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A revenue management approach for tourism logistics optimization

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Abstract

Tourism is a very complex industry and represents one of the most profitable activity in the world. A tourist product requires the execution of several multifaceted activities. Indeed, transportation, accommodation, entertainment, food, and beverages represent only some of the products/services required by a tourist. It is evident that a single enterprise cannot provide all the components of a tourist product, but several interrelated actors should collaborate. This leads to a very complex supply tourist chain that needs to be optimized. The relevant theories and methods of logistics can be used to efficiently manage all the flows that are generated in a tourist chain. The definition of appropriate policies, at the different nodes of the chain, can improve the performance of all the actors involved in tourism logistics. In this paper, we concentrate our attention on tour operators that are relevant in the touristic logistic chain since they are involved in several activities. We introduce different revenue management policies to support tour operators in the decision of accepting the most profitable tourist requests. A request consists of flights and hotel booking, characterized by a starting time of the trip and the length of stay at the destination. We allow for various combinations of flight legs and multiple categories of hotels to accommodate a variety of customer preferences and needs. A computational study is carried out by considering different scenarios, and the performance of the considered revenue management policies is analyzed in detail.

Keywords: tourism logistics; holiday package; revenue management; booking limit policy; bid price policy; buy-up

1. Introduction

Efficient management of the flow of people and goods within the tourism logistics chain is vital for the tourism industry, which ranks among the most profitable sectors in the world (https://www.statista.com/topics/962/global-tourism). The flows need to be properly organized in terms of safety, services, and resources in order to achieve higher profits. The fulfillment of this objective is not trivial and requires the optimization of strategical, tactical, and operational decisions made at any point of the tourism chain (Alkier et al., 2023). It is necessary to organize and coordinate several "logistics nodes," including booking sites, airports, stations, hotels, and places of interest. The quality of the tourist experience is closely related to the management of the overall logistics chain. A crucial role, in the touristic chain, is played by service suppliers, like tour operators, that manage a large set of activities and flow chains (Muhcina and Popovici, 2008). Tour operators combine basic services into packages to be offered to tourists or retailers. Given the narrow profit margins in the tourism industry, the adoption of well-tailored strategies to efficiently manage available capacity and define competitive prices is essential for tour operators (Holloway et al., 2012; Ye et al., 2019). In this scenario, revenue management methodologies (McGill and Ryzin, 1999) can be very effective in supporting companies to define optimal capacity allocation and pricing policies (Yang et al., 2017).

The literature on revenue management is extremely rich and heterogeneous since it includes methods and techniques to model, estimate, and forecast demand; to set prices; and to control capacity, with the aim of improving industries' profits. Exhaustive reviews on revenue management features, techniques, and sectors of application can be found in Chiang et al. (2007), Bell (2012), and Klein et al. (2020). The relevance of revenue management research to optimally manage inventory is underlined in Yeoman (2022), whereas the adoption of revenue management techniques in the tourism and hospitality sector is investigated in Subying and Yoopetch (2023). The scientific literature provides contributions focusing on one sector at a time. This means that only one typology of resource is considered, for example, seats for airlines, rooms for hotels, trucks for car rental, and tables for restaurants.

In this paper, we focus on a capacity control strategy applied to manage two different resources, that is, seats for airlines and rooms for hotels. We consider the problem of selling holiday packages with the aim of maximizing the revenue for a tour operator. We consider a holiday package composed of flights and hotel accommodation with several categories and a certain length of stay. At each decision time, a customer arrives and requires to travel from an origin to a destination at a given time, possibly with more than one leg. Customers also define the hotel category and the length of stay. The tour operator checks for seat availability on the flights and rooms in the hotel. If these resources are available, then the tour operator decides whether to accept the customer's request. The decision is made considering the simultaneous availability of both flight seats and hotel rooms. Indeed, managing the two resources separately can lead to situations where, for example, flights are booked for both departure and return trips, but no room is available in the hotel for the length of stay or vice versa.

In the sequel, we briefly present the most relevant contributions dealing with revenue management applications of capacity control techniques in the hotel and airline industries.

Hotels industry. Among the first works in control capacity, we cite Bitran and Gilbert (1996), where the problem of allocating rooms to a customer is discussed, considering a simple scenario with one room type and one day stay. In Baker and Collier (1999), a simulation model is considered in order to compare the performance of five heuristics under 36 realistic hotel operating environments for the optimal allocation of the hotel rooms. In Goldman et al. (2002), deterministic and stochastic optimization problems are considered. In Badinelli (2000), a dynamic model for finding optimal booking policies for the hotel revenue management problem is presented. The proposed model aims at maximizing the expected revenue for all customer stays that overlap a given booking date. In Rusmevichientong et al. (2023), the authors propose a capacity control strategy tailored for modern hotel types such as Airbnb, boutique hotels, and bed and breakfasts. The reader is referred to Binesh et al. (2021) for a detailed survey of the hotel industry.

Airline industry. The airline industry is one of the first fields in which revenue management techniques have been applied. Among the first contributions, we cite Belobaba (1989) which introduced the concept of booking limits for seat inventory control. Following this seminal work, several contributions have been published. Feng and Xiao (2001) consider a stochastic control model, while An et al. (2021) propose a policy based on robust optimization. Additionally, in Shihab and Wei (2022), the authors introduce a deep reinforcement learning approach for determining the optimal policy for the seat inventory control problem. Gao and Le (2022) address the case in which dynamic pricing and seat inventory control are jointly considered. Other contributions consider customer choice behavior (Talluri and Ryzin, 2004a; Liu and Garrett, 2008; Ryzin and Vulcano, 2008; Bront et al., 2009; Jiang and Miglionico, 2014) and flexible/opaque products (Gallego and Phillips, 2004; Chen et al., 2010; Gonsch, 2020). The reader is referred to Raza et al. (2020) for a recent survey on revenue management for the airline industry.

To the best of our knowledge, this is the first attempt to apply revenue management techniques to the problem of selling services that incorporate a combination of two types of different resources (seats and rooms). Furthermore, we consider the possibility for a customer to upgrade to a product with better features than those requested. We consider two different cases. In the first one, the tour operator can offer a customer a higher hotel category than that requested at the price of the original customer request. In a second case, we consider the possibility of upgrading by associating with each customer a given buy-up probability. Thus, the customers, in case of rejection, can ask, with a given probability, a more expensive hotel accommodation. We give a dynamic programming formulation to the problem of accepting or denying a holiday package request on a given time horizon with the aim of maximizing the total tour operator revenue. Due to "the curse of dimensionality," the dynamic programming model cannot be solved optimally (Talluri and Ryzin, 2004b). In order to provide the decision-maker with a tool useful in taking decisions, we develop some integer programming approximation of the problem (de Boer et al., 2002). Based on the proposed mathematical models, we define several revenue-based policies taking into account the peculiarity of the customer choice behavior.

The rest of the paper is organized as follows. In Section 2, we present the dynamic programming formulation of the holiday packages problem. In Section 3, we propose two integer programming approximations of the problem. The first, described in Section 3.1, refers to the case of a single hotel category, while the second, reported in Section 3.2, is an extension to the multiple hotel categories case. In Section 4, partitioned booking limits and bid price policies, based on the solutions of the integer programming models, are presented. In Section 5, a computational phase is carried out with the aim of evaluating the proposed policies considering a set of meaningful instances. We draw some conclusions in Section 6. The paper ends with the Appendix in which we prove that the constraint matrices associated with the considered problems are totally unimodular.

2. A dynamic programming formulation of the holiday packages problem

We consider the problem faced by a tour operator that offers holiday packages to several customers on a given time horizon. Each package typically includes a round-trip ticket originating from a selection of specific locations to designated destinations, along with accommodation arrangements. At each time of the planning horizon, the tour operator has to decide how to manage the overall capacity in the most profitable way, taking into account that complete information on the future demand is not available. Let $J = \{1, \dots, n\}$ denote the set of possible itinerary indexes. Define $P = \{1, \dots, p\}$ as the set of potential origin/destination (indexes) nodes, and let $E = \{1, \dots, e\} \subseteq P$ denote the subset containing the indexes of the nodes that are exclusively possible destinations. We refer to the jth itinerary with origin O and destination D as (O_j, D_j), $O_j, D_j \in P$ and $j \in J$. Let $IT_l = \{j \in J : D_j = l\},\ l \in E$ be the set of itinerary indexes with the same destination l. Each itinerary is a combination of one or more legs $i \in I = \{1, \dots, m\}$. Customers can be grouped in K distinct classes $k, k = 1, \dots, K$, based on their length of stay requirement. A customer belongs to class k if her/his required length of stay is k nights, indicating the duration of her/his stay at a hotel. We also assume that customers will return to their origin node after the journey. A holiday package is completely defined by a combination of an itinerary and a length of stay. We also assume that customers will arrive at the destination in one day. At each time period $t = 1, \dots, T$ of the booking horizon, the tour operator has to decide on accepting/denying the request of a customer asking for the itinerary $j = (O_j, D_j),\ j \in J$, departure time at $\bar{t}, \bar{t} = 1, \dots, \bar{T}$, and length of stay of k nights.

The objective of tour operators is to maximize the total revenue generated from the accepted requests within the booking horizon. It is also assumed that a customer, with departure time $\bar{t}$, will return at the origin after k nights, that is, at time $\bar{t} + k, k = 1, \dots, K$. In the sequel, we will refer to $1, \dots, \bar{T}$ as the "operational horizon," that is, the horizon where the holiday takes place.

Let B denote a binary matrix with dimension $(2m \times n)$. Each element $b_{ij}, i = 1, \dots, 2m; j = 1, \dots, n$ is equal to 1 if itinerary j uses leg i and zero otherwise. Each column of matrix B contains all the information related to the legs involved in both the outward and the return trip. In particular, the first m rows refer to the outward trip while the last m rows refer to the return trip for the itinerary j. For the sake of clarity, we report in Example 1 an instance of the problem along with the associated sets and the matrix B.

Example 1. We consider four origins/destinations named Rome (ROM), Lamezia (SUF), Madrid (MAD), and Warsaw (WAW). We associate indexes 1, 2, 3, and 4 with ROM, SUF, MAD, and WAW, respectively. Hence $P = \{1, 2, 3, 4\}$. We consider six legs, represented as arcs in Fig. 1. In particular, we have legs 1 (SUF/ROM), legs 2 (ROM/MAD), legs 3 (ROM/WAW), legs 4 (ROM/SUF), legs 5 (MAD/ROM), and legs 6 (WAW/ROM). Hence $I = \{1, 2, 3, 4, 5, 6\}$.

The instance contains four itineraries, whose characteristics are depicted in what follows:

- Itinerary $j = 1 = (SUF, WAW)$ (red arrow) with origin $O_1 = 2$ (SUF) and destination $D_1 = 4 (WAW)$. Therefore, itinerary 1 is constituted by legs 1 and 3 (outward trip) and by legs 6 and 4 (return trip).
- Itinerary $j = 2 = (ROM, MAD)$ (yellow arrow) with origin $O_2 = 1$ (ROM) and destination $D_2 = 3$ (MAD). Therefore, itinerary 2 is constituted by legs 2 (outward trip) and by legs 5 (return trip).

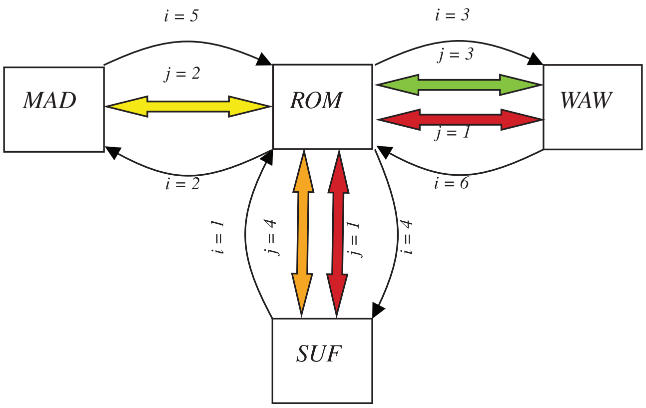

Fig. 1. Graphical representation of the instance of Example 1. (For interpretation of the references to color of this figure, the reader is referred to the web version of this article).

		1	2	3	4
1	*SUF/ROM*	1	0	0	1
2	*ROM/MAD*	0	1	0	0
3	*ROM/WAW*	1	0	1	0
4	*ROM/SUF*	0	0	0	0
5	*MAD/ROM*	0	0	0	0
6	*WAW/ROM*	0	0	0	0
7	*SUF/ROM*	0	0	0	0
8	*ROM/MAD*	0	0	0	0
9	*ROM/WAW*	0	0	0	0
10	*ROM/SUF*	1	0	0	1
11	*MAD/ROM*	0	1	0	0
12	*WAW/ROM*	1	0	1	0

Fig. 2. The matrix B associated with the instance of Fig. 1.

