An optimal control of start-up for nonlinear fire-tube boilers with thermal stress constraints

Stefano Spinelli[1,2], Marcello Farina[2] and Andrea Ballarino[1]

Abstract— **In this paper we propose a Nonlinear Model Predictive Control (NMPC) scheme for the optimization of the start-up procedure of a nonlinear boiler model. The proposed formulation of the MPC problem allows for a significant reduction of the optimization horizon with respect to state of the art - often open loop - optimization approaches (that commonly solve the nonlinear program for a long horizon that includes the whole start-up time), while guaranteeing the recursive feasibility and remarkable performances. A numerically efficient implementation of NMPC is obtained by subsequent linearisation of the system along the predicted trajectory. Simulation results show the advantages of the proposed method with respect to standard manual procedures and to open-loop optimization approaches.**

I. INTRODUCTION

Steam plays a central role in production in many sectors, among the others food, textile, chemical, medical, power, heating, and transport industries. In the past, steam production plants were used to provide base load power. Start-up operations were performed a few times a year, therefore they were seldom optimized and typically performed manually.

Nowadays, steam generation is often integrated with electricity production in cogenerative systems or in combined cycle plants. For a greater integration of such utility plants with the power grid, to exploit price volatility, and to respond to varying steam and electricity demand, in many cases, these production facilities must be operated in a flexible way. The demands imposed by the liberalized electricity market, as well as the intermittent usage and request of steam in other industrial applications, as e.g. in batch production, require start-up procedures to be possibly operated frequently, see e.g., [14]. During the start-up, the system experiences a large temperature and pressure transient, which can be very harmful to the system itself if not opportunely controlled: specifically, the thermal stress on boiler elements, e.g. tubes and shell, has to be limited, as it can reduce the lifetime of the system components.

In the industrial practice, start-up procedures are typically performed manually, while automatic regulation is activated only as the boiler reaches a nominal operating point. On the other hand, model-based automatic start-up optimization approaches are indeed considered strongly necessary to pursue to manifold objective of reducing the start-up time, of limiting the thermal stress, of minimizing the operative cost and the environmental footprint of the boiler, e.g., by limiting the fuel consumption and the emissions.

Many efforts have been recently dedicated to study the optimization of the start-up procedures, while keeping the stressed components under control, with a special focus on Combined Cycle Power Plants (CCPP). In particular, offline optimization approaches have been proposed to manage the entire procedure by defining the - open loop - optimal input trajectories to be implemented on the system: in the paper [1] the authors propose a model-based start-up optimization for a coal-fired power plant; in [2], the start-up optimization on Heat Recovery Steam Generators (HRSG) is discussed, while the papers [3] and [4] focus on the turbine side of the plant. In [5] and [6], the authors solve a Nonlinear Programming (NLP) problem for the open-loop optimization for a CCPP drum boiler, considering the thermal stress model. Similarly, in [7], a Modelica model of the drum boiler is also presented for the start-up optimization. An off-line optimization of the firing curves is also described in [8] and [9], considering an extremely detailed model of the thermal stresses for critical components. As mentioned, all the previously-discussed research works propose open loop optimization approaches. This is primarily due to the fact that the computation of the reference trajectory for the entire procedure for the nonlinear system can be time-consuming and too computationally expensive to be implemented online. On the other hand, closed-loop optimization can allow operating start-up procedures more safely and reliably, and to control the operation in the presence of disturbances or compensate for modelling errors. The closed-loop approach is indeed explored in [10], where the authors propose a Nonlinear Model Predictive Control (NMPC) scheme selecting a prediction horizon that includes the whole start-up. In view of this, although this approach is very promising, computational complexity is still an issue. The choice of the MPC prediction horizon, as a time interval that includes the whole start-up procedure, is mainly due to a twofold reason. Firstly, it allows obtaining globally optimal performances; secondly, methodologically sound implementations of MPC require to define the prediction horizon in such a way that, in the end, the state lies in a suitably-defined invariant set around the steady-state point, and so, in practice, commonly very close to a stationary one.

