

Decoherence without entanglement and quantum Darwinism

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It is often assumed that decoherence arises as a result of the entangling interaction between a quantum system and its environment, as a consequence of which the environment effectively measures the system, thus washing away its quantum properties. Moreover, this interaction results in the emergence of a classical objective reality, as described by quantum Darwinism. In this Rapid Communication, we show that the idea that entanglement is needed for decoherence is imprecise. We propose a dynamical mixing mechanism capable of inducing decoherence dynamics on a system without creating any entanglement with its quantum environment. We illustrate this mechanism by introducing a simple and exactly solvable collisional model that combines both quantum and classical decoherence features. Interestingly, by tuning the model parameters, we can describe the same open system dynamics both with and without entanglement between system and environment. For a finite environment, we show that dynamical mixing can account for non-Markovian recoherence, even in the absence of entanglement. Our results highlight that system-environment entanglement is not necessary for decoherence or information back-flow, but plays a crucial role in the emergence of an objective reality.

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Introduction. The emergence of a classical objective reality from the underlying quantum description of the world is arguably the most studied, debated, and still elusive open problem in the foundations of quantum theory. This is known as the quantum measurement problem and it is generally formulated and addressed using the theory of open quantum systems [1–3]. The starting point is the realization that every realistic quantum system is never completely isolated and, therefore, its quantum description must be seen in a more general framework. Specifically, the system of interest is embedded in a larger quantum system, known as its environment. Due to the inevitable interaction with the latter, quantum superpositions are transformed into a classical statistical mixture of the pointer states, which are unaffected by the interaction with the environment [4]. This dynamical phenomenon goes under the name of environment-induced decoherence [5,6].

The microscopic description of the system-environment interaction generally allows us to identify the pointer states, but in order to explain how different observers obtain a consistent, and therefore objective, description of reality one must invoke the process known as quantum Darwinism (QD) [7,8].

In words, QD predicts that multiple observers having access to different small fragments of the environment retrieve the same information about the system's state if it is redundantly encoded in such fragments. This is known as objectivity of measurement outcomes and it has been recently experimentally demonstrated in Refs. [9–11].

The more general concept of objectivity of observables [12] has been demonstrated in Ref. [13] for finite-dimensional systems, and in Ref. [14] for infinite-dimensional ones, where it was proven that QD is generic, i.e., it occurs independently from the specific model considered (see also Ref. [15]). Note that, while the description of decoherence focuses on the dynamics of the open system only—with the environment generally being traced out—QD promotes the role of the environment from passive to active, since it assumes that it is what we actually observe to indirectly retrieve information on the system. Therefore, a dynamical description in terms of the reduced state of the system is not sufficient anymore, and one needs to look at the combined system-environment (or environmental fragments) state instead.

The very idea that open system dynamics arises from the interaction between two parts of a bipartite total system naturally suggests that entanglement must be established between the two during the time evolution. This is indeed very often the case and it is therefore not surprising that environment-induced decoherence has been frequently associated with the presence of entanglement between system $\mathcal{S}$ and environment $\mathcal{E}$. However, our results show that, as long as we limit our

attention to the reduced dynamics of the system S only, decoherence may take place without entanglement [16,17], even in the presence of a quantum environment initially in a pure state. Specifically, we introduce a simple and exactly solvable collisional model in which collisions occur at random times, which results in two different microscopic evolutions of the total system, one establishing S - $\mathcal{E}$ entanglement while the other one not, leading to the same reduced dynamics for S .

The same model shows that also information back-flow, recently identified as the source of memory effects, i.e., non-Markovian dynamics [18], does not require S - $\mathcal{E}$ entanglement. Also here, non-Markovianity is defined by looking at the properties of the dynamical map describing the reduced system. Finally, we show that entanglement plays a pivotal role in the objectification process, since it appears to be needed for it to take place. Our results suggest that, in order to fully grasp the true nature of the quantum-to-classical transition, and in particular to elucidate the role played by entanglement, a description in terms of the open system only may not be sufficient.

A crucial part of our results is the introduction of a new microscopic collisional model [19–27] allowing us to compute analytically the dynamics not only of the system but also of the system-environment fragments. Within this framework, we propose a mechanism, dynamical mixing, that can induce decoherence dynamics on a system without creating any entanglement with its environment. The key ingredient is, as its name suggests, a random process that drives the interaction times with the environment. The environment is composed of a set of initially uncorrelated ancillae colliding with the system sequentially and only once, at variance with previous work studying QD in collisional models [28]. This interaction mechanism results in pure dephasing of the system qubit, exhibiting both Markovian and non-Markovian dynamics, depending on the relevant parameters. Our analysis reveals that, while dynamical mixing can give rise to exactly the same qubit dephasing as that caused by an entangling interaction, it is not capable of accounting for QD. However, the introduction of a superenvironment acting as the source for the randomness in the collision times elucidates the origin of QD and the role played by entanglement in it.

