# MODELLING SEA CLUTTER IN SAR IMAGES BASED ON LAPLACE-RICIAN DISTRIBUTION

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**ABSTRACT**

This paper presents a novel statistical model for modelling radar echoes measured by a synthetic aperture radar (SAR) sensor back-scattered from the sea surface. The analysis of ocean surface is widely performed using satellite imagery as it produces information for wide areas under various weather conditions. An accurate SAR amplitude distribution model ensures better results for applications of despeckling, ship detection/tracking and so forth. In this paper, we propose the *Laplace-Rician* distribution for modelling amplitude SAR images of the sea surface. The proposed statistical model is based on Rician distribution to model the amplitude of a complex SAR signal, the in-phase and quadrature components of which are assumed to be Laplace distributed. The Laplace-Rician model is evaluated with SAR images of the sea surface from COSMO-SkyMed and Sentinel-1 and with a comparison study to state-of-the-art statistical models such as $\mathcal{K}$ and Weibull distributions. In order to decide the most suitable model, statistical significance analysis via Kullback-Leibler divergence and Kolmogorov-Smirnov statistics is performed. The results show an accurate modelling performance for the proposed model among others for all utilised images.

***Index Terms—*** Sea clutter modelling, SAR Imaging, Laplace-Rician distribution.

## 1. INTRODUCTION

Synthetic aperture radar (SAR) imagery is an important source of information in the analysis of sea surface thanks to its capability to capture wider areas under different weather conditions. Accurate statistical models for the sea surface, when being the area of interest, has crucial importance for applications such as target detection, tracking, classification.

Literature spans numerous statistical models to accurately model SAR images of the sea surface. Among those, $\mathcal{K}$-distribution dominates the literature for sea clutter modelling [1, 2, 3]. It also has several improved versions such as generalised-$\mathcal{K}$ [4], $\mathcal{KK}$ [5] distributions in sea SAR applications. Weibull distribution is another important and robust statistical model for sea clutter modelling for several SAR frequency bands [6, 7] and also in constant false alarm rate (CFAR) based ship detection applications [8, 9]. Moreover, Log-normal [7, 10], Pareto [3], gamma [11], and $\alpha$-Stable based [12] distributions have also been used to model sea clutter for various applications like classification, etc.

All the statistical models given above are developed for the non-Rayleigh case. Similarly, Rician distribution is a non-Rayleigh statistical model and has crucial importance in telecommunications for fading channel modelling. As for SAR modelling, despite having seldom usage, Rician distribution appears, especially in cases when a single scatterer dominates the scene [13, 14]. In the literature, the Rician distribution is used in SAR imagery for automatic target recognition [15] and amplitude modelling as a combination with inverse Gaussian distribution [16]. Sea surface could be seen as a potential/unexplored application area for Rician based statistical models since SAR scenes might be dominated by a target or even by different wave heights in the absence of a target.

In this paper, we present a novel statistical model for modelling the amplitude of complex radar echoes measured by a SAR sensor back-scattered from the sea surface. The proposed model is inspired by Moser et. al. [17], in which the Rayleigh case has been extended with generalised Gaussian distributed components of complex SAR back-scattered signal to cover heavier tailed SAR amplitude data. We, in this study, extend this model by replacing Rayleigh with the Rician case, where the components of the complex signal are Laplace distributed. The proposed Laplace-Rician model is evaluated with various SAR images of the sea surface by compared to state-of-the-art statistical models of $\mathcal{K}$, lognormal, Weibull, Rician as well as the Laplace-Rayleigh of [17]. The most suitable statistical model is selected via statistical significance measures of Kullback-Leibler (KL) divergence and Kolmogorov-Smirnov (KS) statistics.

The rest of the paper is organised as follows: we present first the general background information including the SAR signal model and statistical models. Then, we introduce the proposed Laplace-Rician model and the corresponding parameter estimation method. We next demonstrate the experimental analysis for real SAR data, followed by concluding remarks and future work.

