Statics and stability of bending-optimized double-layer grid shell

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Abstract. Grid shells are unbeatable structures for their aesthetics and lightness, but their efficiency is highly reduced as soon as their shape deviates from a pure membrane. Many contemporary architectures have a freeform shape, which is conceived mostly on aesthetics and functional criteria, and in these cases, finding an efficient grid shell often requires substantial shape modifications. This work addresses a new kind of double-layer structure that aims at not altering the target shape design. The structural system comprises a quad-meshed grid shell aligned to the target shape and enriched with an additional reinforcement layer that adds bending stiffness. This additional layer, going inward and outward of the main surface, presents variable-height and discontinuous elements on the basis of the required bending strength. The obtained structural system differs from both grid shells, as these latter are very deformable in this setup, and from classic double-layer structures or space frames, which are heavier and redundant. In this paper, we show how the present system compares with grid shell and double-layer competitors in terms of statics and stability. We highlight the pros and cons based on a systematic comparative analysis run on selected freeform shapes.

Keywords: Freeform surface · Space frame · Computational design · Structural design · Finite Element Model.

1 From grid shells to space frames and reinforced structures

Grid shells are elegant and lightweight structures used in architecture to enclose large spaces. Their lightness is due to the structural efficiency in developing membrane actions to resist external loads, which flow along a grid of rods. This behavior that relies on in-plane stiffness rather than on bending can be obtained only through a careful shape design, i.e. form-finding, and a proper selection topology of the grid and the structural scheme [1]. However, the shape of the grid shell is sometimes hardly pre-determined based on site constraints, or it is simply sculptured by the architect. Therefore, a structural design that seeks funicular to accomplish membrane behaviour can strongly alter the initial design or can conflict with other requirements. Hence, bending forces inherently arise when transversal forces can not be balanced by in-plane stress. To address bending resistance demands and to increase the robustness, the alternative concept of double-layer structures or space frames can be used [2]. Two interconnected surfaces generate bending inertia from spacing the members of the two surfaces in the out-of-plane direction. These surfaces can be continuous or segmented, i.e., shaped as grids, plates, shells etc. The higher the distance between the layers, the larger the bending inertia.

Classically, a space frame is an assembly of linear elements arranged to transfer forces in a 3D manner. Straight steel tubes, mostly loaded axially, and joints are the basic units of this system. A large variety of space frames exists, but their common feature is to have the form of a surface in space and to be constructed of as-nearly-as-possible identical units arranged with a repeating pattern [3]. One of the two layers is usually filled or equipped with cladding panels since they are employed as roofs, envelopes and slabs. Double-layer plate structures are typically used in timber construction to realize dry-connected lightweight shells in the form of cassettes, hollow boxes or waffle structures [4–6]. This concept can be extended even towards less stiff materials [7].

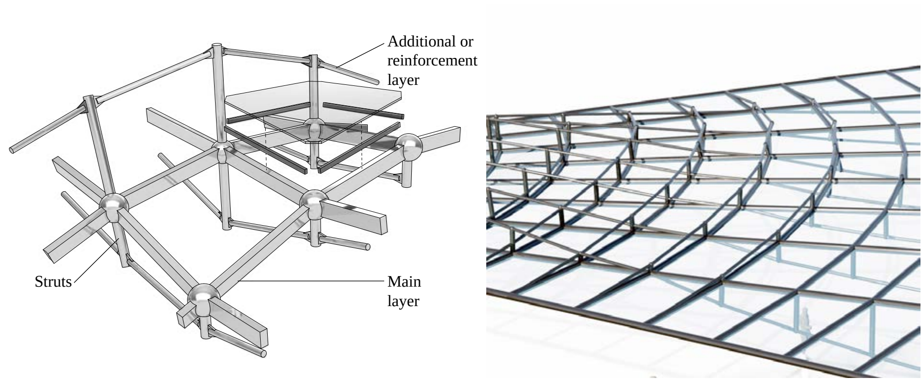
An interesting design option is to vary the distance between the layers following the bending moment plot. This approach has inspired several design concepts for 2D structures such as truss bridges and frames, for instance, the Berlin Central Station roof [8], designed by GMP Architekten and sbp; and the Waterloo station roof [9] in London, designed by Grimshaw Architects and YRM Anthony Hunt Associates. Both structures are translation surfaces where a transversal planar frame is replicated along a path. The main frame is an arch line, stiffened by cables and along-normal struts, oriented inward or outward according to the bending moment pattern profile. This tension stiffening system can be considered an added layer, and it is usually post-tensioned until bending forces are removed and/or compressive forces on the main layer are preserved.