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In this paper, we investigate the application of nonlinear MPC for tracking, inspired by the methodology presented in [11], for control of the start-up procedure. The advantage of this approach is the possibility of selecting an N-steps optimization horizon possibly significantly shorter than the time required to complete the whole start-up procedure. This is due to the fact an additional optimization variable can be included in the MPC problem, representing the "temporary" target steady state, reachable in N discrete-time steps, as the closest one with respect to the desired final state. Such a new steady state is used as a terminal constraint and as a reference value in the cost function. This allows to greatly reduce the optimization horizon - and the corresponding numerical complexity - and, as a byproduct, to avoid the (often complex) computation of a terminal positively invariant set and of a suitable terminal cost and to guarantee recursive feasibility of the MPC optimization problem. Also, in this way, the actual - minimal - duration of the start-up phase is not required to be known a priori.
The proposed approach, clearly, compromises the global optimality of the solution. However, the use of a suitable additional cost penalizing the displacement of the temporary final target state with respect to the final nominal operation point can partially mitigate for such suboptimality, in view of the dynamic programming paradigm. In this paper, we will evaluate numerically this performance loss in the selected case study.
A numerically efficient implementation of NMPC is obtained by resorting to a parameter-varying linearisation of the nonlinear system along the state/input trajectory computed at the previous optimization instant, reducing the optimization program to a Constrained Quadratic one. This approach, referred here to as Linear Parameter-Varying Model Predictive Control (LPV-MPC), is similar to the one proposed in [12]. The main difference with [12] consists of how the trajectory around which the system is linearised is computed. As it will be discussed more in details later in the paper, a strong connection with the Real-Time iteration scheme proposed in [13] also exists.

This work can be regarded as a follow-up of the paper [14]: in the previous work, indeed, a hierarchical control scheme has been proposed to manage and control a cogeneration system, possibly imposing fitful activations of a Fire Tube Boiler (FTB) during the daily horizon, for the unit commitment problem. However, in [14] we discarded the control of the FTB during the start-up phase, which is our goal here. As in [14], a combined heat and power (CHP) Internal Combustion Engines (ICE) is operated for the electrical generation and, in the studied configuration, it can provide an additional source of heat to the boiler.

The paper is structured as follows: Section II describes in detail the model of the FTB system and the thermal stress model, Section III presents the optimal predictive control method, while in Section IV simulation results are reported. Finally, in Section V some conclusions are drawn.

II. MODELLING

A. The Boiler dynamic model

Fire-tube boilers, also known as shell-tube boilers, are essentially composed of a vessel, filled up with water, where hot combustion gasses run in several submerged tubes.
The mathematical model of the fire-tube boiler is inspired by [15] and [16]. Differently from [15], two sources of heat are here considered, i.e., the gas burner and the exhaust gases diverted from the CHP. The set of equations describing the system dynamics is

$$\begin{aligned}\frac{d}{dt}[\rho_{\mathrm{s}}V_{\mathrm{s}}+\rho_{\mathrm{w}}V_{\mathrm{w}}] &= q_{\mathrm{f}}-q_{\mathrm{s}}\\ \frac{d}{dt}[\rho_{\mathrm{s}}e_{\mathrm{s}}V_{\mathrm{s}}+\rho_{\mathrm{w}}e_{\mathrm{w}}V_{\mathrm{w}}] &= Q^{\mathrm{t,GAS}}_{\mathrm{m}\to\mathrm{w}}+Q^{\mathrm{t,CHP}}_{\mathrm{m}\to\mathrm{w}}+q_{\mathrm{f}}h_{\mathrm{f}}-q_{\mathrm{s}}h_{\mathrm{s}}\\ \frac{d}{dt}[m^{\mathrm{t,GAS}}c_pT^{\mathrm{t,GAS}}] &= Q^{\mathrm{B}}_{\mathrm{GAS}}-Q^{\mathrm{t,GAS}}_{\mathrm{m}\to\mathrm{w}}\end{aligned} \quad (1)$$