## 2. BACKGROUND

### 2.1. SAR Signal Model

The general model for the back-scattered complex signal, $R = x_1 + jx_2$, received by a SAR sensor from a given area follows several assumptions: i) there are large number of scatterers, ii) the scatterers are statistically independent, iii) The amplitude and phase of the scatterer are independent random variables, iv) the phase is uniformly distributed in $[0, 2\pi]$ v) reflectors are small when compared to the illuminated area, and vi) there is no dominating scatterers in the whole scene [17, 18].

Considering the first two assumptions which invoke the central limit theorem, the real and imaginary parts of the reflected signal $R$ are assumed to be jointly Gaussian. Under the assumption (vi) along with (i) and (ii), $x_1$ and $x_2$ are turned out to be independent, identically distributed and zero-mean Gaussian random variables with equal variances, hence leads the amplitude distribution to

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be Rayleigh distributed as

$$f(r|\gamma) = \frac{r}{\gamma^2} \exp\left(-\frac{r^2}{2\gamma^2}\right) \tag{1}$$

where $r = \sqrt{x_1^2 + x_2^2}$ refers to the amplitude, $\theta = \arctan(x_1/x_2)$ is the phase and $\gamma$ is the scale parameter.

In the case of having one scatterer dominates the whole illuminated scene, the assumption (vi) will no longer be valid and the signal components $x_1$ and $x_2$ will be independent, identically distributed, and however nonzero-mean Gaussian random variables with equal variances. Thus, the amplitude distribution of $R$ becomes the Rician (or Rice) distribution which is given as

$$f(r|\sigma, \delta) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2 + \delta^2}{2\sigma^2}\right) \mathcal{I}_0\left(\frac{r\delta}{\sigma^2}\right) \tag{2}$$

where $\delta = \sqrt{2}\mu$, $\mathcal{I}_0(\cdot)$ refers to the zeroth-order modified Bessel function of the first kind, and $\mu > 0$ is the non-zero mean of components $x_1$ and $x_2$.

### 2.2. Non-Rayleigh Statistical models

As discussed earlier in the paper, there are several statistical models that have been utilised to model sea surface in SAR images under the assumption that the back-scattered signal components are non-Gaussian, equivalently the amplitude distribution is non-Rayleigh. Among those *Weibull*, $\mathcal{K}$ and *lognormal* distributions have great importance in the literature and are utilised in this study. To this end, please note that this selection of models is not exhaustive, as the list can be extended in a detailed study without page limitations.

The univariate probability density function (pdf) expressions of Weibull, $\mathcal{K}$ and lognormal distributions are expressed as [19, 20]

$$\text{Weibull} \rightarrow \quad f(r|\alpha, \gamma) = \tfrac{\alpha}{\gamma}\left(\tfrac{r}{\gamma}\right)^{\alpha-1} \exp\left(-\left(\tfrac{r}{\gamma}\right)^{\alpha}\right),$$

$$\mathcal{K} \rightarrow \quad f(r|\alpha, \gamma) = \tfrac{2}{\gamma\Gamma(\alpha+1)}\left(\tfrac{r}{2\gamma}\right)^{\alpha+1} K_\alpha\left(\tfrac{r}{\gamma}\right),$$

$$\text{Lognormal} \rightarrow \quad f(r|\mu, \gamma) = \tfrac{1}{r\gamma\sqrt{2\pi}} \exp\left(-\tfrac{(\log r - \mu)^2}{2\gamma^2}\right).$$

### 2.3. Generalised Gaussian Rayleigh Distribution

Similar to [18, 21] which proposes a generalisation of Rayleigh distribution in terms of symmetric $\alpha$-Stable distributions, in the *generalised Gaussian Rayleigh* (GGRay) derivation of [17], it has been assumed that the received signal components, $x_1$ and $x_2$ are non-Gaussian and have heavier tails than Gaussian distribution. Thus, components are assumed to be zero-mean generalised Gaussian (GG) distributed as $x_i \sim \mathcal{GG}(\gamma, \alpha)$, for $i = 1, 2$ where the generalised Gaussian distribution is defined for the shape parameter $\alpha$ and the scale parameter $\gamma$ as

$$f(x_i|\alpha, \gamma) = \frac{\alpha}{2\gamma\Gamma(\frac{1}{\alpha})} \exp\left(-\left|\frac{x_i}{\gamma}\right|^{\alpha}\right), \quad \text{for } i = 1, 2. \tag{3}$$

