Observation of Anderson localization of light in a nonlocal nonlinear medium

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Outline

- Introduction
- Experiments
  - Disordered fiber
  - Multi-color disorder localized states
  - Power dependent localization length
  - Evidence of a nonlocal effect
- Theory

PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

Absence of Diffusion in Certain Random Lattices

P. W. ANDERSON
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received October 10, 1957)

This paper presents a simple model for such processes as spin diffusion or conduction in the "impurity band." These processes involve transport in a lattice which is in some sense random, and in them diffusion is expected to take place via quantum jumps between localized sites. In this simple model the essential randomness is introduced by requiring the energy to vary randomly from site to site. It is shown that at low enough densities no diffusion at all can take place, and the criteria for transport to occur are given.

- Above a certain amount of disorder no transport is possible „Anderson localization"

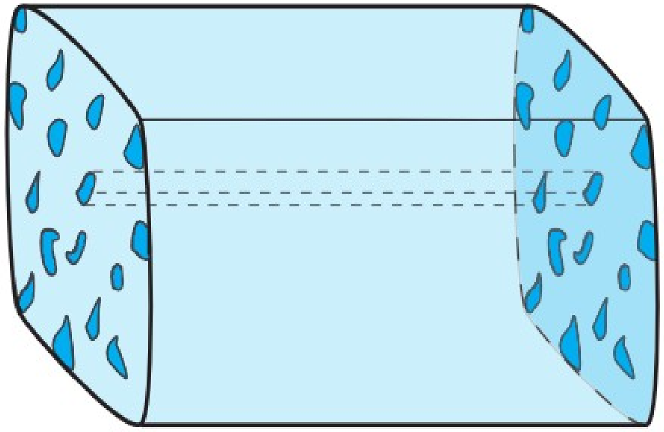
- The reason: localized states due to disorder

Literature

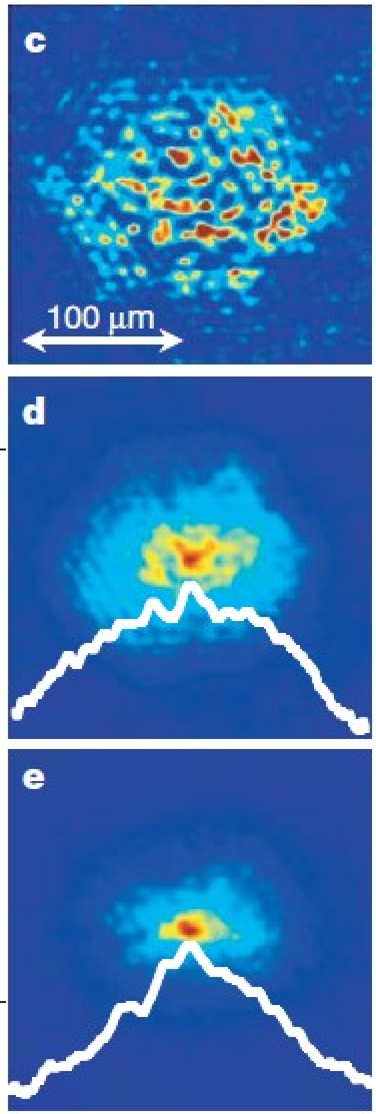
- Observation of Anderson localization in
  - Nonlinear Optics
    - Y. Lahini et al. PRL 100, 013806 (2008)
    - T. Schwartz, G. Bartal, S. Fishman, M. Segev, Nature 446, 52 (2007)
  - Bose-Einstein condensation
    - J. Billy et al. Nature 453, 891 (2008)
    - G. Roati et al. Nature 453, 895 (2008)
    - S. S. Kondov, Science 66, 334 (2011)
  - Linear disordered media (optics)
    - M. Storzer, P. Gross, C. M. Aegerter, G. Maret, PRL 96, 063904 (2006)
    - A. A. Chabanov, M. Stoytchev, A. Z. Genack, Nature 404, 850 (2000)
    - T. Sperling at al, Nature Photonics 7, 48 (2013)

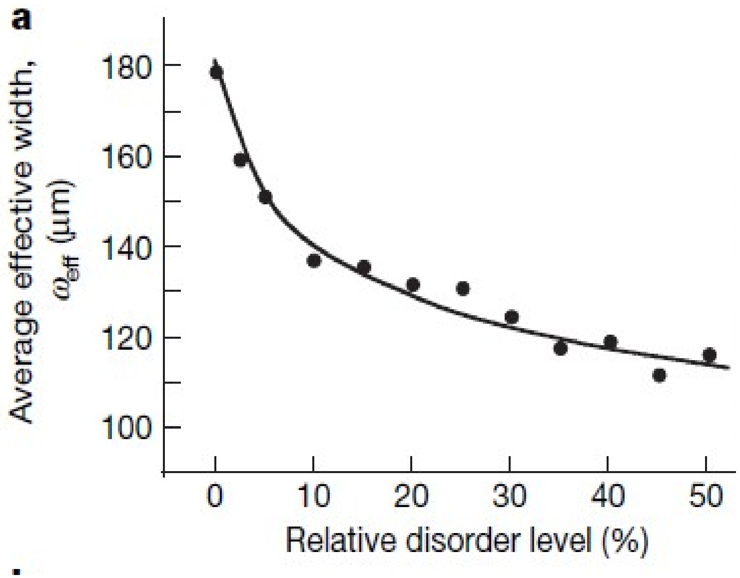
TRANSVERSE Anderson Loc

T. Schwartz, G. Bartal, S. Fishman, M. Segev, Nature 446, 52 (2007)



INDEX CONTRAST 0.0001
PROPAGATION 1cm

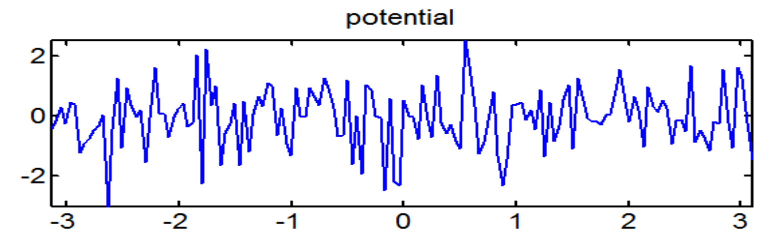




The simplest model

Linearly localized states

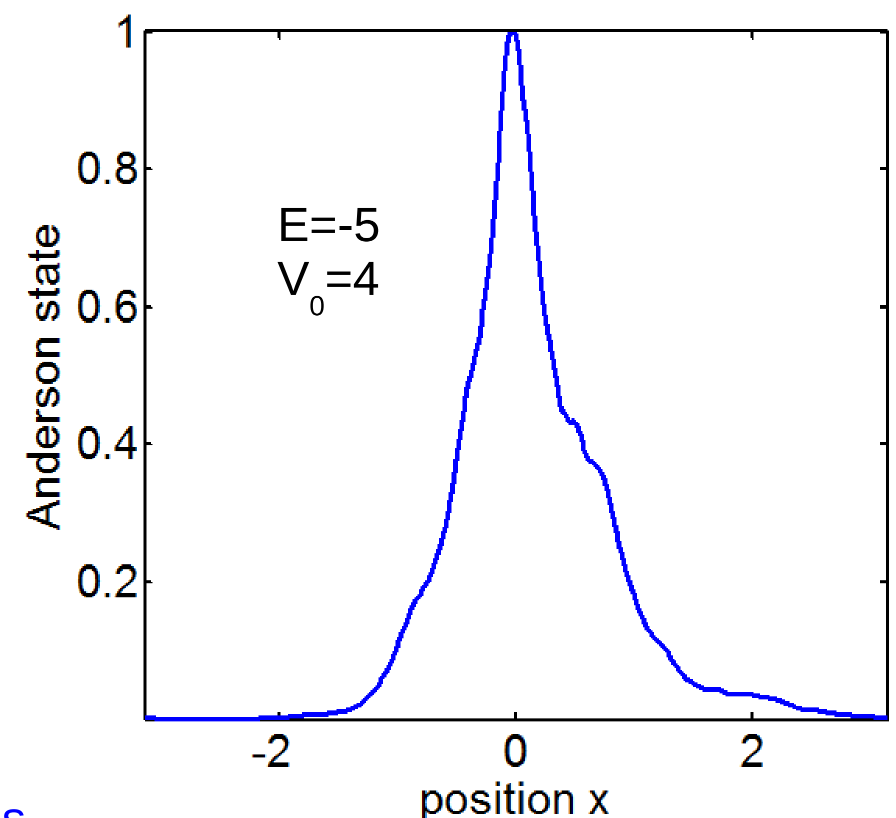
$$-\varphi_{xx} + V(x)\varphi = \mathcal{L}\varphi = E\varphi,$$



- Gaussian potential
- Negative eigenvalues
- Decays as $\exp(-\sqrt{-E}|x|)$
- **Link between**

  localization length and

  eigenvalue



See book
Lifshitz, Gredskul, Pastur
Introduction to theory of disordered systems

Also
CC, PRA 86,061801 (2012)

The statistical distribution of eigenvalues

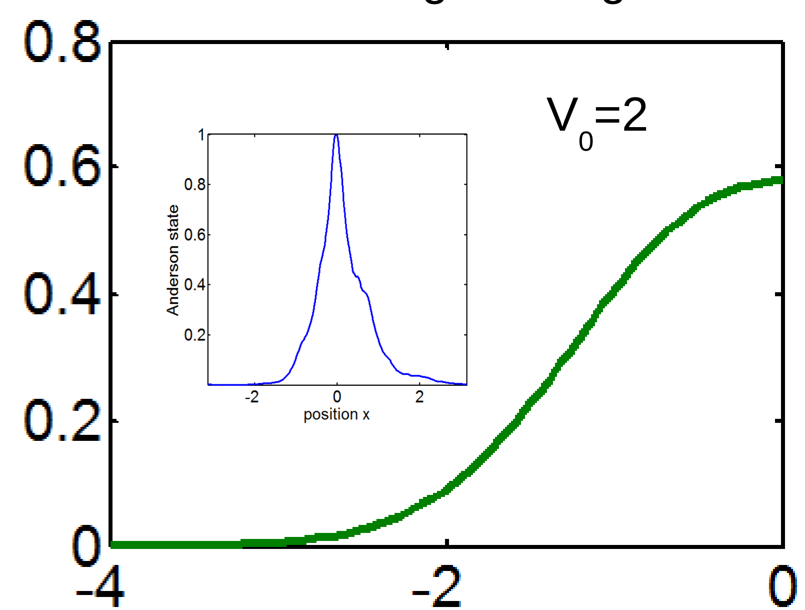
- There is a tail of negative energies corresponding to

exponentially highly localized states

$$\langle V(x)V(x')\rangle = V_0^2\delta(x-x')$$

$$\overline{E}_L \cong -V_0^{4/3}/3$$

Distribution of negative eigenvalues



Energy E

The localization length decreases as the Inverse square root of the |energy|, hence the localization length decreases with the amount of disorder (as observed experimentally)

Including nonlinearity

Nonlinearity : theory

- Effect of nonlinearity and disorder
  - Scattering theory (Gredeskul, Kivshar & others)
  - Chaos (Flach & others)
  - Perturbation theory on Lyapunov exponents (Fishman & others)
  - Spin glass theory (Leuzzi, Conti & others)
  - Self-consistent approaches (Tureci & others)
  - Scaling laws (Skipetrov & others)
  - FDTD (Sebbah, Conti & others)
  - Many others ...
  - **COMPARISON WITH EXPERIMENTAL DATA IS LIMITED**
  - The simplest thing to do:
    - Measure the localization length Versus nonlinearity

Question:

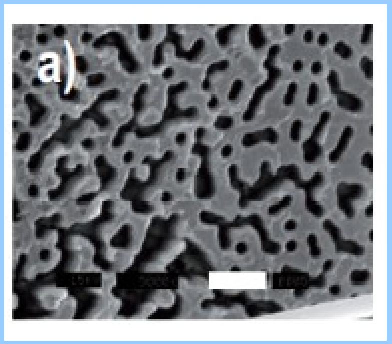
- Disorder induced states are un-coupled (absence of transport, Anderson regime)

- What happens in the presence of a „long range“ interaction?