- Itinerary $j = 3 = (ROM, WAW)$ (green arrow) with origin $O_3 = 1$ (ROM) and destination $D_3 = 4$ (WAW). Therefore, itinerary 3 is constituted by legs 3 (outward trip) and by legs 6 (return trip).
- Itinerary $j = 4 = (SUF, ROM)$ (orange arrow) with origin $O_4 = 2$ (SUF) and destination $D_4 = 1$ (ROM). Therefore, itinerary 4 is constituted by legs 1 (outward trip) and by legs 4 (return trip).

It follows that the set of itineraries J is $J = \{1, 2, 3, 4\}$ and the set of destinations E is $E = \{1, 3, 4\}$, that is, ROM, MAD, and WAW.

The sets $IT_l, l \in E$, containing the itineraries with destinations $l \in E$, are characterized as $IT_1 = \{j = 4\}$, $IT_3 = \{j = 2\}$, and $IT_4 = \{j = 1, j = 3\}$.

The matrix B associated with the considered instance is reported in Fig. 2.

The capacity of the system depends on both the plane capacities and the room availability for the entire length of stay at the destination hotel.

The state of the network at time t is described by a matrix $X(t)$ of resource capacities

$$X(t) = \begin{pmatrix} x_1^1(t) & \cdots & x_1^{\bar{t}}(t) & \cdots & x_1^{\bar{T}}(t) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_m^1(t) & \cdots & x_m^{\bar{t}}(t) & \cdots & x_m^{\bar{T}}(t) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{m+e}^1(t) & \cdots & x_{m+e}^{\bar{t}}(t) & \cdots & x_{m+e}^{\bar{T}}(t) \end{pmatrix},$$

whose generic column $x^{\bar{t}}(t) = (x_1^{\bar{t}}(t), \dots, x_m^{\bar{t}}(t), x_{m+1}^{\bar{t}}(t) \dots, x_{m+e}^{\bar{t}}(t))$, $\bar{t} = 1, \dots, \bar{T}$ represents the resources availability at each time $\bar{t}$ of the "operational horizon." Indeed at each time period $\bar{t}$, $\bar{t} = 1, \dots, \bar{T}$, the first m rows refer to the leg capacities while the latter e to the hotel capacity at each destination. We assume that each flight is operated every day in the whole operational horizon.

We indicate with B_j^1 the vector containing the first m rows of the jth column of B (outward route) and e additional rows equal to zero. We also indicate with B_j^2 a vector containing the remaining m rows of the jth column of B and e additional rows equal to zero. Time is discrete, and there are T booking periods indexed by t, which runs forward so that $t = T$ is the departure time. In each time period t, at most one request for a holiday package can arrive. We denote with $\lambda_{j\bar{t}k}^t$, the probability that at time t one request of class k for itinerary $j \in J$ and departure time $\bar{t} = 1, \dots, \bar{T}$, is made. It holds that $\sum_{k=1}^{K} \sum_{\bar{t}=1}^{\bar{T}} \sum_{j \in J} \lambda_{j\bar{t}k}^t + \lambda_0^t = 1$, where λ_0^t is the probability that no request for holiday packages arrives at time t.

Let us introduce Boolean variables $u_{j\bar{t}k}^t$, with $u_{j\bar{t}k}^t = 1$ if the customer request, arriving at time t of the booking horizon, for a holiday with itinerary j, length of stay of k nights, and departure time at $\bar{t}$ is accepted and $u_{j\bar{t}k}^t = 0$ otherwise.

Let R_j^k be the revenue associated with the holiday package of class k for the itinerary $j \in J$. We assume that, on the given booking horizon, the revenue is independent of time, that is, a revenue of R_j^k is obtained from the holiday package of class k for itinerary $j \in J$ whatever it is the departure time $\bar{t}$ and the time t in which the request arrives.

The problem can be formulated as a dynamic program by defining $V_t(X(t))$ as the maximum expected revenue obtainable from periods $t, t+1, \dots, T$ given that, at time t, the network capacity is $X(t)$. The Bellman equation for $V_t(X(t))$ is reported as follows:

$$V_t(X(t)) = \sum_{k=1}^{K} \sum_{l=1}^{e} \sum_{j \in IT_l} \lambda_{j\bar{t}k}^t \max_{u_{j\bar{t}k}^t \in \{0,1\}} \left[R_j^k u_{j\bar{t}k}^t + V_{t+1}(X(t+1)) \right] + \lambda_0^t V_{t+1}(X(t)) \tag{1}$$

with boundary conditions:

$$V_t(0) = 0, \qquad \forall t, \tag{2}$$

$$V_T(X(T)) = 0, \qquad \text{if } x_r^{\bar{t}}(T) \geq 0 \ \forall r, \bar{t}, \tag{3}$$

$$V_t(X(t)) = -\infty, \qquad \text{if } x_r^{\bar{t}}(t) < 0 \text{ for some } r, \bar{t}, \forall t, \tag{4}$$

where (2) states that, at each time period t, if the capacity equals zero, the expected revenue is also zero; conditions (3) ensure that at the end of the planning horizon, there is no further opportunity for revenue generation. Finally, the boundary conditions (4) discourage the optimization process from exploring infeasible solutions.

We denoted by $X(t+1)$ the matrix obtained by appropriately updating the network capacity. It is worth noting that the update of the network capacity is related to the following event: At time t, a customer requires a holiday package, thereby defining the itinerary j, the length of stay k, and the arrival time $\bar{t}$.

The tour operator can accept or deny the current request. If the request is accepted, we need to update the leg capacities by filling the seats on all the legs involved in itinerary $j \in J$. In particular, the columns $\bar{t}$ and $\bar{t}+k$ of matrix $X(t)$ need to be updated as follows: $x^{\bar{t}}(t+1) = x^{\bar{t}}(t) - B_j^1 u_{j\bar{t}k}^t$ and $x^{(\bar{t}+k)}(t+1) = x^{(\bar{t}+k)}(t) - B_j^2 u_{j\bar{t}k}^t$.

Moreover, we need to modify the hotel capacity for the destination index $l = D_j$ associated with the itinerary $j \in J$. In fact, a new room at the destination indexed by l will be occupied from $\bar{t}$ to $\bar{t}+k-1$, that is, $x_{(m+l)}^{\tilde{t}}(t+1) = x_{(m+l)}^{\tilde{t}}(t) - 1,\ \forall \tilde{t} = \bar{t}, \ldots, \bar{t}+k-1$.

It is worth observing that, if at time t, the tour operator denies the current request or no request arrives, the network capacity available at time $t+1$ remains unchanged (i.e., $X(t+1) = X(t)$).

3. Integer programming formulations

The proposed dynamic programming model is unlikely to be solved optimally due to the curse of dimensionality. In the next sections, we present two integer programming approximations of the problem that we use to define several revenue management policies.

3.1. The holiday package problem

Starting from the dynamic programming problem, in the integer programming approximation, we replace stochastic quantities by their mean values and we assume that capacity and demand are continuous. Let be

- d the random cumulative future demand at time t, and $\bar{d}$ its mean. In particular, $d_{jk}^{\bar{t}}$ is the aggregate number of requests for a holiday package with itinerary $j \in J$, length of stay $k = 1, \ldots, K$, and departure time $\bar{t} = 1, \ldots, \bar{T}$;
- R_j^k the revenue associated with a holiday package with itinerary $j \in J$ and length of stay $k = 1, \ldots, K$;
- $b_{ij},\ i \in I,\ \ j \in J$, equal to 1 if itinerary j uses leg i and zero otherwise. b_{ij} is an element of matrix B introduced in Section 2;
- $x^{\bar{t}}(t) = (x_1^{\bar{t}}(t), \ldots, x_m^{\bar{t}}(t), x_{m+1}^{\bar{t}}(t) \ldots, x_{m+e}^{\bar{t}}(t))$ the resource vector constituted by $m+e$ elements, indicating the leg and room capacity for the operational instant time $\bar{t}$ at booking period t;

- $z^{\bar{t}}_{jk}$ integer variable indicating the number of accepted holiday packages with itinerary $j \in J$, departure time at $\bar{t} = 1, \ldots, \bar{T}$ and length of stay $k = 1, \ldots, K$;
- $y^{\bar{t}}_{lk}$ integer variable indicating the number of rooms occupied at $\bar{t}$ by a customer of class $k = 1, \ldots, K$ at the destination $l \in E$.

The total revenue achievable by the tour operator at booking time period t, when the network capacity is $X(t)$, can be calculated by solving the following optimization problem:

$$R^{HPP}(X(t)) = \max \sum_{\bar{t}=1}^{\bar{T}} \sum_{k=1}^{K} \sum_{j=1}^{n} R^k_j z^{\bar{t}}_{jk}, \tag{5}$$

$$z^{\bar{t}}_{jk} \leq \bar{d}^{\bar{t}}_{jk} \quad \forall\, k, j, \bar{t}, \tag{6}$$

$$\sum_{j=1}^{n} b_{ij} \sum_{k=1}^{K} z^{\bar{t}}_{jk} + \sum_{j=1}^{n} b_{(m+i)j} \sum_{k=1}^{K} \sum_{\{\tilde{t} \leq \bar{t} | \bar{t} = \tilde{t}+k\}} z^{\tilde{t}}_{jk} \leq x^{\bar{t}}_i(t) \quad \forall\, i, \quad \bar{t}, \tag{7}$$

$$\sum_{j \in IT_l} z^{\bar{t}}_{jk} = y^{\bar{t}}_{lk} \quad \forall\, l, \ k, \quad \bar{t}, \tag{8}$$

$$\sum_{k \in K} y^{\bar{t}}_{lk} + \sum_{\tilde{t}=1}^{\bar{t}-1} \sum_{\tilde{k}=(\bar{t}-\tilde{t})+1}^{K} y^{\tilde{t}}_{l\tilde{k}} \leq x^{\bar{t}}_{(m+l)}(t) \quad \forall\, \bar{t}\, l, \tag{9}$$

$$z, y \geq 0, \text{integer}. \tag{10}$$

Constraints (6) state that the tour operator cannot allocate more holiday packages to initial booking requests than the average demand $\bar{d}$. Constraints (7) control the availability of seats in all the legs involved in the itinerary. Equations (8) are link constraints between z and y variables. Constraints (9) control the availability of rooms at each destination for the entire length of stay. Finally, the variable domain constraints are shown in Equation (10).

3.2. The holiday package problem with multiple hotel categories

We consider an extension of the model defined in Section 3.1 by introducing the possibility of different hotel categories. Let $S = \{1, \ldots, \bar{S}\}$ be the set of possible hotel categories, named also starts, associated with a holiday package. In the following, we will also consider the possibility for a customer that requires a hotel with $\tilde{s} \in S$ stars to upgrade to a hotel with a higher number of stars $s' \in S, \ s' > \tilde{s}$. In particular, we consider two types of possible upgrades, dealing with two different versions of the holiday package problem (HPP) with multiple hotel categories (HPPS):

1. The tour operator can assign to customers a hotel category s' greater than that requested, that is, $\tilde{s}$, at the price associated with a hotel of category $\tilde{s}$. The associated problem is referred to as HPPS1;
2. at any time t, the tour operator has to make the following decisions: (D1) accept or reject a booking request for a holiday package with itinerary j, departure time at $\bar{t}$, and length of stay k in a hotel with $\tilde{s}$ stars. (D2) In the case of rejection in D1 and if the customer wants to buy up to a hotel with category s', accept or reject the customer's new booking request. The associated problem is referred to as HPPS2.

3.2.1. Mathematical formulation for HPPS1

To formulate HPPS1, we need to introduce the following parameters and variables:

- d the random cumulative future demand at time t, and $\bar{d}$ its mean. In particular, $d^{\bar{t}}_{jks}$ is the aggregate number of requests for a holiday package with itinerary $j \in J$, length of stay $k = 1, \ldots, K$ in a hotel with $s \in S$ stars and departure time $\bar{t} = 1, \ldots, \bar{T}$;
- R^k_{js} the revenue associated with a holiday package with itinerary $j \in J$ and length of stay $k = 1, \ldots, K$ in a hotel with $s \in S$ stars;
- $\xi^{\bar{t}}_{ls}(t)$ the number of available rooms for $\bar{t} = 1, \ldots, \bar{T}$ at the destination $l \in E$ in a hotel with $s \in S$ stars at the booking time period t;
- $z^{\bar{t}}_{jkss'}$ integer variable indicating the number of the accepted requests for a holiday package with itinerary $j \in J$, length of stay $k = 1, \ldots, K$, departure time $\bar{t} = 1, \ldots, \bar{T}$ in a hotel with $s \in S$ stars by using an hotel with $s' \in S$ stars $s' \geq s$. In particular if $s' > s$ an upgrade takes place.
- $y^{\bar{t}}_{lks}$ integer variable indicating the number of occupied rooms at $\bar{t} = 1, \ldots, \bar{T}$ by customers of class $k = 1, \ldots, k$ at the destination $l \in E$ in a hotel with $s \in S$ stars.