According to [17], having GG distributed components leads the amplitude distribution to be the GGRay distribution, the pdf expression of which can be defined as

$$f(r|\alpha, \gamma) = \frac{\alpha^2 r}{4\gamma^2\Gamma^2(\frac{1}{\alpha})} \int_0^{2\pi} \exp\left[-\frac{|r\cos\theta|^\alpha + |r\sin\theta|^\alpha}{\gamma^\alpha}\right] d\theta. \tag{4}$$

A statistical model of this-type has special members for $\alpha$ equal to 2 and 1, which are the *Rayleigh* and *Laplace-Rayleigh* (L-Ray) distributions, respectively. In particular, it is straightforward to show that (4) can be simplified to Rayleigh distribution for $\alpha = 2$, and in the case of $\alpha = 1$, one can obtain the L-Ray distribution by replacing $\alpha$ with 1 in (4) as

$$f(r|\gamma) = \frac{r}{4\gamma^2} \int_0^{2\pi} \exp\left[-(r/\gamma)\left(|\cos\theta| + |\sin\theta|\right)\right] d\theta. \tag{5}$$

## 3. LAPLACE-RICIAN MODEL FOR SAR IMAGES

In terms of the proposed methodology, we apply the generalisation idea of Rayleigh distribution based on the generalised Gaussian distribution of Moser et. al [17], to the Rician distribution. For non-zero location parameter $\mu$ of the components, the pdf definition in (4) will no longer be valid and needs reconstruction.

In this paper, we first assume that signal components, $x_1$ and $x_2$ are non-zero GG distributed with $\alpha = 1$, which makes each component Laplace distributed as

$$f(x_i|\mu, \gamma) = \frac{1}{2\gamma} \exp\left(-\left|\frac{x_i - \mu}{\gamma}\right|\right), \quad \text{for } i = 1, 2. \tag{6}$$

As long as the components $x_1$ and $x_2$ are independent [17], the joint pdf can be written as

$$f(x_1, x_2|\mu, \gamma) = f(x_1|\mu, \gamma) f(x_2|\mu, \gamma) \tag{7}$$

$$= \frac{1}{4\gamma^2} \exp\left(-\frac{|x_1 - \mu| + |x_2 - \mu|}{\gamma}\right). \tag{8}$$

Thus, the pdf expression for the amplitude can be written by using the identity

$$f(r, \theta|\mu, \gamma) = r f(r\cos\theta, r\sin\theta) \tag{9}$$

as

$$f(r, \theta|\mu, \gamma) = \frac{r}{4\gamma^2} \exp\left(-\frac{|r\cos\theta - \mu| + |r\sin\theta - \mu|}{\gamma}\right). \tag{10}$$

Hence, the corresponding marginal amplitude pdf can be obtained by averaging (10) over $\theta$ and turns out to be

$$f(r|\mu, \gamma) = \frac{r}{4\gamma^2} \int_0^{2\pi} \exp\left(-\frac{|r\cos\theta - \mu| + |r\sin\theta - \mu|}{\gamma}\right) d\theta, \tag{11}$$

which can be basically defined as the *Laplace-Rician* (L-Ric) distribution.

## 4. BAYESIAN ESTIMATION OF LAPLACE-RICIAN DISTRIBUTION PARAMETERS

In this section, a Markov chain Monte Carlo (MCMC) methodology is developed for estimating Laplace-Rician distribution parameters, namely the scale parameter $\gamma$, and the location parameter $\mu$. In particular, the method is a Metropolis-Hastings (MH) algorithm, and in each iteration, it applies one of two different moves: (1) $\mathcal{M}_1$ which updates $\mu$ for fixed $\gamma$, (2) $\mathcal{M}_2$ which updates $\gamma$ for fixed $\mu$. The proposed parameter estimation procedure is given in Algorithm 1.

