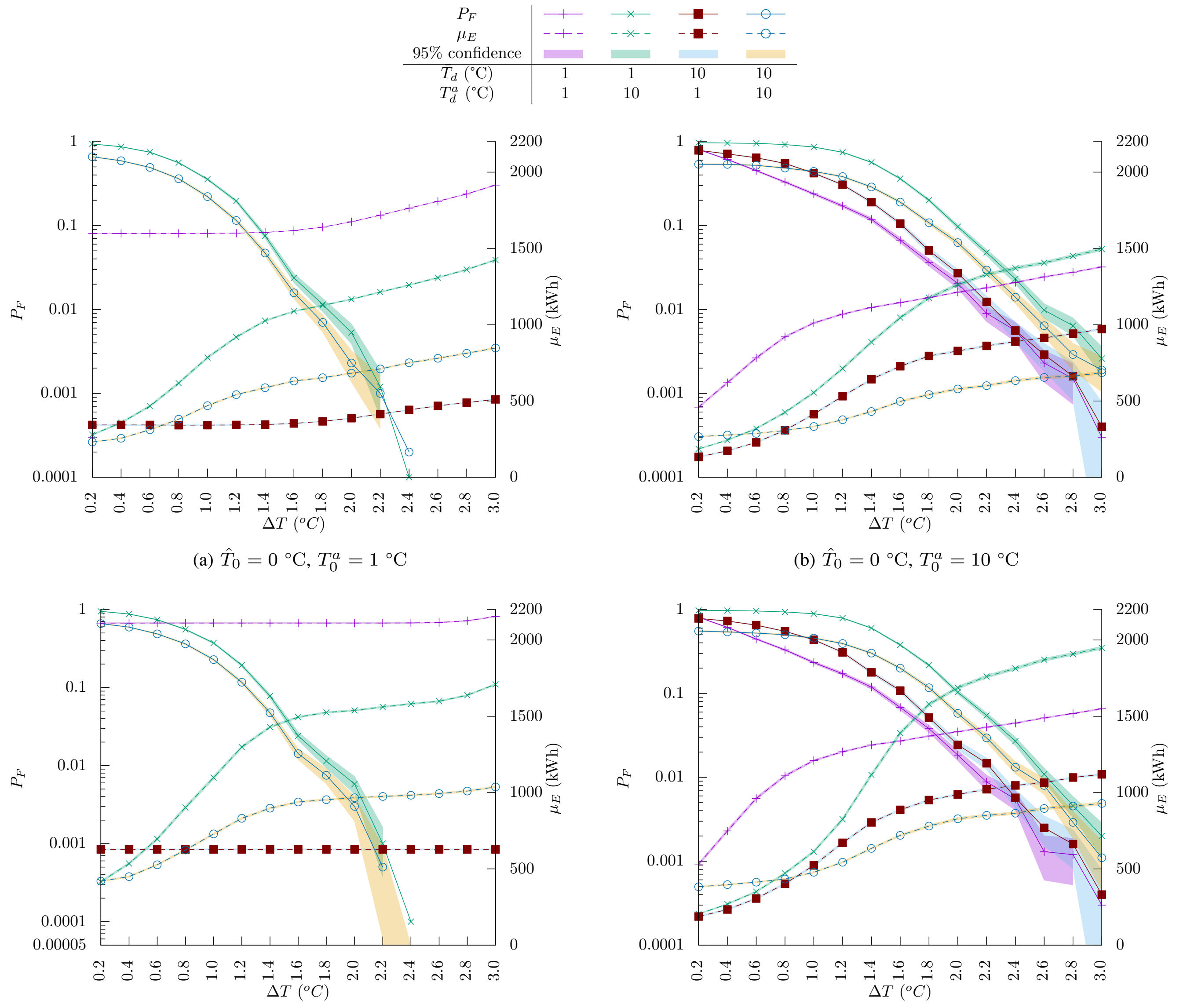


(b) $T_d^a = 10$°C

Figure 2. Traces of the air temperature $T_0(t)$, the temperature at the 9-th switch heater $T_9(t)$, the dew point $T_d(t)$ and $f(t)$ in two scenarios for $\hat{T}_0 = -5$°C$, T_0^a = 10$°C$, \bar{T}_d = 1$°C.

