TAPAS: a Tool for Stochastic Evaluation of Large Interdependent Composed Models with Absorbing States

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Agenda

- Context & objectives
- Addressed models and measures
- Input format: (a restriction of) Stochastic Automata Network
- Descriptor matrices <u>and vectors</u> through Tensor Trains
- Demo
- Next steps

Context & Objectives

Context:

- Reliability CTMC models (with focus on limiting behavior) where there are absorbing states, e.g., system failure states
- Large models, in particular: the system model comprises several synchronized submodels, each with a relatively small state-space

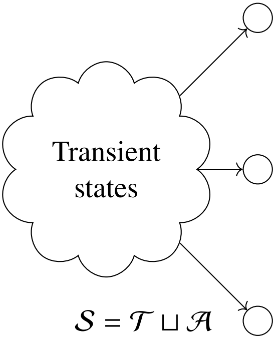
How our tool TAPAS (Tool for Stochastic Evaluation of Large Interdependent Composed Models with Absorbing States) contributes:

- it evaluates performability measures
- it exploits (compressed) implicit representation of all the matrices and vectors

Addressed models and measures

Models & measures of interest

- Consider the CTMC $\{X_t\}_{t\geq 0}$ with absorbing states $\mathcal{A}$, where $\forall t.X_t \in \mathcal{S}$ and $X_0 \notin \mathcal{A}$



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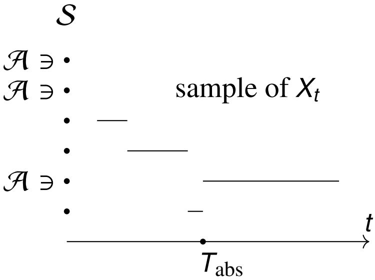

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Infinitesimal Generator Matrix

$$\left[\begin{array}{ccccccc} & Q_{\mathcal{T}} & & v_1 & v_2 & \dots & v_{|\mathcal{A}|} \\ \hline 0 & \dots & 0 & 0 & 0 & 0 & 0 \\ \hline & \vdots & & \vdots & \vdots & \vdots & \vdots \\ \hline 0 & \dots & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Models & measures of interest

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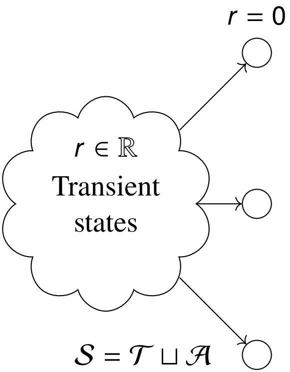
- Define, for $i \in \mathcal{S}$, a reward vector $r = [r_i]$ such that $r_i = 0$ if $i \in \mathcal{A}$



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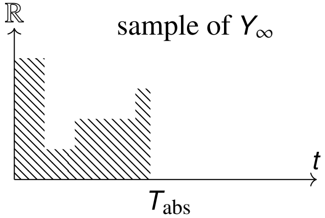
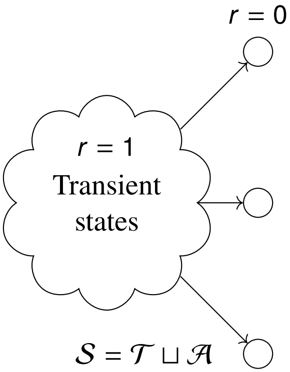
- Define $Y_\infty := \int_0^\infty r_{X_t}\, dt$



Models & measures of interest

- The moments of Y_∞,
  i.e., $\mathcal{M}_k := E[Y_\infty^k]$,
  e.g., MTTA= $E[T_{\text{abs}}] = E[Y_\infty]$
  where $r_i = 1$ for $i \in \mathcal{T}$



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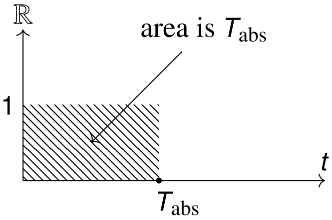

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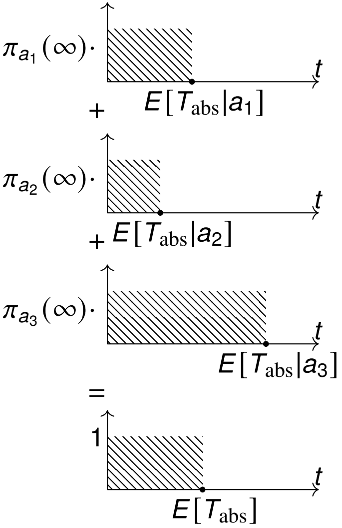
- The moments of Y_∞, i.e., $\mathcal{M}_k := E[Y_\infty^k]$, e.g., MTTA= $E[T_{\text{abs}}] = E[Y_\infty]$ where $r_i = 1$ for $i \in \mathcal{T}$
- $\pi_{\mathcal{B}}(\infty)$, i.e., the probability that X is absorbed in $\mathcal{B} \subset \mathcal{A}$
- $E[Y_\infty|\mathcal{B}]$ for $\mathcal{B} \subset \mathcal{A}$, e.g., for $\{a_1\} \subseteq \{a_1, a_2, a_3\}$ evaluate $E[T_{\text{abs}}|\{a_1\}]$