Assume we have the observed data $y$, the hierarchical model is then expressed by Bayes' theorem as

$$p(\mu, \gamma|y) \propto p(y|\gamma, \mu) p(\gamma) p(\mu) \tag{12}$$

where $p(\mu, \gamma|y)$ is the joint posterior distribution, or namely the MH target distribution, $p(y|\gamma, \mu)$ refers to the likelihood distribution, and $p(\gamma)$ and $p(\mu)$ are priors.

Due to lack of information about conjugate priors, we choose noninformative (Jeffrey's) priors for the location and scale parameters. In particular, we assume the location parameter $\mu$ is equally likely and prior for the scale parameter $\gamma$ is $p(\gamma) = 1/\gamma$, which lead us to $p(\mu, \gamma) \sim 1/\gamma$. The likelihood $p(y|\gamma, \mu)$ is the Laplace-Rician distribution in (11) with parameters $\gamma$ and $\mu$.

Depending on the selected move in iteration $i$, one of the proposal distributions given below is used to sample candidate parameters $\mu^*$ or $\gamma^*$

$$\mu^* \propto q\left(\mu^*|\mu^{(i)}\right) = \mathcal{U}\left(\mu^{(i)} - \nu, \mu^{(i)} + \nu\right), \tag{13}$$

$$\gamma^* \propto q\left(\gamma^*|\gamma^{(i)}\right) = \mathcal{N}\left(\gamma^{(i)}, \xi^2\right) \tag{14}$$

where $\mathcal{U}(\cdot)$ is the uniform, and $\mathcal{N}(\cdot)$ is the Gaussian distribution, both of which are defined in interval $[0, \infty]$ since $\mu$ and $\gamma$ are positive parameters. $\xi$ and $\nu$ are hyperparameters belonging to the proposal distributions. Please note that these selection of proposals are not unique and can be replaced with any other distribution for better performance, faster convergence, etc.

Consequently, the acceptance probability expressions for each move can be constructed as

$$A_{\mathcal{M}_1} = \min\left(1, \frac{p(y|\gamma^*, \mu^*)q\left(\mu^{(i)}|\mu^*\right)}{p(y|\gamma^{(i)}, \mu^{(i)})q\left(\mu^*|\mu^{(i)}\right)}\right), \tag{15}$$

$$A_{\mathcal{M}_2} = \min\left(1, \frac{p(y|\gamma^*, \mu^*)p(\gamma^*)q\left(\gamma^{(i)}|\gamma^*\right)}{p(y|\gamma^{(i)}, \mu^{(i)})p(\gamma^{(i)})q\left(\gamma^*|\gamma^{(i)}\right)}\right). \tag{16}$$

**Algorithm 1** MCMC Parameter Estimation for Laplace-Rician Distribution

**Inputs:** Given data $y$.
**Output:** Joint Posterior $f(\mu, \gamma|y)$
**Initialise:** $\mu^{(1)}$, $\gamma^{(1)}$, $\nu$ and $\xi$.
**for** $i = 1 : N_{iter}$ **do**
  Choose Move, $m^{(i)}$
  **if** $m^{(i)} \rightarrow \mathcal{M}_1$ **then**
    Sample $\mu^* \sim q\left(\mu^*|\mu^{(i)}\right)$
    Set $\gamma^* = \gamma^{(i)}$ and $A = A_{\mathcal{M}_1}$.
  **elseif** $m^{(i)} \rightarrow \mathcal{M}_2$ **then**
    Sample $\gamma^* \sim q\left(\gamma^*|\gamma^{(i)}\right)$
    Set $\mu^* = \mu^{(i)}$ and $A = A_{\mathcal{M}_2}$.
  **end if**
  Sample random variable $R \sim \mathcal{U}(0, 1)$
  **if** $R \leq A$ **then**
    $\mu^{(i+1)} = \mu^*$ and $\gamma^{(i+1)} = \gamma^*$
  **else**
    $\mu^{(i+1)} = \mu^{(i)}$ and $\gamma^{(i+1)} = \gamma^{(i)}$
  **end if**
**end for**

## 5. EXPERIMENTAL RESULTS

The proposed method was tested from two different perspectives using both simulated and real data. In the first simulation case, we used synthetically generated L-Ric data for various parameters and tested the parameter estimation performance of the proposed MCMC method. In the former case, we subsequently conducted experiments to determine the best fitting distribution for given real SAR patches of the sea surface.