where ρ, V, q, T, h, and e denote the density, volume, mass flow rate, temperature, and specific enthalpy and energy, respectively of steam (s), water (w), and feedwater (f). $m^{\mathrm{t,GAS}}$ denotes the mass of the tubes heated by the gas burner, while $T^{\mathrm{t,GAS}}$ denotes their temperature. The term $Q^{\mathrm{t,CHP}}$ represents the heat flux provided by the gasses diverted into the boiler from the CHP and is considered as a known disturbance during the start-up. Since the system works at saturated conditions, the water and steam pressures satisfy the equality $p_{\mathrm{w}}=p_{\mathrm{s}}=p(T_{\mathrm{w}})$. Also,

$$Q^{\mathrm{B}}_{\mathrm{GAS}}=\eta C_{\mathrm{LHV}}q^{\mathrm{B}}_{\mathrm{GAS}} \quad (2)$$

where C_{LHV} is the lower heating value and $q^{\mathrm{B}}_{\mathrm{GAS}}$ is the combustion gas flow rate. The heat transmitted from the metal walls to the water can be modelled as

$$Q^{\mathrm{t,GAS}}_{\mathrm{m}\to\mathrm{w}}=\beta(T^{\mathrm{t,GAS}}-T_{\mathrm{w}}) \quad (3)$$

where $\beta(T_{\mathrm{w}})$ is the heat transfer coefficient depending on the boiling two phase mixture of steam and water, close to the tube walls, that induces a natural recirculation and the interaction of numerous tubes in the bundle, based on Cooper correlation [17].
The mass equation in (1) can be further manipulated, considering in the FTB two separated regions for water and steam, by splitting the equation for the two regions:

$$\begin{aligned}\frac{d}{dt}[\rho_{\mathrm{s}}V_{\mathrm{s}}] &= q^{w\to s}_{\mathrm{s}}-q_{\mathrm{s}}\\ \frac{d}{dt}[\rho_{\mathrm{w}}V_{\mathrm{w}}] &= q_{\mathrm{f}}-q^{w\to s}_{\mathrm{s}}\end{aligned} \quad (4)$$

with $q^{w\to s}_{\mathrm{s}}$ representing the steam released from the water sector into the steam region.
We simplify the system of equations by neglecting steam accumulation in the steam zone $q^{w\to s}_{\mathrm{s}}=q_{\mathrm{s}}$ and by writing the water mass balance as:

$$\frac{d}{dt}[\rho_{\mathrm{w}}V_{\mathrm{w}}]=\rho_{\mathrm{w}}\frac{d}{dt}V_{\mathrm{w}}+V_{\mathrm{w}}\frac{d}{dt}\rho_{\mathrm{w}}\simeq-\rho_{\mathrm{w}}\frac{dV_{\mathrm{s}}}{dl_{\mathrm{w}}}\frac{dl_{\mathrm{w}}}{dt}$$

The above approximation can be done assuming the second term negligible and recalling that $V_{\mathrm{w}}+V_{\mathrm{s}}=V_{\mathrm{Tot}}$ is constant.

It is possible to express the volume of the steam zone V_s as a function of the water level, l_w, by considering the FTB geometry.

An analogous separation between water and steam zones can be considered also for the energy balance in eq. (1). It is assumed a saturated liquid and a thermal equilibrium between the two zones, $T_w = T_s$, therefore it is not necessary to consider the energy equation for the steam region, but just for the water region.

The nonlinear dynamic model of the FTB boiler can be recast in the following form:

$$\begin{aligned} \frac{dT^{t,GAS}}{dt} &= \frac{1}{m^{t,GAS}c_p}[Q^B_{GAS} - \beta(T^{t,GAS} - T_w)] \\ \frac{dl_w}{dt} &= \frac{1}{2RL}[q_f - q_s] \\ \frac{dT_w}{dt} &= \frac{1}{\rho_w V_w(l_w)c_v}[Q^{t,GAS}_{m\to w} + Q^{t,CHP}_{ex} + \\ &\quad + q_f c_p(T_f - T_w) - q_s\lambda_s] \end{aligned} \tag{5}$$

The reader is referred to [15] for the details.