By varying the combination of values assigned to the four weather-related parameters, 16 weather profile scenarios have been analyzed. As already noted, the weather profiles are selected on purpose among those more representative of rather cold geographical zones, since in such conditions the heating is more intensive and a cautious trade-off analysis is expected to reduce significantly the overall energy consumption.

The function $f(t)$ of (2) has been selected equal to $-\sin\left(\frac{2\pi t}{1440 \text{ minutes}}\right)$, so that $\hat{T}_0$ is expected to decrease between midnight and 6 AM, increase between 6 AM and 6 PM, and eventually decrease till midnight. The function $f(t)$ is depicted



(a) $\hat{T}_0 = 0$ °C, $T_0^a = 1$ °C

(b) $\hat{T}_0 = 0$ °C, $T_0^a = 10$ °C

(c) $\hat{T}_0 = -15$ °C, $T_0^a = 1$ °C

(d) $\hat{T}_0 = -15$ °C, $T_0^a = 10$ °C

Figure 3. Probability of failure, i.e., freezing, within one day (P_F, in log scale, continuous lines) vs expected energy consumption (μ_E, dashed lines) at increasing of ΔT, with shaded confidence intervals obtained with a 95% confidence level.

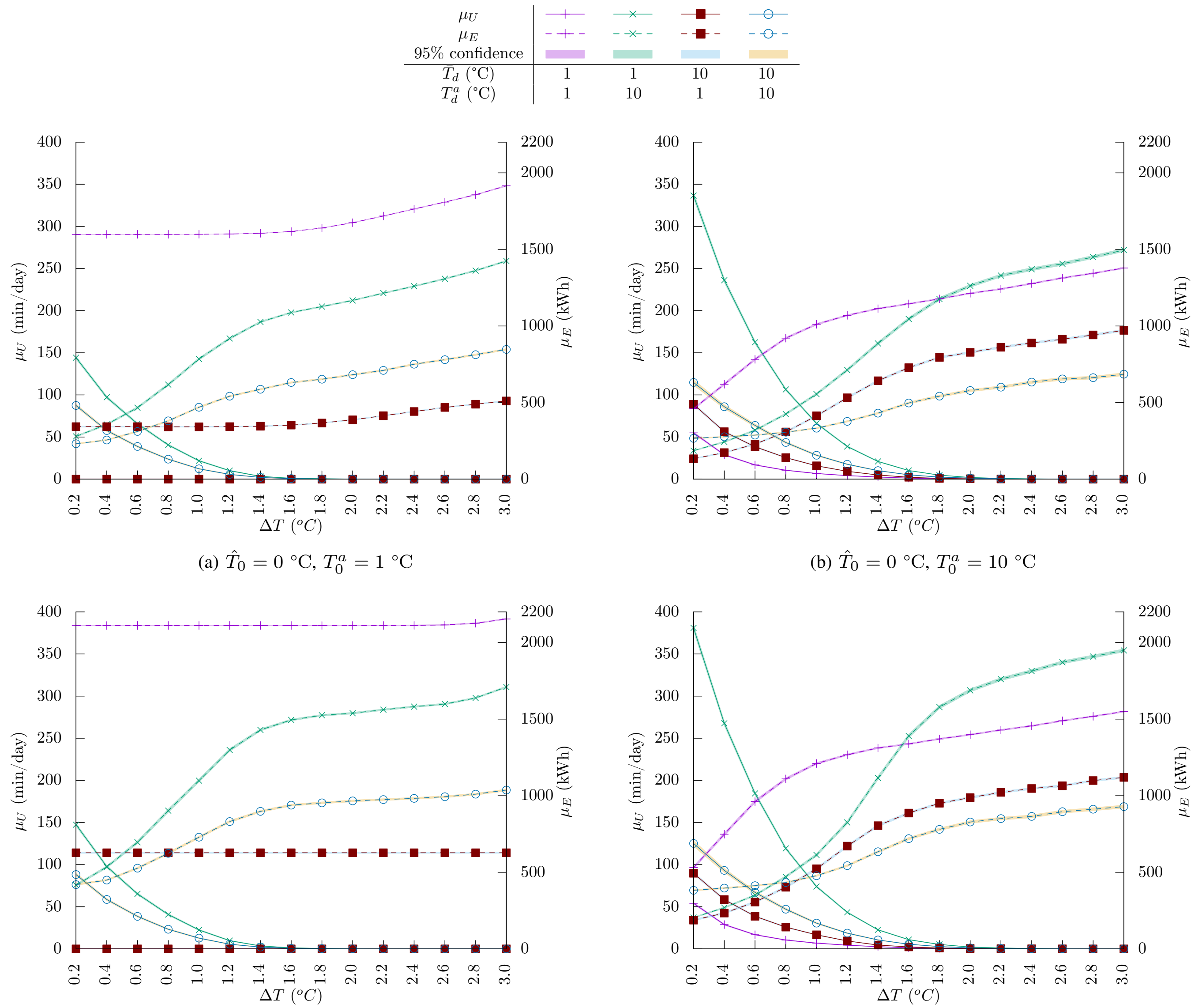
in Figure 2 together with a couple of traces of $T_0(t)$, $T_d(t)$ and $T_9(t)$ (the temperature at the 9-th switch heater) obtained in different scenarios.