We used statistical significance measures of *Kullback-Leibler* (KL) divergence, *Kolmogorov-Simirnov* (KS) score in order to assess the performance of fitting distributions. For KL and KS values the smaller value gives the better modelling performance. KL divergence is to test the performance by considering the estimated pdfs and data histograms, whereas KS score is calculated by evaluating the estimated and the empirical cumulative distribution functions (CDFs).

The number of iterations, $N_{iter}$ in MCMC parameter estimation method was set to 1000 iterations and first 250 iterations were discarded as burn-in period. Initial values for $\mu^{(1)}$ and $\gamma^{(1)}$ were set to 1. For proposal hyperparameters, we chose $\nu = 2.5$ and $\xi = 3$. Moves $\mathcal{M}_1$ and $\mathcal{M}_2$ are equiprobable whilst satisfying $p(\mathcal{M}_1) + p(\mathcal{M}_2) = 1$. For all state-of-the-art statistical models except L-Ray, we used an MCMC based maximum likelihood (ML) methodology to estimate the model parameters. For L-Ray, a similar methodology given for L-Ric was applied by bypassing the move $\mathcal{M}_1$ so as to estimate only $\gamma$.

### 5.1. Synthetically Generated Data

In the first set of simulations, four synthetically generated L-Ric data sets were obtained and the proposed parameter estimation method was used to estimate $\mu$ and $\gamma$ for each data set. The corresponding data sets were generated for $(\mu, \gamma)$ are (1.7, 1.3), (7, 2), (12, 15) and (55, 22). Each data set has 1500 samples, and the results are presented in Table 1.

**Table 1**. Modelling and statistical significance results for synthetically generated Laplace-Rician data sets

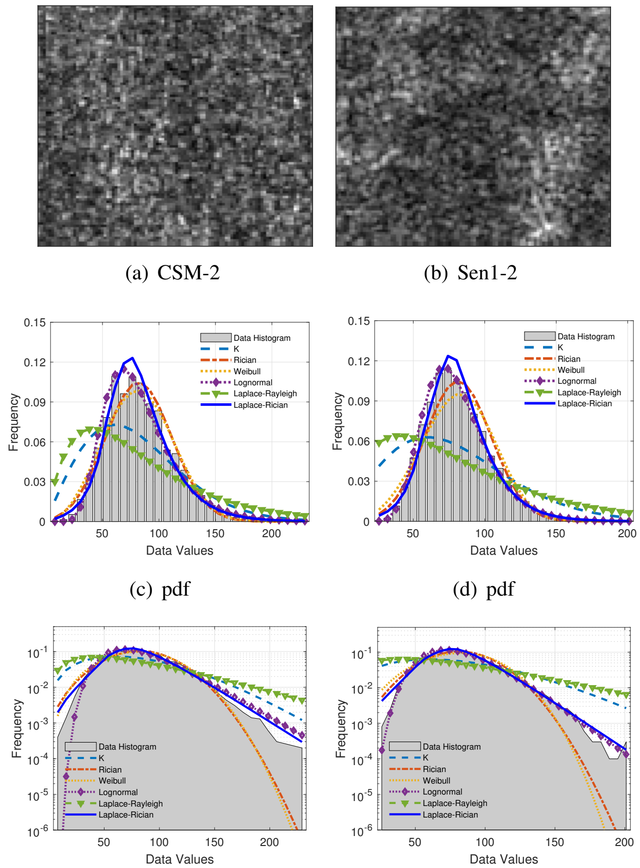
| $(\mu, \gamma)$ | **Est. Location ($\hat{\mu}$)** | **Est. Scale ($\hat{\gamma}$)** | **KL Div.** | **KS Score** |
|---|---|---|---|---|
| (1.7, 1.3) | 1.74 | 1.25 | 0.007 | 0.008 |
| (7, 2) | 7.01 | 2.01 | 0.004 | 0.005 |
| (12, 15) | 12.00 | 14.73 | 0.010 | 0.011 |
| (55, 22) | 54.60 | 22.20 | 0.013 | 0.014 |