The state vector $x = [T^{t,GAS}, l_w, T_w]'$ includes the tube temperature, the water level, and the water temperature, while the manipulable input vector $u = [q_f, q^B_{GAS}, q_s]'$ includes the feedwater flowrate, the combustible gas flowrate, and the steam output flowrate. The exhaust gas heat flux $Q^{t,CHP}_{ex}$ is regarded as an exogenous disturbance variable d, e.g. imposed by a higher control layer, as described in [14].

The parameters of the model are analytically computed based on the physical and geometric properties of the system. A fine tuning has been conducted based on available data.

The general continuous-time nonlinear boiler model is represented by the dynamical system:

$$\dot{x} = f(x, u, d) \tag{6}$$

For later use, the model (6) is discretized using a forth-order Runge-Kutta method, i.e.,

$$x(k+1) = f_{RK4}(x(k), u(k), d(k)) \tag{7}$$

where k represents the discrete time step.

The input and state variables are subject to the following constraints: $q_f \in [0, q_{f,max}]$, $q^B_{GAS} \in [q^B_{min}, q^B_{max}]$, $q_s \in [0, q_{s,max}]$, $T_w \in [T_{w,min}, T_{w,max}]$ and $l_w \in [l_{min}, l_{max}]$.

These are written in compact form (considering discrete-time state and input variables $x(k)$ and $u(k)$, respectively) as $h(x(k), u(k)) \leq 0$.

Note that function h is affine.

B. Thermal stress model and constraint

In boiler start-up, one of the main limitations to the maximum firing of the boiler is related to the thermal stress of the shell and the internal tubes. A high thermal stress, due to a too steep increment of the temperatures, leads to a reduction of components' life-cycle, increasing the costs for inspections and maintenance. The thermal stress σ is modelled as follows.

$$\sigma = k_t(T^c)\frac{dT^c}{dt} \tag{8}$$

where T^c is the temperature of the metal component and k_t is a property of the material that can be either constant or temperature dependent. Specifically, for the pressurized components, as the boiler shell, the maximum temperature rate r_T is computed following the European standard EN 12952-3 [18]:

$$\left|\beta_P(P - P_0)\frac{d_{in} + s}{2s} + \beta_T\frac{E\alpha}{1-\nu}\frac{c_p\rho}{k}s^2\phi_f r_T\right| < \sigma^{max} \tag{9}$$

where P and P_0 are the nominal and initial pressure, d_{in} is the internal diameter, s the wall thickness, the mechanical and thermal material properties -E, ν, α, ρ, k and c_p- are, respectively, Young's modulus, Poisson's ratio, thermal expansion coefficient, density, thermal conductivity and specific heat - and ϕ_f is the cylindrical shape factor, which is a function of the ratio of internal and external diameters, see [8] and reference therein for details.

The coefficient k_t, which can be recovered by inspection from eq. (9), is proportional to the Young's modulus and the thermal expansion coefficient, that are in general temperature dependent. However for the temperature range considered in this specific application, $E = 1.82e5$MPa and $\alpha = 1.35\text{e}{-5}\text{m}^2/\text{s}$ and k_t can be considered constant.

The thermal stress limit can be therefore formulated as a constraint on the rate of change of the component temperature, with r^{max}_T computed from eq. (9) by imposing the maximum allowable stress.

The rate limitation is applied to the temperatures of the submerged tubes and the shell wall. While the tube temperature corresponds to the state $T^{t,GAS}$, the shell temperature is assumed to be in equilibrium with the water temperature and to be equal to T_w. Therefore, the thermal constraint reads as:

$$x_j(k+1) - x_j(k) \leq r^{max}_{T,j} \tag{10}$$

where $j = \{1, 3\}$ is the index of the state vector corresponding to the involved temperatures.

