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Stability of a weakly collisional plasma with runaway electrons

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Abstract. We investigate the problem of the tearing stability of a post-disruption weakly collisional plasma where the current is completely carried by runaway electrons. We adopt here a two fluid model which takes into account also ion sound Larmor radius and electron inertia effects in the description of the reconnection process. In the past, it has been demonstrated in [Helander *et al. Phys. Plasmas* **14**, 12, (2007)] that in the purely resistive regime the presence of runaway electrons in plasma has a significant effect on the saturated magnetic island width. In particular, runaway electrons generated during disruption can cause an increase of 50% in the saturated magnetic island width with respect to the case with no runaway electrons. These results were obtained adopting a periodic equilibrium magnetic field that limited the analysis to small size saturated magnetic islands. Here we present our results to overcome this limitation adopting a non-periodic Harris' type equilibrium magnetic field. Preliminary results on the effects of the ion sound Larmor radius effects will also be presented.

1. Introduction

Runaway electrons are created in a variety of environmental and laboratory plasmas such as thunderstorm clouds [1], solar flares [2], as well as during disruptions of tokamak plasmas [3]. In particular when dealing with tokamak plasmas the role of a runaway electrons current linked with reconnection events becomes crucial. On one side the reconnection process could favor the relativistic electrons impact into the wall, with the risk of significant damage on plasma facing components. On the other side a high runaway current can drive a reconnection event in the post disruption phase altering the stability condition of the plasma. Moreover recent experiments in DIII-D [4] have shown that when the current is entirely driven by a high relativistic runaway current MHD instabilities lead to a rapid and complete loss of the runaway electrons population. For these reasons detailed studies on how this interaction occurs and on its effects are necessary to improve the strategies of runaway electron mitigation for ITER and beyond. First studies on magnetic reconnection driven by a runaway current date back to 2007 [5]. Assuming a fluid description of a runaway current peaked at the rational surface, where reconnection occurs, the linear growth rate and the saturation magnetic island amplitude have been derived analytically and verified numerically in the framework of resistive MHD. More recently, the linear theory has been extended to cylindrical geometry considering runaway current not peaked on the rational surface in [6] and linear and nonlinear simulations of the MHD instabilities occurring in DIII-D have been presented in [7]. Interestingly the work in ref. [6], that deals with a single helicity

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perturbation in 3D configurations, finds a new scale sublayer in the resistive inner layer, whose scale length is comparable with the electron skin depth. However, still in the framework of the slab geometry adopted in Ref. [5], some questions remain open due to the periodic boundary conditions adopted in the radial direction and to the simplified single-fluid description. Indeed, when a periodic magnetic equilibrium is adopted, each linearly unstable mode is characterized by two resonant surfaces, where two identical magnetic islands form and grow. As far as the size of the islands is smaller than the distance between the corresponding resonant surfaces, their mutual interaction can be neglected. On the other hand, the nonlinear evolution of large magnetic islands is affected by their mutual interaction. In paper [5] the choice of a periodic magnetic equilibrium, dictated by the boundary conditions of the numerical solver adopted, prevented to properly check the saturation magnetic island amplitude for values of the standard instability parameter Δ' [8] larger than 0.4. Moreover, in the single fluid approach all the effects related to electron inertia and to Larmor radius are neglected. In this paper we aim to overcome these limitations and present new simulations carried out with the SCOPE3D code, that allows the numerical solutions to vanish at the boundary in the radial direction and, therefore, to adopt a non periodic equilibrium magnetic field and to avoid the influence of the boundary conditions. SCOPE3D solves the two-fluid model equations [9] in two and three dimensional slab configurations. In the pure resistive limit we find that the possibility of the magnetic island to grow without the cross talking problems induced by periodic boundary conditions corrects what previously observed in Ref. [5] concerning the presence of a bifurcation behavior. Indeed here we are able to verify the formula for the saturated magnetic island up to values of Δ' of order 1. Moreover we carry out simulations taking into account also the electron temperature effects, entering into the equations through the ion sound Larmor radius parameter. We show that also in this case the runaway current does not alter significantly the linear growth rate. We find a linear dependence on the stability parameter for the island saturated width as observed in the pure resistive case. The paper is organized as follows. After the description of the model equations, the numerical code and the set-up, we benchmark the results obtained with the new equilibrium against the theory reported in Ref. [5]. Subsequently the results of our simulations including the ion sound Larmor effects are discussed in the linear and nonlinear regimes. Conclusions and future perspectives follow.

