

GIMC SEMNI Workshop

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Non-linear structural analysis of masonry buildings using NOSA-ITACA

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Mechanics of Materials and Structures Laboratory

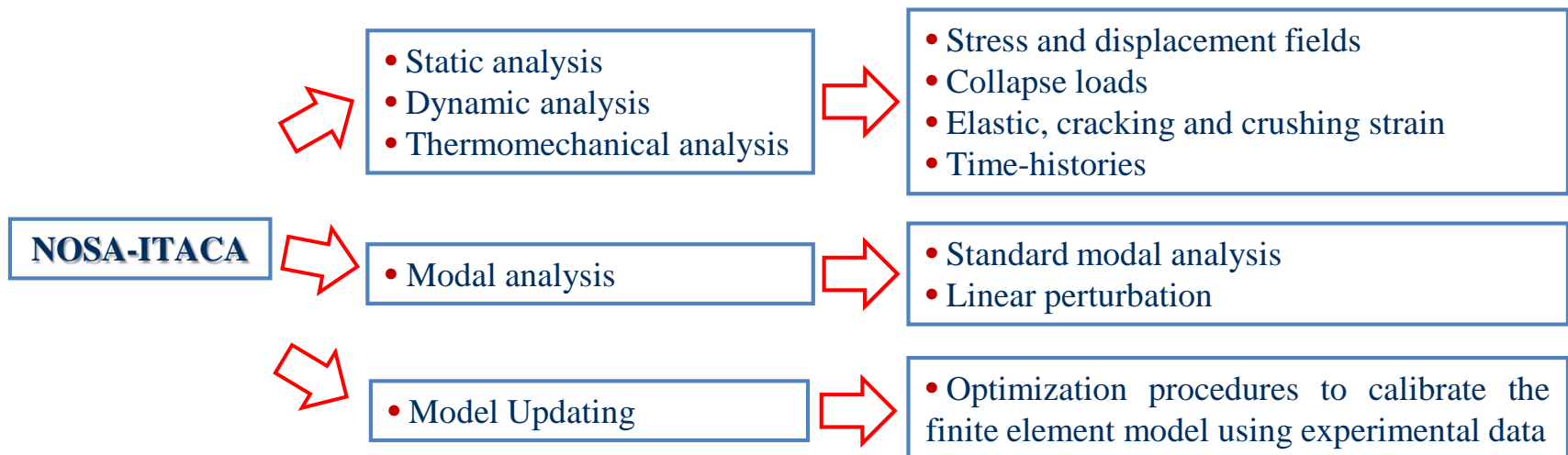


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The NOSA-ITACA code

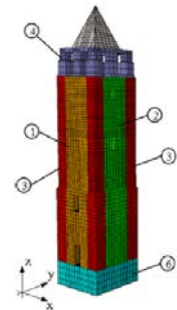
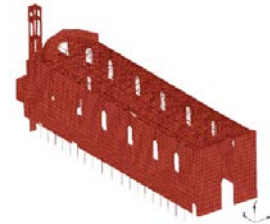
- **NOSA-ITACA** is a freeware software package developed by ISTI-CNR. It is a finite element code that combines NOSA (solver) with the open-source graphic platform SALOME (pre-post processing operations), suitably modified. It has been specifically implemented to study the static and dynamic behavior of masonry constructions, using the constitutive equation of *masonry-like* materials
- **NOSA-ITACA** is distributed via the <http://www.nosaitaca.it/software/>
- **NOSA-ITACA library** has 35 elements including: beam, truss, shell, 2D, 3D elements



- The capability of modelling restoration and consolidation operations makes the code a helpful tool for maintaining historical buildings

The NOSA-ITACA code

- San Francesco Church, Lucca - Italy (2012)
- Il Voltone (The Great Vault), Livorno - Italy (2012-2013)
- San Cerbone Cathedral, Grosseto – Italy (2013)
- The bell Tower of San Frediano, Lucca - Italy (2014-2017)
- The vault of Franchetti Palace, Pisa - Italy (2014)
- The Devil's Bridge, Lucca - Italy (2014)
- The Clock Tower, Lucca – Italy (2016-2018)
- The Matilde Donjon, Livorno – Italy (2016-2018)
- Torre Grossa, Siena – Italy (2016-2018)
- The Mogadouro Clock Tower, Portugal (2018)
- Vicari Tower and Town Hall, Florence - Italy (2020)
- Guimaraes castle´s tower keep, Portugal (2020)
- Carillon Tower, Castel San Pietro Terme – Italy (2021)
- Guinigi Tower, Lucca - Italy (2022)



The constitutive equation of *masonry-like* materials

\mathbf{E}	infinitesimal strain tensor,
\mathbf{T}	Cauchy stress tensor,
$\mathbf{E}^e, \mathbf{E}^f, \mathbf{E}^c$	elastic, cracking and crushing part of the strain tensor,
$\mathbb{C}(\theta)$	isotropic fourth-order tensor of elastic constants
$E(\theta), \nu(\theta)$	Young 's and Poisson's modulus,
$\sigma^t(\theta) \geq 0, \sigma^c(\theta) > 0$	tensile and compressive strength,
α	linear coefficient of thermal expansion,
θ, θ_0	absolute and reference temperature

Given \mathbf{E} and $\theta \in [\theta_1, \theta_2]$, it is possible to obtain $\mathbf{E}^f, \mathbf{E}^e, \mathbf{E}^c, \mathbf{T}$ such that :

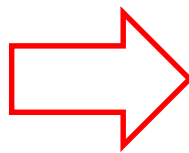
$$\mathbf{E} - \alpha(\theta - \theta_0) \mathbf{I} = \mathbf{E}^e + \mathbf{E}^f + \mathbf{E}^c$$

$$\mathbf{T} = \mathbb{C}(\theta)[\mathbf{E}^e]$$

$$\mathbf{E}^f \cdot (\mathbf{T} - \sigma^t(\theta)) = \mathbf{E}^c \cdot (\mathbf{T} + \sigma^c(\theta)) = 0$$

$$\mathbf{T} - \sigma^t(\theta) \in \text{Sym}^-, \mathbf{E}^f \in \text{Sym}^+$$

$$\mathbf{T} + \sigma^c(\theta) \in \text{Sym}^+, \mathbf{E}^c \in \text{Sym}^-$$

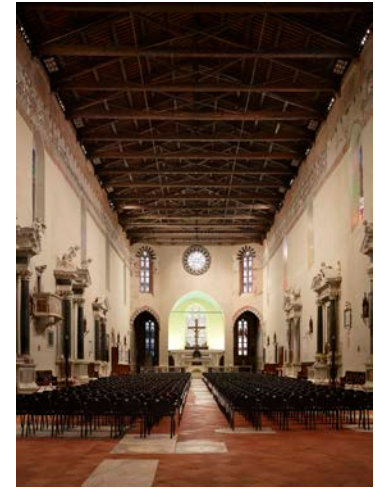
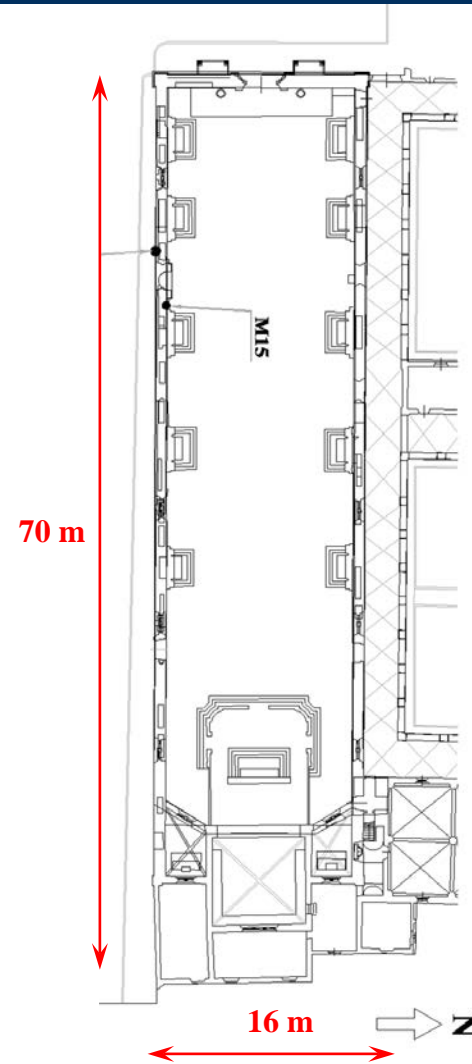


$$\mathbf{T} = \check{\mathbf{T}}(\mathbf{E}, \theta) \quad \text{stress function (explicit expression)}$$

$$\mathbf{D}_{\mathbf{E}} \check{\mathbf{T}}(\mathbf{E}, \theta) \quad \text{derivative of stress function with respect to the strain tensor (explicit expression)} \quad \Rightarrow \quad \mathbf{K}_{\mathbf{T}}$$