Here, the emphasis is on the dynamics of weather profiles, considering other system configuration aspects (e.g., the switches topology) and components conditions (e.g., failure of the communication network) as fixed. Extending this study to analyze the impact of those parameters that assume fixed values in the current evaluation is of course possible with the developed modeling framework, and planned as future work.

To facilitate the comparison between the trends of the curves referred to each analyzed indicator, so to better appreciate the trade-off between them, each figure shows results of both unreliability (or unavailability), whose values are indicated on the left Y-axis (using a logarithmic scale), and energy consumption, with values indicated on the right Y-axis (using a linear scale). Finally, the measures of interest are evaluated simulating the model described in Section Section IV. 10^4 simulation batches are employed to obtain the confidence intervals reported in the figures with a 95% confidence level.

C. Discussion on obtained results

The results of the first study, focusing on the analysis of individual indicators and trade-offs considerations, are discussed in the following. Figure 3 shows the results for the unreliability indicator $P_F(t)$, defined in (4), and the energy consumption indicator $\mu_E(t)$, defined in (3), while Figure 4 depicts the results for the other pair $\mu_U(t)$, i.e., the unavailability related indicator defined in (5), and $\mu_E(t)$. The four sub-figures in both Figures 3 and 4 differentiate from each other for the



(a) $\hat{T}_0 = 0$ °C, $T_0^a = 1$ °C

(b) $\hat{T}_0 = 0$ °C, $T_0^a = 10$ °C

(c) $\hat{T}_0 = -15$ °C, $T_0^a = 1$ °C

(d) $\hat{T}_0 = -15$ °C, $T_0^a = 10$ °C

Figure 4. Expected unavailability time, in minutes, within one day (μ_U, continuous lines) vs expected energy consumption (μ_E, dashed lines) at increasing of ΔT, with shaded confidence intervals obtained with a 95% confidence level.

considered combination of values of $\hat{T}_0$ and T_0^a, characterizing the temperature, while the two sets of four curves inside each of them represent different combinations of values of the dew point parameters. The varying parameter on the X-axis in each figure is the temperature threshold ΔT. It is the crucial parameter of the energy management policy controlling the switch heaters, to be properly tuned on the basis of the analysis outcomes.

A first general comment on both Figures 3 and 4 is that, as expected, increasing ΔT increases energy consumption but improves dependability (both unreliability and unavailability assume lower values). This confirms that the two requirements, energy consumption and dependability, are conflicting with each other, thus motivating the need for carrying on investigations as conducted in this paper to gain further knowledge, necessary to operate the most appropriate tuning of ΔT.