The approach of adding reinforcement-where-needed inspired research based on graphic statics aimed at transforming a generic free-form curve into a funicular shape. In the works [10, 11], the funicularity of 2D and 3D shapes is achieved via an additional out-of-plane cable net using graphic statics. With a similar purpose, in [12], the funicular geometry is exploited to design a tension stiffening system shaped according to the bending moment in a 2D free-form curve.

To solve somehow the same problem in the continuum, the authors in [13] introduced 3D shell surfaces reinforced by ribs, whose height is proportional to the bending. The ribs form a grid aligned with a cross-field that considers the principal directions of the strain in the equivalent continuum. Several years before, in a similar fashion, Nervi patented a rib-stiffening scheme for isostatic concrete slabs that follows the principal bending directions [14]. Adding ribs and reinforcement [15, 16] or aligning the beams of a grid along specific paths [17, 18], i.e. principal stress directions, is a well-known strategy that is inspired by the

Michell-like optimum structures.
In this paper, we analyze the structural behavior of Bending-Optimized Double-Layer Grid Shells introduced in [19], which are quad-based grid shells reinforced with an out-of-plane discrete net of elements that are shaped to increase the bending strength. In this method, firstly, the input surface is tessellated, creating a main layer that aims at maximizing the resistance to out-of-plane bending; then, the location of an additional reinforcement layer is optimized using a novel energy formulation. We compare these structures with state-of-the-art alternatives to prove that they have intermediate features of both the space frames and the grid shells.

2 Bending-optimized double layer grid shells



(a) (b)

Fig. 1. Our Bending-Optimized Double-Layer Grid Shells: (a) section view and nomenclature; (b) rendered closeup.

Our computational pipeline automatically designs a reinforcement structure optimized for a predefined load condition and capable of efficiently supporting out-of-plane loads. The details on method and implementation are in [19]. In the resulting structure, a quad-based grid shell is connected with a reinforcement structure having similar connectivity but whose nodes are placed at variable distances and orientations with respect to the original grid shell, as in Fig. 1. These two interconnected structures can generate bending inertia in 3D from the spacing between the two layers in the out-of-plane direction. To have always tension reinforcements (and compressed struts), we provide the additional layer to go inward or outward the base grid shell. The additional layer can be discontinued if no bending strength is locally needed.

Given an initial surface, as a first step, we build a linear finite element analysis as a thick shell, and we use the strain tensor on the top and bottom nodes

Table 1. Dataset quantitative measures for the three structural cases: model name; size of the axis-aligned bounding box in meters; number of vertices #V; number of edges #E; volume of the structures V

Model	Size	Grid shell			Ours			Space frame		
	$(m \times m \times m)$	#V	#E	V (m^2)	#V	#E	V (m^2)	#V	#E	V (m^2)
Rust	80.35×58.08×19.15	615	1194	46.43	1143	2460	66.10	1230	3003	122.03
Shell1	51.11×81.06×23.56	726	1398	50.62	1222	2587	70.01	1452	3522	136.90
Shell2	90.59×37.08×18.33	379	707	26.01	712	1460	38.55	758	1793	68.69
Vault	42.75×23.32×12.37	619	1184	20.18	1153	2352	30.21	1238	2987	65.91
Wave	43.44×44.01×6.05	431	818	24.94	1239	2626	58.87	1178	2865	77.10

to create a cross-field. This vector field generates a quad mesh, in which its alignment to the cross-field is forced if the cross directions have higher anisotropy or higher magnitude. The quad mesh topology is chosen since it can effectively capture a planar principal stress with the lowest number of elements.

In the second step, we use the internal forces computed with a linear static analysis on the quad mesh to initialize the additional layer. Then, we compute the additional layer's optimal elevation, solving a constrained minimization problem in which the structural efficiency is improved. The optimization variables are the distances from the main layer. In the optimal setup, the stress due to bending would be constant in both layers. In the final step, the structure is modified to conform to fabrication constraints.

The main strength of this work is decoupling the meshing phase from the initialization/optimization of the additional layer, which is, to the best of our knowledge, the first method to automatically derive reinforcement structures for non-membrane stress in a discrete setting. Thus, any quad mesh can be used.