1. Lucchesi, M., Padovani, C., Pasquinelli, G., & Zani, N. (2008). *Masonry constructions: mechanical models and numerical applications*. Springer Science & Business Media
2. Pellegrini, D. (2024). Thermo-mechanical analyses of masonry structures in fire conditions. *Finite Elements in Analysis and Design*, 234, 104128.

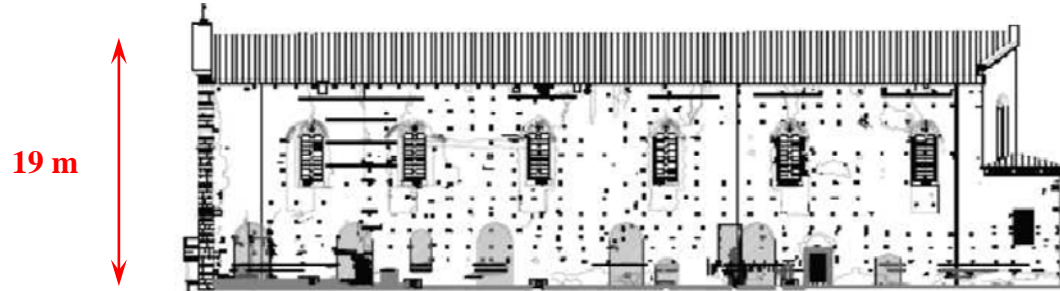
The San Francesco church, Lucca (Italy)



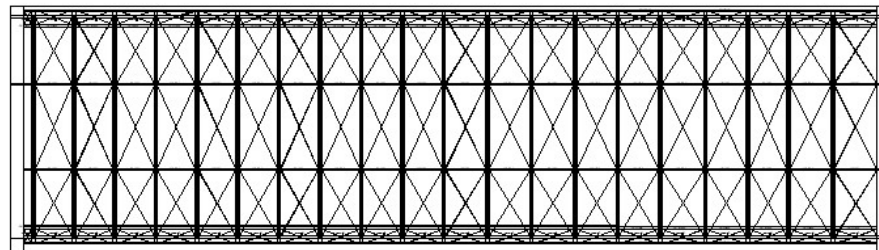
Girardi, M., Padovani, C., & Pasquinelli, G. (2013). Numerical modelling of the static and seismic behaviour of historical buildings: the church of San Francesco in Lucca. In *Proceedings of the Fourteenth International Conference on Civil, Structural and Environmental Engineering Computing*, Civil-Comp Press, Stirlingshire, UK, Paper (Vol. 80).

The San Francesco church, Lucca (Italy)

The southern wall presented large out-of-plane deflections and extensive cracks over the windows and near the facade



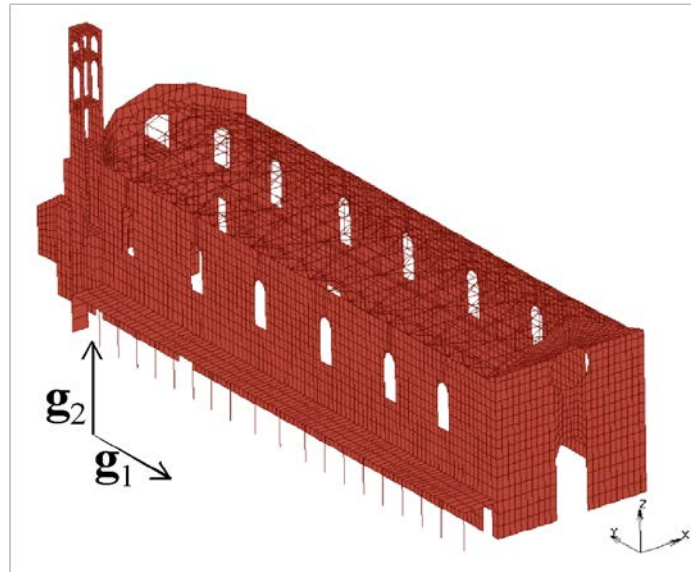
- Reinforcement operations, mainly aimed at improving the quality of the masonry and the connections between the walls, were conducted in 2013.
- Because of the slenderness of the nave walls, it was decided to increase the building's resistance to horizontal actions.
- Thus, a metal framework was constructed at roof level to brace the structure



Numerical simulations were conducted to assess the efficiency of the reinforcement operations

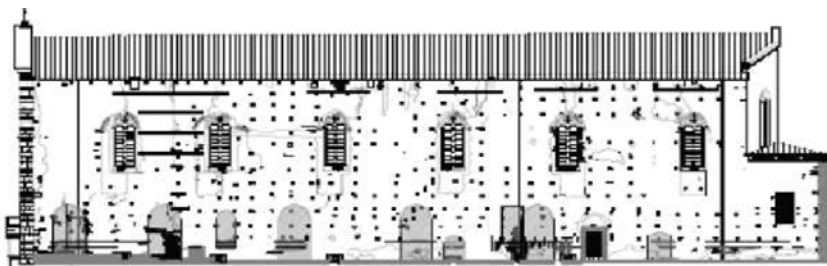
The San Francesco church, Lucca (Italy)

Analysis of the church subjected to the permanent loads without reinforcement



Finite element model:

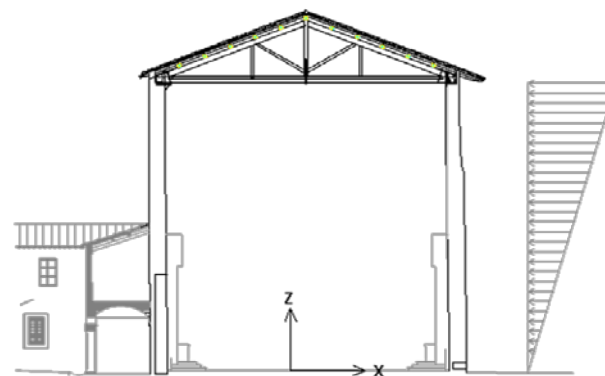
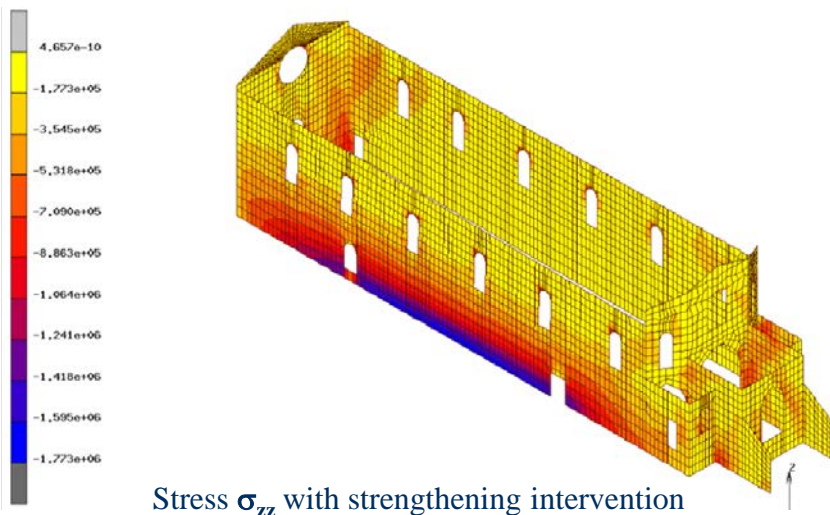
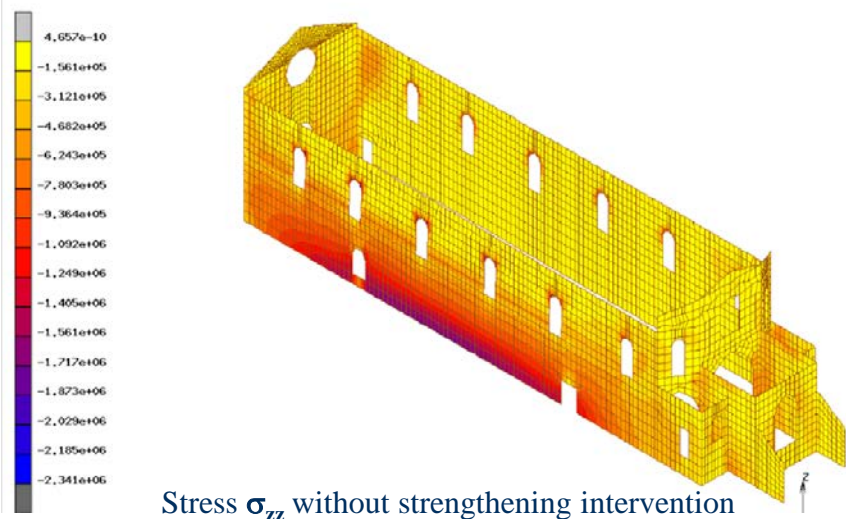
- thick shell and beam elements;
- foundations are assumed to rest on a Winkler spring bed;
- off-plumb deformation of the southern wall has been carefully modelled in the mesh, as have the reinforcement operations on the construction