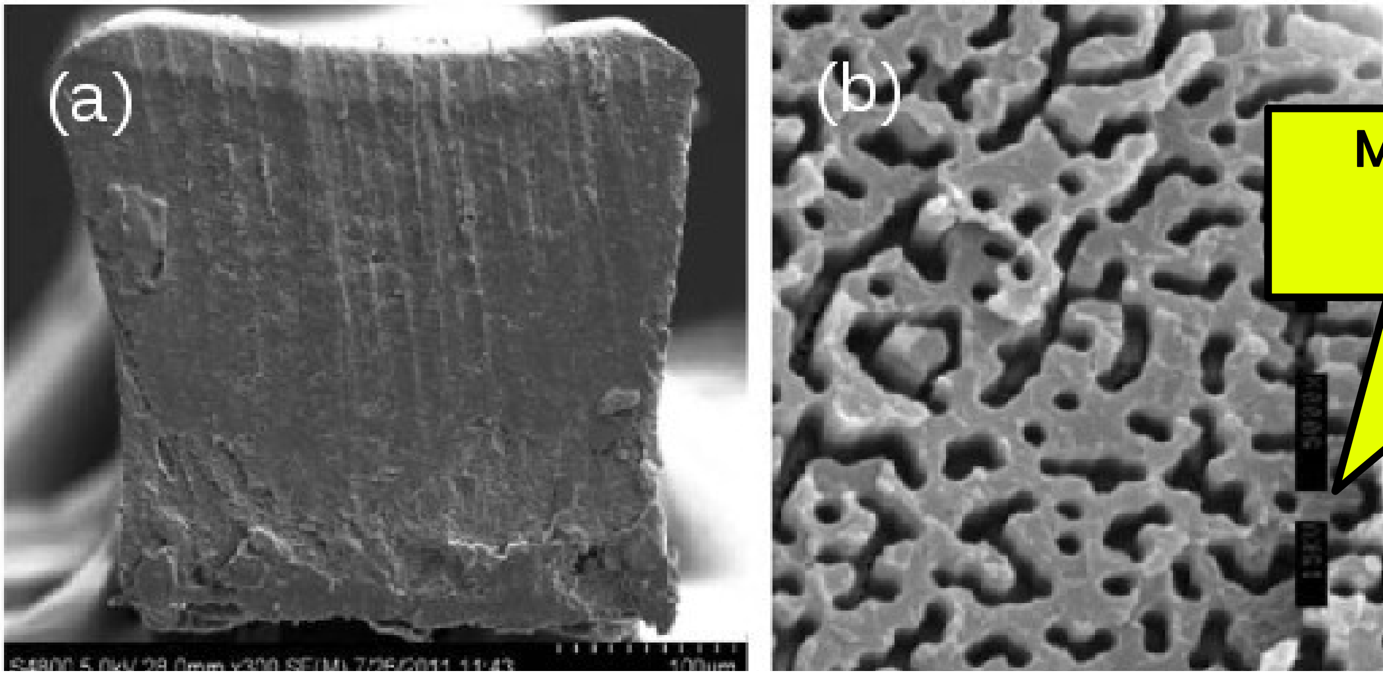
- Hypothesis: localized states interact

- We want an experimental evidence !

Transverse localization in 2D fibers



Our experiments on transverse localization in two dimensional fibers



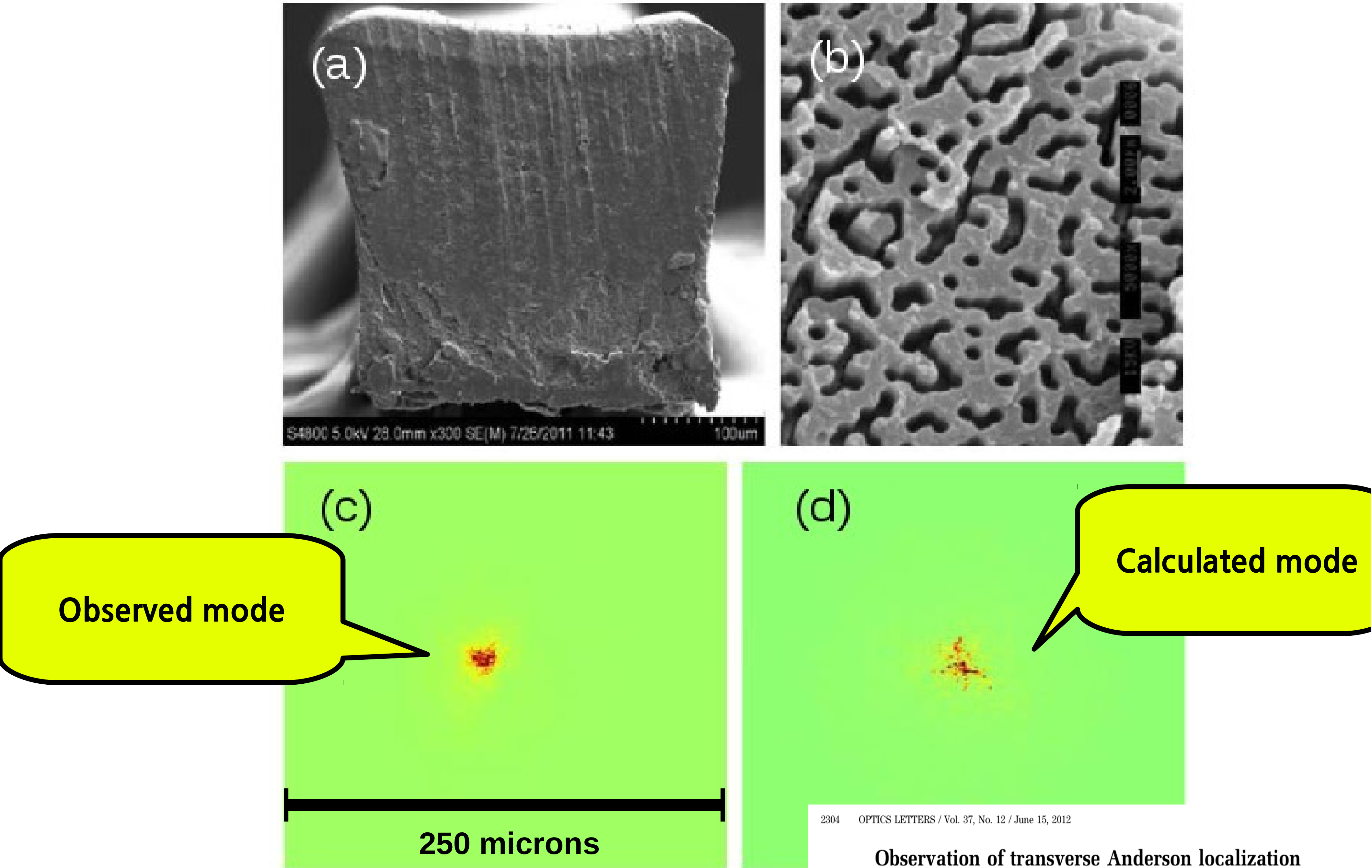
Mixture of PS and PPMA
Index contrast 0.1
Propagation >7 cm

40000 pieces of PMMA and 40000 pieces of PS randomly mixed and fused together
n(PS)=1.59
n(PMMA)=1.49

2304 OPTICS LETTERS / Vol. 37, No. 12 / June 15, 2012

Observation of transverse Anderson localization in an optical fiber

Salman Karbasi,[1] Craig R. Mirr,[1] Parisa Gandomkar Yarandi,[1] Ryan J. Frazier,[1] Karl W. Koch,[2] and Arash Mafi[1,*]



2304 OPTICS LETTERS / Vol. 37, No. 12 / June 15, 2012

Observation of transverse Anderson localization in an optical fiber

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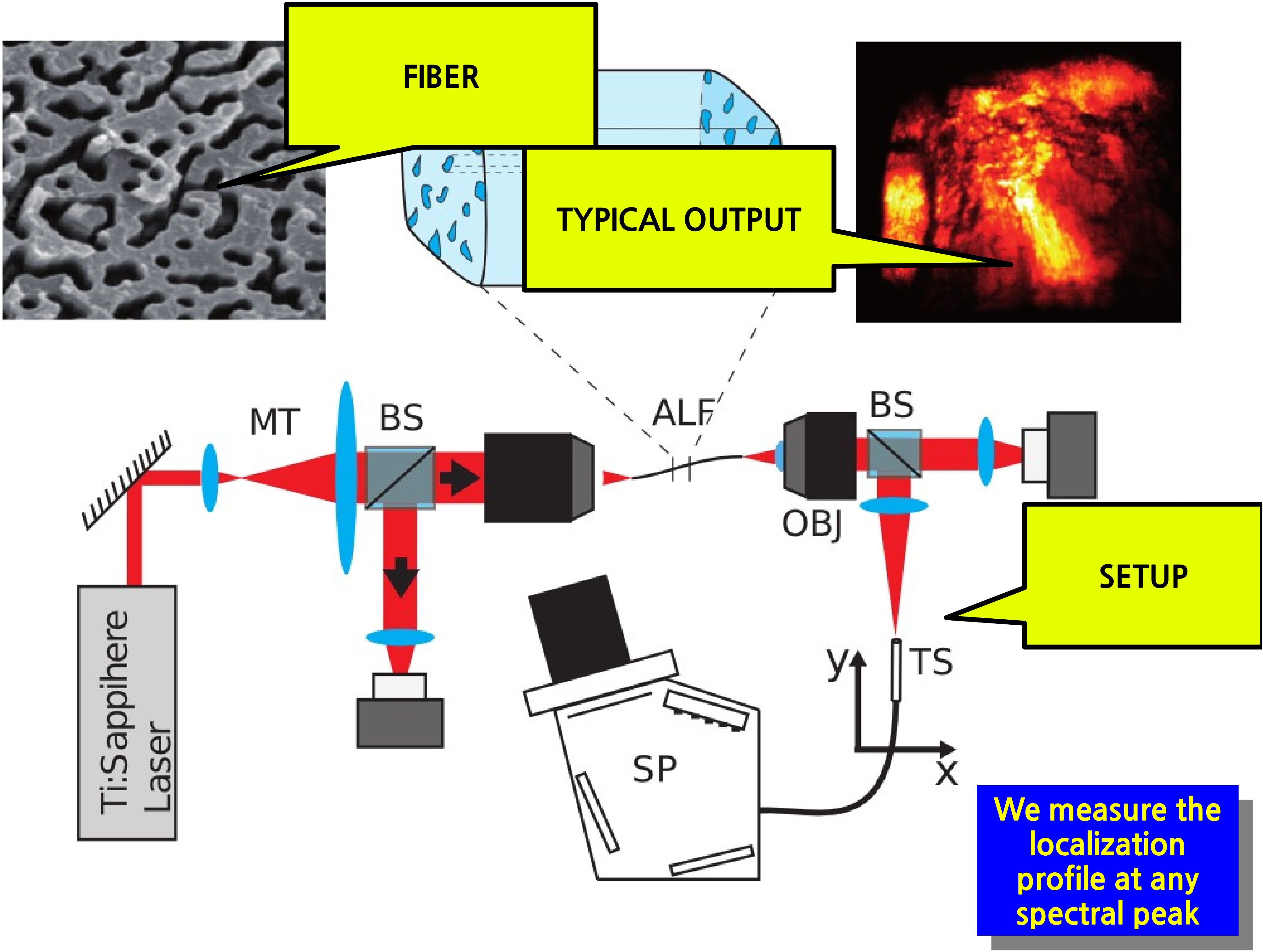
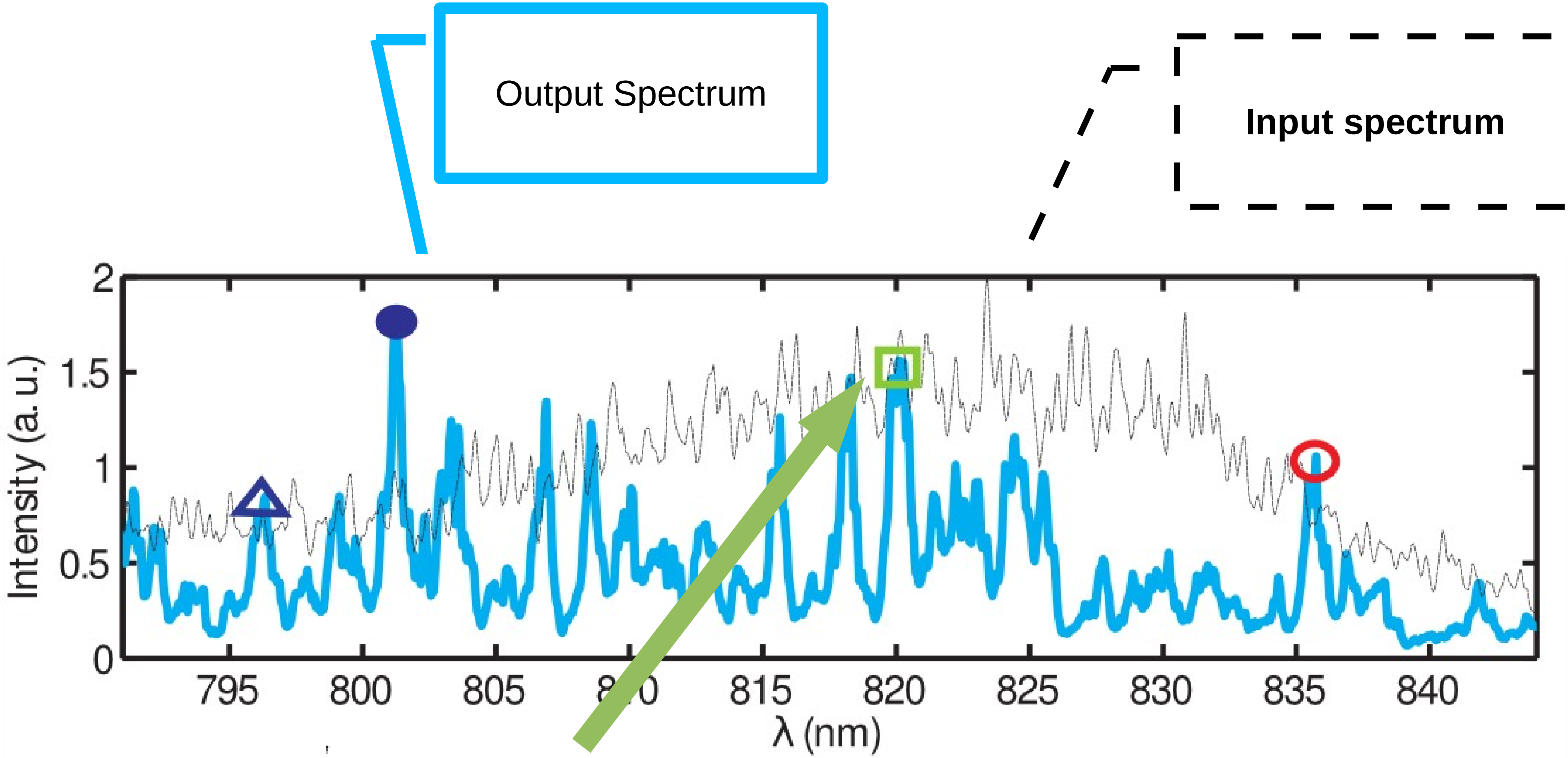
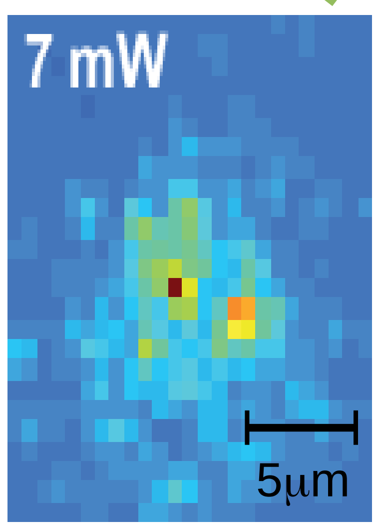
Multicolor transverse Anderson-localization