III. Optimal start-up procedure

A. The nonlinear MPC problem

The NMPC algorithm used in this paper is inspired by [11] and is formulated as a tracking problem towards the target operating point x_T and where additional objectives are included, in particular the minimization of the operating cost.

$$\begin{aligned} \min_{\mathbf{u},\bar{x}_k} & \left\{ \sum_{i=0}^{N_p-1} \left\|x(i) - \bar{x}_k\right\|^2_Q + \left\|\Delta u(i)\right\|^2_R \right\} + \\ & + \|\bar{x}_k - x_T\|^2_{Q_T} \\ s.t. \quad & x(i+1) = f_{RK4}(x(i), u(i), d(i)) \\ & h(x(i), u(i)) \leq 0 \qquad i = 0, ..., N_p - 1 \\ & x_j(i+1) - x_j(i) \leq r^{max}_{T,j} \quad j \in \{1, 3\} \\ & x(0) = x_k \\ & x(N_P) = \bar{x}_k \\ & h(\bar{x}_k, \bar{u}_k) \leq 0 \end{aligned} \tag{11}$$

In the optimization problem (11), N_p is the optimization/prediction horizon $\Delta u(i) = u(i) - u(i-1)$, and x_k is the measure of the state of the system at time k. The supplementary decision variable $\bar{x}_k$ has a twofold role: it is indeed both the set point for the state vector in the cost function and the terminal state condition. As such, it must be reachable in N_p steps from the initial condition $x(0) = x_k$. Also, to guarantee recursive feasibility of the optimization problem, it is defined as an admissible steady state condition: letting $\bar{u}_k$ be the corresponding input such that $\bar{x}_k = f_{\text{RK4}}(\bar{x}_k, \bar{u}_k, \bar{d}_k)$ (where $\bar{d}_k$ is the forecasted disturbance value), the constraint $h(\bar{x}_k, \bar{u}_k) \leq 0$ is enforced to guarantee that, from time step $k+1$ on, an admissible solution to the NMPC problem exists. To enforce the asymptotic convergence of $\bar{x}_k$ to the target state x_{T}, in line with [11], the term $\|\bar{x}_k - x_{\text{T}}\|^2_{Q_T}$ is included in the cost function to minimize, at each time step, the distance of the temporary target $\bar{x}_k$ from the final one. This additional cost plays the role of the cost-to-go under a dynamic programming viewpoint.

Note that, also, this strategy allows to converge to an optimal solution also in case the target state is non-admissible, pushing the system automatically to the best (closest) feasible point with respect to the target.

B. Linear Parameter-Varying implementation

In this paper, we propose to use a numerical solution based on the reformulation of model (7) with its Linear Parameter-Varying (LPV) approximation, obtained performing a sequential linearisation. The reference input-output trajectory $(x^{\text{r}}(i), u^{\text{r}}(i))$ used to linearise the model (7) for the implementation of the MPC algorithm at time instant k is obtained as follows:

- For all $i = 0, \ldots, N_P - 2$, $u^{\text{r}}(i) = u(i+1|k-1)$, being $u(0|k-1), \ldots, u(N_P-1|k-1)$ the optimal input trajectory obtained as a result to the MPC optimization problem at time instant $k-1$.
- $u^{\text{r}}(N_P - 1) = \bar{u}_{k-1}$, i.e., the steady-state admissible input such that $\bar{x}_{k-1} = f_{\text{RK4}}(\bar{x}_{k-1}, \bar{u}_{k-1}, \bar{d}_{k-1})$.
- Set $x^{\text{r}}(0) = x_k$.
- Recursively compute, for all $i = 0, \ldots, N_P - 1$, $x^{\text{r}}(i+1) = f_{\text{RK4}}(x^{\text{r}}(i), u^{\text{r}}(i), d(i))$, being $d(i)$ the forecasted value of the disturbance - which is known in advance.

Then, by defining

$$A_i = \left.\frac{\partial f_{\text{RK4}}}{\partial x}\right|_{x^{\text{r}}(i), u^{\text{r}}(i)} \qquad B_i = \left.\frac{\partial f_{\text{RK4}}}{\partial u}\right|_{x^{\text{r}}(i), u^{\text{r}}(i)} \tag{12}$$

$$\zeta_i = f_{\text{RK4}}(x^{\text{r}}(i), u^{\text{r}}(i), d(i)) - A_k x^{\text{r}}(i) - B_k u^{\text{r}}(i) \tag{13}$$

the required time-varying linearised model

$$\xi(i+1) = A_i \xi(i) + B_i u(i) + \zeta_i \tag{14}$$

is obtained, to be used in place of the nonlinear one in the optimization problem (11). The approach is sketched in Figure 1.