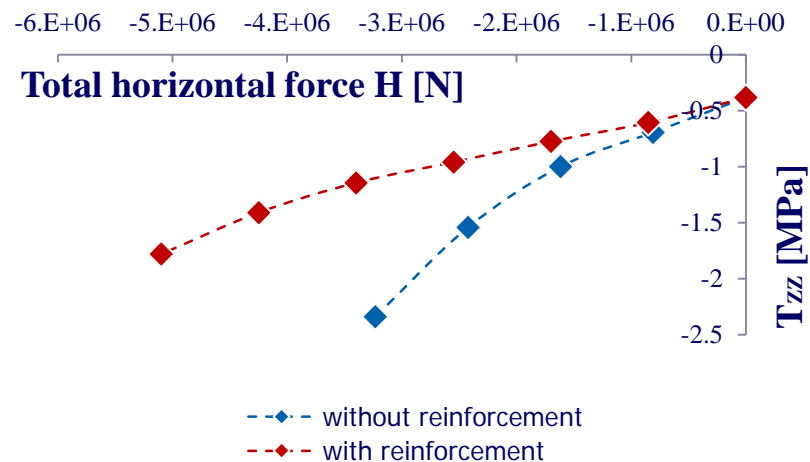
The southern wall fracture strain

The San Francesco church, Lucca (Italy)

The church is subjected to the permanent loads and horizontal out-of-plane loads modelling seismic actions

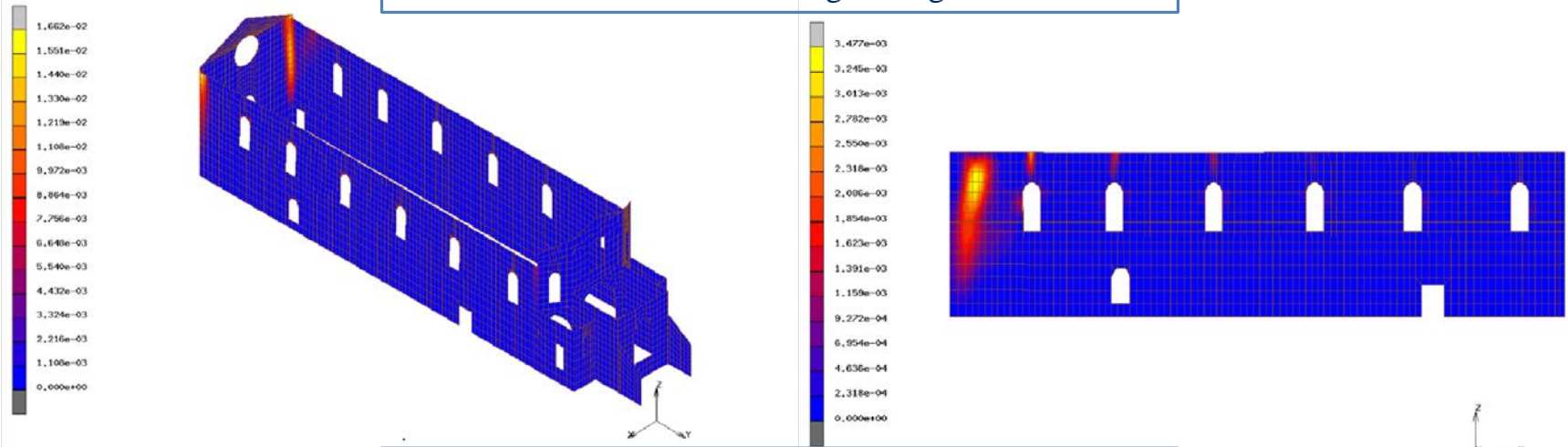


Maximum values of compressive stress

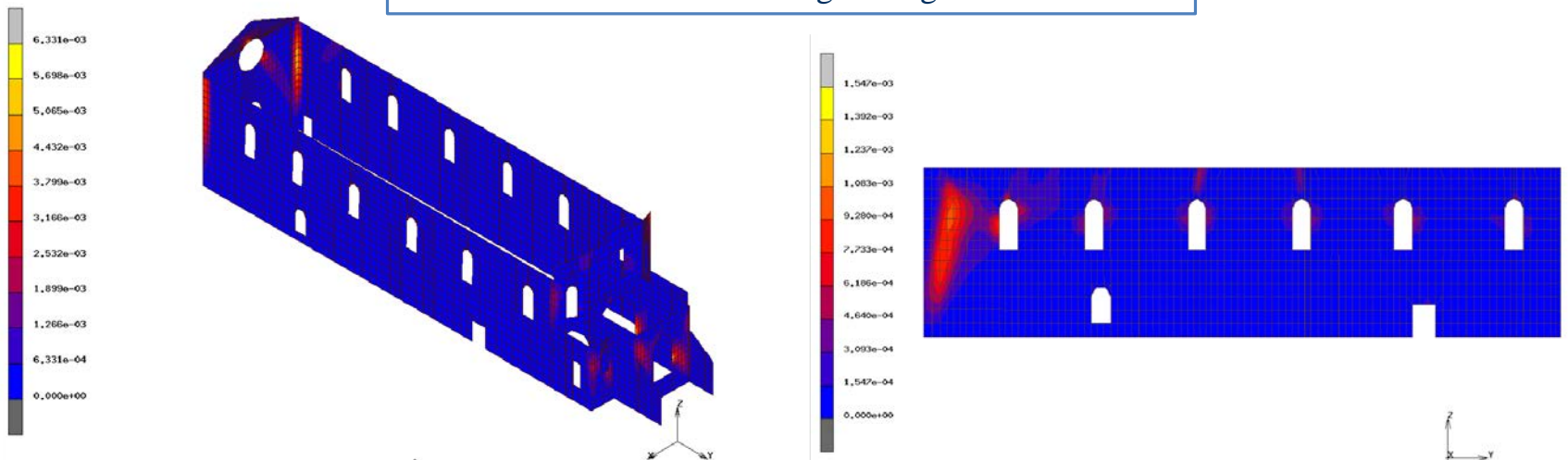


The San Francesco church, Lucca (Italy)