The total revenue achievable by the tour operator at time t, when the network capacity is $X(t)$, can be calculated by solving the following holiday package problem with S categories, that is, HPPS1:

$$R^{HPPS1}(X(t)) = \max \sum_{\bar{t}=1}^{\bar{T}} \sum_{k=1}^{K} \sum_{j=1}^{n} \sum_{s=1}^{S} \sum_{s'=s}^{S} R^k_{js} z^{\bar{t}}_{jkss'}, \tag{11}$$

$$\sum_{s'=s}^{S} z^{\bar{t}}_{jkss'} \leq \bar{d}^{\bar{t}}_{jks} \quad \forall\, k,\ j,\ \bar{t},\ s, \tag{12}$$

$$\sum_{j=1}^{n} b_{ij} \sum_{k=1}^{K} \sum_{s=1}^{S} \sum_{s'=s}^{S} z^{\bar{t}}_{jkss'} + \sum_{j=1}^{n} b_{(m+i)j} \sum_{k=1}^{K} \sum_{\{\tilde{t} \leq \bar{t} \mid \bar{t} = \tilde{t}+k\}} \sum_{s=1}^{S} \sum_{s'=s}^{S} z^{\tilde{t}}_{jkss'} \leq x^{\bar{t}}_i(t) \quad \forall\, i\ \bar{t}, \tag{13}$$

$$\sum_{j \in IT_l} \sum_{s'=1}^{s} z^{\bar{t}}_{jks's} = y^{\bar{t}}_{lks} \quad \forall\, l,\ k,\ \bar{t},\ s, \tag{14}$$

$$\sum_{k \in K} y^{\bar{t}}_{lks} + \sum_{\tilde{t}=1}^{\bar{t}-1} \sum_{\tilde{k}=(\bar{t}-\tilde{t})+1}^{K} y^{\tilde{t}}_{l\tilde{k}s} \leq \xi^{\bar{t}}_{ls}(t) \quad \forall \ \bar{t}, \ l, \ s, \tag{15}$$

$$z, y \geq 0, \text{ integer.} \tag{16}$$

Constraints (12) state that the tour operator cannot allocate more holiday packages to the initial booking requests than the average demand $\bar{d}$. Constraints (13) control the availability of seats in all the legs involved in the itinerary. Equations (14) are link constraints between z and y variables. Constraints (15) control the availability of rooms at each destination for the entire length of stay. Indeed, it states that the number of rooms occupied at $\bar{t}$ plus those filled before $\bar{t}$ but still busy have to be less than or equal to the number of rooms available at $\bar{t}$.

3.2.2. Mathematical formulation for HPPS2

To formulate HPPS2, we need to introduce the following parameters and variables:

- $Q_{jkss'}$ the probability that a customer (with a request characterized by itinerary $j \in J$ and length of stay $k = 1, \ldots, K$), whose initial request to stay at a hotel with $s \in S$ stars is rejected, wishes to buy up to a hotel with $s' \in S, \ s' > s$ stars;
- $z^{\bar{t}}_{jks}$ integer variable indicating the number of accepted holiday packages with itinerary $j \in J$, departure time at $\bar{t} = 1, \ldots, \bar{T}$ and length of stay $k = 1, \ldots, K$ in a hotel with $s \in S$ stars;
- $z^{\bar{t}}_{jkss'}$ integer variable indicating the number of upgraded holiday packages from a hotel with $s \in S$ stars to a hotel with $s' \in S, \ s' > s$ stars with itinerary $j \in J$, departure time at $\bar{t} = 1, \ldots, \bar{T}$ and length of stay $k = 1, \ldots, K$; $z^{\bar{t}}_{jkss'}$ is the upgraded booking limit of capacity when a customer (with a request characterized by itinerary $j \in J$ and length of stay $k = 1, \ldots, K$), whose initial request of booking a hotel with s star is rejected, wishes to upgrade to a hotel with s' star with probability $Q_{jkss'}$.

The total revenue achievable by the tour operator at time t, when the network capacity is $X(t)$, can be calculated by solving the following holiday package problem with S categories, that is, HPPS2:

$$R^{HPPS2}(X(t)) = \max \sum_{\bar{t}=1}^{\bar{T}} \sum_{k=1}^{K} \sum_{j=1}^{n} \sum_{s=1}^{S} (R^k_{js} z^{\bar{t}}_{jks} + \sum_{s'=s}^{S} R^k_{js'} z^{\bar{t}}_{jkss'}), \tag{17}$$

$$z^{\bar{t}}_{jks} \leq \bar{d}^{\bar{t}}_{jks} \quad \forall \, k, \, j, \ \bar{t}, \ s, \tag{18}$$

$$z^{\bar{t}}_{jkss'} \leq (\bar{d}^{\bar{t}}_{jks} - z^{\bar{t}}_{jks}) Q_{jkss'} \quad \forall \, k, \, j, \ \bar{t}, \ s, \ s' = s, \ldots, S, \tag{19}$$

$$\sum_{j=1}^{n} b_{ij} \sum_{k=1}^{K} \sum_{s=1}^{S} (z^{\bar{t}}_{jks} + \sum_{s'=s}^{S} z^{\bar{t}}_{jkss'}) + \sum_{j=1}^{n} b_{(m+i)j} \sum_{k=1}^{K} \sum_{\{\tilde{t} \leq \bar{t} | \bar{t} = \tilde{t}+k\}} \sum_{s=1}^{S} (z^{\tilde{t}}_{jks} + \sum_{s'=s}^{S} z^{\tilde{t}}_{jkss'}) \leq x^{\bar{t}}_{i}(t) \quad \forall\ i\ \bar{t}, \tag{20}$$

$$\sum_{j \in IT_l} (z^{\bar{t}}_{jks} + \sum_{s'=1}^{s} z^{\bar{t}}_{jks's}) = y^{\bar{t}}_{lks} \quad \forall\ l,\ k,\ \ \bar{t},\ s, \tag{21}$$

$$\sum_{k \in K} y^{\bar{t}}_{lks} + \sum_{\tilde{t}=1}^{\bar{t}-1} \sum_{\tilde{k}=(\bar{t}-\tilde{t})+1}^{K} y^{\tilde{t}}_{l\tilde{k}s} \leq \xi^{\bar{t}}_{ls}(t) \quad \forall\ \bar{t},\ l,\ s, \tag{22}$$

$$z, y \geq 0, \text{integer}. \tag{23}$$

Constraints (18) state that the tour operator cannot allocate more holiday packages to the initial booking requests than the average demand $\bar{d}$. Constraints (19) state that the capacity allocated to the buy-up booking requests must not exceed the number of upgrade booking requests for each product. Constraints (20), (21), and (22) are of the same type as (13), (14), and (15), respectively.

It can be proven that the sets of constraints related to HPP and HPPS1 are defined over a totally unimodular matrices (see the Appendix). Hence, being the right-hand side of each constraint an integer number, solving the linear relaxation of both HPP and HPPS1 we obtain integer optimal solutions. The constraint matrix of HPPS2 is not totally unimodular since the variable $z^{\bar{t}}_{jks}$ is multiplied by $Q_{jkss'}$. In the sequel, we define primal and dual policies. Hence, we consider the linear relaxations for HPP and HPPS1. For HPPS2, we have to deal with the integer linear formulation when considering the primal policy in order to consider optimal integer solutions, whereas the linear relaxation has to be considered for the dual policy in order to retrieve dual information.

4. Revenue-based primal and dual policies

In this section, we provide partitioned booking limits and bid price controls to accept or reject a request (Talluri and Ryzin, 2004b), based on the resolution of the formulations proposed in Section 3.

In the partitioned booking limit control, a fixed amount of capacity for each resource is allocated to every product offered. The demand for each product has access only to its allocated capacity, and no other product may use this capacity.

In contrast, a bid price control policy sets a threshold price or bid price for each resource (Talluri and Ryzin, 1998). Roughly speaking, this bid price is an estimate of the marginal cost of consuming the next incremental unit of the resources capacity. When a booking request for a product arrives, the revenue of the request is compared to the sum of the bid prices of all the resources required by the product. If the revenue exceeds the sum of the bid prices, the request is accepted provided

Algorithm 1. $\mathcal{BLP}_{\mathcal{HPP}}$ scheme.

for $t = 1, \ldots, T$ **do**
  Solve the linear relaxation of $R^{HPP}(X(t))$ to obtain optimal solution $z^{\star\bar{t}}_{jk}$;
  for *each arrived request* $(j, k, \bar{t})$ *at time t* **do**
    if $z^{\star\bar{t}}_{jk} > 0$ **then**
      Accept the request;
      Set $z^{\star\bar{t}}_{jk} = z^{\star\bar{t}}_{jk} - 1$;
      Update the capacity,.that is;
      Set $x^{\bar{t}}_{i}(t) = x^{\bar{t}}_{i}(t) - b_{ij}, \forall i = 1, \ldots, m$;
      Set $x^{\bar{t}+k}_{i}(t) = x^{\bar{t}+k}_{i}(t) - b_{(m+i)j}, \forall i = 1, \ldots, m$;
      Set $x^{\tilde{t}}_{m+l}(t) = x^{\tilde{t}}_{m+l}(t) - 1, \forall \tilde{t} = \bar{t}, \ldots, \bar{t} + k - 1, l = D_j$;
      Calculate the revenue obtained from accepting the request;
    else
      Deny the request;
  Set $X(t+1) = X(t)$;

that all the resources associated with the requested product are still available; if not, the request is rejected.

It is worth noting that the linear relaxations of the defined formulations have to be solved in order to implement the bid price policy.

4.1. Booking limit and bid prices policies for the HPP

With reference to the HPP defined in Section 3.1, optimal solutions $z^{\star\bar{t}}_{jk},\ j \in J,\ k = 1, \ldots, K,\ \bar{t} = 1, \ldots, \bar{T}$, give partitioned booking limits while bid prices are formed from optimal dual variables $\pi^{\bar{t}}_{i}, \bar{t} = 1, \ldots, \bar{T}, i = 1, \ldots, m$, of constraints (7) and optimal dual variables $\rho^{\bar{t}}_{l}, \bar{t} = 1, \ldots, \bar{T},\ l \in E$ associated with constraints (9). We highlight that since the constraint matrix of HPP is totally unimodular and the right-hand side of each constraint is an integer number, the linear relaxation of HPP can be solved by obtaining optimal integer solutions. The partitioned booking limit policy and the bid price policy based on the dual formulation can be formally stated as follows.

At a certain instant time t of the booking horizon, decisions about either accepting or denying holiday package requests are to be made. In particular, for each instant time $t = 1, \ldots, T$, the linear relaxation of $R^{HPP}(X(t))$ is solved obtaining the optimal primal solution, that is, $z^{\star\bar{t}}_{jk}, y^{\star\bar{t}}_{lk}$ and dual solution, that is, $\pi^{\star\bar{t}}_{i}, \rho^{\star\bar{t}}_{l}$. For each request, arrived at instant time t, policies are applied in order to accept or deny such a request.

From a primal viewpoint, the strategy to be adopted is a partitioned booking limits policy ($\mathcal{BLP}$, for short). The procedure $\mathcal{BLP}$ based on HPP, named $\mathcal{BLP}_{\mathcal{HPP}}$, is depicted in Algorithm 1.

From a dual viewpoint, it is necessary to solve the linear relaxation of problem $R^{HPP}(X(t))$ and to deal with the dual variables associated with the capacity constraints. We will indicate with $\mathcal{BPP}$ the bid price policy associated with the dual formulation.

The procedure of $\mathcal{BPP}$ based on HPP, named $\mathcal{BPP}_{\mathcal{HPP}}$, is depicted in Algorithm 2.