Examining estimated values in Table 1, we can state that $\gamma$ and $\mu$ values are estimated in relation to the exact values. For all four example data sets, statistical significance values are obviously low which specifies that the model parameters are successfully estimated.

### 5.2. Real SAR Data

In the second set of simulations, modelling performance of the proposed statistical model was tested on real SAR images of the sea surface. The L-Ric distribution was compared to the statistical models of $\mathcal{K}$, Rician, Weibull, lognormal and L-Ray [17] distributions in terms of KL divergence and KS statistics along with the visual demonstrations.

Five different sea surface patches with size of 100×100 pixels were cropped from SAR images of two different satellite platforms, namely COSMO-SkyMed and Sentinel-1. The five corresponding patches were then modelled by all statistical models mentioned above by estimating their required parameters. The estimated parameters for each model and each patch are given in Table 2. By




(a) CSM-2 (b) Sen1-2

(c) pdf (d) pdf

(e) log-pdf (f) log-pdf

**Fig. 1**. Visual demonstration for SAR sea clutter modelling. First row depicts sea surface patches. The second and third rows show estimated pdfs in numerical and logarithmic scales, respectively.

using the estimated parameters for each model, fitted pdfs were obtained and statistical significance measures were calculated. The corresponding modelling results are presented in Table 3 and Figure 1.

When examining the estimated parameters in Table 2, the proposed model, L-Ric, always have lower scale parameter values then the classical Rician estimations. For all the data sets, Weibull estimates are above Rayleigh (Weibull distribution is equal to Rayleigh for $\alpha = 2$) as expected, and similarly $\mathcal{K}$ has relatively high shape parameter estimates even though it falls short to model all example SAR patches.

The statistical significance analysis in Table 3 clearly shows that the most suitable distribution to model sea surface patches is the proposed L-Ric distribution. In terms of both KL divergence and KS score, L-Ric has lowest values for all three COSMO-SkyMed sea surface patches. For Sentinel-1 results, despite not having the lowest KL values, for KS values the proposed methodology can be defined as the most suitable choice.

Notwithstanding not having a shape parameter, it is obvious that the generalisation presented in this paper in terms of Laplace and Rician distributions addresses the sea surface characteristics better than state-of-the-art models and is highly flexible for various sea conditions, SAR platform, operating frequency, etc. Fitting results demonstrated in Figure 1 provide visual support to the numerical results presented in Table 3, where L-Ric outperforms the reference models and follows the data histogram better than the others.

**Table 2**. Modelling results for real SAR data

| Statistical Models | Parameters | Images | | | | |
|---|---|---|---|---|---|---|
| | | CSM-1 | CSM-2 | CSM-3 | Sen1-1 | Sen1-2 |
| $\mathcal{K}$ | $\hat{\alpha}$ | 9.68 | 10.98 | 8.63 | 10.88 | 12.45 |
| | $\hat{\gamma}$ | 13.69 | 13.16 | 9.83 | 11.57 | 12.25 |
| Rician | $\hat{\delta}$ | 73.36 | 76.54 | 49.79 | 68.52 | 77.97 |
| | $\hat{\gamma}$ | 29.34 | 30.22 | 21.84 | 23.46 | 23.56 |
| Weibull | $\hat{\alpha}$ | 3.02 | 3.06 | 2.84 | 3.44 | 3.69 |
| | $\hat{\gamma}$ | 88.91 | 92.50 | 61.56 | 80.77 | 90.20 |
| Lognormal | $\hat{\mu}$ | 4.31 | 4.35 | 3.94 | 4.23 | 4.35 |
| | $\hat{\gamma}$ | 0.37 | 0.37 | 0.40 | 0.33 | 0.28 |
| L-Ray | $\hat{\gamma}$ | 49.53 | 51.51 | 34.16 | 45.19 | 50.85 |
| L-Ric | $\hat{\mu}$ | 50.62 | 52.81 | 34.25 | 47.72 | 54.16 |
| | $\hat{\gamma}$ | 22.69 | 23.37 | 16.72 | 18.26 | 17.77 |