Another general observation is that energy consumption is higher when the temperature variability with respect to the average value is low (that is, $T_0^a = 1$°C). This is explained by the fact that the temperature remains rather close to zero all the time, and for values of threshold ΔT greater than 1, the heaters are always activated. But, as positive effect, the switches are well working for longer time (as shown by the curves related to both $P_F(t)$ in Figure 3 and $\mu_U(t)$ in Figure 4). Note that in the left sub-figures of Figure 3, only 2 curves related to $P_F(t)$ are shown, since the other 2 curves assume too low values to be displayable (a logarithmic scale is used).

Looking more specifically on trade-off behaviors, it can be observed that there are scenarios for which higher reliability can be achieved without a significant increase in the energy consumption, as curves identified by $(\bar{T}_d = 10°\text{C}, T_d^a = 10°\text{C})$ in Figures 3b and 3d, respectively. The same trend characterizes also some unavailability scenarios, e.g., in Figure 4a where the curve relative to energy consumption identified by $(\bar{T}_d = 10°\text{C}, T_d^a = 1°\text{C})$ remains quite flat.

Also, depending on the weather profiles, increasing ΔT higher than a certain value does not bring any benefit but only adds cost. In Figure 3, this trend can be observed in all the left sub-figures, where a very low unreliability is obtained already for $\Delta T = 2.4°\text{C}$. In Figure 4, this effect is visible in all the analyzed scenarios, although reaching unavailability close to zero requires a higher ΔT values when $T_0^a = 10°\text{C}$ ($\Delta T = 2.0°\text{C}$ on the right sub-figures, instead of $\Delta T = 1.6°\text{C}$ for the sub-figures on the left).

As a final consideration, we remark that, although it would have been helpful to compare these results with field data to better assess their quality, unfortunately there is a lack of availability of field data to be exploited for such a comparison. In fact, only aggregated and averaged estimates for energy consumption and dependability indicators are typically provided by railway operators (such as values for energy consumption reported in Section I). However, we also remind that the utility of the assessment investigation performed in this study is to gain knowledge relevant to enhance effectiveness of the switch heaters management policy, by finding the proper tuning of the heating threshold that assures a satisfactory trade-off between energy consumption and dependability. In this respect, the developed modeling framework allows to analyse a large variety of system configurations and weather scenarios, and the aim is to conduct comparative analysis among variants of energy policies, not assessment of energy/dependability figures in absolute terms.

D. Introducing a cost function

An interesting addition to the analyses performed on the individual considered metrics is the introduction of a cost function, to obtain an overall estimate of running the switch heating system adopting a given temperature threshold. To this purpose, costs need to be assigned to the energy consumed, to the time interval the switches have been unavailable and to the event that the switches get frozen.

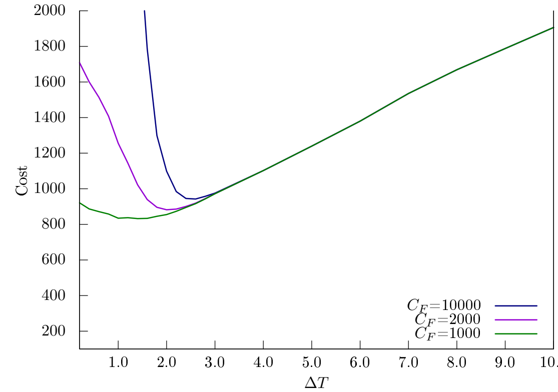
Let's indicate with C_E the unitary cost of energy, C_U the unitary cost of having a switch unavailable, and C_F the cost of having at least a frozen switch at some point during the day. Then, the overall cost afforded when energy consumption and unreliability of railroad switches are the two cost contributors can be formulated as:

$$\text{Cost } = C_F \cdot P_F + C_E \cdot \mu_E. \tag{6}$$

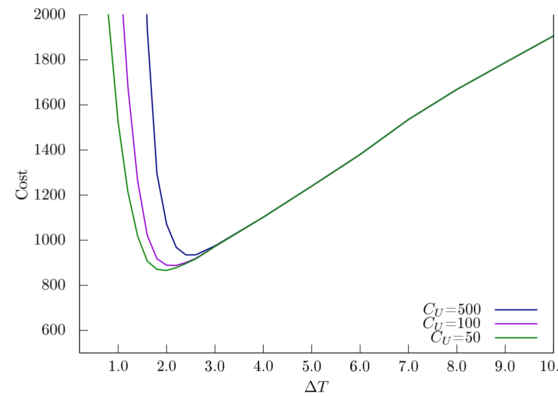
Similarly, the overall cost afforded when energy consumption and unavailability of railroad switches are the two cost contributors can be formulated as:

$$\text{Cost } = C_U \cdot \mu_U + C_E \cdot \mu_E. \tag{7}$$

Then, observing the behavior of the cost function at varying the temperature threshold parameter, the value of such threshold ΔT for which the cost is minimum can be selected as the most convenient one. Examples of cost curves are shown in Figure 5, where it is interesting to note the “U-shaped” trend. To better appreciate the impact of P_F and μ_U, in Figure 5 the value of C_E is set to 1 and three different values of C_F and C_U are considered. In both sub-figures, starting from the left, the high values of the cost function are mainly due to the impact of unreliability (or unavailability), but such impact decreases at increasing the value of ΔT. After a minimum is reached, the cost function has again a raising trend, but this time it is the cost of energy that prevails. Note that the higher are the unitary costs involved, and the greater is the value of ΔT for which the cost function results in a lower value.



(a) Cost function as defined in (6)



(b) Cost function as defined in (7).

Figure 5. Cost function where $C_E = 1$, $\hat{T}_0 = 0$ °C, $T_0^a = 10$ °C, $\bar{T}_d = 1$ and $T_d^a = 10$.

VI. Conclusions and Future Work

This paper presented a stochastic model based framework, tailored to the analysis of trade-offs between energy consumption and dependability related indicators of a critical subsystem in the railway sector: the railroad switch heating system. A trade-off analysis is very helpful in presence of multi-requirements, with contrasting effect on each other (as it is the case for the dependability-related and energy consuming indicators considered in this paper). The ability to represent generic weather conditions (indicative of different geographical zones, from extremely cold to milder areas) by properly tuning the characterizing four parameters, is a strength of the approach, since makes it a powerful and widely applicable instrument. The evaluation results, although relative to the analyzed case study, demonstrate the applicability and usefulness of the framework in tuning energy supply policies through understanding the dynamics of energy consumption and dependability-related measures, under a multiplicity of climate conditions. Since dependability and energy consumption are conflicting requirements (that is, high dependability, which is desirable, goes together with high consumption, which instead is undesirable), such kind of analysis support provides valuable insights on the dynamics and interplay between variability of climate conditions and effectiveness of temperature thresholds piloting the activation/deactivation of the heaters. For example, the conducted analyses provided evidence of the ability of the framework to identify, when it exists, the most convenient value of the temperature threshold ΔT, such that reliability (or availability) are not penalized (that is, increasing ΔT does not bring relevant advantages), instead reduction in energy consumption shows noticeable.

Finally, the view of the large scale distribution of railroad switches should be taken into account when making consideration of the degree of energy saving in a specific study. As reported in [17], there are on average 200-400 switches per 100 KM of network, so even a (very) small energy saving at the level of a single heater can lead to significant amounts, when the focus is on a National geographic area (especially in Northern Countries).

Further extensions are foreseen in several directions, including: i) to investigate the impact of parameters that have been kept constant in this analysis, especially the duration of the interval of time between two activations of the control system, the communication channels failure rate and the repair time of failed channel; ii) to consider other topology configurations, inducing different dependency among failures of communication channels; iii) to introduce the concept of priorities among the switches, such as distinguishing among switches on rail tracks devoted to high-speed trains from other categories of trains, and investigate aspects such as different costs for unavailability or unreliability, as well as the potential benefits of future energy management policies, based on different temperature thresholds to take into account the criticality of the different kinds of switches.

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