Algorithm 2. $\mathcal{BPP}_{\mathcal{HPP}}$ scheme

for $t = 1, \dots, T$ **do**
  Solve the linear relaxation of $R^{HPP}(X(t))$ to obtain the dual variables $\pi_i^{\bar{t}}$ and $\rho_l^{\bar{t}}$;
  for *each arrived request* $(j, k, \bar{t})$ *at time t* **do**
    if $R_j^k \geq \sum_{i=1}^{m} b_{ij}\pi_i^{\bar{t}} + \sum_{i=1}^{m} b_{(m+i)j}\pi_i^{\bar{t}+k} + \sum_{\tilde{t}=\bar{t}}^{\bar{t}+k} \rho_l^{\tilde{t}}$ *and there is enough capacity* **then**
      Accept the request;
      Update the capacity;
      Set $x_i^{\bar{t}}(t) = x_i^{\bar{t}}(t) - b_{ij}, \forall i = 1, \dots, m$;
      Set $x_i^{\bar{t}+k}(t) = x_i^{\bar{t}+k}(t) - b_{(m+i)j}, \forall i = 1, \dots, m$;
      Set $x_{m+l}^{\tilde{t}}(t) = x_{m+l}^{\tilde{t}}(t) - 1, \forall \tilde{t} = \bar{t}, \dots, \bar{t} + k - 1, l = D_j$;
      Calculate the revenue obtained from accepting the request;
    else
      Deny the request;
  Set $X(t+1) = X(t)$;

For both Algorithms 1 and 2, the control about the capacity's availability requires that $x_i^{\bar{t}}(t) \geq b_{ij},\ \forall i = 1, \dots, m$; $x_i^{\bar{t}+k}(t) \geq b_{(m+i)j}, \forall i = 1, \dots, m$; $x_{m+l}^{\tilde{t}}(t) > 0, l = D_j,\ \forall\ \tilde{t} = \bar{t}, \dots, \bar{t} + k - 1$.

4.2. Booking limit and bid prices policies for the HPPS

In this section, we provide partitioned booking limits and bid price policies for both HPPS1 and HPPS2 defined in Section 3.2.

Revenue policies for HPPS1. It is worth noting that since the constraint matrix of HPPS1 is totally unimodular and the right-hand side of each constraint is an integer number, the linear relaxation of HPPS1 can be solved by obtaining optimal integer solutions. We recall that the tour operator can assign to customers, requiring hotel category s, a higher hotel category at the price of category s.

From a primal viewpoint, the strategy adopted is a partitioned booking limits policy. The procedure of $\mathcal{BLP}$ based on HPPS1, named $\mathcal{BLP}_{HPPS1}$, is depicted in Algorithm 3.

Let us indicate with $\pi_i^{\bar{t}}, \bar{t} = 1, \dots, \bar{T}, i \in I$ and $\rho_{ls}^{\bar{t}}, \bar{t} = 1, \dots, \bar{T},\ l \in E\ s \in S$ the dual variables associated with constraints (13) and (15), respectively.

A bid price policy $\mathcal{BPP}$ based on HPPS1, named $\mathcal{BPP}_{HPPS1}$, is depicted in Algorithm 4.

For both Algorithms 3 and 4, the control about the capacity's availability requires that $x_i^{\bar{t}}(t) \geq b_{ij},\ \forall i = 1, \dots, m$; $x_i^{\bar{t}+k}(t) \geq b_{(m+i)j},\ \forall i = 1, \dots, m$; $\xi_{ls'}^{\tilde{t}}(t) > 0, l = D_j, \forall\ \tilde{t} = \bar{t}, \dots, \bar{t} + k - 1$.

Revenue policies for HPPS2. Now, we discuss the possibility that the customer can upgrade with a certain probability, that is, we consider HPPS2. In this case, the variables $z_{jks}^{\bar{t}}$ are multiplied by the probability of buy-up $Q_{jkss'}$ (see constraint (19)). It follows that solving the linear relaxation does not provide the optimal integer solution. Hence, when considering the bid prices policy, we do not use optimal dual information, but rather a dual lower bound on the optimal integer solution. A deep study of the buy-up case with reference to the airlines can be found in Jiang and Miglionico (2014).

Algorithm 3. $\mathcal{BLP}_{HPPS1}$ scheme

for $t = 1, \ldots, T$ **do**
    Solve the linear relaxation of $R^{HPPS1}(X(t))$ to obtain optimal solution $z^{\star\bar{t}}_{jkss'}$;
    for *each arrived request* $(j, k, s, \bar{t})$ *at time t* **do**
        Set $accepted = false$;
        for $s' = s, \ldots, S$ **do**
            if $accepted$ = *false* **then**
                if $z^{\star\bar{t}}_{jkss'} > 0$ **then**
                    Accept the request;
                    Set $accepted = true$;
                    Set $z^{\star\bar{t}}_{jkss'} = z^{\star\bar{t}}_{jkss'} - 1$;
                    Update the capacity;
                    $x^{\bar{t}}_i(t) = x^{\bar{t}}_i(t) - b_{ij}, \forall i = 1, \ldots, m$;
                    $x^{\bar{t}+k}_i(t) = x^{\bar{t}+k}_i(t) - b_{(m+i)j}, \forall i = 1, \ldots, m$;
                    $\xi^{\tilde{t}}_{ls'}(t) = \xi^{\tilde{t}}_{ls'}(t) - 1, \forall\ \tilde{t} = \bar{t}, \ldots, \bar{t} + k - 1, l = D_j$;
                    Calculate the revenue obtained from accepting the request;
        if $accepted$ = *false* **then**
            Deny the request;
    Set $X(t+1) = X(t)$;

At any instant time t of the booking horizon, the tour operator has to make the following decisions:

(D1) Accept or reject a booking request of class k in a hotel with s stars and itinerary j, with departure at time $\bar{t}$.
(D2) In the case of rejection in D1 and if the customer wants to buy up to a hotel with $s' > s$ stars, accept or reject the customer's new booking request.

We next formally describe the partitioned booking limit and bid price policies based on the HPPS2, named $\mathcal{BLP}_{HPPS2}$ and $\mathcal{BPP}_{HPPS2}$, respectively. It is worth noting that the $\mathcal{BLP}_{HPPS2}$ and $\mathcal{BPP}_{HPPS2}$ schemes, depicted in what follows, have to be performed at each instant time $t = 1, \ldots, T$ of the booking horizon. For each instant time t, the problem $R^{HPPS2}(X(t))$ is solved when considering $\mathcal{BLP}_{HPPS2}$, whereas its linear relaxation is solved for $\mathcal{BPP}_{HPPS2}$.

$\mathcal{BLP}_{HPPS2}$ **scheme**

Step BL1. A new booking request of product of class k in a hotel with s stars and itinerary j, with departure at time $\bar{t}$ arrives. Go to Step BL2.
Step BL2. If the number of bookings of the product j accepted from the first booking requests is less than or equal to $z^{*\bar{t}}_{jks} - 1$ and there are enough seats and rooms resources, then accept the booking request, update the capacity, and go to Step BL1. Otherwise, reject the booking request and go to Step BL3.
Step BL3. If the rejected customer does not wish to upgrade to any other product, go to Step BL1. Otherwise, the rejected customer wishes to upgrade to a hotel with $s' > s$ stars. If the number of accepted bookings of product s' which has been upgraded from product s

Algorithm 4. $\mathcal{BPP}_{HPPS1}$ scheme

for $t = 1, \dots, T$ **do**
  Solve the linear relaxation of $R^{HPPS1}(X(t))$ to obtain the dual variables $\pi_i^{\bar{t}}$ and $\rho_{ls}^{\bar{t}}$;
  for *each arrived request* $(j, k, s, \bar{t})$ *at time t* **do**
    Set $accepted = false$;
    for $s' = s, \dots, S$ **do**
      if $accepted$ = *false* **then**
        if $R_{js}^k \geq \sum_{i=1}^{m} b_{ij}\pi_i^{\bar{t}} + \sum_{i=1}^{m} b_{(m+i)j}\pi_i^{\bar{t}+k} + \sum_{\tilde{t}=\bar{t}}^{\bar{t}+k} \rho_{ls'}^{\tilde{t}}$ *and there is enough capacity* **then**
          Accept the request;
          Set $accepted = true$;
          Update the capacity;
          $x_i^{\bar{t}}(t) = x_i^{\bar{t}}(t) - b_{ij}, \forall i = 1, \dots, m$;
          $x_i^{\bar{t}+k}(t) = x_i^{\bar{t}+k}(t) - b_{(m+i)j}, \forall i = 1, \dots, m$;
          $\xi_{ls'}^{\tilde{t}}(t) = \xi_{ls'}^{\tilde{t}}(t) - 1, \forall\, \tilde{t} = \bar{t}, \dots, \bar{t} + k - 1, l = D_j$;
          Calculate the revenue obtained from accepting the request;
    if $accepted$ = *false* **then**
      Deny the request;
  Set $X(t+1) = X(t)$;

to product s' is less than or equal to $z^{\bar{t}}_{jkss'} - 1$, and there are enough seats and rooms resources, then accept the upgraded booking request, update the capacity, and go to Step BL1. Otherwise, reject the upgraded booking request and go to Step BL1.

$\mathcal{BPP}_{HPPS2}$ scheme

Step BP1. A new booking request of product of class k in a hotel with s stars and itinerary j with $j \in IT_l$, with departure at time $\bar{t}$ arrives. Go to Step BP2.

Step BP2. If $R_{js}^k - \left(\sum_{i=1}^m b_{ij}\pi_i^{\bar{t}} + \sum_{i=1}^m b_{(m+i)j}\pi_i^{\bar{t}+k} + \sum_{\tilde{t}=\bar{t}}^{\bar{t}+k} \rho_{ls}^{\tilde{t}}\right) >$ $\sum_{s'=s}^{S} \max\left\{R_{js'}^k - \left(\sum_{i=1}^m b_{ij}\pi_i^{\bar{t}} + \sum_{i=1}^m b_{(m+i)j}\pi_i^{\bar{t}+k} + \sum_{\tilde{t}=\bar{t}}^{\bar{t}+k} \rho_{ls'}^{\tilde{t}}\right), 0\right\} Q_{jkss'}$, and there are enough seats and rooms resources, then accept the booking request, update the capacity, and go to Step BP1. Otherwise, reject the booking request and go to Step BP3.

Step BP3. If the rejected customer does not wish to upgrade to any other product, go to Step BP1. Otherwise, the rejected customer wishes to upgrade to another product s'. If $R_{js'}^k \geq \sum_{i=1}^m b_{ij}\pi_i^{\bar{t}} + \sum_{i=1}^m b_{(m+i)j}\pi_i^{\bar{t}+k} + \sum_{\tilde{t}=\bar{t}}^{\bar{t}+k} \rho_{ls'}^{\tilde{t}}$ and there are enough seats and rooms resources, then accept the upgraded booking request, update the capacity, and go to Step BP1. Otherwise, reject the upgraded booking request and go to Step BP1.

5. Computational experiments

In this section, we present the numerical results obtained by testing the policies described in Section 4. All the models and policies have been implemented by using AIMMS 4.27, with Cplex 12.7

Table 1
Scenarios characteristics

Hub	α	$\|E\|$	$\|J\|$	m
H_1	20	21	420	40
H_2	40	41	1640	80
H_3	60	61	3660	120
H_4	80	81	6480	160

as the solver, on an intel(R) core(TM) i7-4720HQ CPU, 2.60 GHz, 8.00 GB RAM machine, with the Microsoft 10 operating system.

The computational analysis is divided into two parts. In the first one, we evaluate and compare the policies in terms of effectiveness. The related numerical results are reported in Section 5.2. The second part focuses on the efficiency of the proposed formulations. In particular, we analyze the computational effort in order to provide empirical evidence of the applicability of the proposed models to support the decision process of tour operators. The related numerical results are reported in Section 5.3. The computational experiments have been carried out on several scenarios, whose generation is described in Section 5.1

5.1. Instances generation

We generate different scenarios. In particular, as for the flights, we assume a hub and spokes airport configuration. Possible destinations can be all the cities where an airport is located. Assuming that α represents the number of spokes, each scenario is characterized by $|E| = \alpha + 1$ destinations, $|J| = \alpha(\alpha + 1)$ itineraries, and $m = 2\alpha$ legs.

Table 1 reports the number of spokes α, destinations $|E|$, itineraries $|J|$, and legs m for each considered scenario.

For each scenario, the length of stay has been set equal to $k \in \{1, 2, 3\}$ and the departure time to $\bar{t} \in \{1, 2, 3, 4, 5, 6, 7\}$. Thus, $\bar{T} = 7$. In addition, we consider a booking horizon $T = 2$. The leg and hotel capacities are generated using a uniform distribution in the range $[0, c]$, where $c \in \{25, 50, 75\}$. We considered three categories for the hotels, that is, $s \in \{1, 2, 3\}$. The revenue R^k_{js}, $j = 1, \ldots, n$, $k = 1$ is set equal to 5, 11, and 16 for s equal to 1, 2, and 3, respectively. It is incremented by a factor of 1.8 and 2.6 when the length of stay k is equal to 2 and 3, respectively.