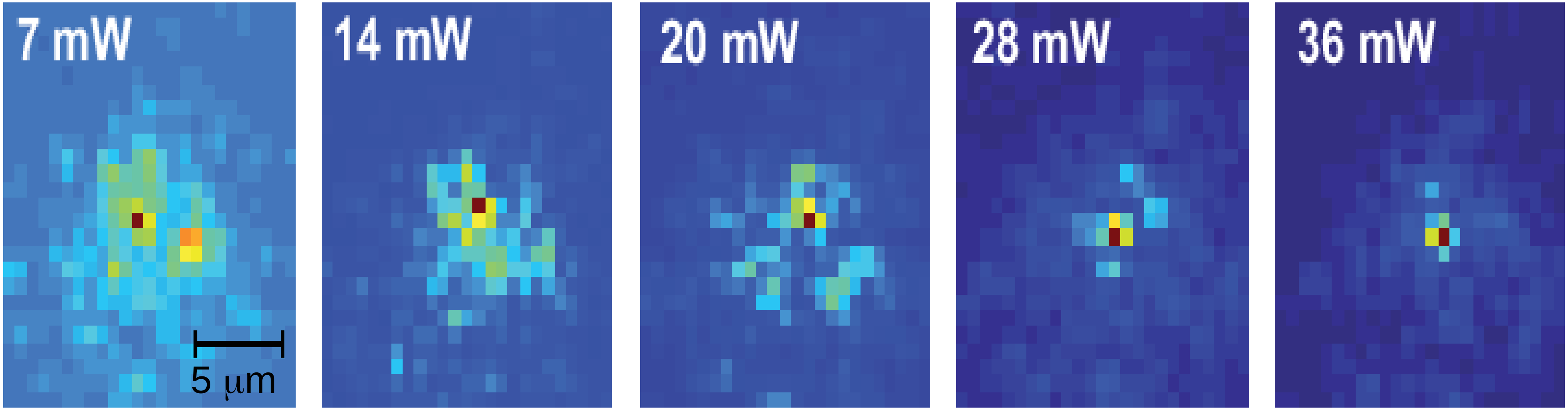
- we excite several localizations at different wavelengths simultaneously


FIBER
TYPICAL OUTPUT
MT
BS
ALF
BS
OBJ
SETUP
Ti:Sapphiere Laser
SP
TS
y
x
We measure the localization profile at any spectral peak

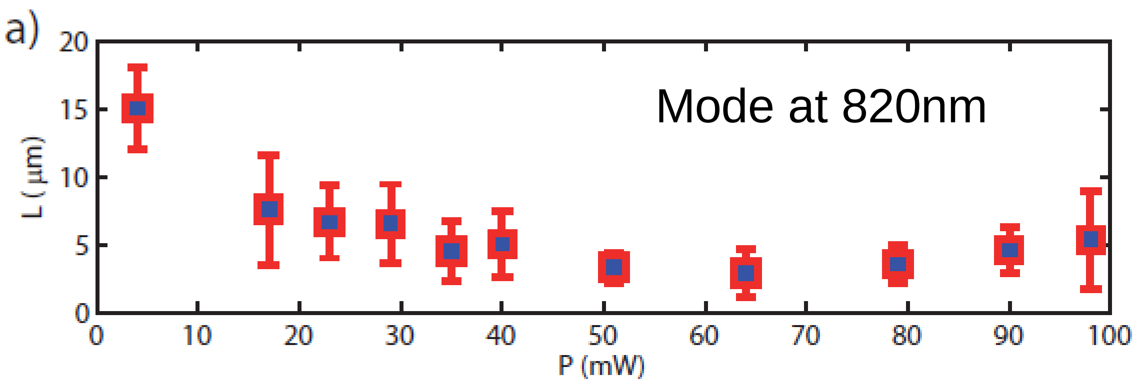




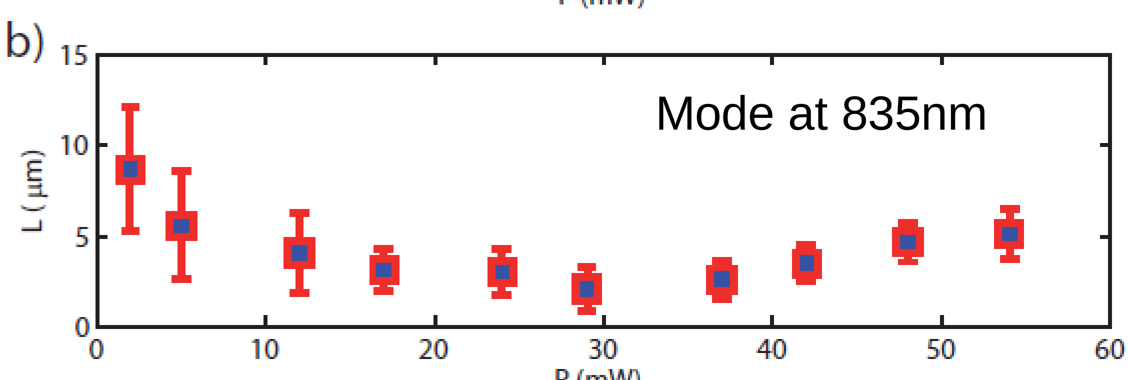
At any spatial location there are several localized modes at different frequencies