2. Model equations

To describe the reconnection process we adopt a refined version of the resistive MHD model adopted in Ref. [5], based on a two-fluid approach which retains the electron inertia and ion sound Larmor radius effects. The model is the one used for the studies of reconnection in weakly collisional regimes in Refs. [9], where, in the dissipative term, the equilibrium current is replaced by the runaway current J_r. Although post disruption plasmas are often characterized by high collisionality, Liu *et al.* [6] find in their linear analysis calculations the existence of a sublayer in the resistive layer that accounts for a real frequency in the growth of the magnetic island. The estimation of the width of this sublayer is comparable to the electron skin depth. This justify the choice of retaining the electron inertia contribution in the plasma Ohm's law. In slab geometry the equations normalized on the Alfvén time and on the equilibrium magnetic field scale length are:

$$\frac{\partial \psi}{\partial t} + [\varphi, \psi] - d_e^2 \frac{\partial \nabla^2 \psi}{\partial t} - d_e^2 [\varphi, \nabla^2 \psi] - \rho_s^2 [U, \psi] + \eta (J - J_r) + \frac{\partial \varphi}{\partial z} - \rho_s^2 \frac{\partial U}{\partial z} = 0 \qquad (1)$$

$$\frac{\partial U}{\partial t} + [\varphi, U] + [\nabla^2 \psi, \psi] + \frac{\partial \nabla^2 \psi}{\partial z} = 0. \qquad (2)$$

Here, ψ and φ are the magnetic flux and stream function respectively. The current density and the vorticity are $J = -\nabla^2 \psi$ and $U = \nabla^2 \varphi$, where laplacian depends only on the coordinates

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perpendicular to the guide field, as $\nabla = \partial_x^2 + \partial_y^2$. The Poisson brackets $[f, g]$ are defined as $\partial_x f \partial_y g - \partial_x g \partial_y f$. The parameter η is the resistivity, $d_e = c/\omega_{pe}$ the electron skin depth, ω_{pe} the plasma frequency, $\rho_s = \sqrt{(T_e/m_i)}/\omega_{ci}$ the ion sound Larmor radius, accounting for the parallel electron temperature contribution. The magnetic field is given as $\mathbf{B} = B_0 \mathbf{e_z} + \nabla\psi \times \mathbf{e_z}$, where B_0 is the uniform magnetic guide field and is set to 1.

The runaways move practically at the speed of light along the magnetic field, and according to [5] the equation for the normalized runaway current density J_r, which closes the system of equations, is:

$$\frac{\partial J_r}{\partial t} + [\varphi, J_r] + \frac{c}{v_A}\Big([\psi, J_r] - \frac{\partial J_r}{\partial z}\Big) = 0. \tag{3}$$

3. The numerical tool SCOPE3D

The model equations given by 1-3 are solved by means of SCOPE3D a code specifically designed for simulations of magnetic reconnection in low collisional plasmas in a 3D slab geometry. This code is explicit in time, assuming a third order Adams-Bashforth scheme and parallelized along both the periodic directions y and z, while a compact finite difference scheme [10] is implemented on a nonequispaced grid along the magnetic field shear direction, allowing for a high mesh refinement in the reconnection region. In order to remove short length scales that form due to the nonlinear interactions, numerical filters [10] are implemented along the periodic directions, while the physical dissipation is sufficient to control the numerical noise along the radial direction. For the purpose of this paper, the code has been run in the 2D limit, avoiding the dependence on the z-direction, while a resolution of $ny = 96$ grid points has been adopted along the y-direction. For the x-direction, $nx = 1200$ grid points have been adopted on the nonequispaced grid so as to guarantee a resolution of $dx = 0.0038$ around $x = 0$, where the tearing instability takes place. Since the first goal of this work is to benchmark our results against the ones in [5], we will assume here $d_e = 0$ and consider only the limit $c/v_A = 1$. Although preliminary simulations, not shown here, have already been run with values of $c/v_A \gg 1$. Equations are integrated over the domain $[-L_x, L_x] \times [-L_y, L_y]$ with $L_x = 3\pi$ while L_y was varied in order to consider different degrees of instability. The initial condition on the fields φ, ψ , J_r corresponds to:

$$\varphi(x, y, 0) = 0 \tag{4}$$

$$\psi(x, y, 0) = \psi_{eq}(x) + \hat{\psi}(x) \exp ik_y y + c.c. \tag{5}$$

$$J_r(x, y, 0) = J_{eq}(x, y, 0) = -\nabla^2\psi_{eq}(x), \tag{6}$$

where $k_y = \pi m/L_y$, with m an integer number, is the wave vector along the y-direction and the last condition follows from the assumption that all the current is carried by the runaway electrons.

These equations were solved in ref. [5] adopting a periodic function for the in-plane equilibrium magnetic field, which implied the presence of two magnetic islands, one at the center of the integration box and the other at the boundary. The interaction of these two islands prevented the analysis on long nonlinear times. Here we adopt the standard Harris equilibrium [11] for the in-plane component of the magnetic field $\mathbf{B}_{eq} = \tanh(x)\mathbf{e}_y$, which allows the investigation of a single growing magnetic island all the way to saturation. This magnetic field gives an equilibrium current that has been discussed in MHD theory as the most probable profile [12]. The corresponding magnetic flux function is $\psi_{eq} = -\ln\cosh(x))$. The standard Δ' parameter for the adopted equilibrium is $\Delta' = 2(1/k_y - k_y)$.

In ref. [5] it has been shown that the linear properties of the tearing mode are similar to those in a plasma without runaways to leading order in the smallness of η.

4. The $\rho_s = 0$ limit

Here we reproduce the numerical results of Ref. [5] with the new equilibrium. We choose for the resistivity the value of $\eta = 3e-4$ and vary the Δ' parameter in the range $[0.1, 0.5]$. In fig.1 we compare the linear growth rates with the expected theoretical values for cases with and without runaways. We recall that due to the relatively high value of resistivity the theoretical prediction

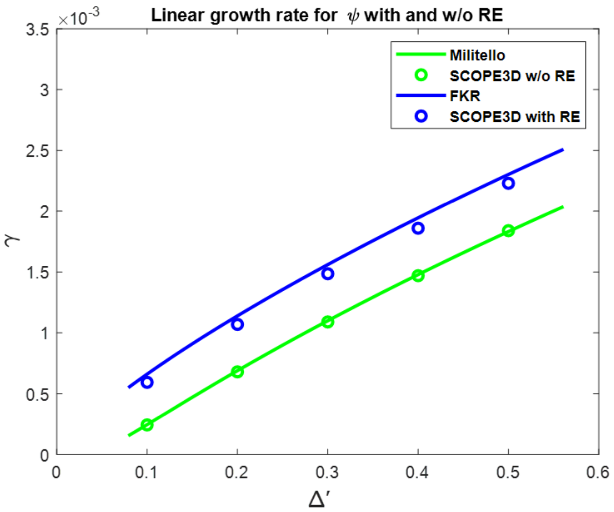

Figure 1. Linear growth rates for a pure resistive reconnecting mode driven by a runaway current. As a comparison also the case without runaway current is shown.

must account for the finite resistivity correction [13]. The linear dispersion relation has been derived in slab geometry in Ref. [5] with and without runaways electrons, obtaining:

$$\frac{(\gamma)^{5/4}}{\eta^{3/4} k_y^{1/2}} = 0.47\Delta' \qquad \text{with runaways} \tag{7}$$

$$\frac{(\eta\gamma)^{1/4}}{k_y^{1/2}}[\gamma - \eta b] = 0.47\eta\Delta' \qquad \text{without runaways} \tag{8}$$

where $b = \frac{\psi_{eq}^{IV}}{\psi_{eq}^{II}} |_{x=0}$. Equation 7 is the standard Furth, Killeen and Rosenbluth (FKR) growth rate in the small Δ' regime, defined by the inequality $\Delta'\eta^{1/3} \ll 1$. While equation 8 takes into account the corrections due to the higher order derivatives of the current density at the resonant surface [13]. We indeed find a very good agreement in both cases.