For each test problem, the booking process was simulated 20 times. In each simulation run, the holiday package requests are randomly generated by applying a two-phase procedure. In the first phase, for each origin-destination pair, departure time, length of stay, and the number of stars (for the correspondent model), the number of holiday package requests is randomly generated according to a normal distribution, with a given expected demand and a given coefficient of variation, chosen randomly from the interval $[0, d]$, with $d \in \{10, 20, 30\}$ and $[0, 1]$, respectively. The requests generated by the procedure outlined above are then processed. In particular, at each time instant t in the booking horizon, a holiday package request, for which the booking arrival time is less than or equal to the considered booking instant, is chosen and the accept/deny decision is made based on one of the proposed policies. The resource availability is then updated and another booking

Table 2
Average revenue and number of accepted requests for each scenario, by varying the demand d for HPP

		Revenue					Requests				
d		H_1	H_2	H_3	H_4	Avg.	H_1	H_2	H_3	H_4	Avg.
10	$\mathcal{PK}$	19766	42806	63765	85398	52934	1361	2853	4216	5464	3473
	$\mathcal{FCFS}$	14218	28080	43973	54196	35117	1346	3050	4462	5338	3549
	$\mathcal{BLP}$	12752	**41333**	**59978**	**79112**	**48294**	847	2774	3991	5087	3175
	$\mathcal{BPP}$	**16150**	37159	57290	76559	46790	1125	2746	4065	5141	3269
20	$\mathcal{PK}$	21078	43194	64776	86522	53893	1427	2907	4306	5612	3563
	$\mathcal{FCFS}$	14805	27450	42973	54608	34959	1473	3258	4706	5739	3794
	$\mathcal{BLP}$	14044	**42340**	**61740**	**81740**	**49966**	930	2838	4113	5298	3295
	$\mathcal{BPP}$	**16561**	36693	56389	75968	46403	1189	2737	4100	5194	3305
30	$\mathcal{PK}$	21535	43299	65118	86894	54212	1444	2926	4352	5682	3601
	$\mathcal{FCFS}$	15143	26882	42262	54166	34613	1550	3343	4854	6002	3937
	$\mathcal{BLP}$	14607	**42551**	**62889**	**83016**	**50766**	963	2857	4210	5435	3366
	$\mathcal{BPP}$	**16778**	36300	55882	74701	45915	1221	2711	4151	5209	3323

request is processed. We move to the next booking time period when there are no more requests, arrived before t, that need to be evaluated.

It is worth observing that the value of the revenue is affected by the order in which the booking requests are processed. In our experiments, we solve the models, used to define the policies, a number of times equal to the length of the booking horizon. In each of the 20 simulation runs, all the requests for each test problem, are processed considering all the policies defined in Section 4.

5.2. *Effectiveness of the policies*

In this section, we analyze the revenue and the number of accepted requests obtained by considering a perfect knowledge of realized demand ($\mathcal{PK}$), the first-come first-served ($\mathcal{FCFS}$), the $\mathcal{BLP}$, and the $\mathcal{BPP}$ policies, for each problem, that is, HPP, HPPS1, and HPPS2. For HPPS2, we consider two different values for the probability of buy-up, that is, $Q \in \{0.4, 0.8\}$. In particular, $Q_{jkss'} = Q$ if $s' = s + 1$ (one category buy-up), and $Q_{jkss'} = Q/2$ if $s' = s + 2$ (two categories buy-up).

The analysis is conducted in order to compare the policies, for each problem, by considering the revenue obtained and the requests accepted by varying the demand d and the legs and hotels capacity c. In addition, we compute the percentage of revenue obtained by each policy with respect to that returned by $\mathcal{PK}$, that is, $\%R_{\text{pol}} = 100 \times \left(R_{\text{pol}}/R_{\mathcal{PK}}\right)$ with pol $\in \{\mathcal{FCFS}, \mathcal{BLP}, \mathcal{BPP}\}$, where R_{pol} is the revenue obtained by policy pol. We also analyze the behavior of the policies by varying the load factor LF computed as c/d. Hence, for the values of d and c considered, we have $LF \in \{0.1, 0.2, 0.3, 0.4, 0.6, 0.8, 1.2\}$.

Tables 2–7 report the average revenue and number of accepted requests for each considered problem and policy by varying the demand d and the legs and hotels capacity c. The bold entries indicate the highest obtained revenue among the proposed policies excluding the $\mathcal{PK}$ one.

5.2.1. Main insights from the computational analysis

For HPP, $\mathcal{BPP}$ behaves the best only for a scenario with a number of spokes α at least equal to 20. On average, $\mathcal{BLP}$ is the best performing version, especially for a high value of demand d. In addition, the higher the LF the higher the benefit of $\mathcal{BLP}$ with respect to $\mathcal{BPP}$.

For HPPS1, $\mathcal{BLP}$ behaves the best and the benefit increases with increasing demand d. The revenue obtained with $\mathcal{BPP}$ is lower than that obtained by $\mathcal{BLP}$ for each value of demand d and legs and hotels capacity c considered in this paper. In addition, the gain of $\mathcal{BLP}$ with respect to $\mathcal{BPP}$ increases at increasing values of LF. The revenue provided by $\mathcal{BLP}$ is higher than that returned by $\mathcal{BPP}$ for each value of LF considered in this paper.

For HPPS2, $\mathcal{BLP}$ remarkably outperforms $\mathcal{BPP}$ for each value of the demand d, legs and hotels capacity c, and LF. The bad performance of $\mathcal{BPP}$ can be attributed to the inaccurate dual information derived from the linear relaxation of HPPS2 that provides an upper bound on the optimal solution.

For all the considered problems, on average, $\mathcal{BPP}$ accepts a higher number of requests than those accepted by $\mathcal{BLP}$. However, the latter is able to accept more profitable requests compared to those accepted by $\mathcal{BPP}$. This result is mainly due to the characteristics of the considered problems. We recall that we consider two types of resources, that is, the number of seats for the legs and the number of rooms for the hotels, both of which are time dependent. In addition, the resources used by each request depend on the instant time in which the trip associated with the itinerary starts and on the length of stay. Hence, the decision to accept a request, when considering $\mathcal{BPP}$, is influenced by the estimated opportunity reward derived from a combination of the resources, which depend on both the departure and the return time for the legs as well as the outward time and the length of stay for the hotels.

In Sections 5.2.2, 5.2.3, and 5.2.4, we provide a detailed analysis of the effectiveness of the policies considering HPP, HPPS1, and HPPS2, respectively.

5.2.2. Analysis for HPP

The numerical results collected in Tables 2 and 3 highlight that for scenario H_1 (the smallest one) $\mathcal{BPP}$ behaves the best.

Looking at Table 2, $\mathcal{BPP}$ returns an average revenue that is 1.27, 1.18, and 1.15 times higher than that obtained by $\mathcal{BLP}$ for d equal to 10, 20, and 30, respectively. As observed, the gain of $\mathcal{BPP}$ with respect to $\mathcal{BLP}$ slightly decreases for increasing values of d. The same trend is observed by comparing $\mathcal{BPP}$ and $\mathcal{FCFS}$, where the former provides an average revenue that is 1.14, 1.12, and 1.11 times higher than that obtained by the latter, for d equal to 10, 20, and 30, respectively. Table 3 shows the same behavior by varying the legs and hotels capacity c. $\mathcal{BPP}$ is the best policy, for each value of c, considering scenario H_1. $\mathcal{BPP}$ gives an average revenue, that is 1.08, 1.19, and 1.24 times higher than that provided by $\mathcal{BLP}$. In this case, the higher the value of c the better $\mathcal{BPP}$ than $\mathcal{BLP}$. The same trend is observed by comparing $\mathcal{BPP}$ with $\mathcal{FCFS}$.

For the scenarios H_2, H_3, and H_4, $\mathcal{BLP}$ behaves the best, for each value of d and c. Even considering all the scenarios, the average revenue obtained with $\mathcal{BLP}$ is the highest (see column Avg. of Tables 2 and 3). In particular, $\mathcal{BLP}$ provides an average revenue that is 1.03, 1.08, and 1.11 (1.38, 1.43, and 1.47) times higher than that obtained with $\mathcal{BPP}$ ($\mathcal{FCFS}$) for values of d equal to 10, 20, and 30, respectively (see the Avg. column of Table 2). The gain in revenue of $\mathcal{BLP}$ with respect to

Table 3
Average revenue and number of accepted requests for each scenario, by varying the legs and hotels capacity c for HPP

c		Revenue					Requests				
		H_1	H_2	H_3	H_4	Avg.	H_1	H_2	H_3	H_4	Avg.
25	$\mathcal{PK}$	11337	22617	34013	45474	28360	751	1519	2257	2971	1875
	$\mathcal{FCFS}$	8014	13892	21823	28194	17981	835	1765	2582	3222	2101
	$\mathcal{BLP}$	8113	**21914**	**32949**	**43742**	**26680**	527	1478	2191	2858	1764
	$\mathcal{BPP}$	**8764**	18985	28575	38652	23744	638	1412	2118	2698	1716
50	$\mathcal{PK}$	21047	43170	64786	86527	53882	1423	2904	4313	5606	3562
	$\mathcal{FCFS}$	14889	27424	43041	54460	34954	1468	3249	4702	5746	3791
	$\mathcal{BLP}$	13919	**42371**	**61991**	**81947**	**50057**	923	2840	4134	5315	3303
	$\mathcal{BPP}$	**16616**	36284	56357	75630	46222	1167	2715	4104	5194	3295
75	$\mathcal{PK}$	29995	63513	94860	126813	78795	2057	4261	6304	8181	5201
	$\mathcal{FCFS}$	21263	41097	64344	80316	51755	2065	4637	6738	8111	5388
	$\mathcal{BLP}$	19371	**61939**	**89667**	**118179**	**72289**	1290	4150	5988	7646	4768
	$\mathcal{BPP}$	**24110**	54882	84630	112945	69142	1730	4067	6094	7652	4886

both $\mathcal{BPP}$ and $\mathcal{FCFS}$ increases with increasing values of demand d. Looking at the Avg. column of Table 3, we observe that the revenue achieved by $\mathcal{BLP}$ is 1.12, 1.08, and 1.05 (1.48, 1.43, and 1.40) times higher than that obtained by $\mathcal{BPP}$ ($\mathcal{FCFS}$), for values of c equal to 25, 50, and 75, respectively. In this case, the higher the value of c the lower the gain in terms of revenue of $\mathcal{BLP}$ with respect to both $\mathcal{BPP}$ and $\mathcal{FCFS}$.

Looking at the number of accepted requests by varying the demand d (Table 2), it is evident that, on average, $\mathcal{BLP}$ accepts a lower number of requests compared to $\mathcal{BPP}$, for each value of d except when d is equal to 30. The high revenue obtained by $\mathcal{BLP}$, for each value of d, can be justified by the fact that $\mathcal{BLP}$ is able to accept more profitable requests compared to those accepted by $\mathcal{BPP}$. Indeed, $\mathcal{BLP}$ accepts requests with an average revenue equal to 15.21, 15.17, and 15.08, whereas for $\mathcal{BPP}$ we have 14.31, 14.03, and 13.82, for d equal to 10, 20, and 30, respectively.

The same trend is observed by varying the legs and hotels capacity c (see Table 3). In this case, $\mathcal{BLP}$ accepts a higher number of requests than those accepted by $\mathcal{BPP}$, for each value of c except when c is equal to 75. $\mathcal{BLP}$ accepts requests with an average revenue equal to 15.13, 15.15, and 15.16, whereas for $\mathcal{BPP}$ we have 13.83, 14.03, and 14.15, for c equal to 25, 50, and 75, respectively.

Figure 3 shows the average percentage of revenue obtained by each policy with respect to $\mathcal{PK}$, that is, $\%R_{\text{pol}}$, by varying both d (see Fig. 3a) and c (see Fig. 3b).

The average $\%R_{\text{pol}}$ is equal to 93%, 86%, and 65% for $\mathcal{BLP}$, $\mathcal{BPP}$, and $\mathcal{FCFS}$, respectively. $\%R_{\mathcal{BLP}}$ tends to increase at increasing values of d. An opposite trend is observed by varying c. For both $\mathcal{BPP}$ and $\mathcal{FCFS}$, it is observed a decrease for increasing values of d, whereas the higher the value of c, the higher the average $\%R_{\mathcal{BPP}}$ and $\%R_{\mathcal{FCFS}}$. It is worth to be observed that $\mathcal{BLP}$ shows the best average percentage of revenue for each value of both d and c.

Figure 4 shows the trend of $\%R_{\text{pol}}$ by varying the load factor LF.

The results suggest that $\%R_{\mathcal{BLP}}$ is higher than $\%R_{\mathcal{FCFS}}$ for each value of LF. Figure 4 shows that $\%R_{\mathcal{BLP}}$ and $\%R_{\mathcal{BPP}}$ present the same value for $LF = 0.1$. $\%R_{\mathcal{BLP}}$ is higher than $\%R_{\mathcal{BPP}}$ for all other values of LF. In addition, $\%R_{\mathcal{BLP}}$ increases at increasing values of LF, whereas the higher LF, the lower both $\%R_{\mathcal{BPP}}$ and $\%R_{\mathcal{FCFS}}$.