Future work will be devoted to the address the approximation error between (14) and (7). As this may theoretically compromise the admissibility and the recursive feasibility properties of the solution, the activity will characterize the modelling approximation error and propose a rigorous robust implementation based, e.g., on [19]. Also, it is worth pointing out that the LPV-MPC implementation is basically equivalent to the first iteration of a sequencial quadratic programming (SQP) scheme for NMPC: this approach, in the control-oriented optimization context, is also known as Real-Time Iteration (RTI), see [13] and [20]. Future work will be also devoted to establish a sound connection with the RTI approach.

IV. Simulation results

In this section we validate the proposed approach through simulation, using the nonlinear model of the GU presented in Section II.

The FTB is a three-pass 10 bar steam generator with maximum steam flowrate $\bar{q}_s = 12000$ kg/h and the CHP is a 12 valve natural gas ICE producing up to $\bar{P}^{\text{CHP}}_{\text{El}} = 1200$ kW.

The boiler start-up must be optimized, minimizing contextually the time to reach the target operating point and the fuel consumption, satisfying the permitted ranges summarized in Table I, where the variables are adimensionalized with nominal values, identified with the circumflex diacritic.

In particular, the thermal stress is constrained in the prescribed range.

The continuous-time model (5) is discretized with a Runke-Kutta method of the fourth order, with a sampling time of $T_s = 6s$.

In the discretized approach, the thermal stress constraint, given by the model (8) and described in II-B, is recast as a limit on the temperature rate of change, scaled on the discrete grid, $T^c(k+1) - T^c(k) \leq \hat{\sigma}_{max}$, where the superscript c is the component of interest.

The start-up optimization is obtained by solving the QP problem (11), with a prediction horizon $N_p = 50$ and the following weighting matrices: $Q = \text{diag}(0.1, 5, 20) \circ w_x$, $R = \text{diag}(0.01, 0.01, 0) \circ w_u$ and $Q_T = \text{diag}(0.1, 5, 30) \circ w_x$,

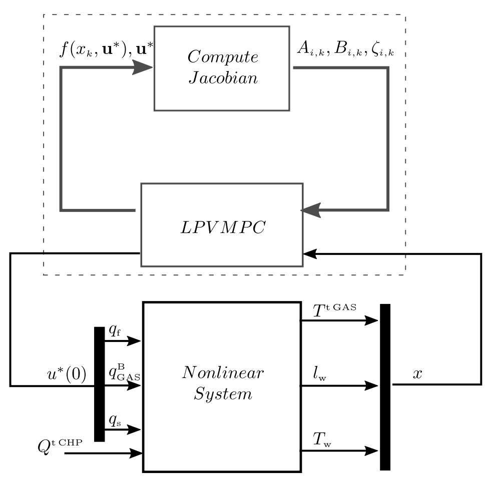

Fig. 1. Linear Parameter-Varying MPC scheme of the FTB system

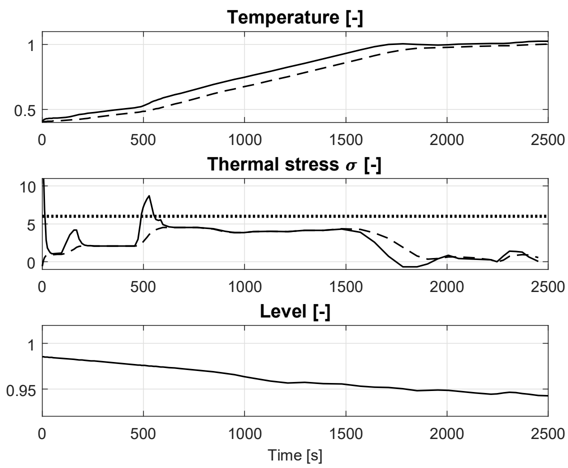

Fig. 2. FTB start-up manual procedure - output variables. Top panel: temperatures of fire tubes (solid line) and of the water (dashed line). Middle panel: thermal stress on tubes (solid line), on the shell (dashed line) and upper bound (dotted line). Bottom panel: water level. All graphs are adimensionalized with nominal values, for confidentiality.