3 Experimental setup and analyses

We tested several input freeform surfaces having different features (Tab. 1). As we want to assess the structural performances of our bending-optimized double-layer grid shells, we set up a comparison with other state-of-the-art structures. For each model, we build and analyze three structural system cases:

- *Grid shell*, directly assembled from the quad mesh by converting edges into beams and vertices into structural nodes;
- *Our bending-optimized double-layer grid shells*, using a multi-chords approach [19];
- *Space frame*, by duplicating the grid shell at 2.5 m along the surface normal and connecting corresponding nodes.

The performance of discrete spatial structures, and of grid shells in particular, is a function of topology, grid spacing and the total weight of the elements [20, 21]. For the sake of comparing different structural systems, we chose to keep fixed the grid spacing of the base mesh. So, the comparison criterion for the structure

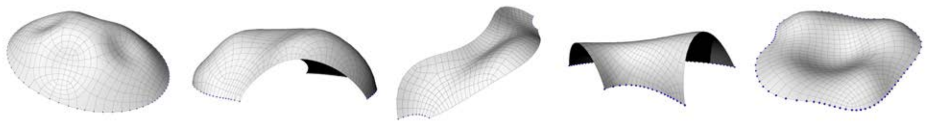


Fig. 2. Experimental dataset of surfaces, from left: Rust, Shell1, Shell2, Vault, Wave. All models are tessellated with the actual quad mesh used in the simulations; blue spheres indicate fixed supports.

is to be initialized on the same quad mesh of Fig. 2. The quad mesh is computed using the approach of our bending-optimized double-layer grid shells. As a result, the topology is inherently different for the three structural systems, as so is the weight: our structures are heavier with respect to the grid shells (about 27 to 57 % more) and can have nodes with various valence (larger or equal to 4); space frames are heavier than our structures (about 23 to 54 % more) and than grid shells (about 62 to 70 % more). The mode of node valence for space frames is 5.

All these finite element models rely on some uniform assumptions:

- No stiffening systems are considered (i.e. provided by bracing cables), so all nodes are modeled as rigid;
- A subset of boundary nodes are supposed to be rigidly fixed (all degrees of freedom are fixed, 3 translations and 3 rotations);
- We used common S355 structural steel;
- The elements' cross-section are CHS of 0.2 m external diameter and 0.025 m thickness for the main beams, for the grid shell and for both layers of the space frame; solid circular sections 0.15 m diameter for the struts; solid circular sections of 0.1 m diameter for the added layer for our double-layer structures;
- The main layer of beams is kept fixed in all three models;
- The dead load is computed from the material density and geometry of the cross sections;
- We used a common uniform load $f_{ext} = 2\ kN/m^2$, representing a typical service load including non-structural load (e.g., cladding) and environmental load (e.g., snow), which is applied as a lumped load on the nodes of the grid shell and of the main layer of the other cases;
- Material non-linearity is neglected as the analyses already involve many variables;
- Each beam is modeled as a single finite element to reduce the computational time, even though this simplification prevents from highlighting single member buckling.

This work aims not to solve real-world design cases, but to evaluate our reinforcement strategy in bending-optimized double-layer grid shell in comparison with grid shell, which often would not have enough stiffness, and space frame, which usually has more redundancy. We establish a two-level assessment: (a) serviceability limit state conditions (SLS) to check internal forces and displacements; (b) stability to control the failure modes. We do not analyze the strength at the ultimate limit state (ULS), even if yielding could sometimes be a realistic failure mode. Instead, we are interested in estimating the ultimate capacity on a comparative basis. We compute geometrically non-linear analyses (GNA) on commercial software [22]. Additionally, we compare the load factors obtained with the GNA with the first eigenvalue from a linear buckling analysis (LBA).