Decoherence without entanglement. Let us first describe the model under consideration, which is also depicted in Fig. 1. The system is a single qubit with free Hamiltonian $H_S = \frac{\omega}{2}\sigma_z$, where ω is the qubit frequency, subject to collisions with qubit ancillae at random times. The collision times follow a Poisson process with rate λ , meaning that the intercollision time is exponentially distributed. The initial state of all the ancillae is $\rho_a = |0\rangle\langle 0|$, henceforth uncorrelated. When an ancilla collides with the system, it interacts with it with Hamiltonian $H_I = \frac{\eta}{2}\sigma_x^a \otimes \sigma_z^S$, where the superscripts stand for ancilla and system, respectively. As is customary in collisional models, the interaction time is considered to be extremely short, so the collision can be regarded as instantaneous, and its effect amounts to a unitary transformation applied to both the system and the ancilla. Here we denote by interaction strength the limit $\theta = \lim_{t \rightarrow 0} t\eta$, where t is the duration of the collision. The resulting unitary transformation for the collision is $U_\theta = e^{-i(\theta/2)\sigma_x^a \otimes \sigma_z^S}$.

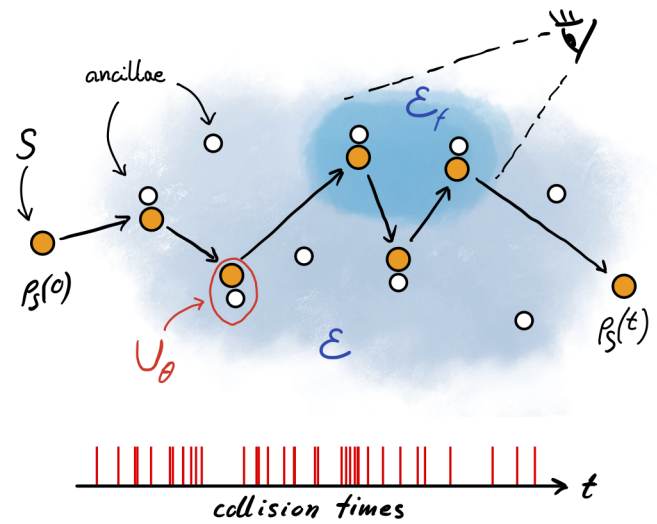


FIG. 1. Sketch illustrating the model. The orange dots represent the system S colliding with different ancillae (white dots) at exponentially distributed random times, depicted as red vertical lines on the t axis. During each collision, the system and the corresponding ancilla undergo a unitary transformation U_θ . The set of all ancillae defines the environment $\mathcal{E}$, whereas $\mathcal{E}_f$ represents a randomly chosen fraction f of the ancillae, to which an observer might have access in order to acquire information about the state of the system.

Since all the ancillae are initially uncorrelated with each other and with the system, and they collide with the system only once, we can describe the change in the system's state in terms of the collision channel $\Phi_c[\rho_S] = \text{Tr}_a[U_\theta \rho_a \otimes \rho_S U_\theta^\dagger]$. In the eigenbasis of the interaction Hamiltonian H_I , the collision channel can be cast in Kraus form as $\Phi_c[\rho_S] = K \rho_S K^\dagger + K^\dagger \rho_S K$, with

$$K = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{pmatrix}. \quad (1)$$

As a result, the effect of the channel is a factor $\cos \theta$ multiplying the coherences of the qubit state [29]. The state of the system at time t is given by the convex sum of all possible stochastic realizations of the ancillary dynamics (trajectories), each of them weighted by its probability. As shown in the Supplemental Material (SM) [29], the dynamics of the state of the system is described by the master equation

$$\dot{\rho}_S(t) = -i[H_S, \rho_S(t)] + \lambda\{\Phi_c[\rho_S(t)] - \rho_S(t)\}, \quad (2)$$

which is in Gorini-Kossakowski-Sudarshan-Lindblad form, with K and $K^\dagger$ the Lindblad operators [1]. Hence, we conclude that the system undergoes Markovian dynamics.