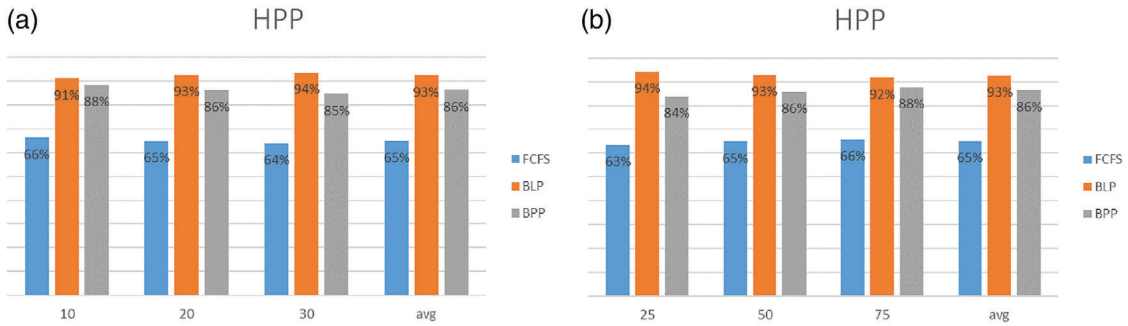

Fig. 3. Average percentage of revenue obtained by each policy with respect to $\mathcal{PK}$ for HPP.

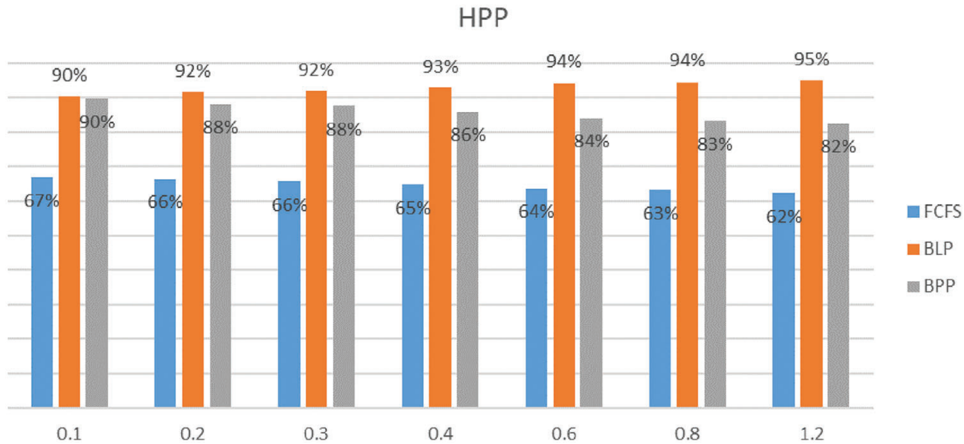

Fig. 4. Average percentage of revenue obtained by each policy with respect to $\mathcal{PK}$ for HPP by varying the load factor LF.

5.2.3. *Analysis for HPPS1*

Tables 4 and 5 show that $\mathcal{BLP}$ is the best performing version by varying both the demand d and the legs and hotels capacity c. In particular, the revenue obtained with $\mathcal{BLP}$ is 1.11, 1.21, and 1.27 (1.87, 2.19, and 2.42) times higher than that returned by $\mathcal{BPP}$ ($\mathcal{FCFS}$), for d equal to 10, 20, and 30, respectively. It is worth to be observed that the higher the value of d, the higher the gain of $\mathcal{BLP}$ with respect to both $\mathcal{BPP}$ and $\mathcal{FCFS}$.

The average number of accepted requests when $\mathcal{BLP}$ is applied is lower than that observed for $\mathcal{BPP}$, for each value of d but 30. The good behavior of $\mathcal{BLP}$ in terms of revenue is justified by the average revenue per accepted request. Indeed, this value is equal to 31.96, 31.52, and 31.35 for d equal to 10, 20, and 30, respectively. Whereas, for $\mathcal{BPP}$ we have 28.00, 25.89, and 24.84 for d equal to 10, 20, and 30, respectively. These results suggest that $\mathcal{BLP}$ is able to accept more profitable requests than those accepted by $\mathcal{BPP}$ for each value of d.

When considering the value of c (see Table 5), we observed that the higher c the higher the gain of $\mathcal{BLP}$ with respect to both $\mathcal{BPP}$ and $\mathcal{FCFS}$. In particular, the revenue obtained with $\mathcal{BLP}$ is 1.30, 1.20, and 1.15 (2.57, 2.19, and 1.99) times higher than that returned by $\mathcal{BPP}$ ($\mathcal{FCFS}$).

Table 4
Average revenue and the number of accepted requests for each scenario, by varying the demand d for HPPS1.

		Revenue					Requests				
d		H_1	H_2	H_3	H_4	Avg.	H_1	H_2	H_3	H_4	Avg.
10	$\mathcal{PK}$	66797	131795	190830	255903	161331	2120	4213	6052	7977	5091
	$\mathcal{FCFS}$	36927	68696	101565	125045	83058	2359	4965	7053	9244	5905
	$\mathcal{BLP}$	**64080**	**125974**	**184377**	**246744**	**155294**	2038	4024	5819	7556	4859
	$\mathcal{BPP}$	59693	114097	167550	218167	139877	2122	4166	5990	7704	4995
20	$\mathcal{PK}$	68997	134404	194111	259501	164253	2177	4330	6249	8196	5238
	$\mathcal{FCFS}$	33371	61022	90492	107721	73152	2536	5222	7414	9601	6193
	$\mathcal{BLP}$	**67037**	**130424**	**190053**	**253549**	**160266**	2132	4211	6085	7911	5085
	$\mathcal{BPP}$	57485	107349	160047	206544	132856	2197	4261	6154	7918	5132
30	$\mathcal{PK}$	69656	135350	195233	260526	165191	2203	4369	6322	8264	5290
	$\mathcal{FCFS}$	30932	56223	82731	98456	67085	2589	5290	7506	9646	6258
	$\mathcal{BLP}$	**68050**	**132269**	**192647**	**256512**	**162370**	2174	4271	6208	8062	5179
	$\mathcal{BPP}$	54299	101678	154102	201899	127994	2173	4270	6232	7938	5153

Table 5
Average revenue and number of accepted requests for each scenario, by varying the legs and hotels capacity c for HPPS1

		Revenue					Requests				
c		H_1	H_2	H_3	H_4	Avg.	H_1	H_2	H_3	H_4	Avg.
25	$\mathcal{PK}$	36207	70410	101629	135551	85949	1135	2246	3242	4247	2718
	$\mathcal{FCFS}$	15176	27489	40529	48518	32928	1344	2727	3875	4991	3234
	$\mathcal{BLP}$	**35481**	**69105**	**100408**	**133700**	**84673**	1121	2210	3194	4163	2672
	$\mathcal{BPP}$	27485	52108	78188	103564	65336	1109	2192	3204	4104	2652
50	$\mathcal{PK}$	68837	134401	193905	259339	164120	2178	4330	6237	8184	5232
	$\mathcal{FCFS}$	33378	60935	90026	107948	73072	2525	5210	7382	9558	6169
	$\mathcal{BLP}$	**66842**	**130182**	**189581**	**253717**	**160080**	2130	4192	6076	7911	5078
	$\mathcal{BPP}$	56880	107676	158709	208579	132961	2183	4237	6139	7896	5114
75	$\mathcal{PK}$	100406	196738	284640	381040	240706	3188	6335	9144	12006	7668
	$\mathcal{FCFS}$	52676	97517	144233	174756	117295	3616	7539	10715	13942	8953
	$\mathcal{BLP}$	**96844**	**189381**	**277089**	**369388**	**233175**	3093	6103	8842	11455	7373
	$\mathcal{BPP}$	87112	163340	244803	314467	202431	3199	6268	9032	11560	7514

The average number of accepted requests when $\mathcal{BLP}$ is applied is lower than that observed for $\mathcal{BPP}$, for each value of c but 25. The good behavior of $\mathcal{BLP}$ in terms of revenue is justified by the average revenue per accepted request. Indeed, this value is equal to 31.69, 31.53, and 31.62 for c equal to 25, 50, and 75, respectively. Whereas, for $\mathcal{BPP}$ we have 24.63, 26.00, and 26.94 for c equal to 25, 50, and 75, respectively. These results suggest that $\mathcal{BLP}$ is able to accept more profitable requests than those accepted by $\mathcal{BPP}$, for each value of c.

Figure 5 shows the average value of $\%R_{\text{pol}}$ by varying both d (see Fig. 5a) and c (see Fig. 5b).

The average $\%R_{\text{pol}}$ is equal to 97%, 82%, and 45% for $\mathcal{BLP}$, $\mathcal{BPP}$, and $\mathcal{FCFS}$, respectively. $\%R_{\mathcal{BLP}}$ tends to increase at increasing values of d. An opposite trend is observed by varying c. For both $\mathcal{BPP}$ and $\mathcal{FCFS}$, it is observed a decrease of $\%R_{\mathcal{BPP}}$ and $\%R_{\mathcal{FCFS}}$ for increasing values of d, whereas the higher the value of c, the higher the average $\%R_{\mathcal{BPP}}$ and $\%R_{\mathcal{FCFS}}$. It is worth to be observed that $\mathcal{BLP}$ shows the best average percentage of revenue for each value of both d and c.

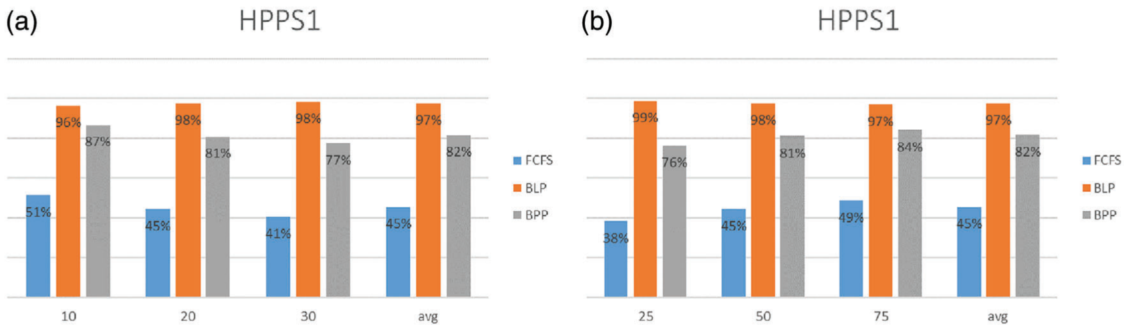

Fig. 5. Average percentage of revenue obtained by each policy with respect to $\mathcal{PK}$ for HPPS1.

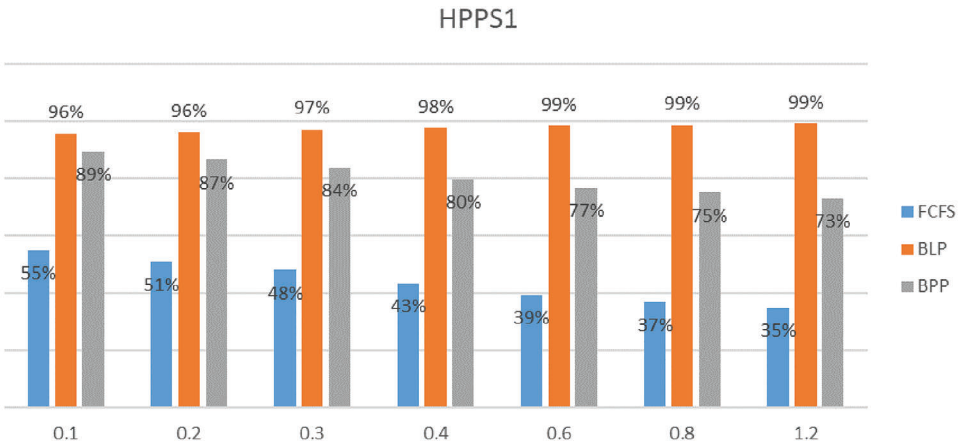

Fig. 6. Average percentage of revenue obtained by each policy with respect to $\mathcal{PK}$ for HPPS1 by varying the load factor LF.

Figure 6 shows the trend of $\%R_{\text{pol}}$ by varying the load factor LF.

The results suggest that $\%R_{\mathcal{BLP}}$ is higher than both $\%R_{\mathcal{BPP}}$ and $\%R_{\mathcal{FCFS}}$. In addition, $\%R_{\mathcal{BLP}}$ increases at increasing values of LF, whereas the higher LF, the lower both $\%R_{\mathcal{BPP}}$ and $\%R_{\mathcal{FCFS}}$.