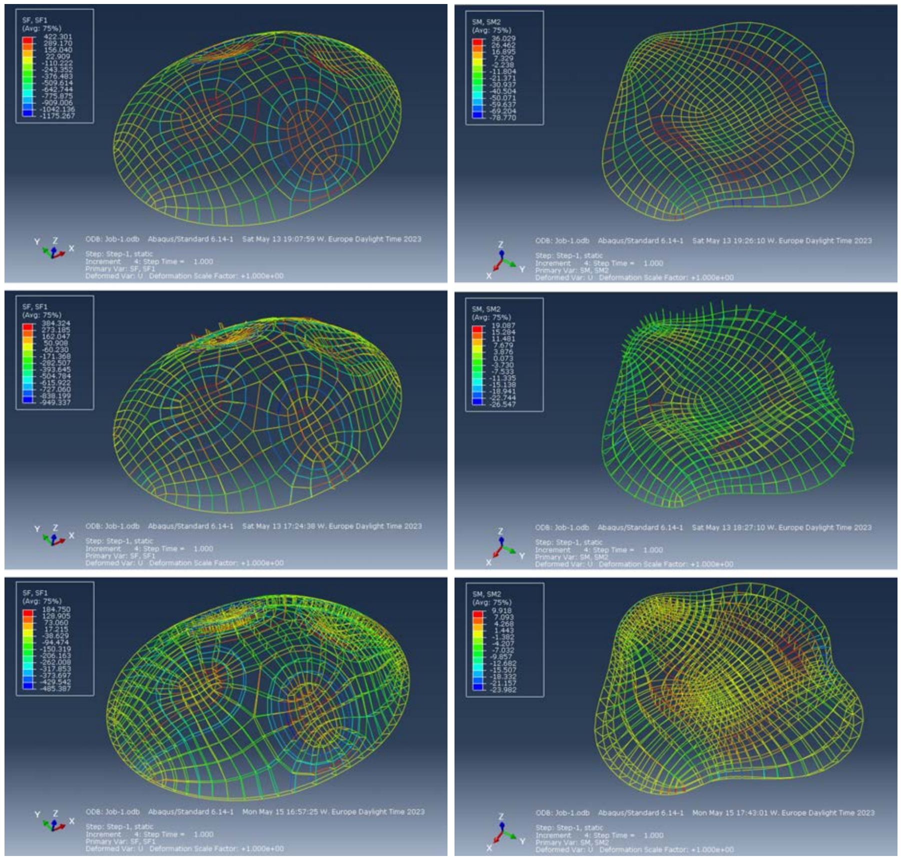
4 Results and discussion

At the SLS, our double-layer grid shell has an intermediate stiffness between a grid shell and a space frame, as evidenced in Tab. 2. Some models, like Shell1 and Shell2, would not have enough stiffness as grid shells since they overshoot the usual allowable displacement. Therefore, it is at least recommended to change the structural system for these kinds of shapes. 'Well'-shaped surfaces like Vault are obviously good performing as grid shells. Concerning internal forces, the utilization of the elements is larger in grid shells, as expected. However, the more the shape is far from a membrane, the higher the risk of yielding. Additionally, it is worth converting grid shells to other structural systems despite the weight increase (Fig. 3a). Indeed, the load transfer mechanism changes and the elements are prevalently employed in the axial regime, which is more efficient. Similarly to a space frame, the bending moments, which our double-layer grid shells are supposed to capture, reduce considerably (Fig. 3b). However, remarkably, the utilization of the elements is more uniform in our case.

Table 2. Maximum displacement magnitude d at the SLS.

Model	Grid shell d (m)	Ours d (m)	Space frame d (m)
Rust	0.167	0.085	0.043
Shell1	0.445	0.235	0.100
Shell2	3.047	1.964	0.741
Vault	0.026	0.011	0.006
Wave	0.075	0.040	0.031

Under incremental load, the grid shell has the lowest load factor. As expected, our double-layer grid shells performance is in between grid shells and space frames (Tab. 3). Except for the Vault model, which has a more efficient shape, all grid shells do not exhibit a pure membrane behavior, and therefore they show a softening trend prior to collapse. Instead, our double-layer grid shells are often characterized by a post-critical stiffening, where the additional layer balances unstable displacements of the main layer. The critical shape of our double-layer grid shells and space frames is different. The stiffness and the load transfer mechanisms of the two systems is quite different. In our double-layer grid shell,



(a) (b)

Fig. 3. Internal forces for the three structural systems (from top: Grid shell, ours, space frame): (a) axial force, Rust model; (b) out-of-plane bending moment, Wave model.

the additional layer is mostly tensioned and can be located inward or outward of the main layer; and it has a smaller cross section. In space frames, the two layers build a more uniform stiffness and could be stressed either in tension or compression.