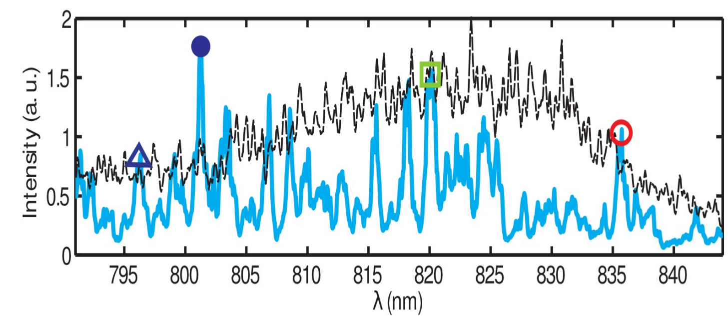


Mode profile at 820nm

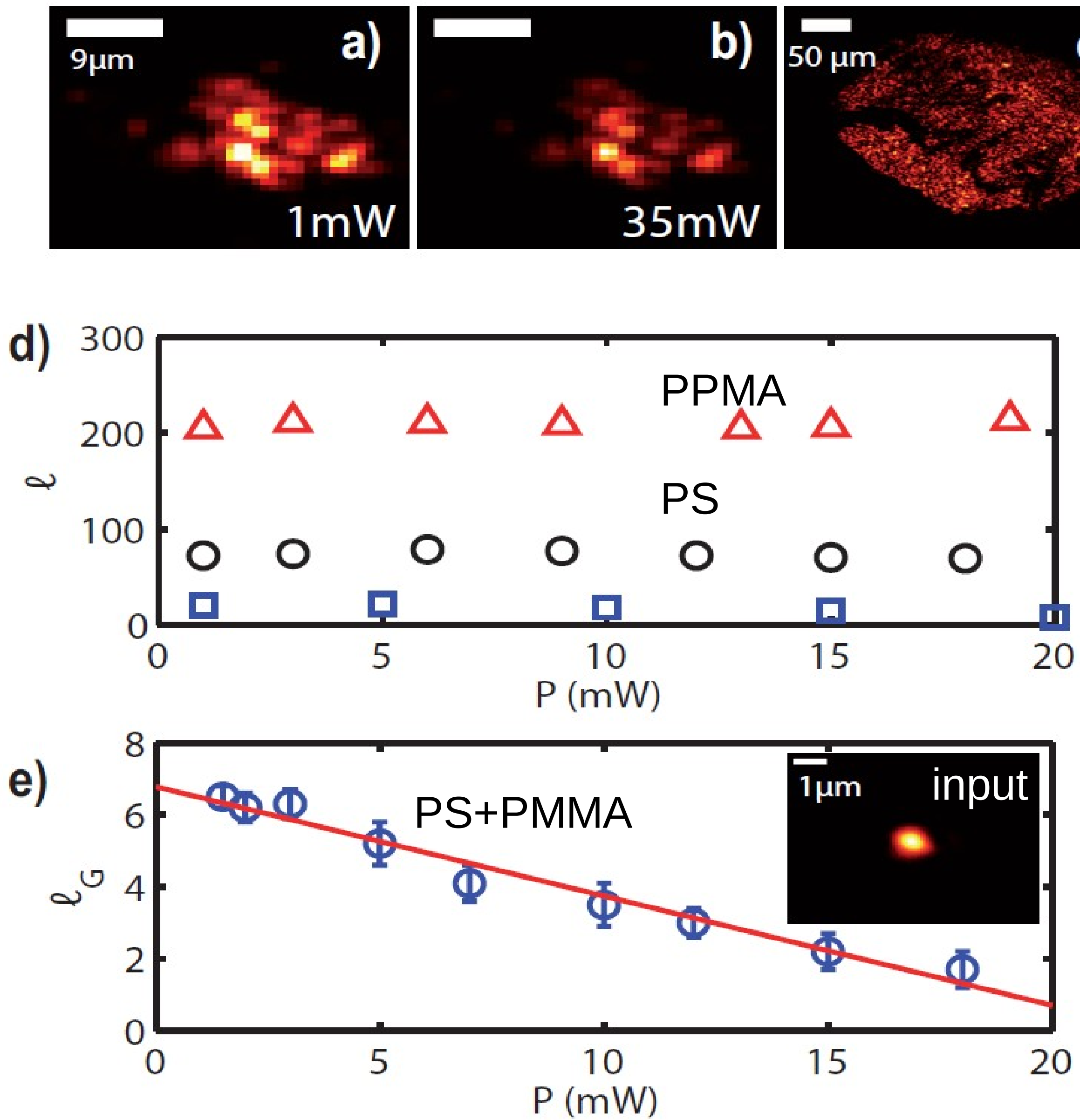


We observe focalization of any of the localized mode when incresing power





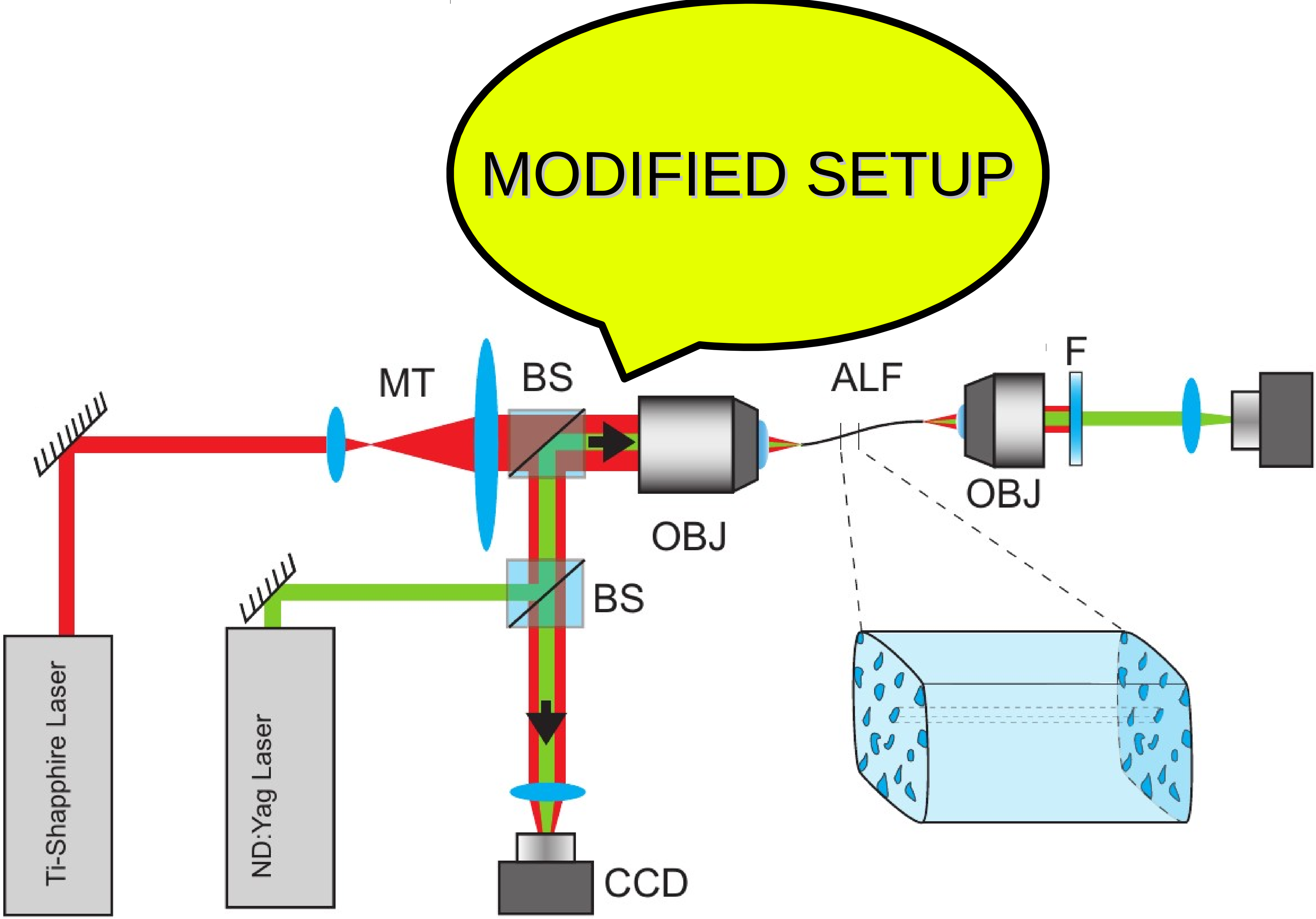
Comparison with ordered fibers

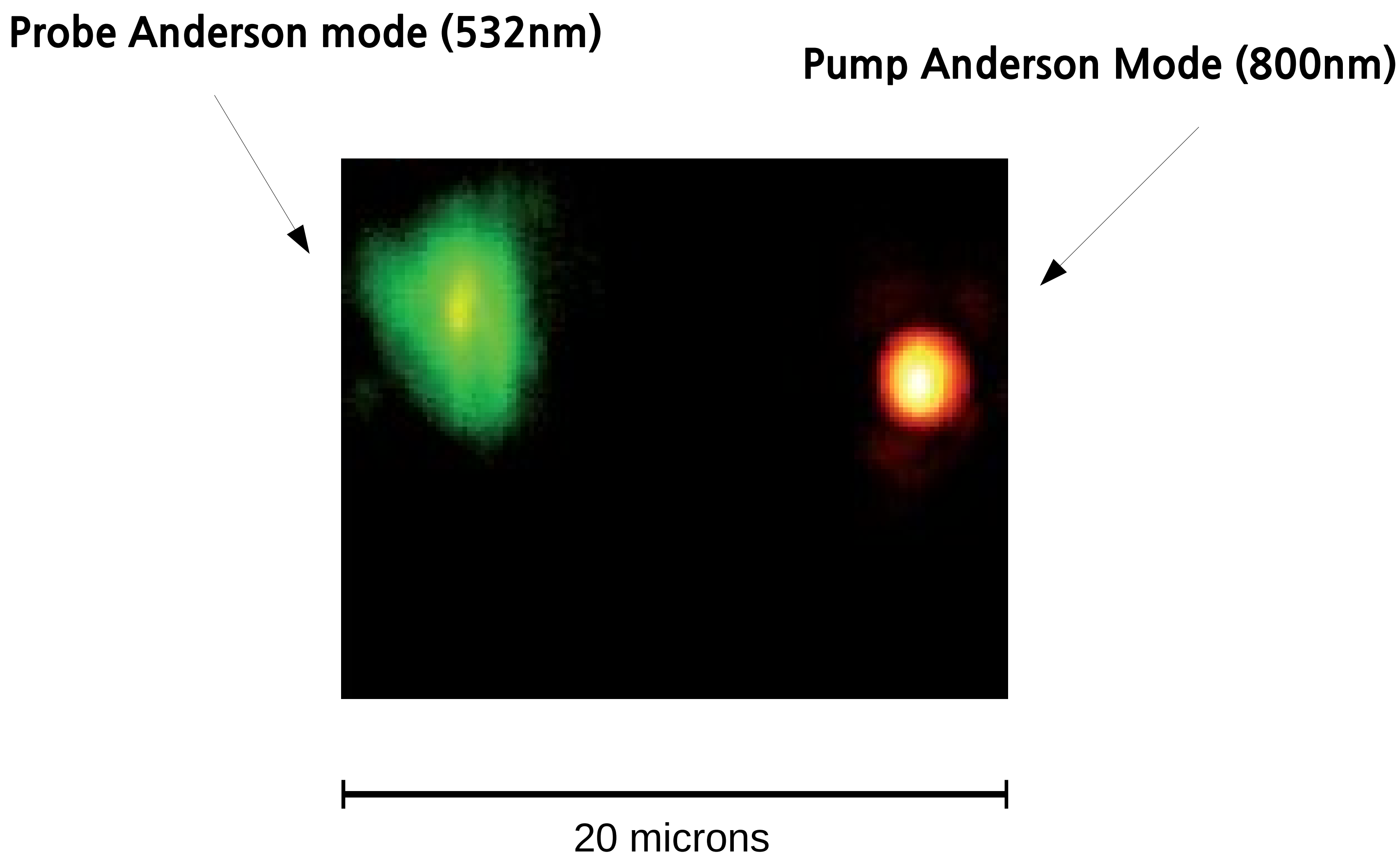


Here
Wavelength 1064nm

Action at a distance between Anderson localizations in nonlinear nonlocal media

- thermal nonlinearity is nonlocal!

