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Editors

Shape Analysis and Structuring

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1 Introduction

The idea of composing this book arose from the desire to enrich and systematise the extensive state-of-the-art studies that were carried out within the framework of AIM@SHAPE, a Network of Excellence funded by European Commission under the FP6-IST [1]. The main goal of the network is to develop new methodologies for modelling and processing knowledge embedded in digital *shapes*.

Current approaches to modelling are focused on the geometry of shapes, while their semantics, e.g., meaning or functionality in a given context, is still overlooked. This is partly due to the lack of methods for the automatic extraction of the semantic content from digital shapes, known as the process of *semantic annotation* in research areas related to the development of the Semantic Web, and partly to the evolution of research on shape modelling which in the past years was highly focused on the geometric aspects of shapes. The shift from a purely geometric to a semantic-aware level of representation of digital shapes is the ultimate scientific objective of AIM@SHAPE.

In this scenario, a crucial role is played by geometry processing methods that are aimed to *preserve and enhance shape information* as well as to effectively *capture the structure of a shape* by identifying relevant shape components and their mutual relationships. Each chapter of the book provides a detailed state-of-the-art review on a specific topic, which is crucial for shape analysis and structuring, contains a classification of the techniques developed in the area, and discusses open problems.

Structural analysis play a fundamental role in the automatic extraction of semantic information. While shapes are fully characterized by a specific geometry, shape information is treated differently by the human brain with respect to several other forms of information. At a geometric level, a digital shape is a computational structure which defines a geometric representation. Different types of geometric models can be used to describe the same object. Examples are polygons, surface models (e.g., splines, NURBS surfaces), or solid models (e.g., triangle or tetrahedral meshes, boundary representations, constructive volumetric representations). The structural level in the representation is reached by organizing geometric information to reflect, or make explicit, the association between the various components of the shape. A structural description is the basis for developing semantic-based shape representations, since it abstracts from the low-level, detailed, description provided by a geometric model.

Several techniques have been developed in the literature for processing different aspects of the geometry of shapes, in particular shape interrogation and re-meshing techniques enhance a shape description with information which can be effectively used to attach semantics to the shape. *Shape interrogation* is the process of extracting information from a geometric model. Geometric models need to be analyzed with respect to different aspects, such as visual pleasantness, technical smoothing, geometric constraints, or surface intrinsic properties. The various methods devel-

[1] AIM@SHAPE, http://www.aimatshape.net

oped in the literature are used to detect surface imperfections, to analyze shapes, or to visualize different forms. Such methods are reviewed in *Chapter one*.

Re-meshing is often used for efficient shape approximation, and it consists of repartitioning a set of primitives so that they best fit the original shape. Re-meshing preserves the shape, in the sense that it still approximates the shape after re-meshing, and it can be designed to enhance the shape. Every shape feature is locally fit with a primitive that minimally characterizes the shape. In addition, some recent re-meshing techniques operate through a careful analysis based on multi-scale discrete differential geometry so as to estimate the main (and detailed) axis of symmetry of the shape. The shape is, thus, locally classified as spherical, parabolic, elliptic, and hyperbolic in order to drive the re-meshing process. Such classification may be also used for shape enhancement. *Chapter two* presents a survey on re-meshing techniques.

A first way of structuring shape information is provided by those techniques that organize a geometric shape description defined by a function, by a mesh, or by a set of points into a representation of the shape at different levels of resolutions, from which concise and adaptive shape descriptions can be extracted. This topic has received considerable attention in recent years in many fields of computer graphics, geometric modelling and visualization, and numerous research efforts have been devoted to it. This book contains two chapters on focused on multi-resolution analysis, and the other on subdivision surfaces.

Multi-resolution analysis provides a powerful tool for efficiently representing functions at multiple levels of detail. Herein, a complex function is decomposed into a coarser low-resolution part, together with a collection of detail coefficients, necessary to recover the original function. Multi-resolution analysis has many inherent advantages, including compression, progressive transmission, visualization and editing at different levels of detail. An overview of methods for multi-resolution analysis is presented in *Chapter three*.

Subdivision surfaces define the basis for generating a smooth surface from a coarse mesh, and, thus, they have bees extensively used in geometric modeling for creating, editing and transmitting a shape. The surface is defined by the initial coarse mesh plus a subdivision scheme to progressively subdivide the mesh by inserting new vertices and connecting them to the edges and faces until a smooth surface is obtained in the limit. *Chapter four* contains a review of surface subdivision schemes and their application in geometric modeling.

The third part of the book is devoted to structural shape representations. In the above framework, many research efforts have been devoted to study concise, structural representations of a shape based on *skeletal structures*, such as the medial axis, or the Reeb graph. Skeletal structures provide an abstract shape representation by idealized lines that retain the connectivity of the original shape. In advanced fields, such as virtual human modeling, available modeling tools to represent structured geometry focus on adding a skeleton to the 3D geometry in order to animate it and provide different degrees of realism. A survey of different skeletal structures is presented in *Chapter five*.

Another class of structural representations is provided by *morphology-based descriptions for scalar and vector fields*. There has been a considerable amount of work

in the literature on extracting critical features (point, integral lines, etc.) from two-dimensional scalar fields describing grey-level images and terrains, and, more recently, some work has been done on volume data on extracting critical features and for representing the topological structure of the field iso-surfaces. A survey of morphological representations for two-dimensional and three-dimensional scalar fields is presented in *Chapter six*.

Topological methods based on features, like critical points or separatrix lines, have also been applied for the analysis of vector fields. The basic idea is to use such features for segmenting the flow into areas of different flow behaviour, and use this as a tool for understanding complex phenomena described by the vector fields. After introducing topological features for two-dimensional and three-dimensional vector fields, *Chapter seven* presents a survey of methods for extracting topological features from vector fields and using them as visualization tools for complex flow phenomena, represented both as static and dynamic fields. Applications of topological methods for compressing, simplifying, comparing, and constructing vector fields are also discussed.

The last part of the book, namely *Chapter eight*, provides a review on the use of structural data for modelling shapes with a high semantic characterization, e.g., *virtual humans*. In this case, the structural model, called the *control skeleton*, has by itself a specific role in the evaluation of the many different shapes associated with all the possible postures that the body model can reach. The first part of the Chapter discusses the control articulated skeleton structure and different approaches to build skeletons and bind it to the shape geometry. The second part addresses the generation of level-of-detail models for virtual humans, in terms of the geometry and of the articulated skeleton.

We would like to acknowledge the work of all the authors of the various chapters in this book, that contributed with their expertise and energy to assemble a substantial part of state-of-the-art reports on a variety of interesting topics. Our special thanks go to Bianca Falcidieno, the coordinator of the AIM@SHAPE Network of Excellence, for her enthusiasm and support in this work and for having inspired much of the motivations of this collection, and to all partners in AIM@SHAPE. Many people contributed to the preparation of the book, and we would like to thank specifically Emanuele Danovaro for his invaluable help in editing and preparation.

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