5.2.4. *Analysis for HPPS2*

For HPPS2, we observed that the results with Q equal to 0.4 and 0.8 have small differences and the analysis gives us the same conclusion. Hence, we present the numerical results averaged over the two values of Q.

Tables 6 and 7 show that $\mathcal{BLP}$ is the best performing policy. In particular, $\mathcal{BLP}$ presents a revenue that is 1.49, 1.61, and 1.69 (1.50, 1.54, and 1.58) times higher than that observed with $\mathcal{BPP}$ ($\mathcal{FCFS}$) for the value of d equal to 10, 20, and 30, respectively. The gain of $\mathcal{BLP}$ with respect to both $\mathcal{BPP}$ and $\mathcal{FCFS}$ increases at increasing values of d.

Considering the results by varying the values of legs and hotels capacity -c, Table 7 shows that, on average, $\mathcal{BLP}$ is 1.74, 1.61, and 1.53 (1.62, 1.54, and 1.51) times higher than that returned by

Table 6
Average revenue and number of accepted requests for each scenario, by varying the demand d for HPPS2

		Revenue					Requests				
d		H_1	H_2	H_3	H_4	Avg.	H_1	H_2	H_3	H_4	Avg.
10	$\mathcal{PK}$	68297	133833	193446	259064	163660	2149	4297	6182	8149	5194
	$\mathcal{FCFS}$	43360	85188	128371	161920	104710	1899	4055	5867	7662	4871
	$\mathcal{BLP}$	**64501**	**128358**	**187029**	**250234**	**157530**	2033	4082	5913	7688	4929
	$\mathcal{BPP}$	45409	87387	127547	162223	105641	2297	4846	6899	9017	5765
20	$\mathcal{PK}$	70113	135745	195854	261291	165751	2211	4393	6357	8303	5316
	$\mathcal{FCFS}$	44864	86962	129135	160737	105424	2130	4470	6391	8333	5331
	$\mathcal{BLP}$	**67330**	**132689**	**192272**	**256627**	**162229**	2132	4267	6198	8050	5162
	$\mathcal{BPP}$	43887	83159	122900	152711	100664	2498	5156	7342	9474	6117
30	$\mathcal{PK}$	70594	136308	196479	261747	166282	2225	4430	6393	8344	5348
	$\mathcal{FCFS}$	45385	85917	125458	157874	103658	2259	4655	6611	8638	5541
	$\mathcal{BLP}$	**68109**	**134168**	**193957**	**258970**	**163801**	2155	4334	6296	8184	5242
	$\mathcal{BPP}$	42098	80434	119137	146968	97159	2559	5255	7479	9575	6217

Table 7
Average revenue and the number of accepted requests for each scenario, by varying the legs and hotels capacity c for HPPS2.

		revenue					requests				
c		H_1	H_2	H_3	H_4	avg	H_1	H_2	H_3	H_4	avg
25	$\mathcal{PK}$	36605	70778	102128	136050	86390	1140	2267	3277	4271	2739
	$\mathcal{FCFS}$	23111	43447	63667	80463	52672	1197	2438	3477	4541	2913
	$\mathcal{BLP}$	**35518**	**69769**	**100925**	**134529**	**85185**	1111	2226	3229	4205	2693
	$\mathcal{BPP}$	21113	40418	59857	74265	48913	1330	2714	3866	4966	3219
50	$\mathcal{PK}$	69962	135718	195621	261161	165616	2205	4394	6333	8294	5306
	$\mathcal{FCFS}$	45044	86488	127862	160283	104919	2128	4459	6367	8311	5316
	$\mathcal{BLP}$	**67107**	**132389**	**191899**	**256429**	**161956**	2126	4258	6176	8034	5148
	$\mathcal{BPP}$	43654	83294	122583	152902	100608	2479	5140	7313	9435	6092
75	$\mathcal{PK}$	102437	199391	288030	384890	243687	3240	6460	9321	12231	7813
	$\mathcal{FCFS}$	65455	128131	191434	239786	156201	2964	6283	9027	11781	7514
	$\mathcal{BLP}$	**97315**	**193056**	**280434**	**374873**	**236419**	3083	6198	9002	11683	7491
	$\mathcal{BPP}$	66626	127269	187143	234734	153943	3545	7403	10542	13664	8788

$\mathcal{BPP}$ ($\mathcal{FCFS}$) for values of c equal to 25, 50, and 75, respectively. In this case, the gain of $\mathcal{BLP}$ with respect to both $\mathcal{BPP}$ and $\mathcal{FCFS}$ decreases at increasing values of c.

Figure 7 shows the average value of $\%R_{\text{pol}}$, by varying both d (see Fig. 7a) and c (see Fig. 7b).

The values of $\%R_{\text{pol}}$ clearly highlight that $\mathcal{BLP}$ remarkably outperforms $\mathcal{BPP}$. The average $\%R_{\text{pol}}$ is equal to 99%, 61%, and 63% for $\mathcal{BLP}$, $\mathcal{BPP}$, and $\mathcal{FCFS}$, respectively. $\mathcal{BPP}$ gives less revenue than that provided by $\mathcal{FCFS}$. The worst performance of $\mathcal{BPP}$ is justified by the fact that it uses no optimal dual information. Indeed, the linear relaxation of HPPS2 does not provide an integer solution, but rather an upper bound on the optimal revenue. Hence, to decide whether to accept a request, $\mathcal{BPP}$ uses lower bound information of the dual problem of HPPS2. This allows $\mathcal{BPP}$ to accept not profitable requests.

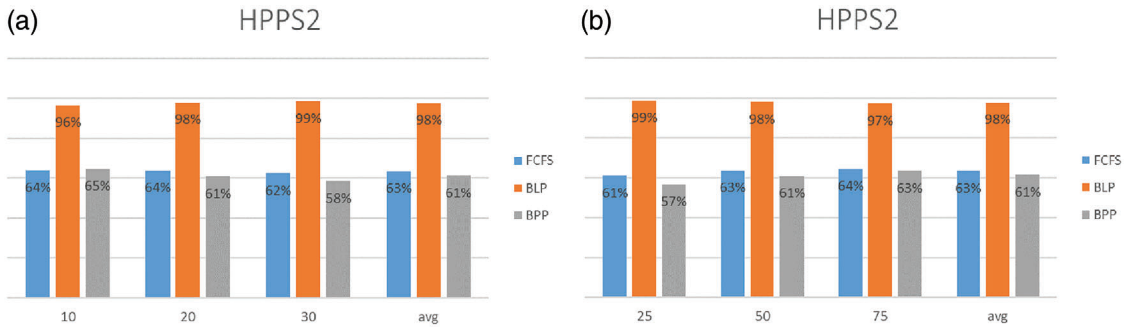

Fig. 7. Average percentage of revenue obtained by each policy with respect to $\mathcal{PK}$ for HPPS2.

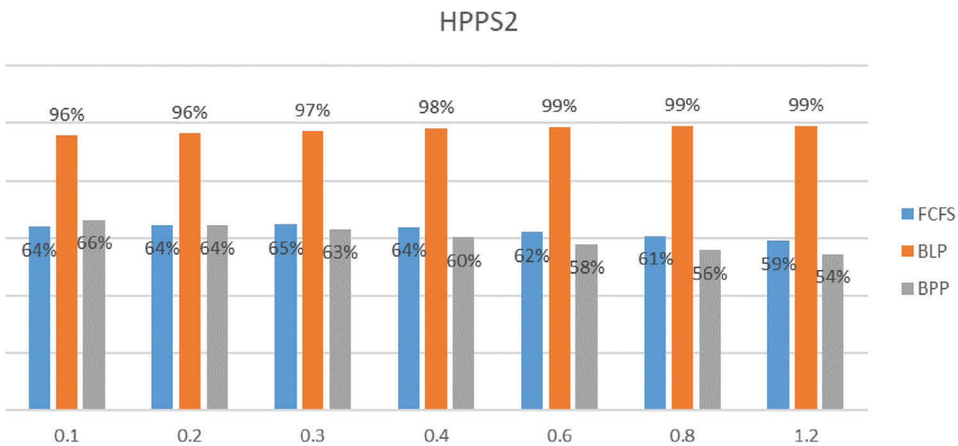

Fig. 8. Average percentage of revenue obtained by each policy with respect to $\mathcal{PK}$ for HPPS2 by varying the load factor LF.

$\%R_{\mathcal{BLP}}$ increases at increasing values of d, whereas an inverted trend is observed by varying the value of c. However, $\mathcal{BLP}$ provides a value of $\%R_{\mathcal{BLP}}$ at least equal to 96%, reaching a value of 99% for d equal to 30 and c equal to 25

Figure 8 shows the trend of $\%R_{\text{pol}}$ by varying the load factor LF.

The results suggest that $\mathcal{BLP}$ behaves the best with increasing values of $\%R_{\mathcal{BLP}}$ at an increase of LF. In addition, Fig. 8 confirms the bud performance of $\mathcal{BPP}$ with a value of $\%R_{\mathcal{BPP}}$ lower than $\%R_{\mathcal{FCFS}}$ for $LF \geq 0.3$.

5.3. *Efficiency of the proposed formulations*

In this section, we analyze the behavior of the proposed integer programming formulations. In particular, we show the execution time required to solve the proposed formulations by varying the parameters c and d, used to define the legs and hotel capacity and the demand, respectively.

We maintain the booking horizon T equal to 2. Hence, the formulations are solved twice, one for each time period, for each considered policy, that is, $\mathcal{BLP}$, $\mathcal{BPP}$, and each problem, that is, HPP,

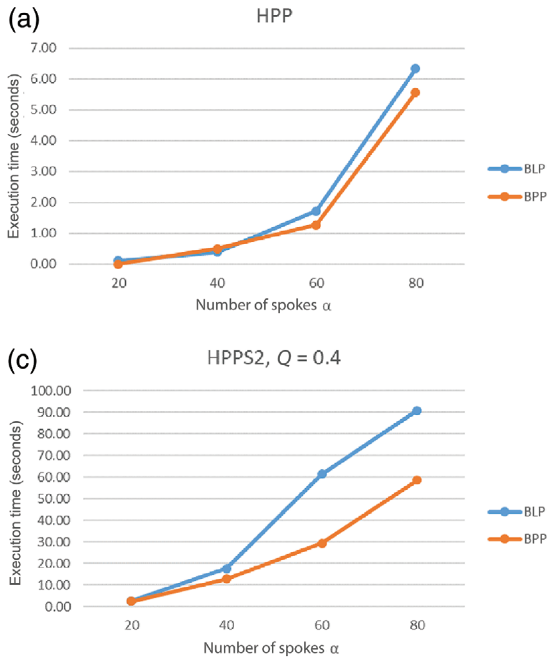

Fig. 9. Average execution time, is seconds, to solve the formulation by varying the parameter α, that is, the scenarios dimension.

HPPS1, and HPPS2. Each formulation is solved by considering the initial demand and capacity at the first time period. Whereas, for the second time period, the capacity is modified based on the requests accepted at the first time period and the demand is a half of the initial one.

In our analysis, we consider the execution time averaged over the two computational efforts due to the first and the second run (instant time of the booking horizon T).

We remark that the $\mathcal{FCFS}$ policy does not require any information on the resolution of the formulations.

Figure 9 reports the average execution time by varying the dimension of the scenarios defined by the number of spokes α involved.

The numerical results clearly show that the execution time grows at increasing scenario dimensions. This trend is observed for each problem. However, it is worth to be observed that the two policies, that is, $\mathcal{BLP}$ and $\mathcal{BPP}$, show the same behavior for HPP and HPPS1, whereas, for HPPS2 the former presents a higher increase in the computational effort than that observed for $\mathcal{BPP}$ for increasing values of α. This is mainly due to the fact that for $\mathcal{BPP}$ the linear relaxation of HPPS2 is solved. We note that, in the worst case (HPPS1), the formulations are solved within 200 seconds. This is an acceptable computational effort for the case under study. We recall that the tour operator run the formulation for each day of the booking horizon in order to retrieve the information from the optimal solution. This information is used to decide whether to accept the requests that arrive during the day. At the end of each day of the booking horizon, the operator runs the formulation which information is used the day after. Hence, the formulation is solved offline during the night, for instance.