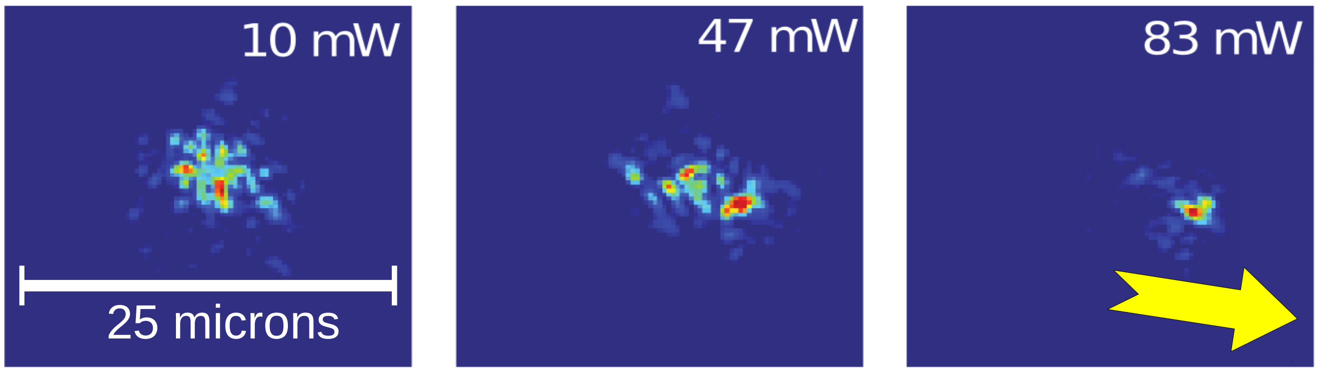

MODIFIED SETUP
MT
BS
ALF
F
OBJ
OBJ
BS
Ti-Shapphire Laser
ND:Yag Laser
CCD

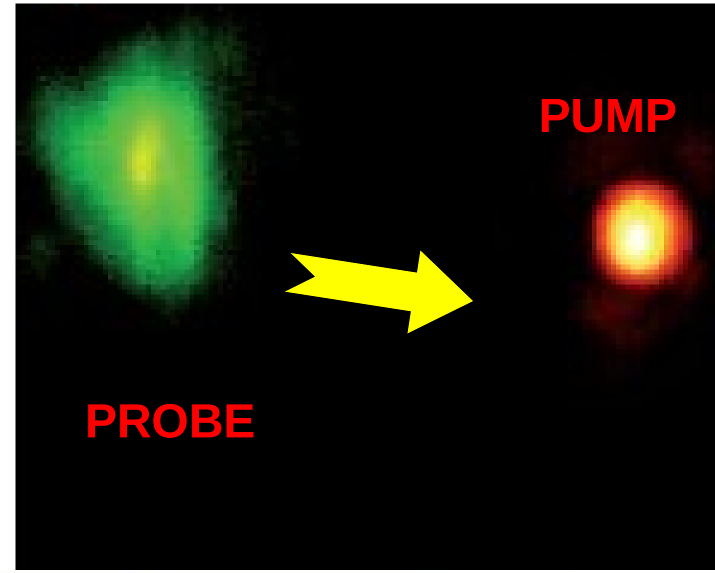


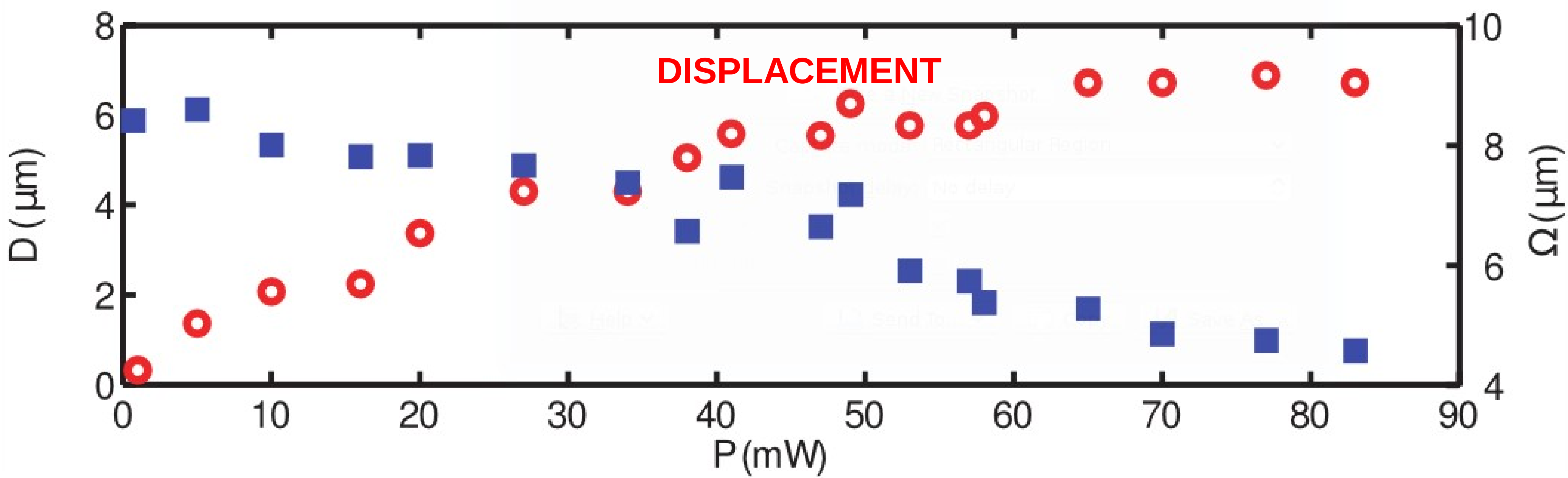
Probe Anderson mode (532nm)
Pump Anderson Mode (800nm)
20 microns

The size of the probe changes with the pump power !

Probe Anderson mode (532nm)







THEORY

(transverse) Anderson localization in nonlocal media

- Link between localization length and

$$i\psi_t + \psi_{xx} = V(x)\psi - s\psi \int_{-\infty}^{+\infty} \chi(x' - x)|\psi(x')|^2 \mathrm{d}x', \quad (1)$$

$$l(P) \cong \frac{l}{\sqrt{1 + sP/\langle P_c \rangle}},$$

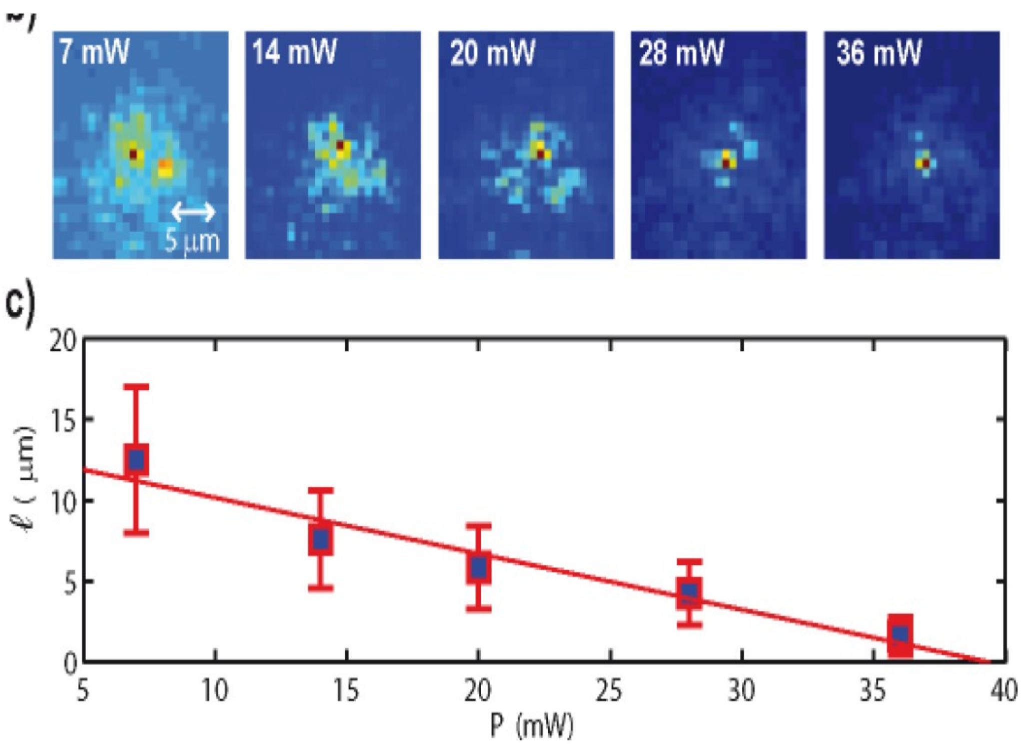
OPTICS LETTERS / Vol. 37, No. 3 / February 1, 2012

Anderson localization in nonlocal nonlinear media

Viola Folli[1,2,4] and Claudio Conti*,[1,2]

Comparison with experiments

- At low power : linear trend $l(P) = l(0)(1 - \frac{P}{2P_C})$



Modelling the action at a distance

- Using collective coordinates in the highly nonlocal approximation

$$2ik\frac{\partial A}{\partial z} + \nabla^2_{x,y}A + 2k^2\frac{\Delta n}{n_0}A = 0,$$

$$\Delta n = n_{PS} - n_{PMMA} = \Delta n_R + \Delta n_{NL}$$

$$\Delta n_{NL} = \int K(x - x', y - y')|A|^2(x', y')dx'dy'.$$

$$\Delta n_{NL} \cong K(x, y)\int |A|^2 d\mathbf{r} \cong P\left(\Delta n_1 + \frac{r^2}{2}\Delta n_2\right)$$

Equation for the positions

- Ehrenfest theorem (CC, PRE 72, 066620)

$$P_p \frac{d^2\mathbf{r}_p}{dz^2} = \int I_p(\mathbf{r}-\mathbf{r}_p)\nabla_{x,y}\frac{\Delta n_{NL}}{n}d\mathbf{r},$$

The position of the localization p varies because of nonlinearity

$$\Delta n_{NL} = \sum_{q=1}^{N} \Delta n_{NL,q} \cong \sum_{q=1}^{N} \frac{P_q \Delta n_2}{2}(\mathbf{r}-\mathbf{r}_q)^2.$$

The localizations are incoherent

$$P_p \frac{d^2\mathbf{r}_p}{dz^2} = -\nabla_{x_p,y_p}\sum_{q=1}^{N}\frac{|\Delta n_2| P_q P_p}{2n_0}|\mathbf{r}_p-\mathbf{r}_q|^2.$$

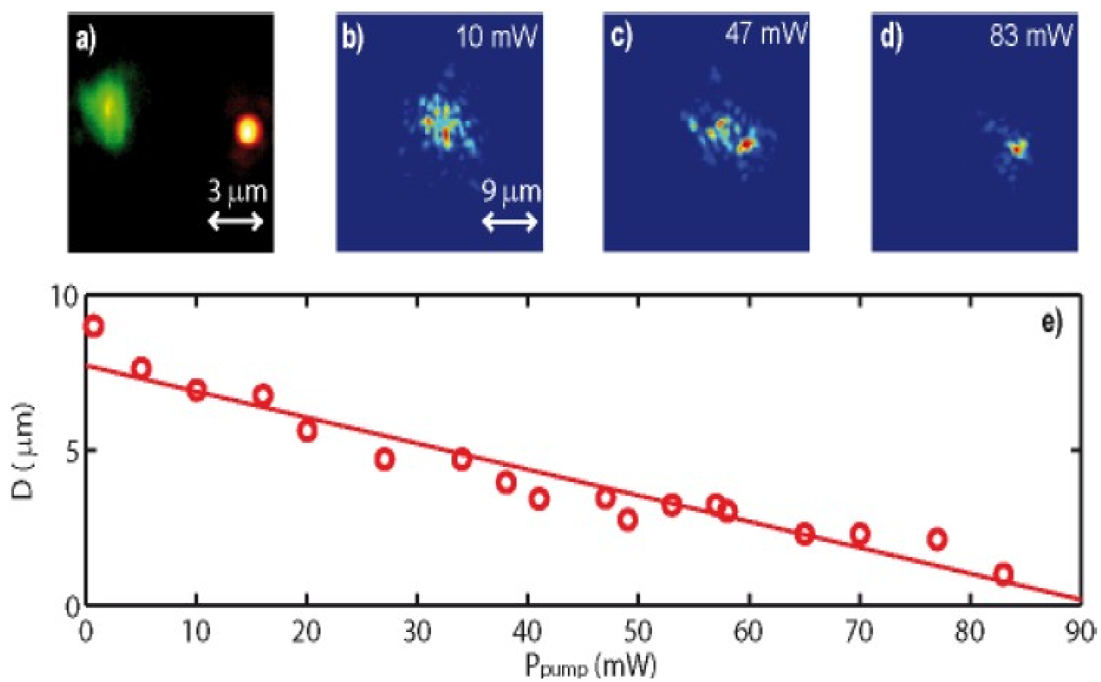
Pairwise interaction potential

Leonetti et al, Phys. Rev. Lett., 112: 193902 (2014)

Comparison with experiments

- Pump and probe Anderson states
  - We consider two states $P_{pump} >> P_{probe}$

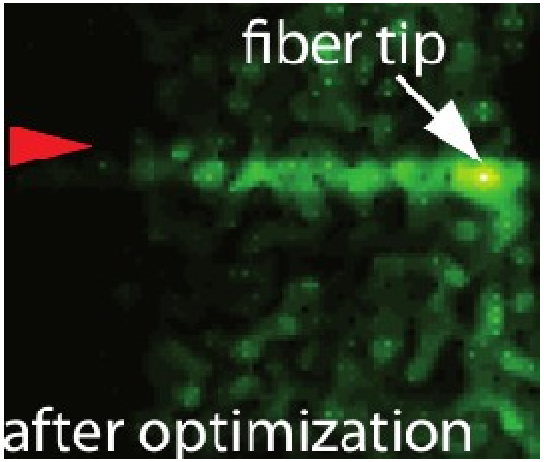
$$D(z) = D(0)\left(1 - \frac{|\Delta n_2| z^2}{2n_0} P_{pump}\right)$$



Leonetti et al, Phys. Rev. Lett., 112: 193902 (2014)

Conclusions

- Nonlinearity and nonlocality in 2D disorder fibers
- Action at a distance
- Transport in the Anderson regime
- Incoherent Anderson states
- Variational theoretical approaches



THANKS !