Fracture strain without strengthening intervention



Fracture strain with strengthening intervention

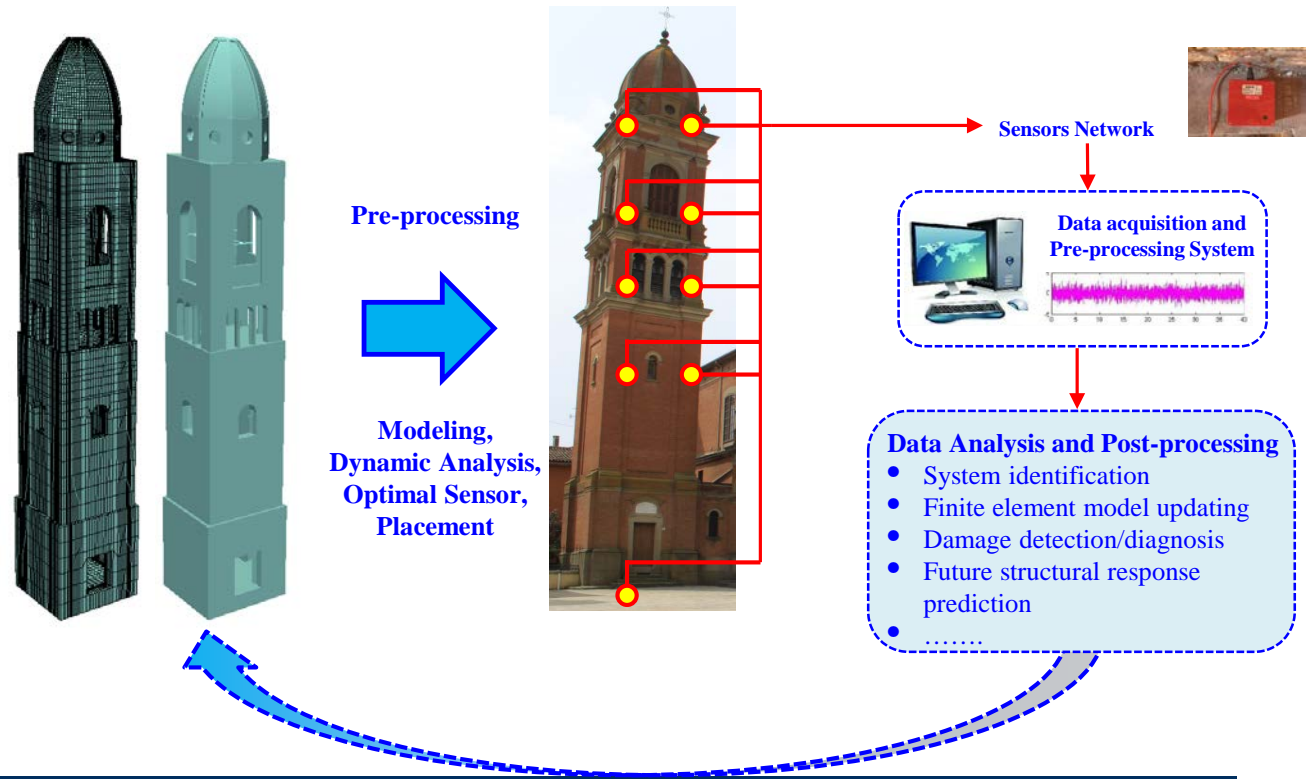


The figures highlight the benefits of the reinforcements, which reduced the values of the fracture strain and the extension of the cracked regions

Ambient vibration test and Finite Element Model Updating

In order to assess the structural behaviour of historical monuments, **numerical model can be integrated by experimental tests.**

In the ambient vibration tests the vibrations induced by natural and anthropic sources (earthquakes, wind, traffic, crowd movements, etc.) are recorded via a sensor network (accelerometers, velocimeters) installed on the historical building and processed to obtain the dynamic properties of the structure, such as **frequencies and mode shapes.**



Ambient vibration test and Finite Element Model Updating

The goal of model updating is to determine the unknown parameters of a structure (material properties, boundary conditions, etc.) by matching numerical and experimental frequencies and mode shapes of a structure (**Inverse problem**)

- Measurement of q experimental frequencies \hat{f}_i and mode shapes \hat{v}_i
- Construction of a **Finite Element** model
- Unknown parameters vector \mathbf{x}

$$\mathbf{K} = \mathbf{K}(\mathbf{x}), \quad \mathbf{M} = \mathbf{M}(\mathbf{x}) \qquad \mathbf{x} \in \mathbf{R}^l, \quad \Omega = [a_1, b_1] \times [a_2, b_2] \times \dots \times [a_l, b_l]$$

$$\mathbf{K}(\mathbf{x})\mathbf{v}(\mathbf{x}) = \omega^2(\mathbf{x})\mathbf{M}(\mathbf{x})\mathbf{v}(\mathbf{x})$$

➔

Frequencies $f(\mathbf{x})$ and mode shapes $\mathbf{v}(\mathbf{x})$ of the **FE** model (modal analysis)

Minimize the objective function **(NLSP)**

$$\phi(\mathbf{x}) = \sum_{i=1}^q w_i^2 \left[\underbrace{\hat{f}_i - f_i(\mathbf{x})}_{\text{frequencies}} \right]^2$$

$$+ w_{q+i}^2 \left[\underbrace{1 - \gamma_i(\mathbf{x})}_{\substack{\text{mode shapes} \\ \gamma_i^2 = \text{MAC}}} \right]^2$$

➔

Optimal value of parameter \mathbf{x}

Ambient vibration test and Finite Element Model Updating

The numerical procedure implemented in NOSA-ITACA to minimize the objective function is based on

- **model reduction** (Lanczos based projection) to reduce FE size to a more manageable order
- **trust region scheme**

The algorithm is characterized by high efficiency (large FE models) and provides information on the reliability of the solution.

In many applications reported in the literature, the FE code is used as black box by a general-purpose optimizer (fmincon, GA, etc.)

The calibrated FE model can be used for further static and dynamic analyses

1. Girardi, M., Padovani, C., Pellegrini, D., Porcelli, M., & Robol, L. (2020). Finite element model updating for structural applications. *Journal of Computational and Applied Mathematics*, 370, 112675.
2. M. Girardi, C. Padovani, D. Pellegrini, L. Robol (2019). “Model Updating Procedure to Enhance Structural Analysis in FE Code NOSA-ITACA”. *Journal of Performance of Constructed Facilities (ASCE)*, 33(4).
3. Girardi, M., Padovani, C., Pellegrini, D., & Robol, L. (2021). A finite element model updating method based on global optimization. *Mechanical Systems and Signal Processing*, 152, 107372.

Ambient vibration test and Finite Element Model Updating

Case study: the Clock Tower, Lucca (Italy)



Bell chamber
Material 1

Stone blocks and bricks

Tower
Material 2

Stone blocks

Four experimental frequencies and mode shape have been calculated during a monitoring campaign in 2016

Finite element model:
11383 8-node brick elements
45511 degrees of freedom

Ambient vibration test and Finite Element Model Updating

Case study: the Clock Tower, Lucca (Italy)

$$\phi(\mathbf{x}) = \sum_{i=1}^q w_i^2 \left[\hat{f}_i - f_i(\mathbf{x}) \right]^2 + w_{q+i}^2 \left[1 - \gamma_i(\mathbf{x}) \right]^2$$

$$q = 4 \quad w_i = \hat{f}_i^{-1}, i = 1 \dots 4, \quad w_5 = w_6 = 0.1, w_7 = w_8 = 0.$$

$$\rho_1 = 1,700 \text{ kg/m}^3, \quad \rho_2 = 2,100 \text{ kg/m}^3 \quad \text{Poisson's ratio } 0.2$$

$$\mathbf{x}_1 = E_1$$

$$\mathbf{x}_2 = E_2$$

$$1 \text{ GPa} \leq E_1 \leq 6 \text{ GPa},$$

$$1 \text{ GPa} \leq E_2 \leq 6 \text{ GPa}$$

$$E_{1\text{opt}} = 1.9288 \text{ GPa.}$$

$$E_{2\text{opt}} = 3.0451 \text{ GPa.}$$

Young's modulus of full
bricks and lime mortar

Young's modulus of
stone masonry

Ambient vibration test and Finite Element Model Updating

Case study: the Clock Tower, Lucca (Italy)