The master equation (2) describes a pure dephasing dynamics, where the coherence ρ_{01} evolves as $\rho_{01}(t) = c(t)\rho_{01}(0)e^{-i\omega t}$, with decoherence factor $c(t) = \exp[-\lambda(1 - \cos \theta)t]$. We notice that $c(t)$ is invariant with respect to changes of the interaction strength θ upon a proper modification of the collision rate. In particular, the system undergoes the same temporal evolution for $\lambda(1 - \cos \theta) = C$, where C is constant. This is an interesting observation, since the interaction strength θ regulates the level of entanglement between the system and a given ancilla after a collision has taken place. For instance, for $\theta = (2m + 1)\pi$, $m \in \mathbb{Z}$, $U_\theta |0\rangle_a \otimes |+\rangle_S =$

$e^{-i\pi/2}|1\rangle_a \otimes |-\rangle_S$, yielding a product state, while for $\theta = (2m+1)\pi/2$, $m \in \mathbb{Z}$, the two become maximally entangled. Hence, we can conclude that, in the former case, the system undergoes pure dephasing while remaining in a separable state with respect to the environment.

The source of randomness in the collision times can be seen as originating from a quantum process where the particle is emitted, for instance, as a result of a spontaneous emission process. This would reintroduce entanglement, in this case with an effective superenvironment, in the overall picture. We will analyze the consequences of such an effective description in more detail when focusing on QD. At this point, it is sufficient to stress that a quantum superenvironment does not need to enter the description, since the collisions with the ancillae can be triggered by some classical and largely macroscopic stochastic process.

Non-Markovianity with fresh ancillae. The model introduced above describes the situation in which the system undergoes Markovian pure dephasing dynamics while remaining in a separable state with the environment. We now show that dynamical mixing can induce non-Markovian behavior as well. To this end, we modify the previous model by limiting the number of ancillae to a finite amount n . Here, every ancilla's collision time has an exponential probability density with rate λ/n , while the effect of the collisions is not altered. The integrated dynamics reduces to that of infinitely many ancillae at short times, $\lambda t \ll n$ [30]. In the integrated dynamics, coherences are multiplied by the factor

$$c_{\text{NM}}(t) = [1 + (\cos \theta - 1)(1 - e^{-\lambda t/n})]^n. \quad (3)$$

In the particular case in which the system and the ancilla entangle maximally after a collision ($\cos \theta = 0$), the system dephases monotonically with $c_{\text{NM}}(t) = e^{-\lambda t}$, exactly like in the model with infinitely many ancillae. In the case of entanglement-free interaction ($\cos \theta = -1$), however, the off-diagonal elements of the density matrix are multiplied by the factor $c_{\text{NM}}(t) = (2e^{-\lambda t/n} - 1)^n$, which is equal to zero at $t_m = n \ln 2/\lambda$ (mixture time) and tends to $(-1)^n$ as $t \rightarrow \infty$. As a consequence, if the initial state of the system is, e.g., $\rho_S(0) = |+\rangle\langle+|$, it becomes maximally mixed at $t = t_m$ and it gradually recovers its purity thereafter. Moreover, the system remains in a highly mixed state for longer periods as the environment size n increases (see Fig. 2), since $c_{\text{NM}}(t)$ scales as $(-\lambda \epsilon/n)^n$ for times t close to t_m , i.e., $t = t_m + \epsilon$ with $\epsilon \ll 1$.

Remarkably, this phenomenon of recoherence takes place despite the fact that the system never collides with the same ancilla more than once, and despite the absence of interactions [22,31] or initial correlations [26] among ancillae; instead, it is merely due to the stochastic nature of the collision times. Since the effect of a single collision is a change of sign of the off-diagonal elements of the density matrix, the latter vanish if the probability for odd and even collisions is equal (this is the case at $t = t_m$). At long times $t \rightarrow \infty$, on the other hand, all ancillae eventually collide and, as a result, the collisional parity is simply given by the parity of n , hence resulting in the aforementioned recoherence. Notice that this behavior is not necessarily exclusive of an environment with a fixed number of ancillae, but can also hold in the case in which n is a random variable (for instance, a distribution that yields