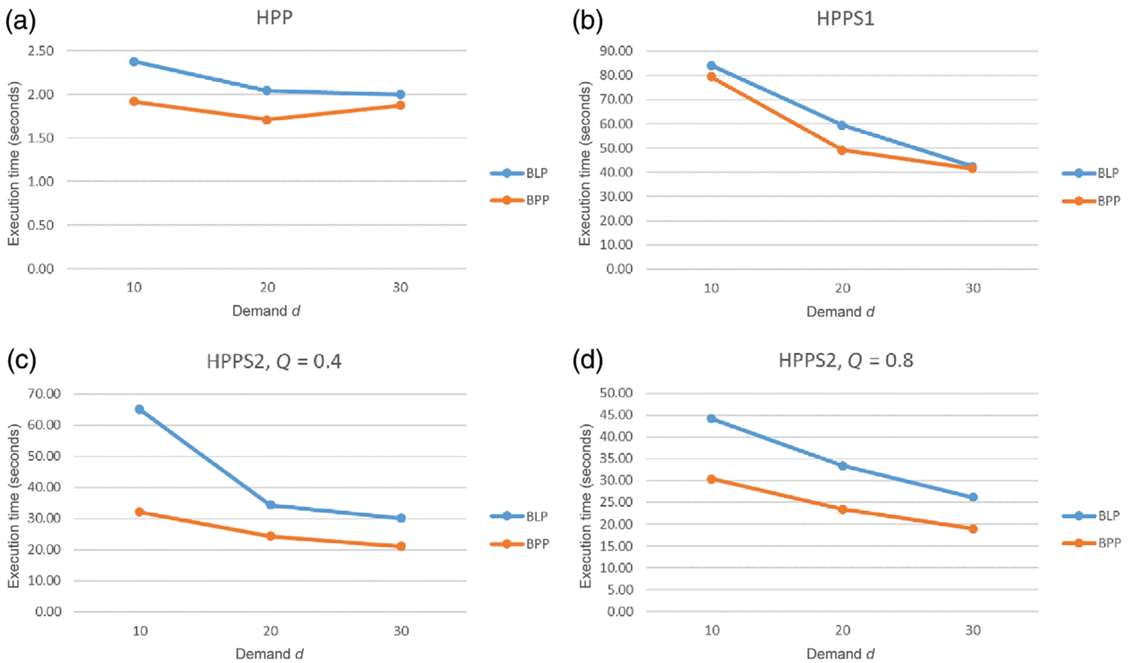

Fig. 10. Average execution time in seconds to solve the formulation by varying the parameter d, that is, the demand.

Figure 10 shows the average execution time, in seconds, by varying the demand, that is, parameter d.

Figure 10 shows that the execution time decreases at increasing values of the parameter d. This trend is observed for each problem. It is worth to be observed that for HPP $\mathcal{BPP}$ shows a similar execution time for each value of d.

Figure 11 shows the average execution time, in seconds, by varying the capacity of both the legs and the hotels, that is, parameter c.

Figure 11 shows that the execution time remains almost unchanged by varying the capacity c when considering HPP. Whereas, an almost linear increasing trend is observed for HPPS1 and HPPS2.

6. Conclusions

In the last few decades, revenue management techniques have been applied to several logistic problems arising mainly, but not only, in airline, hotel, and car rental industries.

In this paper, we focused on an operational problem faced by tour operators that are known to be strategical nodes of the tourism logistic chain. Tour operators have to decide whether to accept or reject a booking request from their customers with the aim of maximizing the total expected revenue. The products they sell, which we have called holiday packages, are complex since they are the combination of different resources, at least a return flight, and a certain number of nights in a hotel.

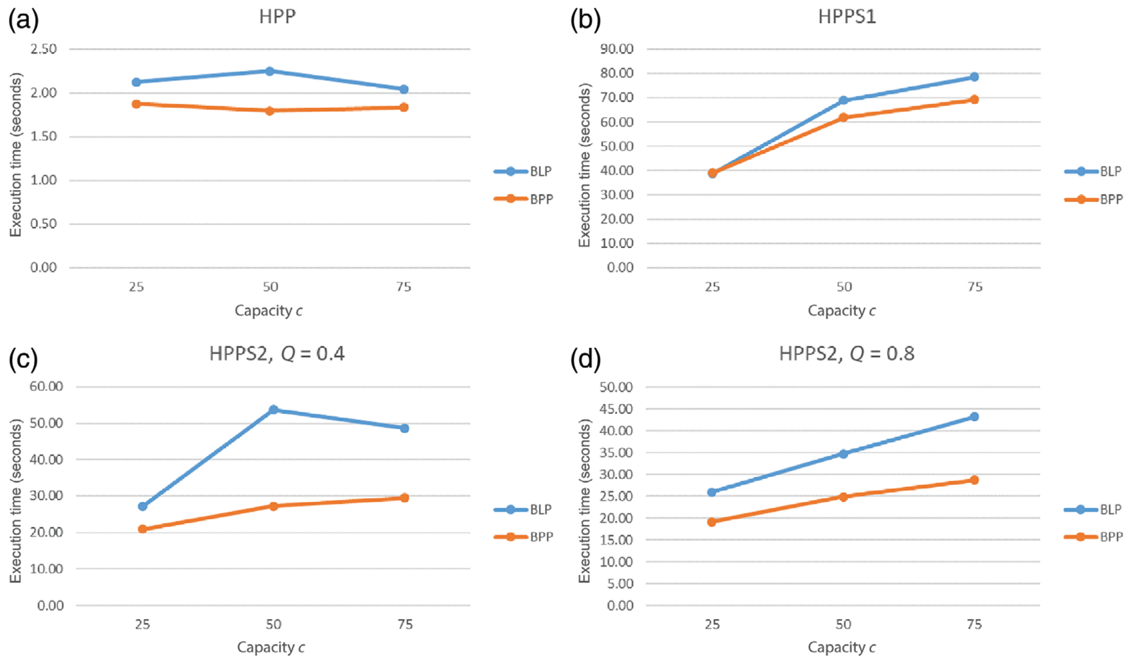

Fig. 11. Average execution time, in seconds, to solve the formulation by varying the parameter c, that is, the capacity of both legs and hotels.

A dynamic programming formulation and integer programming approximations of the problem under consideration have been defined. Based on the proposed integer programming models, borrowing revenue management techniques and primal and dual acceptance policies have been defined, which use partitioned booking limits and bid price controls. Models incorporating different hotel categories have been discussed together with the possibility of upgrading to a hotel of a higher category than that requested.

The performances of the different booking control policies are evaluated and the numerical results show that all the booking control policies, on average, perform better than the simple first-come first-served policy, opening the possibility of exploiting new policies in the case of more complex holiday package typologies.

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Appendix A

Lemma 1. *The constraints matrix of HPP is totally unimodular.*

Proof. To prove the totally unimodularity of the matrix, we may elaborate on the rows in order to obtain values equal to -1, 0, or 1 when sum over all rows, for each column.

We may express the constraints (6)–(9) related to the HPP as a matrix notation as follows:

$$M = \begin{pmatrix} 0 & I_J \\ 0 & B_z \\ I_E & A \\ B_y & 0 \end{pmatrix}$$

where the first row is related to the demand constraints (6), the second row is related to the leg capacity constraints (7), the third is related to the constraints (8) linking variables y and z, and the fourth row is related to the room capacity constraints (9). Recalling that $n = |J|$ is the number of itinerary, $m = |I|$ is the number of legs, $e = |E|$ is the number of destinations, K is the highest length of stay, and $\bar{T}$ is the operational horizon, we have that I_J is identity matrix with dimension $(n \times \bar{T} \times K) \times (n \times \bar{T} \times K)$, matrix B_z has dimension $(m \times \bar{T}) \times (n \times \bar{T} \times K)$, I_E has zero elements but -1 in the diagonal with dimension $(e \times \bar{T} \times K) \times (e \times \bar{T} \times K)$, matrix A has dimension $(e \times \bar{T} \times K) \times (n \times \bar{T} \times K)$, and matrix B_y has dimension $(e \times \bar{T}) \times (e \times \bar{T} \times K)$. The first column of M is related to variables y, whereas the second column is related to variables z. We first elaborate on the first column, that is, matrices I_E and B_y.

Elaboration on I_E and B_y. Matrix B_y has the following form:

$$B_y = \begin{pmatrix} B_y^{l_1} & 0 & \cdots & 0 \\ 0 & B_y^{l_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & B_y^{e} \end{pmatrix}$$

where each matrix B_y^l is related to the destination l. In the sequel, we elaborate on a specific submatrix B_y^l. All the considerations can be extended to all submatrices. Each column (l, t^c, k) of B_y^l

has a value equal to 1 derived from the first summation of constraints (9). Considering the second summation, in each row (l, t^r), presents a value equal to 1 if (1) $t^c + k > t^r$, meaning that at time t^r a room is occupied by a request arrived at time t^c for k time periods, and (2) $t^c < t^r$, hence a zero value is present for each row such that $t^c > t^r$. It follows that, given a column (l, t^c, k) and a row (l, t^r) of the matrix B^l_y, we have

1 if $t^c = t^r$;

1 if $t^c + k \leq t^r$;

0 if $t^c > t^r$ or $t^c + k > t^r$.

Hence, each column (l, t^c, k) has at most $\bar{T} - 1$ element equal to 1 associated with the rows (l, t) with $t = t^c, \ldots, t^c + k - 1$. We also observe that each column (l, t, k) with $k = 1$ has only one element equal to 1.

We can multiply by -1 the rows (l, t) of B_y for odd values of t. In order to obtain values equal to 1, 0, and -1 when sum over all rows associated with matrices B_y and I_E, we can elaborate on I_E as follows:

Each row (l, t, k) of I_E is multiplied by -1 if the following conditions hold:

(1) t is odd and $t + k \leq \bar{T}$;
(2) t is even and $t + k \leq \bar{T}$ with $k > 1$.

We observe that the modifications that occurred for matrix I_E affect matrix A.

Elaboration on I_J, B_z, and A. Let us consider matrix B_z that represents the leg capacity constraints (7). We remark that the case considered in this paper takes into account a structure of hub-spokes type for the airports. Hence, we have two legs connecting each spoke to the hub, named leg^1 and leg^2. We consider the set of legs ordered in such a way the first $|I|/2$ legs are of the type leg^1 and the remaining ones are of the type leg^2. We note that the set I contains an even number of legs.

Let I_j be the set of legs involved in the itinerary j. Then, each column (j, t^c, k) has one element equal to 1 for each row (u, t^r), $\forall u \in I_j$ with $t^r = t^c$, derived from the first summation (outward trip), and an element equal to 1 for each row (u, t^r), $\forall u \in I_j$ with $t^r = t^c + k$, derived from the second summation (return trip). Hence, we have exactly $|I_j|$ values equal to 1 for each column (j, t^c, k) such that $t^c + k > \bar{T}$, meaning that any return trip takes place out of the operational horizon. We can elaborate on matrix B_z multiplying by -1 the first $|I|/2$ rows. Summing over all rows of matrix B_z we have a value equal to 0 for each column (j, t, k) such that $t + k \leq \bar{T}$, whereas we have a value equal to either $-|I_j|$ if the itinerary j is composed of outward legs in the first $|I|/2$ positions of the set I or $|I_j|$ if the outward legs of the itinerary j are in the last $|I|/2$ positions of the set I, for each column (j, t, k) such that $t + k > \bar{T}$. Summing up, we have the following situation when sum all rows of matrix B_z.

(1) 0 for column (j, t^c, k) such that $t^c + k \leq \bar{T}$;
(2.1) $-|I_j|$ for column (j, t^c, k) such that $t^c + k > \bar{T}$ and j is composed of outward legs in the first $|I|/2$ positions of the set I;
(2.2) $|I_j|$ for column (j, t^c, k) such that $t^c + k > \bar{T}$ and j is composed of outward legs in the last $|I|/2$ positions of the set I.

Let now consider the matrices I_J and A. We remark that matrix A is modified according to the elaborations applied for matrix I_E. We multiply by -1 all rows (j, t^r, k) of I_J such that t^r is even,

$k = 1$, and $t^r + k < \bar{T}$ and by an integer number v_j all rows (j, t^r, k) of I_J such that $t^r + k > \bar{T}$. Summing over all rows of both I_J and A, we obtain

(1) 0 for column (j, t^c, k) such that $t^c + k \leq \bar{T}$;
(2) $1 + v_j$ for column (j, t^c, k) such that $t^c + k > \bar{T}$.

Recalling that the sum over all rows of matrix B_z is equal to 0 for column (j, t^c, k) such that $t^c + k \leq \bar{T}$, and $-|I_j|$ (or $|I_j|$) for column (j, t^c, k) such that $t^c + k > \bar{T}$, summing over all rows associated with matrices I_J, A, and B_z, we obtain the following situations:

(1) 0 for column (j, t^c, k) such that $t^c + k \leq \bar{T}$;
(2) $1 + v_j - |I_j|$ (or $1 + v_j + |I_j|$) for column (j, t^c, k) such that $t^c + k > \bar{T}$.

Hence, we can always choose a value for v_j such that $1 + v_j - |I_j| = -1$, or 0, or 1 (or $1 + v_j + |I_j| = -1$, or 0, or 1).

This concludes the proof. □

Applying the same rationale for HPPS1, we can prove that the associated constraints matrix is totally unimodular.