**Table 3**. Statistical significance of the estimates for real data

| Image | Performance Measures | Statistical Models | | | | | |
|---|---|---|---|---|---|---|---|
| | | $\mathcal{K}$ | Rician | Weibull | Lognormal | L-Ray | L-Ric |
| CSM-1 | KL Div. | 0.2417 | 0.0270 | 0.0302 | 0.0266 | 0.4375 | **0.0200** |
| | KS Score | 0.1542 | 0.0417 | 0.0412 | 0.0270 | 0.2256 | **0.0236** |
| CSM-2 | KL Div. | 0.2534 | 0.0263 | 0.0284 | 0.0245 | 0.4712 | **0.0234** |
| | KS Score | 0.1562 | 0.0323 | 0.0334 | 0.0277 | 0.2348 | **0.0243** |
| CSM-3 | KL Div. | 0.2135 | 0.0323 | 0.0300 | 0.0377 | 0.4066 | **0.0264** |
| | KS Score | 0.1385 | 0.0421 | 0.0414 | 0.0366 | 0.2048 | **0.0250** |
| Sen1-1 | KL Div. | 0.3186 | **0.0285** | 0.0359 | 0.0473 | 0.5051 | 0.0360 |
| | KS Score | 0.1871 | 0.0374 | 0.0354 | 0.0378 | 0.2509 | **0.0264** |
| Sen1-2 | KL Div. | 0.3703 | 0.0319 | 0.0582 | **0.0036** | 0.5482 | 0.0151 |
| | KS Score | 0.1824 | 0.0496 | 0.0570 | 0.0267 | 0.2233 | **0.0180** |

## 6. CONCLUSION

In this paper, we proposed a novel statistical model, the *Laplace-Rician* (L-Ric) distribution, for modelling SAR images of the sea surface, which is based on a generalisation of Rician distribution in terms of Laplace distribution. An MH-based Bayesian parameter estimation method was proposed and estimation performance was first tested in fitting several synthetically generated L-Ric random sequences. The performance of L-Ric was then evaluated in modelling SAR images of the sea surface from two satellite platforms namely the COSMO-SkyMed and Sentinel-1, compared to state-of-the-art statistical models, which are $\mathcal{K}$, Rician, Weibull and lognormal distributions. The proposed statistical model showed the best fitting performance among all models for all images utilised in this paper.

The future work will extend the proposed model into more general cases covering GG and S$\alpha$S distributions in modelling other types of scenes, such as urban, forest as well as see surface. Developing log-cumulants based parameter estimation method is also our one of current endeavours.

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## 8. REFERENCES


[1] C. Baker, "K-distributed coherent sea clutter," in *IEE Proceedings F (Radar and Signal Processing)*, vol. 138, no. 2. IET, 1991, pp. 89–92.

[2] T. Eltoft and K. A. Hogda, "Non-Gaussian signal statistics in ocean SAR imagery," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 36, no. 2, pp. 562–575, 1998.

[3] S. Angelliaume, L. Rosenberg, and M. Ritchie, "Modeling the amplitude distribution of radar sea clutter," *Remote Sensing*, vol. 11, no. 3, p. 319, 2019.

[4] G. Ferrara, M. Migliaccio, F. Nunziata, and A. Sorrentino, "Generalized-K (GK)-based observation of metallic objects at sea in full-resolution synthetic aperture radar (SAR) data: A multipolarization study," *IEEE Journal of Oceanic Engineering*, vol. 36, no. 2, pp. 195–204, 2011.