In fig.2 we compare the nonlinear saturated magnetic island widths for Δ' in the range $[0.1, 2]$. We recall that the saturated island width, w, is given in ref. [5] as:

$$w = -\frac{1}{b}\frac{\Delta'}{0.272} \qquad \text{with runaways} \tag{9}$$

$$w = -\frac{1}{b}\frac{\Delta'}{0.411} \qquad \text{without runaways} \tag{10}$$

Since $b = -2$ for the equilibrium adopted here we get $w = 1.85\Delta'$ and $w = 1.22\Delta'$ respectively for the case with and without runaways. The results of our numerical campaign, in agreement with these formulas, confirm that the presence of a runaway current generated during disruption

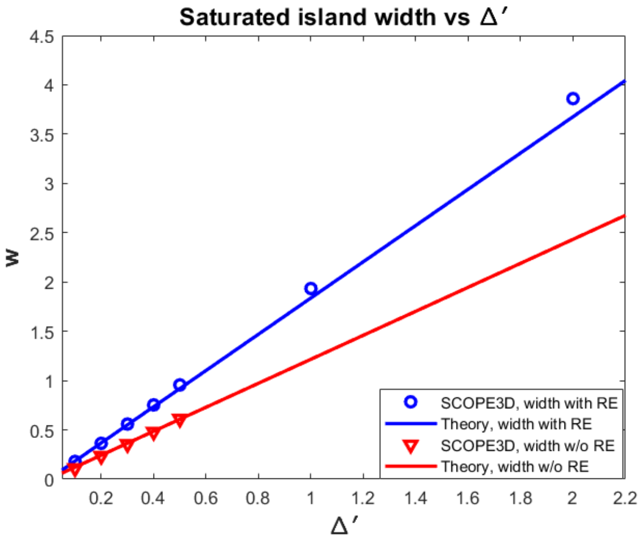

Figure 2. Saturation island width for the pure resistive reconnecting mode driven by a runaway current.

lead to an increase of 50% in the saturated magnetic island width with respect to the case with no runaway electrons. The difference observed in the runaways cases for values of Δ' of order unity is due to the fact thai the simulation parameters begin to depart from the asymptotic limit of the small Δ' regime, where the analytical theory is valid. Nonetheless, we performed these runs to check if the bifurcation in the sequence of saturated equilibria found in ref. [5] was real or induced by the periodic boundary conditions adopted in that paper. We can state here that no bifurcation occurs and the magnetic island can grow up to saturation for values of Δ' of order 1.
In fig.3 we show the total current density accounting for both the thermal and runaway electrons contribution at saturation for the simulation with $\Delta' = 0.5$. On the left panel the 2D distribution of the current is given with superimposed the closed magnetic surfaces highlighting the topology if the magnetic field. On the right panel the profiles of the current density are plotted as a function of the radial coordinate for two different y sections. The blue curve refers to a profile across the magnetic island O-point, the red curve across the X-point, while the green curve refers to the initial equilibrium. We can here appreciate the homogenization of the current density inside the magnetic island, typical of resistive tearing modes, while the peak current at the X-point decreases due to the loss of runaway electrons escaping along the open magnetic surfaces.

5. The $\rho_s \neq 0$ limit

Here we show the new results taking into account the effect of electron compressibility along magnetic field lines. This effect introduces into the equations the ion sound Larmor radius scale length, ρ_s.
In fig.4 we show the linear growth rates for values of Δ' in the range $[0.1, 0.8]$. As reference case we plot also the linear growth rates for a case without runaway current. The linear growth rate of the tearing mode predicted by the linear theory gives $\gamma = (\rho_s \Delta'/\pi)^{2/3} \eta^{1/3} k_y^{2/3}$. We observe that as in the pure resistive case the presence of the runaway current does not alter significantly the linear growth rates. The small discrepancy between theory and numerical results can be ascribed to the fact that for the parameters chosen here we do not fall exactly in the range of