Measures evaluation for explicit models

TAPAS works with the following formalization:

measure	evaluation
$\mathcal{M}_k$	$\begin{cases}(Q-S)x^{(1)} = r, \\ (Q-S)x^{(i)} = \mathrm{diag}(r)x^{(i-1)}, \quad \text{for } i = 2,\dots,k,\end{cases}$ then $\mathcal{M}_k = k!(-1)^k \pi(0)\cdot x^{(k)}$
$\pi_{\mathcal{B}}(\infty)$	$(Q-S)x^{(1)} = Qe_{\mathcal{B}}$, then $\pi_{\mathcal{B}}(\infty) = -\pi(0)\cdot x^{(1)}$,
$\mathrm{MRTA}_{\vert\mathcal{B}}$	$\begin{cases}(Q-S)x^{(1)} = Qe_{\mathcal{B}}, \\ (Q-S)x^{(2)} = \mathrm{diag}(r)x^{(1)},\end{cases}$ then $\mathrm{MRTA}_{\vert\mathcal{B}} = \big(\pi(0)\cdot x^{(2)}\big)/\big(-\pi(0)\cdot x^{(1)}\big)$

where the shift matrix S is defined so that, being $r_a = 0$ for $a \in \mathcal{A}$,

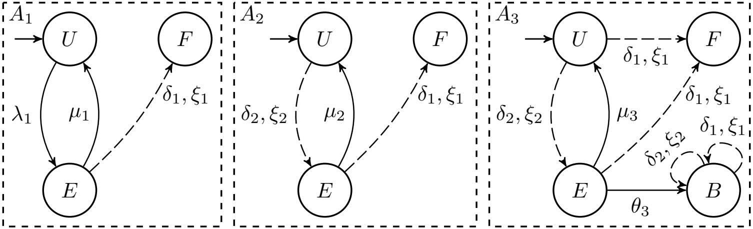
$$\pi(0)\cdot(Q-S)^{-1}\cdot r = \pi_{\mathcal{T}}(0)\cdot Q_{\mathcal{T}}^{-1}\cdot r_{\mathcal{T}}$$

there is no need to distinguish transient from absorbing states in the labeling

Stochastic Automata Network (SAN)

SAN models

$A_1, \ldots, A_n$ define the stochastic process $\tilde{X} = \left(X^{(1)}, \ldots, X^{(n)}\right)$, that is indistinguishable from X, where a synchronization transition (dashed arrow) is enabled if and only if it is enabled in all the automata, e.g., ξ_2 can "fire" if and only if A_2 is in U and $\left(A_3 \text{ is in } U \text{ or in } B\right)$



The state-space (small) exploration of each automaton is performed independently from the others

To evaluate the measures of interest it is required the existence of a path from each state to the absorbing states

Addressed technical challenge

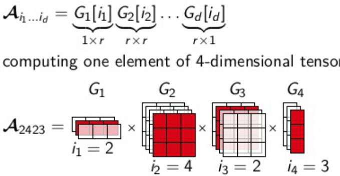
All the quantities involved in the computations $\left(\tilde{Q}, \tilde{S}, \tilde{x}^{(i)}, \tilde{\pi}(0), \tilde{r}\right)$ can be expressed as a sum of Kronecker products (see demo) but

> the issue is that each matrix-vector multiplication squares the number of addends in $\tilde{x}^{(i)}$. Thus, there is an exponential growth of memory consumption in iterative linear system solvers.

Tensor Trains (TT)

Proposed solution

- Compressed matrices and vectors were already investigated in the literature for CTMC performance and availability models (no absorbing states)
- Novelty of TAPAS: TT-based representation for CTMC reliability models (with absorbing states)



- Standard iterative methods to evaluate $\tilde{x}^{(i)}$ fail because in

$$\tilde{x}^{(i,j+1)} = \tilde{x}^{(i,j)} + \Delta\tilde{x}^{(i,j)}$$

the TT-ranks can grow too quickly

Proposed solution

- Thus, ad hoc solution methods have been investigated (6 so far):

Method	Transposed	Published	TT	Exponential Sums
tt-regular-splitting			✓	✓
amen(t)		✓	✓	✓
	✓		✓	✓
gmres(t)			✓	
	✓		✓	
tt-expsumst	✓		✓	✓

- Each $\tilde{x}^{(i,j+1)}$ update has to be re-compressed (exploiting TT round, that is based on SVD), and the TAPAS user can set the corresponding tolerance `ttol`. This parameter impacts mainly on memory occupancy
- The result tolerance, i.e., `tol`, can also be set. This parameter impacts mainly on time and accuracy

Demo

Next steps

- Exploit the tool to evaluate additional performability measures of interest, with focus on specific application domains
- Fully adhere to the Stochastic Automata Network (SAN) formalism, resorting to generalized Kronecker algebra theory
- Develop new features, e.g. to allow steady-state analysis, so to emphaddress availability-related measures, and to import the model description, as elaborated by other tools
- Integration of TAPAS in other tools, in addition to MATLAB, possibly open source ones, to promote wider usability

Thank you
Questions?