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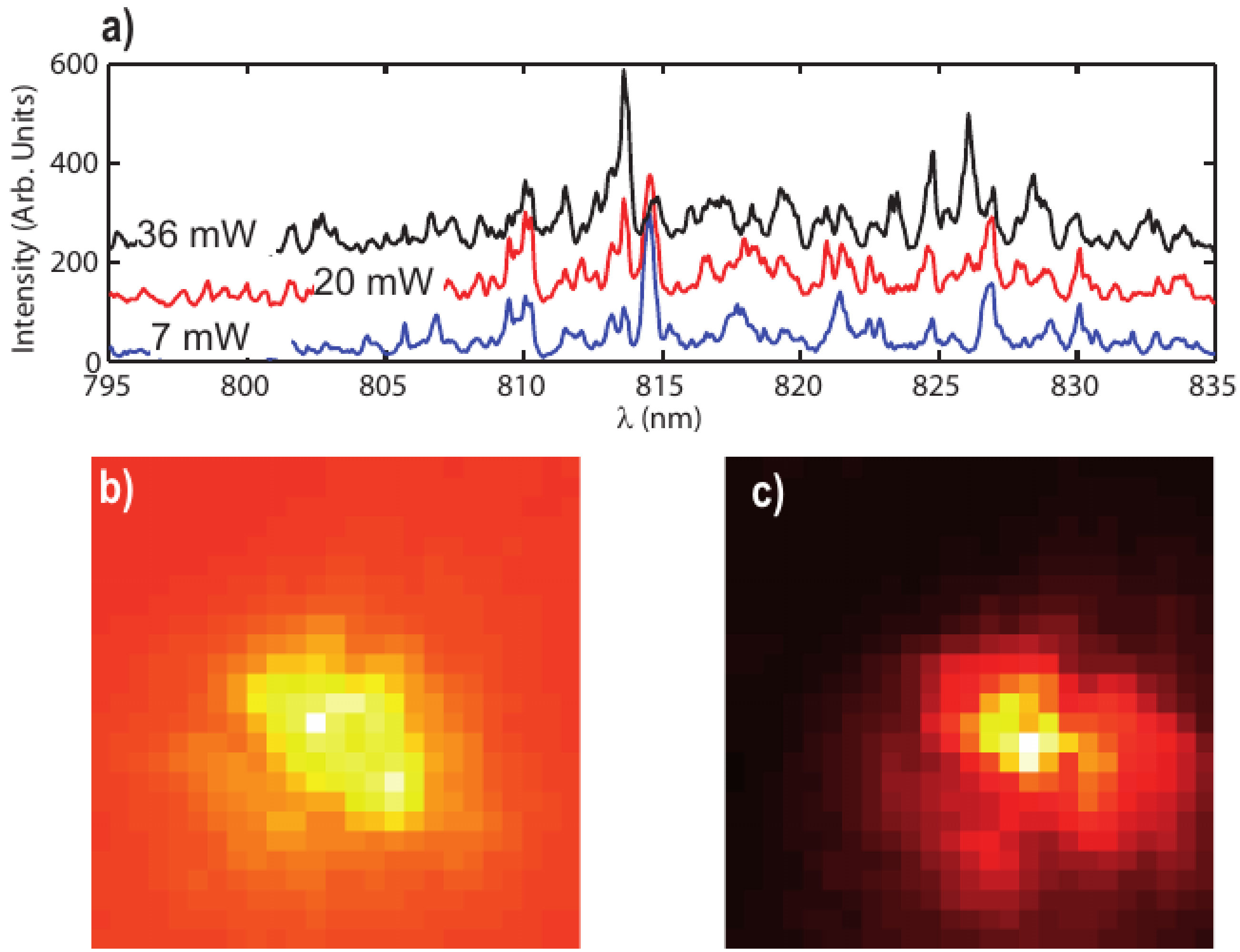
Leonetti et al, Phys. Rev. Lett., 112: 193902 (2014) ; Nature Communications 5, 5534 (2014)



a)
600
400
200
0
Intensity (Arb. Units)
36 mW
20 mW
7 mW
795
800
805
810
815
820
825
830
835
λ (nm)
b)
c)

(transverse) Anderson localization in nonlocal media

- Link between localization length and

$$i\psi_t + \psi_{xx} = V(x)\psi - s\psi \int_{-\infty}^{+\infty} \chi(x' - x)|\psi(x')|^2 \mathrm{d}x', \quad (1)$$

$$l(P) \cong \frac{l}{\sqrt{1 + sP/\langle P_c \rangle}},$$

OPTICS LETTERS / Vol. 37, No. 3 / February 1, 2012

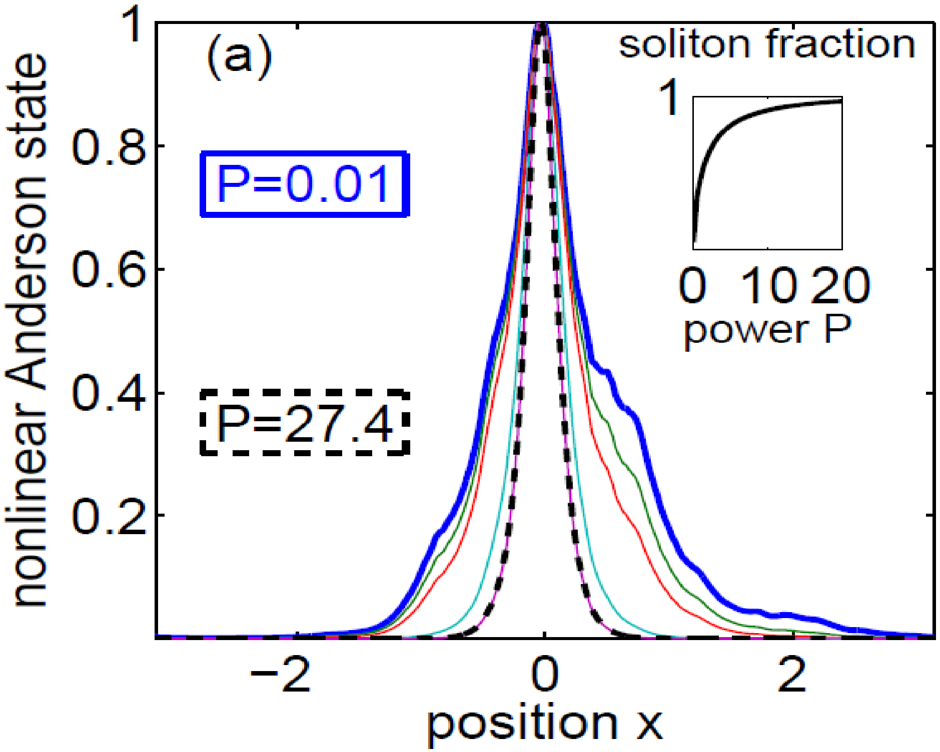
Anderson localization in nonlocal nonlinear media

Viola Folli[1,2,4] and Claudio Conti*,[1,2]

A non-perturbative theoretical analysis

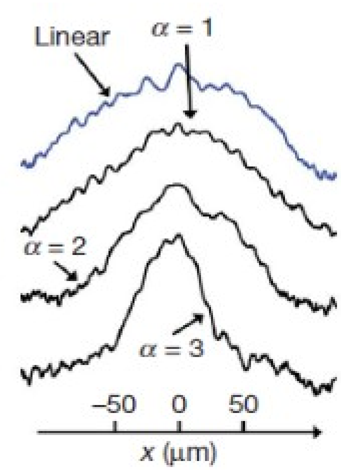
$$i\psi_t = -\psi_{xx} + V(x)\psi - \chi|\psi|^2\psi$$

FOCUSING CASE



$$-\varphi_{xx} + V(x)\varphi - \chi\varphi^3 = E\varphi,$$

„SOLITONIZATION“ of the ANDERSON LOCALIZATION



Swartz et al Nature 08

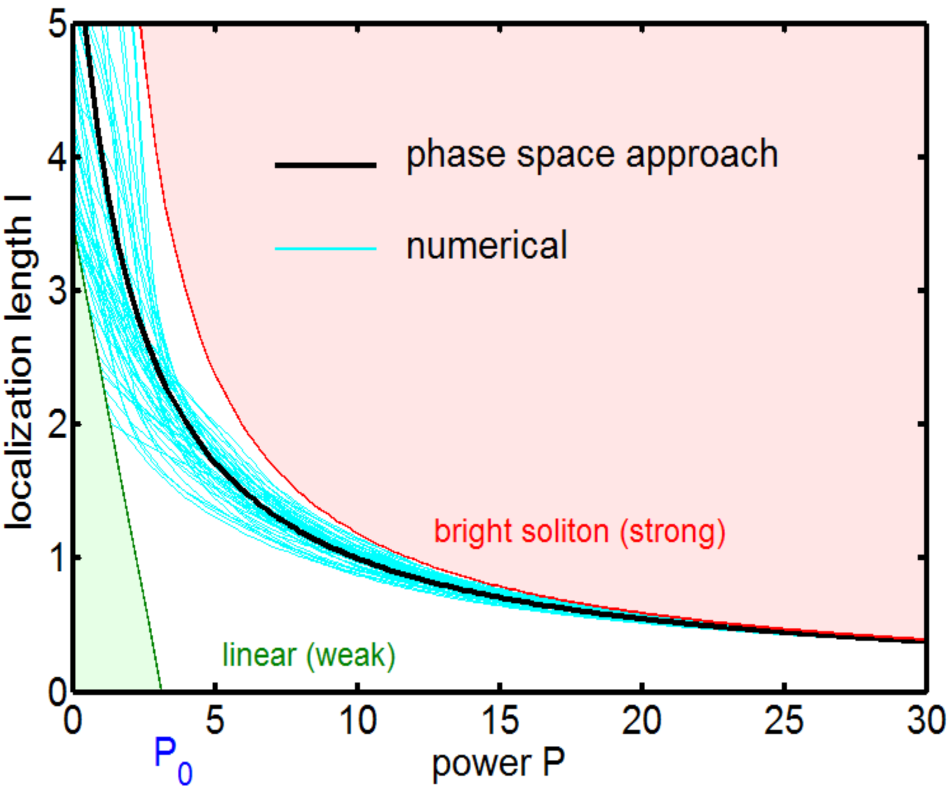
Conti, PRA 86, 061801 (2012)

Variational average

$$-\psi_{xx} - \left(1 + \frac{2LV_0^2}{P^2}\right)|\psi|^2\psi = E\psi$$

EFFECTIVE NLS FOR THE NONLINEAR ANDERSON STATE (FOCUSING CASE) „AVERAGE SOLITON EQ“

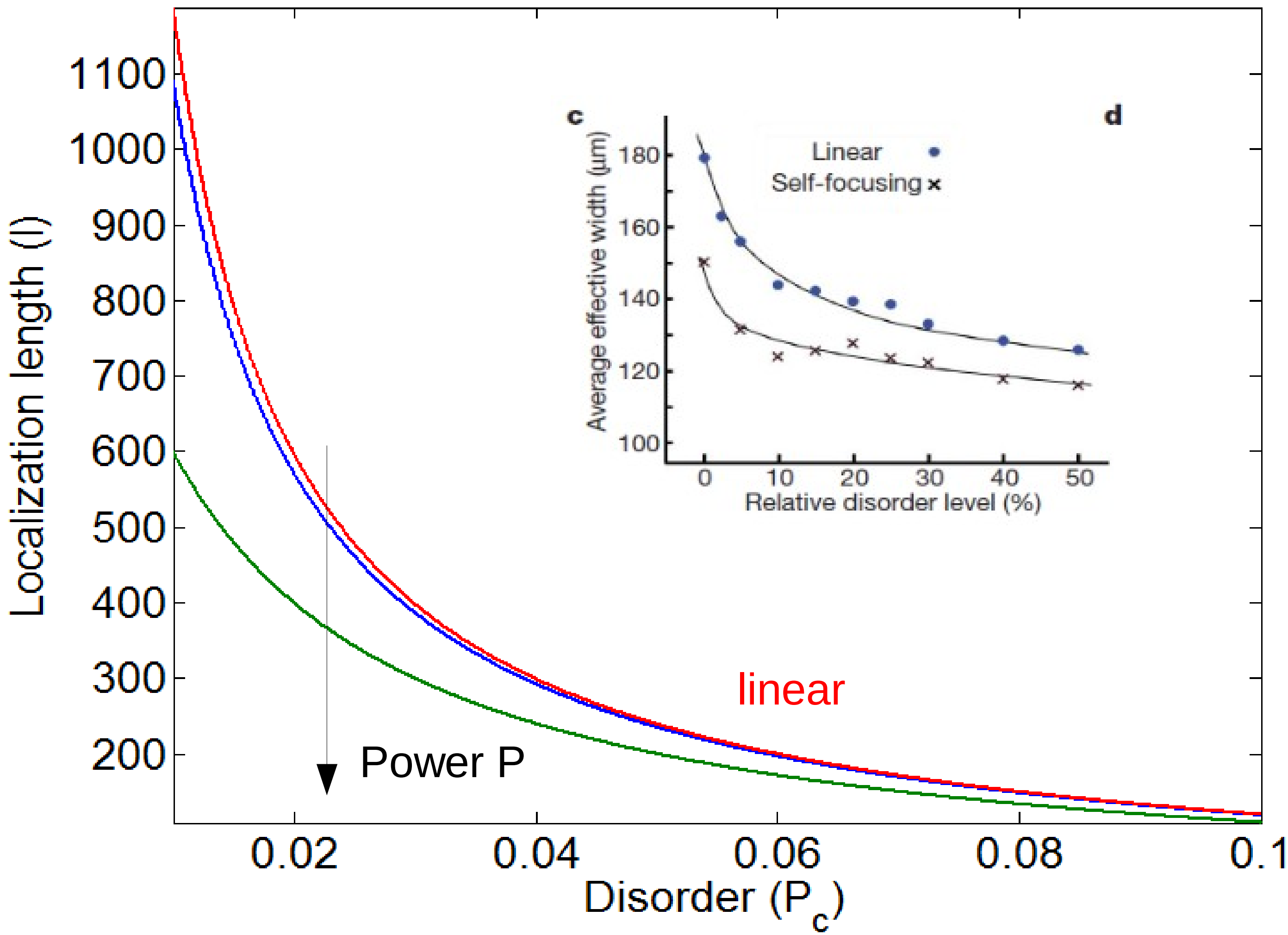
$$l_C = \frac{12/P}{(1 + P_C/P)}$$



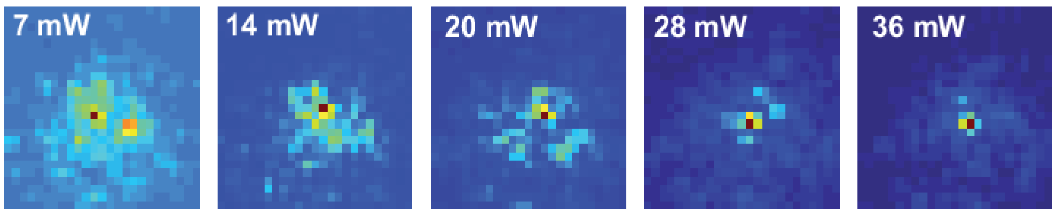
Conti, PRA 86, 061801 (2012)