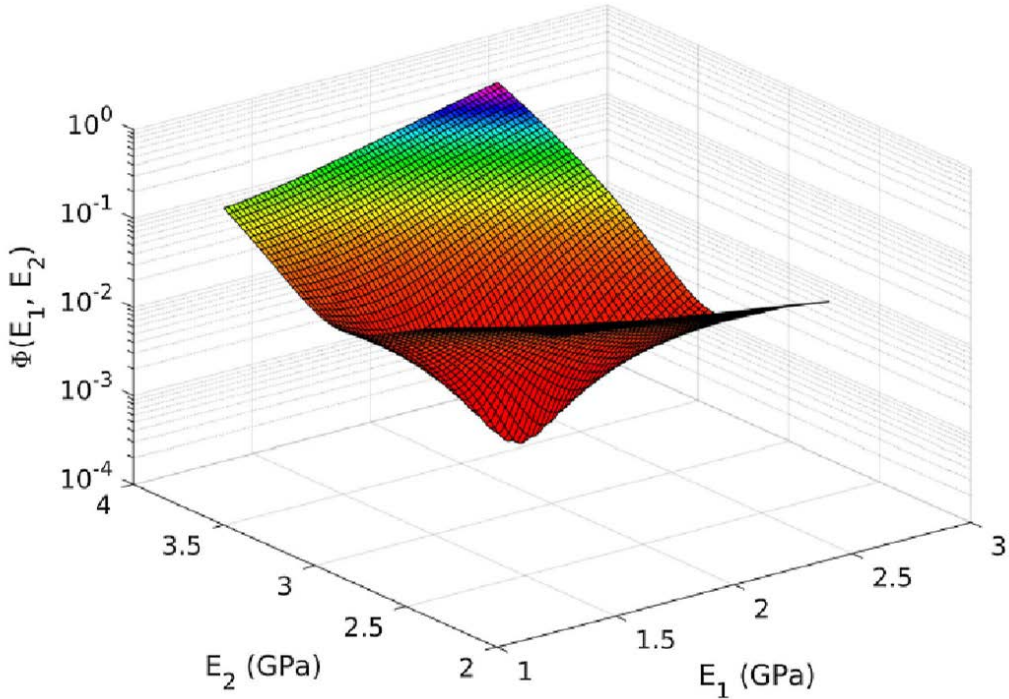
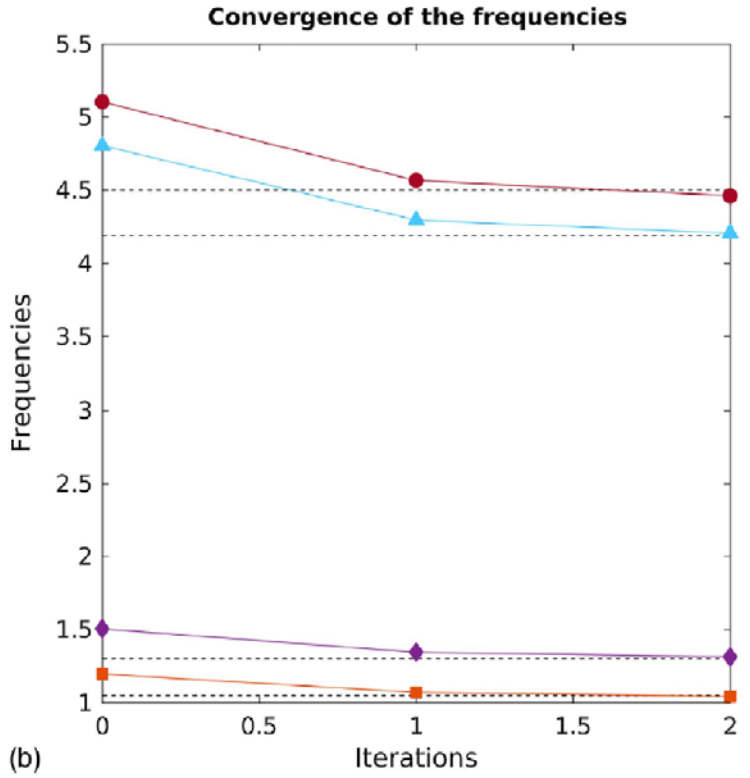


Fig. 13. Clock Tower, Mesh 2: objective function $\Phi(E_1, E_2)$ versus E_1 and E_2 .

Convergence of the model's frequencies to the experimental values (dashed line) during the process

Plot of objective function



Ambient vibration test and Finite Element Model Updating

Case study: the Clock Tower, Lucca (Italy)

Mode shape	Experimental frequencies (Hz)	Numerical frequencies (Hz)	Relative errors (%)	γ_i
1	1.05	1.0440	0.57	0.9862
2	1.3	1.3129	0.99	0.9782
3	4.19	4.1884	0.04	0.2731
4	4.50	4.4522	1.06	0.8950

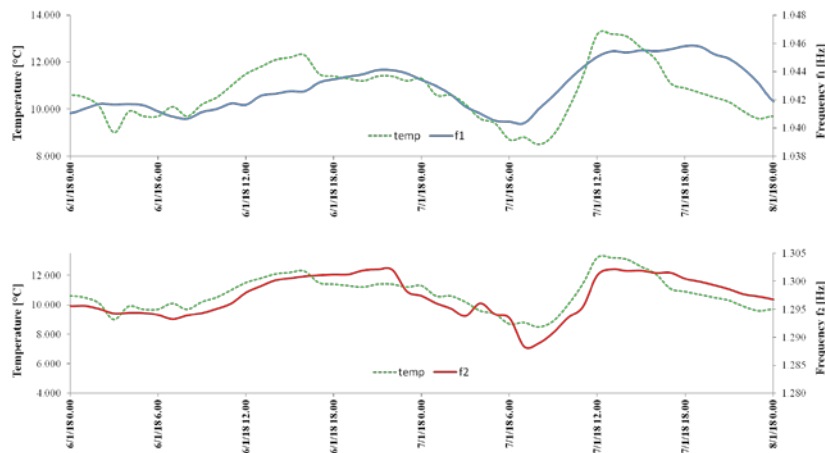
RM	GPO
Numerical frequencies (Hz)	
1.0440	1.0441
1.3129	1.3130
4.1884	4.2067
4.4522	4.4611
Optimal parameters (GPa)	
$E_1 = 1.9288$	$E_1 = 1.9309$
$E_2 = 3.0451$	$E_2 = 3.0444$
Computation times (s)	
27.15	175.41

RM=Procedure implemented
in NOSA-ITACA

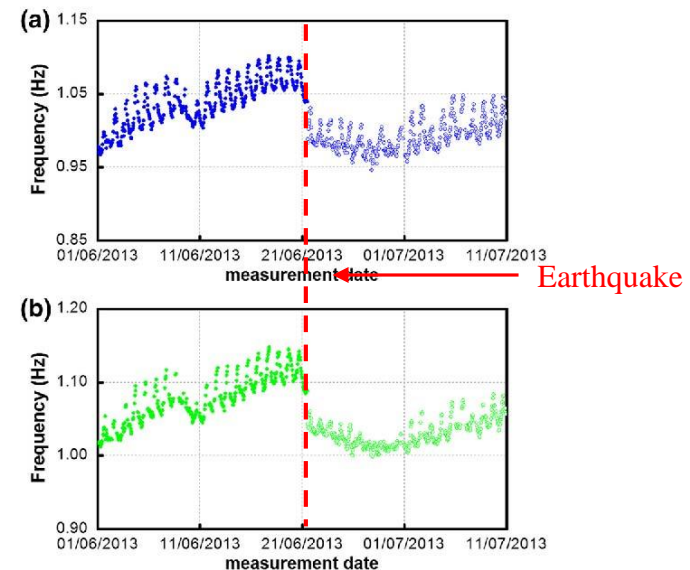
GPO= General Purpose Optimizer (using
NOSA-ITACA as a black box and the SPQ
solver of Matlab)

Linear perturbation analysis and structural health monitoring

- In recent years, continuous long term vibration monitoring turned out to be an effective non-destructive technique to investigate the dynamic behaviour and check the health status of historical buildings.
- Long-term monitoring campaigns have shown that changes in the dynamic properties of a building, such as frequencies and mode shapes, can represent effective damage indicators.
- Experimental frequencies are sensitive to environmental events and structural changes.