The collapse shapes obtained in LBA are very similar to GNA for our double-layer grid shells and space frames (Fig. 4). On the other hand, this is not the case for grid shells, in which probably a more complex combination of buckling modes occurs for the GNA.

5 Conclusions and future developments

This paper assesses the structural performance of bending-optimized double-layer grid shells introduced in [19]. These structures can represent a good al-

Table 3. Comparison of load factors from the LBA (first eigenvalue) and from the GNA for the three structural cases (* indicates tension stiffening, - indicates not run).

Model	Grid shell		Ours		Space frame	
	LB	GNA	LB	GNA	LB	GNA
Rust	6.98	5.97	11.06	8.99	37.81	34.92
Shell1	3.89	*4.51	6.51	*7.52	40.89	*42.56
Shell2	0.63	0.39	1.35	1.03	18.67	14.49
Vault	75.27	46.99	149.36	162.20	722.72	-
Wave	22.92	18.71	52.40	35.29	121.71	*

ternative to classic lightweight spatial structures, such as grid shells and space frames. Space frames are more robust and can achieve a higher prefabrication rate. However, having all modular elements is nowadays not a primal request if digital fabrication tools and machines are employed. Hence, producing labelled pieces of any dimension comes at a low cost.

The role of imperfections in the GNA is still an open topic to investigate further. However, we adopted models and a meshing strategy that led to non-symmetric and non-regular structures. Thus, even in the case of grid shells, the membrane-to-bending strain energy ratio is lowered so that, in theory, structures would be less sensitive to imperfections. Another research direction could be introducing a discretization into multiple finite elements per beam to catch possible local buckling phenomena that may lower the load factors.

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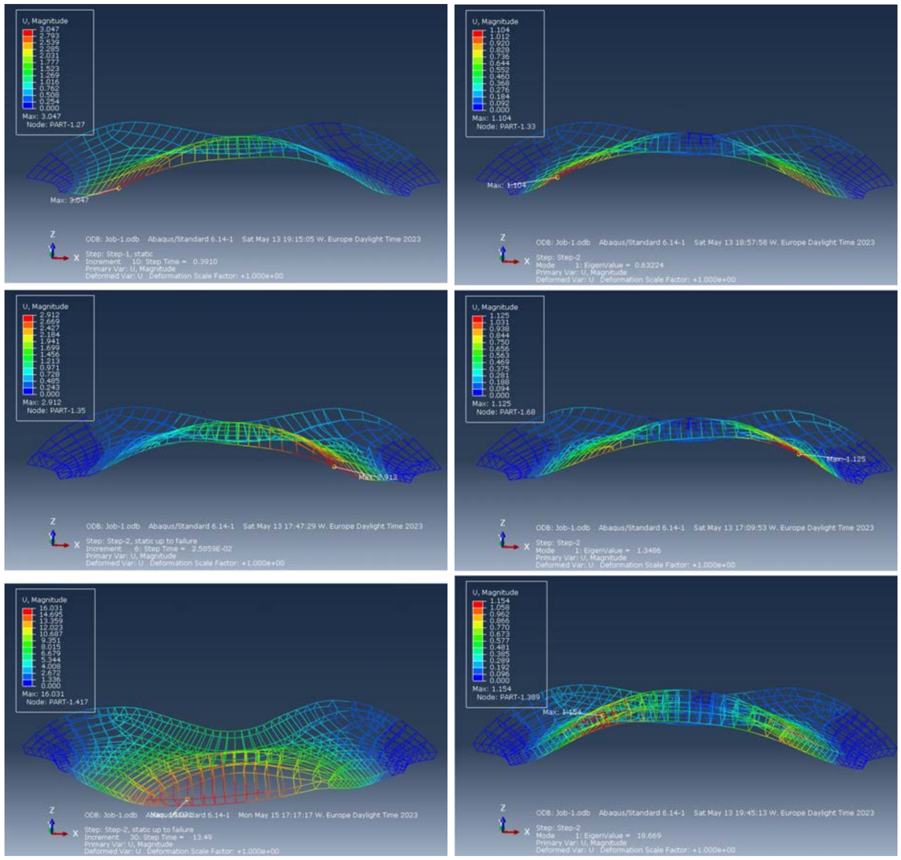

(a) (b)

Fig. 4. Buckling shape, Shell2 model (from top: Grid shell, ours, space frame): (a) GNA; (b) LBA.

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