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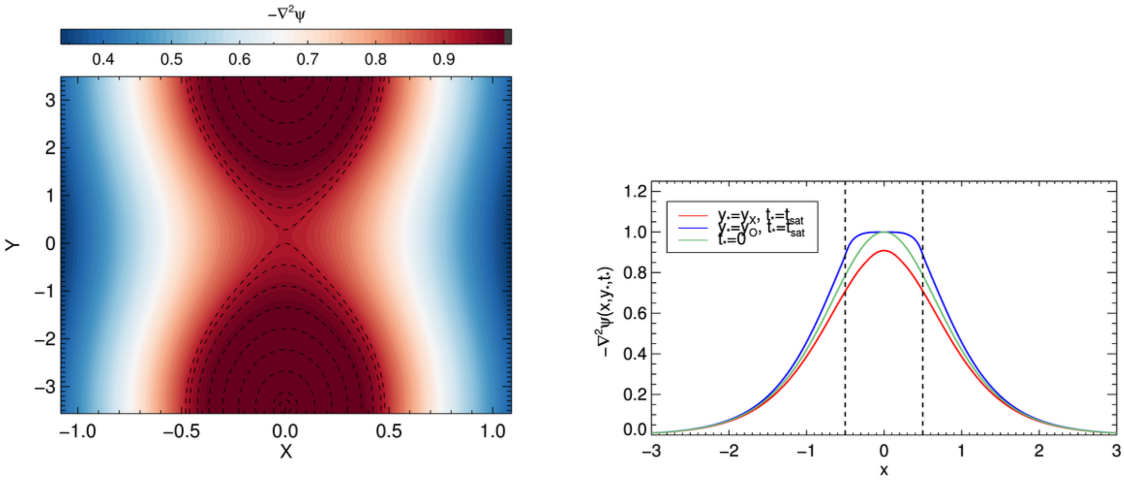

Figure 3. Total current density at saturation for the case $\Delta' = 0.5$. Left panel: contour plot with superimposed magnetic surfaces. Right panel: radial profiles for different poloidal position compared with the equilibrium profile. The dashed line define the width of the magnetic island.

validity of the analytical theory. Preliminary results with smaller values of ρ_s show a better agreement.

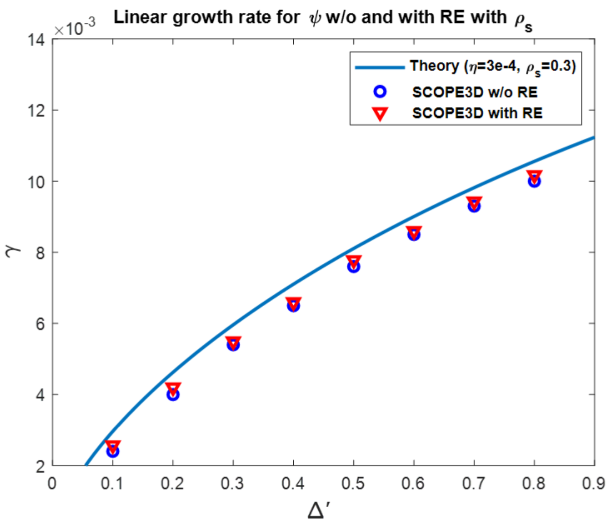

Figure 4. Linear growth rates for a resistive reconnecting mode with and without a runaway current in presence of electron temperature effects.

In fig.5 we show the saturated magnetic island widths. Here we compare the numerical results with the same formulas 9-10 since, as expected, the microphysics at the ion sound Larmor radius scale plays no role in the saturated island width, which should depend only on the free energy available for the reconnection process. Indeed we find a very good agreement also in this case.