‘Clock Tower’ first two frequencies variation and temperature trend



Frequencies variation of the Gabbia Tower *

* Gentile, C., Guidobaldi, M., & Saisi, A. (2016). One-year dynamic monitoring of a historic tower: damage detection under changing environment. *Meccanica*, 51(11), 2873-2889.

Linear perturbation analysis and structural health monitoring

The goal of **linear perturbation** analysis is to model the influence of the nonlinear behaviour of masonry materials and the presence of cracks on the dynamical properties of a masonry structure

The method implemented in NOSA-ITACA consists of two steps:

Step 1. the prescribed loads and boundary conditions are applied to the FE model and the resulting nonlinear equilibrium problem is solved through an iterative scheme.

Step 2. a modal analysis **about the equilibrium solution** is performed, by using the tangent stiffness matrix \mathbf{K}_T calculated in the last iteration before convergence is reached.

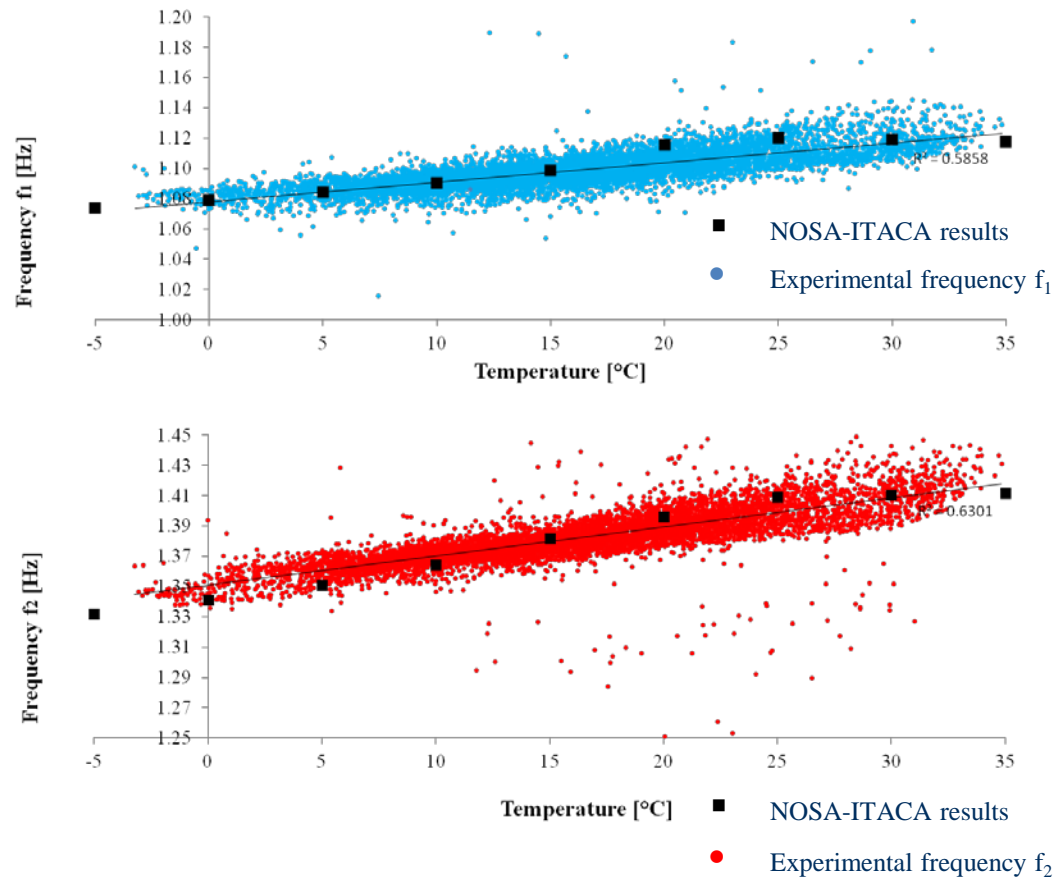
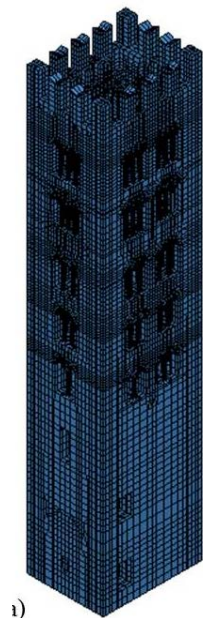
$$\mathbf{K}_T \mathbf{v} = \omega^2 \mathbf{M} \mathbf{v}$$

The procedure can provides for more realistic simulations of the dynamic behaviour of masonry structures in the presence of cracks and, when combined with experimental data, can be helpful in assessing damage. Moreover, the dependence of the natural frequencies on external loads (both mechanical and thermal) can be assessed and used to interpret data from long-term monitoring campaigns.

Girardi, M., Padovani, C., & Pellegrini, D. (2019). Modal analysis of masonry structures. *Mathematics and Mechanics of Solids*, 24(3), 616-636.

Linear perturbation analysis and structural health monitoring

Linear perturbation analysis conducted on the San Frediano bell tower (Lucca, Italy) considering the temperature variations experimented by the structure during the monitoring period (October 2015 – October 2016).



Azzara RM, Girardi M, Iafolla V, Padovani C and Pellegrini D (2020). Long-Term Dynamic Monitoring of Medieval Masonry Towers. *Front. Built Environ.* 6:9. doi: 10.3389/fbuil.2020.00009

Linear perturbation analysis and structural health monitoring

Mogadouro clock tower, Portugal, XVI century



Mode	$f_{i,exp}$ [Hz]
1	2.15
2	2.58
3	4.98
4	5.74

Experimental frequencies

Linear perturbation approach

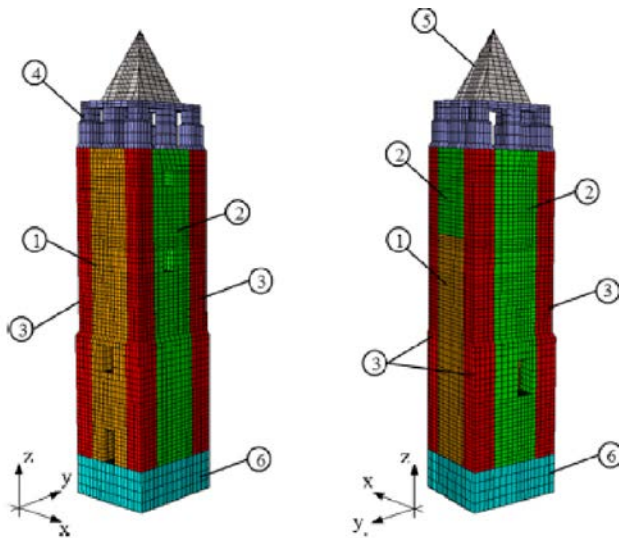
1. Solution to the equilibrium problem
2. Modal analysis