Comparison with Schwartz et al

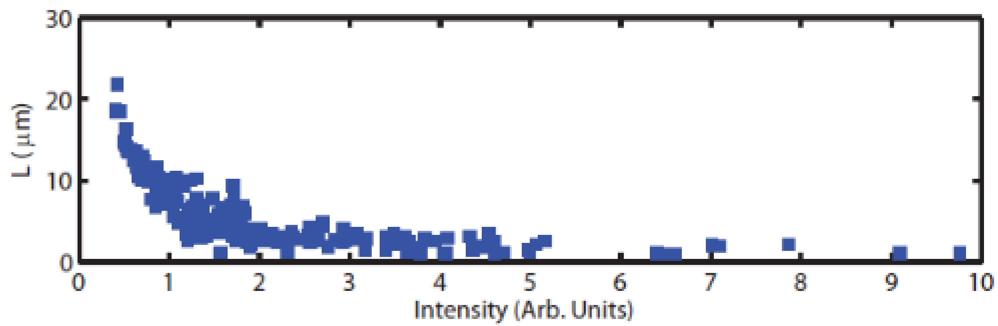
- Loc length versus strength of disorder (analytical)



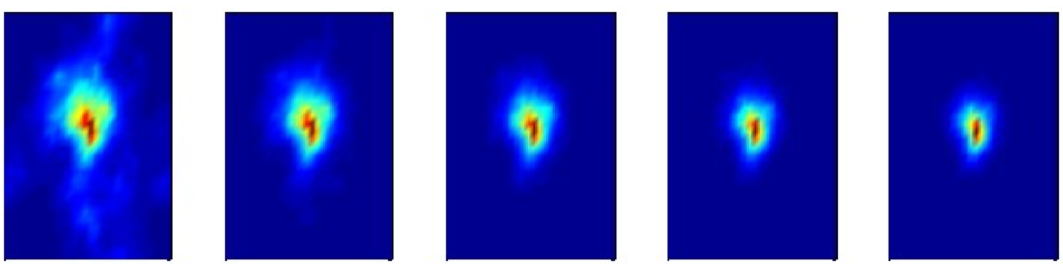
FOCUSING CASE (2D)



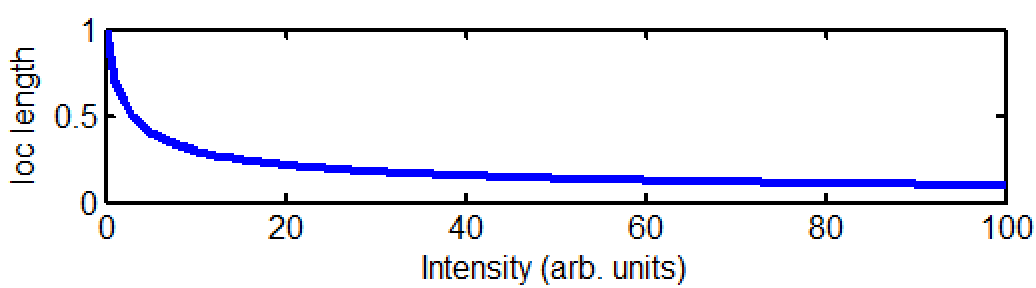
Experiments



Localization length
Versus
Intensity



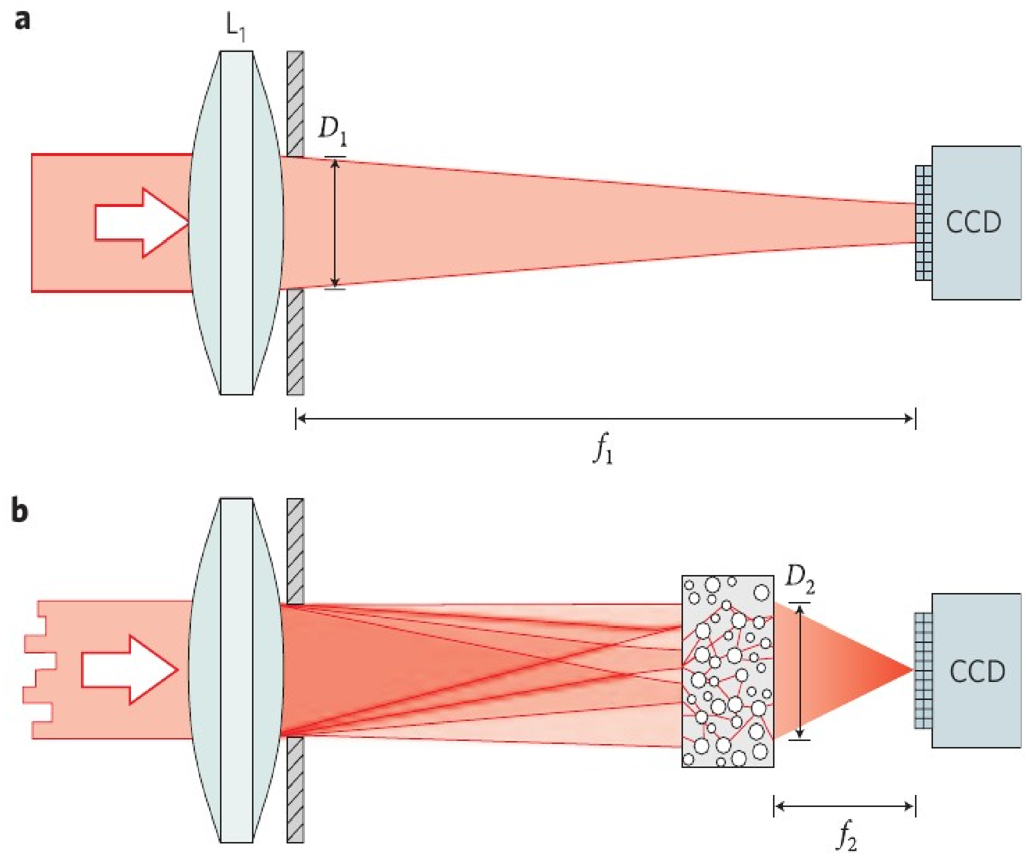
Numerically calculated
bound states of the 2D-NLS
with Gaussian disorder



Theory from
the variational approach

Light focusing through disordered media

The experiment of Vellekov et al



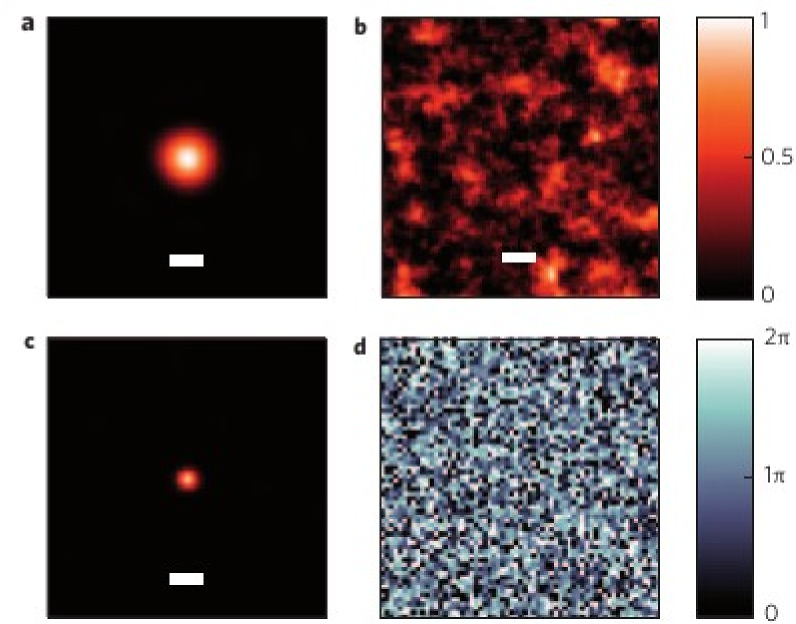


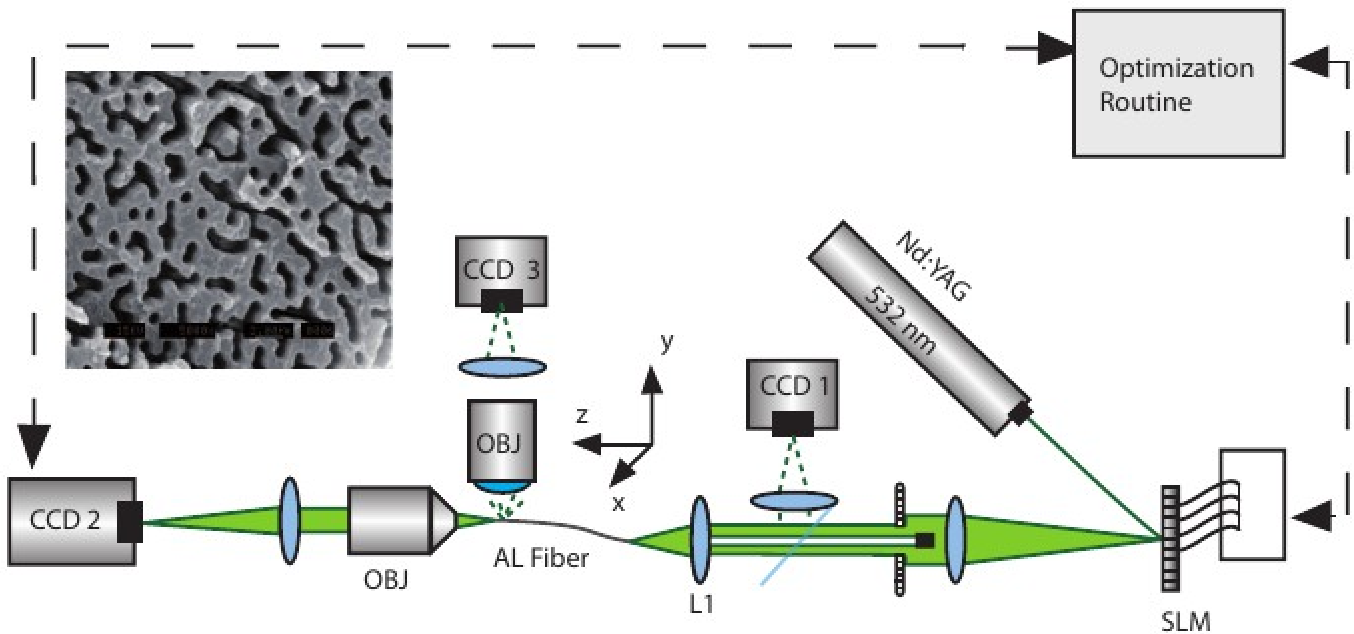
Figure 2 | Measured intensity distribution in the focal plane at 200 ± 3 mm from the glass lens. a, Clean system with an unmodified incident wavefront. The focal width is of the order of the diffraction limit (62 μm, white bar). **b**, Intensity transmission of a 6-μm layer of airbrush paint for the unmodified incident wavefront. No focus is discernible. **c**, System with the sample present, and the wave shaped to achieve constructive interference in the target. A high-contrast, extremely sharp focus is visible. **d**, Pattern on the spatial phase modulator for the set-up in **c**. The intensity plots are normalized to the brightest point in the image.