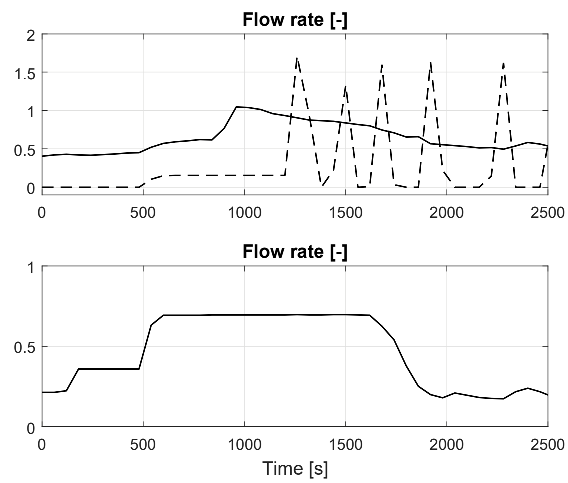

Fig. 3. FTB start-up manual procedure - input variables. Top panel: steam flow-rate (solid line), feedwater flow-rate (dashed line). Bottom panel: gas flow-rate. All graphs are adimensionalized with nominal values, for confidentiality.

where $\circ$ denotes the Hadamard product, while w_x, w_u contain the inverse of the square of the maximum values of state and input components.

As explained in Section III, the LPV-MPC is implemented in a receding horizon fashion, where only the first input $u(0|k)$ of the optimal control sequence is held constant for the sampling period T_s and applied to the system at each control instant. For simulation purposes, the nonlinear model (5) is used.

The typical manual procedure is first described for completeness, as extracted by an historical dataset: the output variables are shown in Figure 2, with the thermal stress shown in addition in the central panel, while the given inputs are displayed in Figure 3. The standard manual procedure imposes three gradual steps on the gas input, in order to reduce the thermal stress on the components. This conservative approach conduces to relatively long start-up phases and does not ensure the fulfillment of the thermal stress constraints, which are slightly violated, as it can be seen in the middle panel of Figure 2. At the same time, the water level loop is loosely controlled, as the level is allowed to drift away from the nominal level: this is moderated by several discontinuous inputs of feed-water inflows.

The optimal solution based on LPV-MPC control overcomes the limitations of the manual procedure, by addressing directly the process constraints. The LPV-MPC solution is presented in Figures 4 and 5, respectively for the output trajectories and optimal inputs. With respect to manual operation, the approach provides better performances in terms of start-up duration without incurring in thermal stress constraint violation. The overall time required to reach the nominal operating condition is reduced by more than 30% with respect to the conservative manual procedure. This time reduction is attained by driving quickly the natural gas input closer to the maximum value, while guaranteeing the respect of the constraints. As typical of MPC approaches, the improved performance is obtained by pushing the system closer to the prescribed operating limits, forcing their compliance throughout the time. Moreover, in the optimal start-up both the water temperature and level are controlled towards their nominal values.

Also, in Figures 4 and 5, the LPV-MPC is compared with the open-loop nonlinear optimization of the overall procedure, showing the solution optimality of the proposed method. The LPV-MPC approach not only reduces the NLP to a Quadratic Program by the linearisation along the predicted trajectory, but it is solved in receding horizon on an optimization window much smaller than the whole start-up duration, e.g. which would require a prediction horizon of at least 172 steps. By letting the terminal state of the LTV-MPC iteration an optimization variable, as described in III, the proposed formulation can drive the system to an optimal solution very close to the nonlinear overall optimization. A small difference is present in the management of the water level loop in the LPV-MPC and in the full nonlinear optimization.

V. Conclusions

In this paper, we have proposed a Nonlinear Model Predictive Control approach for the optimization of the start-up procedure of a nonlinear boiler model.