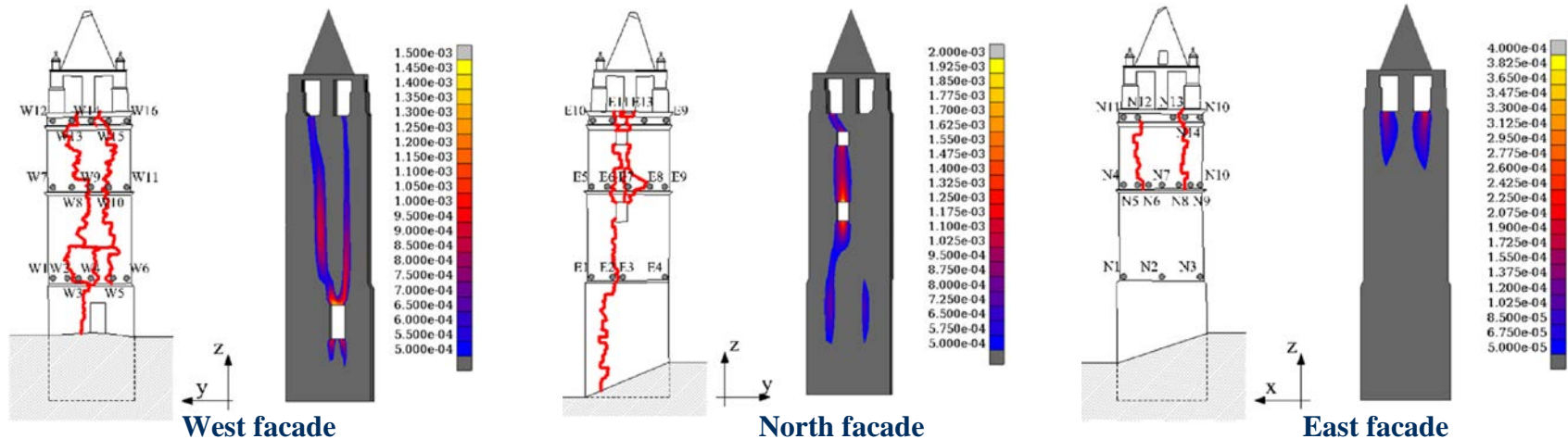
Model calibration

Optimal values of the material mechanical properties before rehabilitation.

Mat.n°	Tower's portion	ρ [kg/m ³]	E [GPa]	σ_t [kPa]
1 (orange)	façades South and North (bottom)	2200	2.500	15.0
2 (green)	façades East, West and North (top)	2200	2.500	0.0
3 (red)	corners	2400	3.500	15.0
4 (indigo)	pillars	2200	1.210	-
5 (grey)	roof	2000	0.195	-
6 (cyan)	foundation	2200	3.500	-



Linear perturbation analysis and structural health monitoring



Before rehabilitation

Mode	$f_{i,exp}$ [Hz]	$f_{i,N}$ [Hz]	Δ_f [%]
1	2.15	2.15	0.00
2	2.58	2.60	-0.78
3	4.98	4.92	1.20
4	5.74	5.88	-2.44

Same FE model and mechanical properties as before
Tensile strength = 10 kPa for the restored walls (material 2)

Mode	$f_{i,exp}$ [Hz]	$f_{i,N}$ [Hz]	Δ_f [%]
1	2.56	2.59	-1.17
2	2.76	2.75	0.36
3	7.15	8.39	-17.34
4	8.86	9.32	-5.19

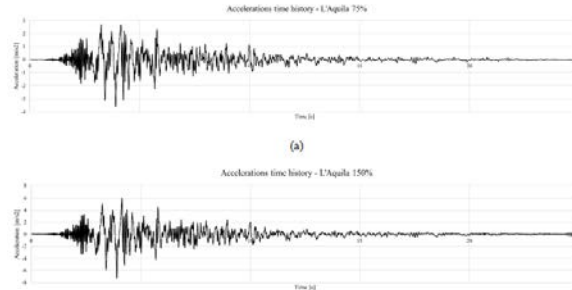
Equilibrium problem + modal analysis

After rehabilitation

Pellegrini, D., Girardi, M., Lourenço, P. B., Masciotta, M. G., Mendes, N., Padovani, C., & Ramos, L. F. (2018). Modal analysis of historical masonry structures: Linear perturbation and software benchmarking. *Construction and Building Materials*, 189, 1232-1250.

Non-linear dynamic analysis

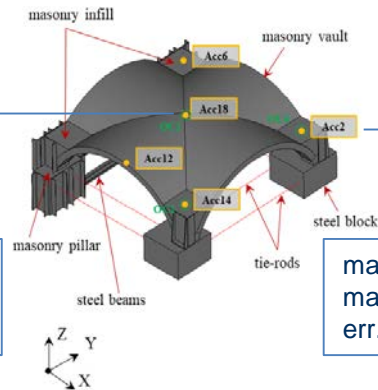
Blind and post diction within the framework of the SERA.ta project



- **Finite model updating calibration**
- **Zero tensile strength**
- **L'Aquila earthquake** as input for the nonlinear dynamic analysis

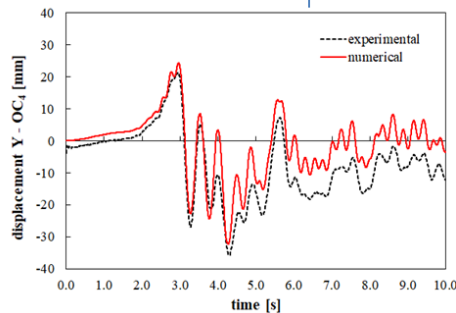
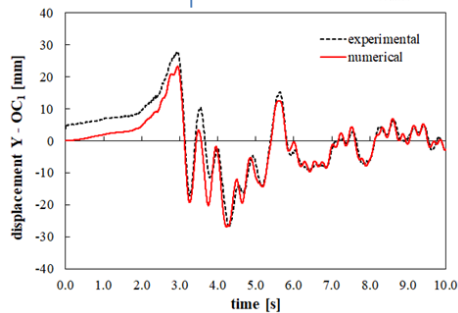
Mode	f_{exp} (Hz)	f_{LPA} (Hz)	Δf_{LPA} (%)	MAC_{LPA}
1	6.15	6.15	0.00	0.92
2	11.62	18.29	-57.40	0.18
3	19.39	20.02	-3.25	0.71