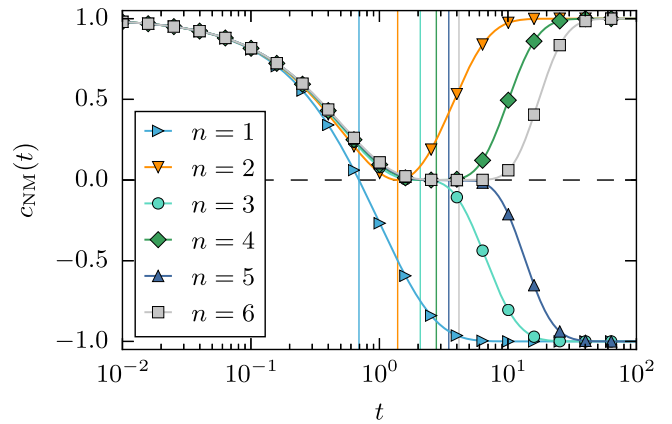


FIG. 2. Coherence factor for the qubit, Eq. (3), as a function of time for different numbers of ancillae with nonentangling interaction strength, $\theta = \pi$ and $\lambda = 1$. The vertical lines indicate the corresponding mixture time t_m , at which the coherence vanishes. As the number of ancillae increases, the coherence remains close to zero for longer periods of time. Notice that the final state depends on the parity of the number of ancillae.

different probabilities for n being odd or even would also result in non-null coherence at long times).

Quantum Darwinism. Our results so far reveal that it is possible for a system to undergo exactly the same decoherence dynamics whether or not it becomes entangled with its environment, or even to exhibit non-Markovian dynamics, as a consequence of dynamical mixing. Needless to say, this raises the question of what role does entanglement play in the quantum-to-classical transition. In what follows, we address this issue in the context of QD. As we will show, decoherence without entanglement does not allow for the encoding of information about the system's state into the environment, whereas this is possible when one considers a superenvironment giving a quantum origin to the randomness in the collision times.

QD explains the emergence of objective reality through the mutual information between system and environment. In particular, if several observers that measure different parts of the environment gather the same information about the state of the system, they can consider such information as objective reality. For that to happen, however, there must be some redundancy in how that information is distributed across the environment, and the typical way to quantify it is by calculating the mutual information I_f between the system and a randomly chosen fraction $\mathcal{E}_f$ of the environment, as a function of the fraction's size f . Such curve reveals the presence of objective reality through a plateau that spans over a wide range of environmental fraction sizes, and whose value is approximately equal to the von Neumann entropy of the system. In order to assess whether this phenomenon is present in the model introduced in this Rapid Communication, we need to calculate $I_f = H_S + H_{\mathcal{E}_f} - H_{S\mathcal{E}_f}$, where H stands for the von Neumann entropy.

We focus first on the case in which the interaction is nonentangling, namely, $\theta = \pi$ and the initial state of the system is $\rho_S(0) = |+\rangle\langle+|$ [32]. By tracing out $k = (1-f)n$ ancillae from the total state of system and environment $\rho_{S\mathcal{E}}(t)$, one can

calculate the reduced state when only a fraction f of the latter is considered, $\rho_{S\mathcal{E}_f}(t) = \text{Tr}_k[\rho_{S\mathcal{E}}(t)]$, while further tracing out the system S yields the reduced state of the fraction of the environment, $\rho_{\mathcal{E}_f}(t)$. The simplicity of our model allows us to perform these calculations analytically, furthermore resulting in density operators in diagonal form, from which computing their von Neumann entropy is straightforward. All the details of the calculations are given in the Supplemental Material [29]. The resulting mutual information is

$$I_f = H_b[P_n^e(t)] - H_b[P_k^e(t)], \quad (4)$$

where $f = 1 - \frac{k}{n}$, $H_b(x) = -x \log x - (1-x) \log(1-x)$ is the binary entropy function, and $P_m^e(t)$ is the probability for m ancillae to yield an even number of collisions at time t . This quantity can be computed exactly and reads

$$P_m^e(t) = \frac{1}{2} [1 + (2e^{-\lambda t/n} - 1)^m]. \quad (5)$$

In Fig. 3(a), we show this curve for different dynamical regimes. Despite an almost linear dependence in some periods, it is mostly flat around null mutual information except for $f \approx 1$ when the system is highly mixed, which implies the absence of objective reality upon which observers can agree.

We now study the case in which the ancillae are emitted as a consequence of a quantum process. In particular, we consider n emitters initially excited that relax to their ground state emitting an ancilla in the process. Moreover, we further assume that the ancilla is emitted in state $\rho_a = |0\rangle\langle 0|$ and, once emitted, it immediately collides with the system, flipping its state (since $\theta = \pi$). Hence, the emitter-ancilla-system dynamics is such that, for $n = 1$ and for the system initially in the $|+\rangle$ state, their joint state at time t can be written as $\sqrt{e^{-\lambda t/n}}|1\rangle_{\text{em}} \otimes |0\rangle_a \otimes |+\rangle + \sqrt{1 - e^{-\lambda t/n}}|0\rangle_{\text{em}} \otimes |1\rangle_a \otimes |-\rangle$ with $|\pm\rangle = e^{-iH_S t}|\pm\rangle$. In this new setting, tracing out the emitters yields exactly the same reduced state as in the previous situation. However, when considering as environment fraction $\mathcal{E}_f$ pairs comprising both the emitter and the corresponding ancilla, the mutual information takes the form