6. Conclusions and discussion

Adopting a new non periodic equilibrium magnetic configuration, which allows for the magnetic island to grow all the way to saturation in the small Δ' regime, we have confirmed the linear

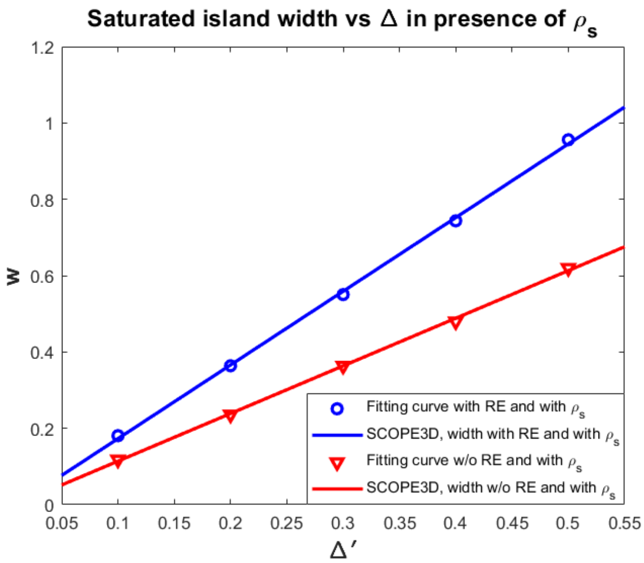

Figure 5. Saturated magnetic island widths in presence of electron temperature effects.

results on the growth rate [5] of a reconnecting mode driven by a runaway electron current. We have also verified the saturation width amplitude found in [5] up to values of Δ' of order 1. These results have been extended to consider regimes that take into account the effect of the electron temperature. No substantial difference between the growth rate without runaways has been found. The saturation width amplitude is insensitive of the microphysics and the formula in ref. [5] is valid also in this regime. Future work foresees to consider reconnecting mode taking into account also the z-dependence. A first step will be to consider single helicity modes. Based on the results of ref. [6] we expect the electron inertia terms in the generalized Ohm's law to become relevant in the sublayer physics found in this regime. A further step of this work will be to consider multiple helicity reconnection modes introducing magnetic field lines stochasticity expected to play a significant role in the runaway electron dispersion [14, 15].

References

[1] Gurevich, A., Milikh, G. and Roussel-Dupre, R. *Phys. Lett. A* **165**, 463 (1992).
[2] Haerendel, G., *Astrophys. J.* **847**, 113 (2017).
[3] Reux, C. *et al.*, *Nucl. Fusion* **55**, 093013 (2015).
[4] Paz-Soldan, C., Eidietis, N.W., Liu, Y.Q., Shiraki, D., Boozer, A.H., Hollmann, E.H., Kim, C.C. and Lvovskiy, A., *Plasma Phys. Control. Fusion*, **61**, 054001 (2019).
[5] Helander, P., Grasso D., Hastie, R.J. and Perona, A., *Phys. Plasmas* **14**, 12, (2007).
[6] Liu, C., Zhao, C., Jardin, S.C., Bhattacharjee, A., Brennan, D.P., and Ferraro, N.M., *Phys. Plasmas* **27**, 9, (2020).
[7] Liu, C., Zhao, C., Jardin, S.C., Ferraro, N.M. Paz-Soldan, C., Liu, Y. and Lyons, B.C., *Plasma Phys. Control. Fusion*, **63**, 125031 (2021).
[8] Furth, H.P., Killeen, J. and Rosenbluth, M.N., *Phys. Fluids* 6, 459 (1963).
[9] Schep, T.J., Pegoraro, F., and Kuvshinov, B.N., *Phys. Plasmas*, **1** (9), 2843–2851, (1994).
[10] Lele, S. K., *J. Comput. Phys.* **103** (1992).
[11] Harris, E.G., *Nuovo Cimento* **23**, 115 (1962).
[12] Biskamp, D., *Magnetic Reconnection in Plasmas* (Cambridge University Press, Cambridge, UK, 2000).
[13] Militello, F., Huysmans, G., Ottaviani, M. and Porcelli, F., *Phys. Plasmas* **11**, 125 (2004).
[14] Bandaru, V., Hoelzl, M., Reux, C., Ficker, O., Silburn, S., Lehnen, M., Eidietis, N., and JOREK Team 7 and JET Contributors, *Plasma Phys. Control. Fusion*, **63**, 035024 (2021).
[15] Liu, Y., Alenykova, K., Paz-Soldan, C., Alenykov, P., Lukash, V. and Khayrutdinov, R., *Nucl. Fusion*, **62**, 066026 (2022)