$E_{opt} = 0.8440$ GPa

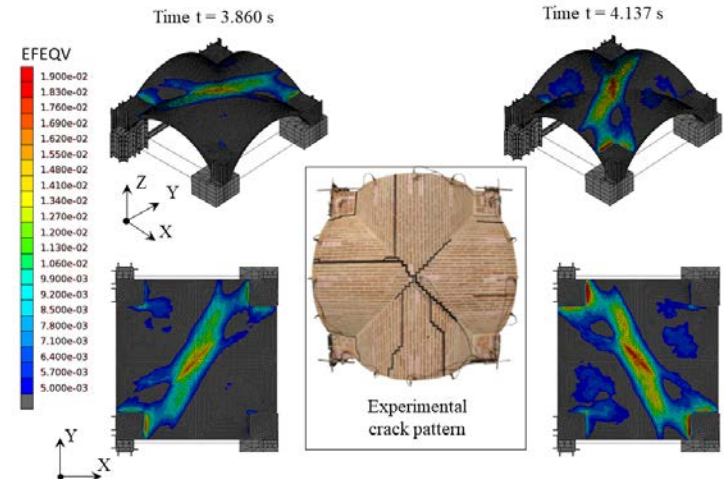


max |Exp| = 30.74mm
max |Num| = 27.0mm
err. Rel. = 12.33%

max |Exp| = 32.05mm
max |Num| = 32.40mm
err. Rel. = -1.09%



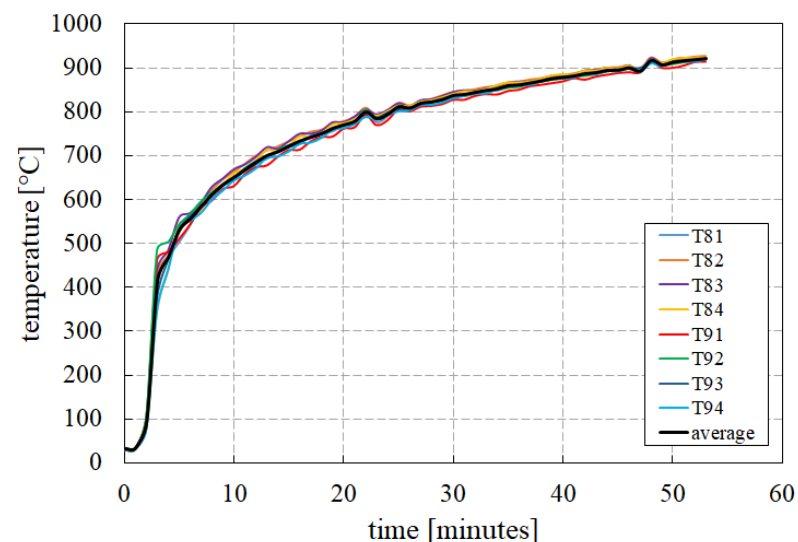
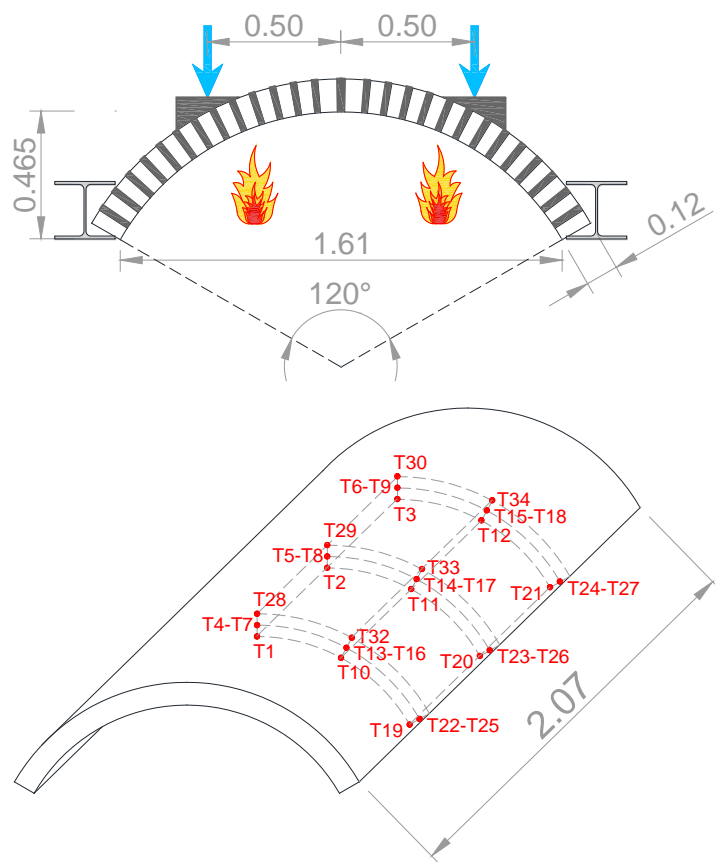
Crack pattern



Pellegrini, D. (2023). Pre- and Post-Diction Simulation of the Seismic Response of a Masonry Cross Vault Tested on a Shaking Table. *International Journal of Architectural Heritage*, 18(12), 1852–1872. <https://doi.org/10.1080/15583058.2023.2242812>

Masonry structures in fire conditions

The case study

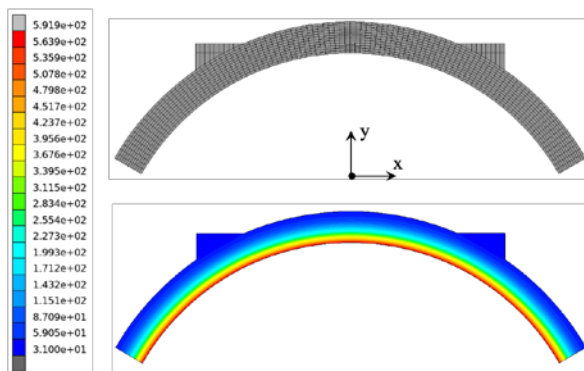


A barrel masonry vault tested in fire conditions by the Italian Fire Department.

The thermo-mechanical analysis, carried out by the NOSA-ITACA code, is performed assuming that the displacement gradient, thermal expansion and its derivative with respect to temperature, strain rate and temperature rate are small. The thermoelastic **equilibrium equations** governing the problem are **uncoupled and can be integrated separately.**

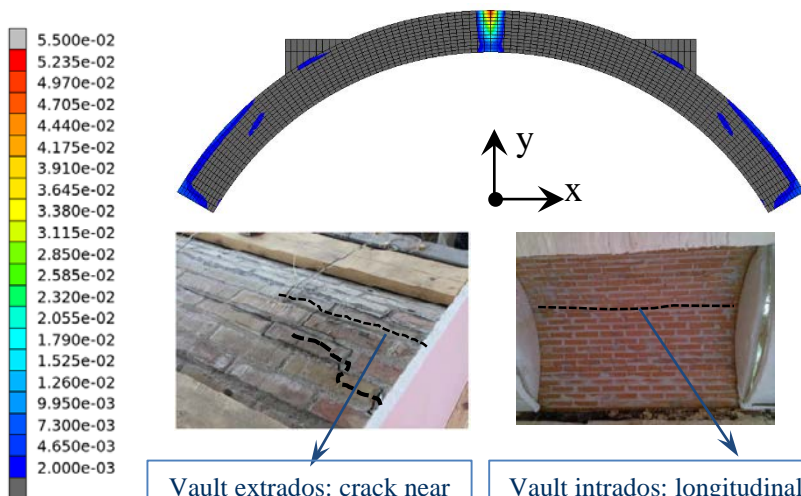
Daniele Pellegrini, Thermo-mechanical analyses of masonry structures in fire conditions, *Finite Elements in Analysis and Design*, Volume 234, 2024, 104128, ISSN 0168-874X, <https://doi.org/10.1016/j.finel.2024.104128>

Masonry structures in fire conditions



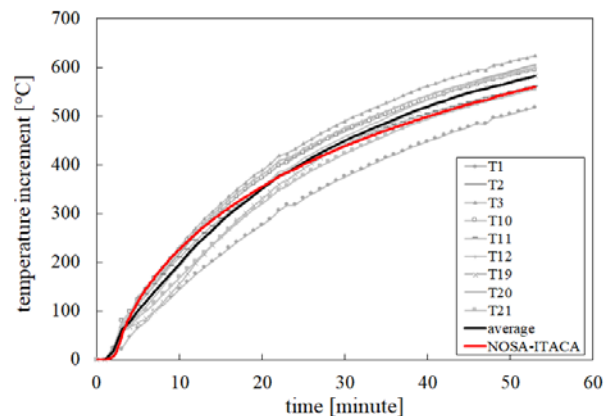
Numerical thermal field, time $t = 52$ minutes

Numerical crack pattern

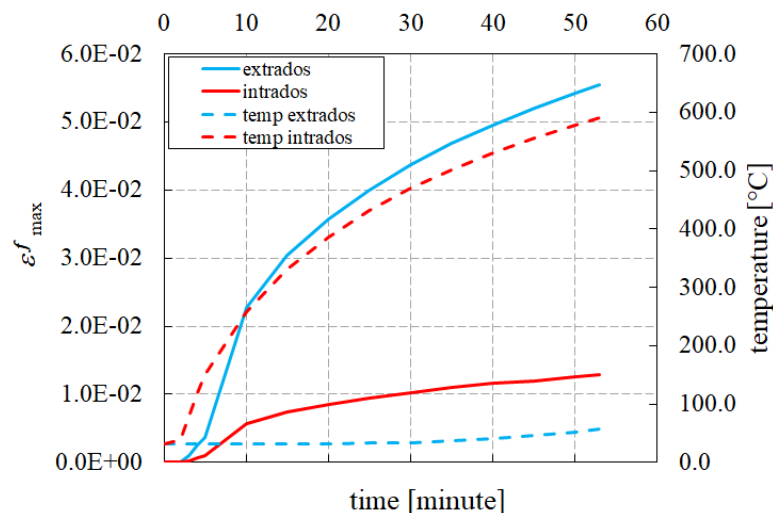


Vault extrados: crack near the keystone

Vault intrados: longitudinal crack near the midsection



Comparison between numerical and experimental temperature at intrados



Evolution of the cracking strain at extrados and intrados

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