$$I_f = H_b[P_n^e(t)] + H_b[P_{n-k}^e(t)] - H_b[P_k^e(t)]. \quad (6)$$

Comparing this result with Eq. (4), we see that the presence of the emitters introduces the term $H_b[P_{n-k}^e(t)]$, which corresponds to the entropy of the environment, $H_{\mathcal{E}_f}$ (see SM). Figure 3(b) depicts the mutual information in this new setting. In this case, there is a clear plateau at $I_f/H_S = 1$, revealing a structure of the total state of system and environment compatible with QD. The fact that the plateau is only present at times in which the reduced state of the system is highly mixed, along with our previous discussion regarding the separability of the system and the ancillae alone, can be interpreted as further evidence that the emergence of objective reality requires entanglement.

Conclusions. We have investigated the emergence of classical reality from its quantum substrate by means of a collisional model, for which we have derived analytically not only the master equation and the dynamical map, but also the relevant system-environment dynamical properties. The stochastic element introduced in the microscopic description of the collisions lends itself to an interesting generalization in terms of a superenvironment that keeps track of

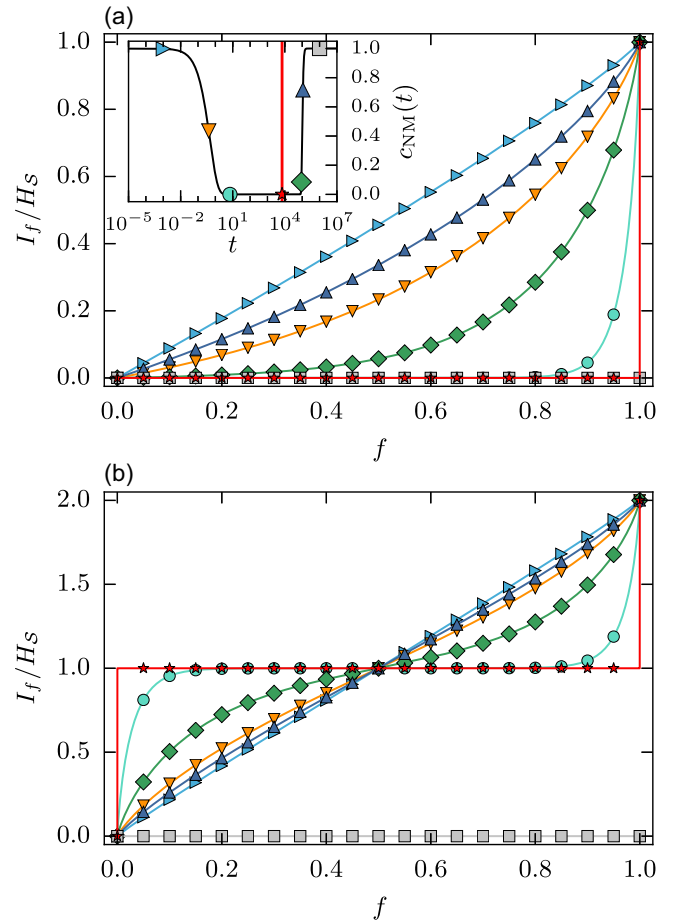


FIG. 3. Mutual information over system entropy as a function of environment fraction for the two settings at different times, for $n = 10^4$ ancillae and $\lambda = 1$. To indicate the state of the system at the time to which each curve corresponds, they have been colored matching the corresponding dot in the inset, which shows the coherence factor as in Fig. 2. The times have been chosen to cover the different dynamical regimes undergone by the system. (a) The origin of the randomness in the mutual information occurs for $I_f = 0$, meaning that the ancillae alone barely carry any information. (b) The emitters are part of the quantum state as well. The effect of the emitters is the appearance of QD while the system is in a highly mixed state.

the occurrence of the collisions. Due to these features, the underlying dynamics is dominated by simple fundamental physical mechanisms allowing us to shed new light on the role of quantum entanglement in three crucial phenomena: decoherence, non-Markovianity as information back-flow, and quantum Darwinism. The study of this model highlights that system-environment entanglement is not necessary for decoherence or information back-flow, but plays a crucial role in the emergence of an objective reality.

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