TABLE I

LOWER AND UPPER BOUNDS ON THE FTB VARIABLES

Variable	Minimum	Maximum
$q^{\mathrm{B}}_{\mathrm{GAS}}$	$0.125\ \hat{q}^{\mathrm{B}}_{\mathrm{GAS}}$	$\hat{q}^{\mathrm{B}}_{\mathrm{GAS}}$
q_{f}	0	$1.667\ \hat{q}_{\mathrm{s}}$
q_{s}	0	$2.223\ \hat{q}_{\mathrm{s}}$
T_{w}	$0.25\ \hat{T}_{\mathrm{w}}$	$1.05\ \hat{T}_{\mathrm{w}}$
l_{w}	$\hat{l}_{\mathrm{w}} - 0.5\%$	$\hat{l}_{\mathrm{w}} + 0.5\%$
σ_j	0	6

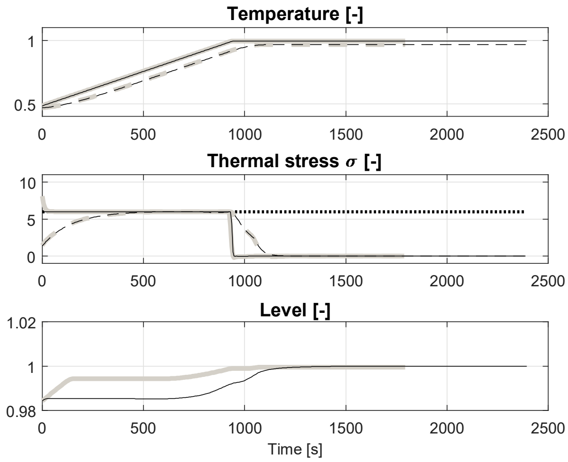

Fig. 4. Optimal FTB start-up - output variables. The graphs show the comparison of the LTV-MPC approach (thin black lines) and the nonlinear open-loop optimization (thick grey lines). Top panel: temperature of fire tubes (solid line) and of the water (dashed line). Middle panel: thermal stress on tubes (solid line), on the shell (dashed line) and upper bound (dotted line). Bottom panel: water level. All graphs are adimensionalized with nominal values, for confidentiality.

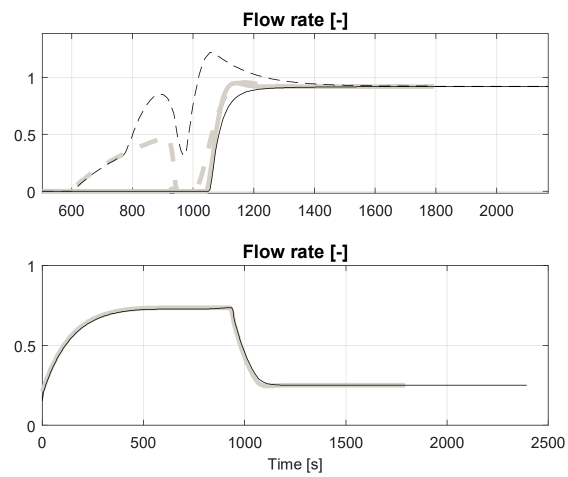

Fig. 5. Optimal FTB start-up - input variables. The graphs show the comparison of the LTV-MPC approach (thin black lines) and the nonlinear open-loop optimization (thick grey lines). Top panel: steam flow-rate (solid line), feedwater flow-rate (dashed line). Bottom panel: gas flow-rate. All graphs are adimensionalized with nominal values, for confidentiality.

The adopted approach consists of the introduction of an intermediate admissible steady-state as a supplementary decision variable to guarantee recursive feasibility of optimization problem, even considering a prediction horizon much smaller than the time window required to reach the terminal target. The adopted numerical method exploits the linearisation of the system along the predicted trajectory.

The simulations show the remarkable performances of the proposed scheme, especially in comparison with the standard manual approach and state-of-the-art open-loop optimization methods.

Future work will be devoted to the characterization of the modelling approximation and of the proposal of a rigorous robust implementation based on tube-based MPC or similar approaches. A theoretically sound analysis of the connections of the adopted numerical approach with the Real-Time Iteration (RTI) method will also be conducted.

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