[5] L. Rosenberg, D. Crisp, and N. Stacy, "Analysis of the KK-distribution with medium grazing angle sea-clutter," *IET radar, sonar & navigation*, vol. 4, no. 2, pp. 209–222, 2010.

[6] S. Ishii, S. Sayama, and K. Mizutani, "Effect of Changes in Sea-Surface State on Statistical Characteristics of Sea Clutter with X-band Radar," *Wireless Engineering and Technology*, vol. 2, pp. 175–183, 2011.

[7] Z. Chen, X. Liu, Z. Wu, and X. Wang, "The Analysis of Sea Clutter Statistics Characteristics Based on the Observed Sea Clutter of Ku-Band Radar," in *2013 Proceedings of the International Symposium on Antennas & Propagation*, vol. 2. IEEE, 2013, pp. 1183–1186.

[8] J. R. M. Fernández, "Estimation of the relation between Weibull sea clutter and the CA-CFAR scale factor," *Revista Ingeniería*, vol. 25, no. 2, pp. 19–28, 2015.

[9] O. Pappas, A. Achim, and D. Bull, "Superpixel-level CFAR detectors for ship detection in SAR imagery," *IEEE Geoscience and Remote Sensing Letters*, vol. 15, no. 9, pp. 1397–1401, 2018.

[10] X. Xing, Z. Chen, H. Zou, and S. Zhou, "Statistical assessment of model fit for SAR sea clutter," in *MIPPR 2009: Multispectral Image Acquisition and Processing*, vol. 7494. International Society for Optics and Photonics, 2009, p. 74940R.

[11] J. Martín-de Nicolás, M.-P. Jarabo-Amores, D. Mata-Moya, N. del Rey-Maestre, and J.-L. Bárcena-Humanes, "Statistical analysis of SAR sea clutter for classification purposes," *Remote Sensing*, vol. 6, no. 10, pp. 9379–9411, 2014.

[12] H. Ding, J. Guan, N. Liu, and G. Wang, "Modeling of heavy tailed sea clutter based on the generalized central limit theory," *IEEE Geoscience and Remote Sensing Letters*, vol. 13, no. 11, pp. 1591–1595, 2016.

[13] J. W. Goodman, "Statistical properties of laser speckle patterns," in *Laser speckle and related phenomena*. Springer, 1975, pp. 9–75.

[14] G. Gao, "Statistical modeling of SAR images: A survey," *Sensors*, vol. 10, no. 1, pp. 775–795, 2010.

[15] M. D. DeVore, A. D. Lanterman, and J. A. O'Sullivan, "ATR performance of a Rician model for SAR images," in *Automatic Target Recognition X*, vol. 4050. International Society for Optics and Photonics, 2000, pp. 34–45.

[16] T. Eltoft, "The Rician inverse Gaussian distribution: a new model for non-Rayleigh signal amplitude statistics," *IEEE Transactions on Image Processing*, vol. 14, no. 11, pp. 1722–1735, 2005.

[17] G. Moser, J. Zerubia, and S. B. Serpico, "SAR amplitude probability density function estimation based on a generalized Gaussian model," *IEEE Transactions on Image Processing*, vol. 15, no. 6, pp. 1429–1442, 2006.

[18] E. E. Kuruoglu and J. Zerubia, "Modeling SAR images with a generalization of the Rayleigh distribution," *IEEE Transactions on Image Processing*, vol. 13, no. 4, pp. 527–533, 2004.

[19] C. Walck, "Hand-book on statistical distributions for experimentalists," Tech. Rep., 1996.

[20] D. R. Iskander and A. M. Zoubir, "Estimation of the parameters of the K-distribution using higher order and fractional moments [radar clutter]," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 35, no. 4, pp. 1453–1457, 1999.

[21] A. Achim, E. E. Kuruoglu, and J. Zerubia, "SAR image filtering based on the heavy-tailed Rayleigh model," *IEEE Transactions on Image Processing*, vol. 15, no. 9, pp. 2686–2693, 2006.