LETTERS
PUBLISHED ONLINE: 14 FEBRUARY 2010 | DOI: 10.1038/NPHOTON.2010.3
nature photonics

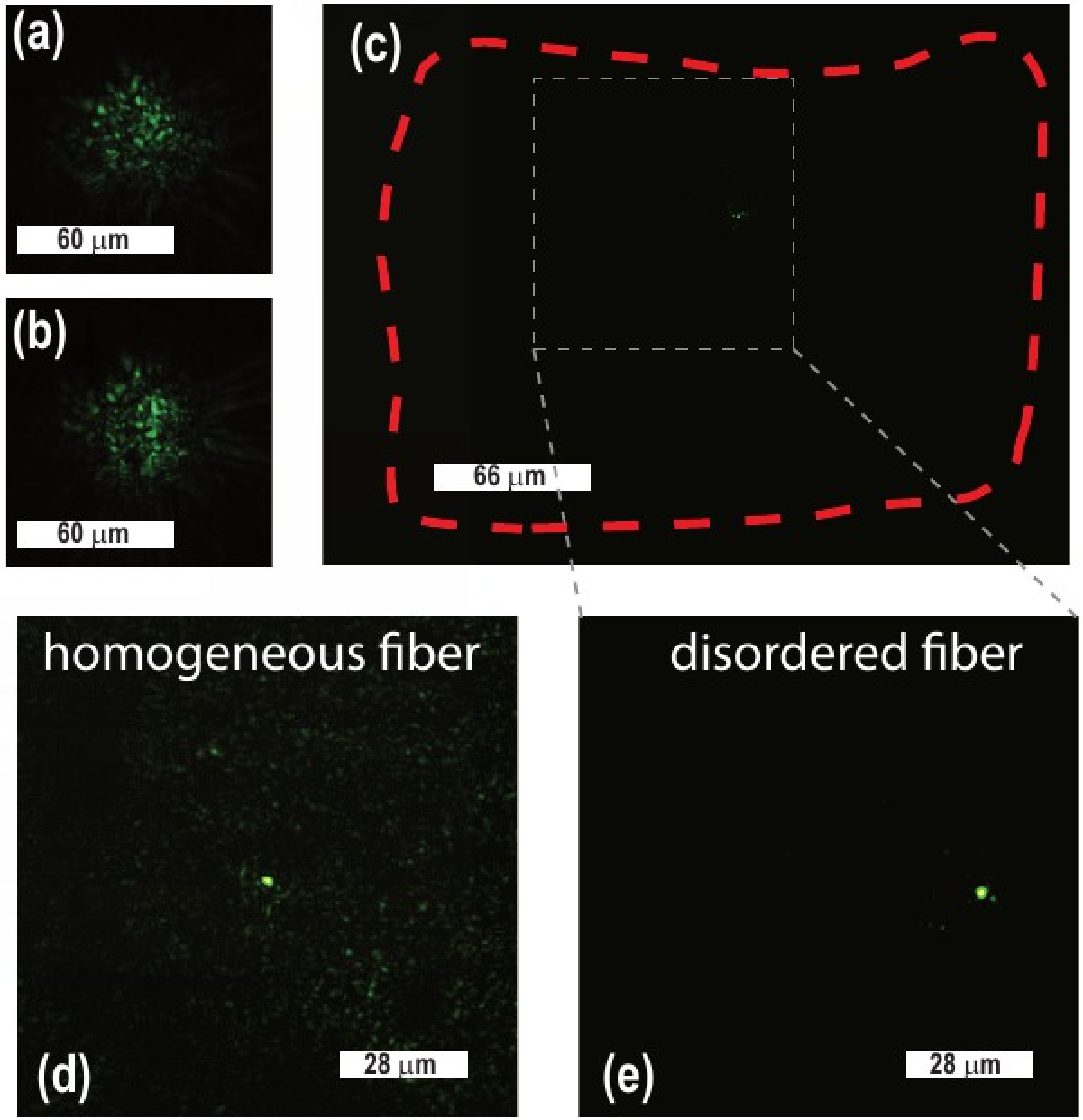
Exploiting disorder for perfect focusing

I. M. Vellekoop[1†], A. Lagendijk[2] and A. P. Mosk[1*]

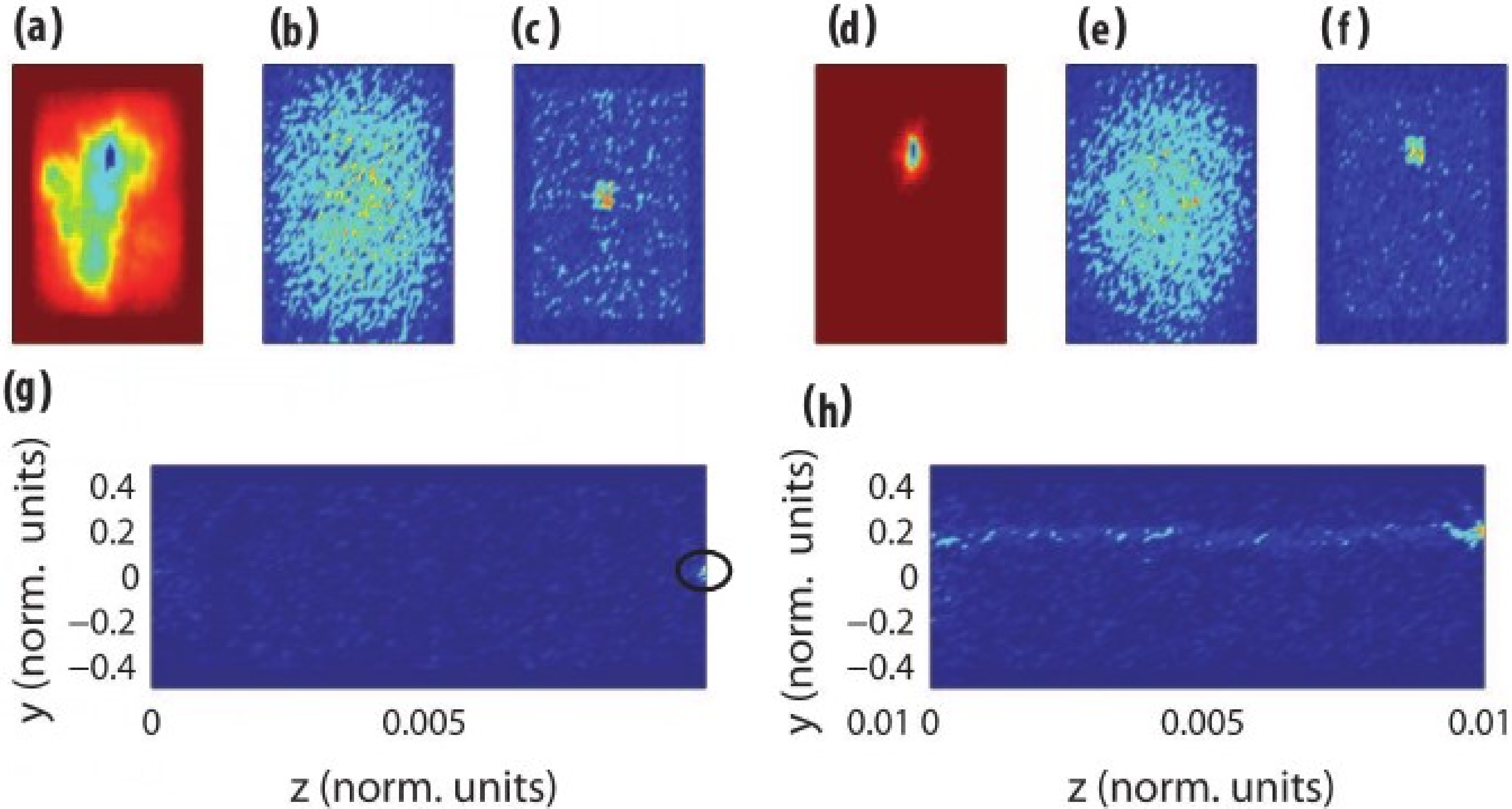
Focusing in the Anderson regime



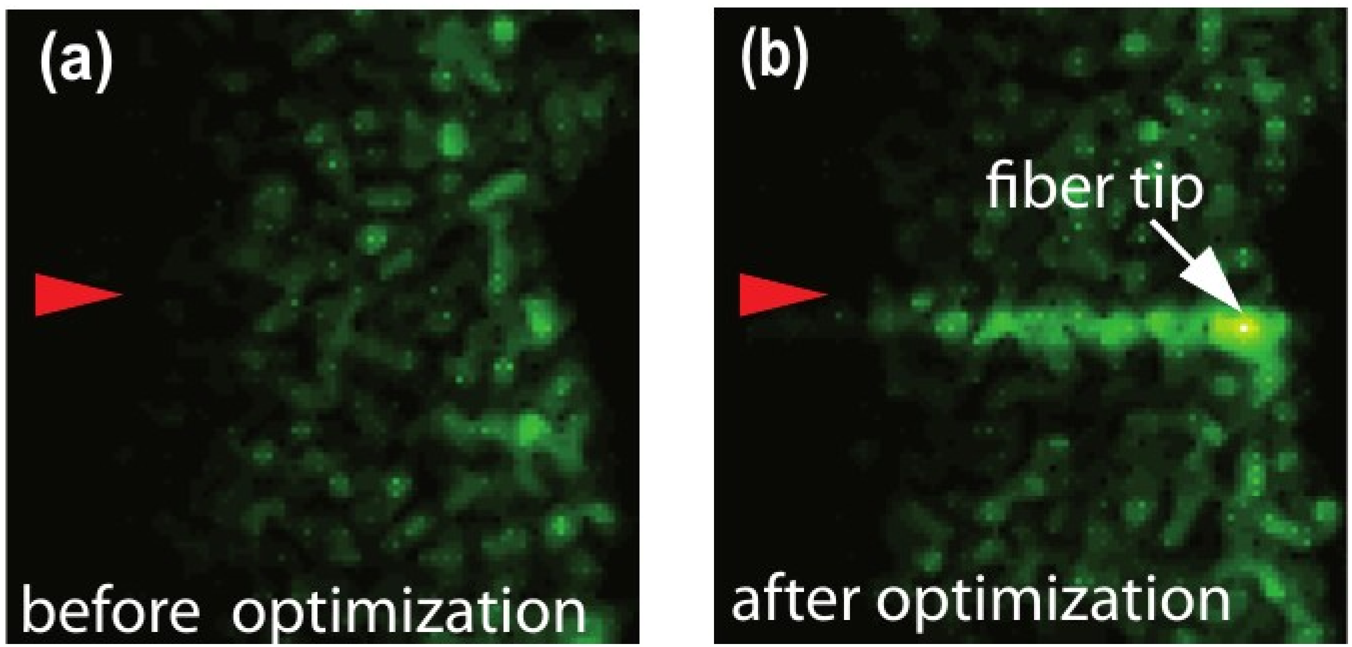
Focusing multi-mode fibers



Some numerical simulations



Measures from the top



Conclusions

- Nonlinearity, nonlocality and transistor/switching in random lasers and 2D disorder optical fibers
- Action at a distance between Anderson localizations
- Non-perturbative variational approach to nonlinear Anderson states

THANKS !

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www.openscholar.org.uk

More sophisticated ideas

- The landscape
- The link with spin-glass theory
- Conti, Leuzzi, PRB 2011

- Predicts
  - The existence of several competing states (Bose/Glass)
  - Out-of-equilibrium regimes
  - A Phase-Diagram for nonlinear waves and lasers

A brief mention to the defocusing case

- The variational approach predicts that above a critical power there are no more localized states

$$-\psi_{xx} + \left(1 - \frac{2LV_0^2}{P^2}\right)|\psi|^2\psi = E\psi$$

- At a certain power the nonlinear coefficient changes sign (no more bright soliton)
- This corresponds to the absence of a single minimum of the phase space
- There exist several minima
- A landscape of localized states with not vanishing „**complexity**“

Stability of nonlinear Anderson states in the focusing case

- Vakhitov – Kolokolov formulation (1973)

$$\psi = (\varphi + \delta\psi)\exp(-iEt)$$

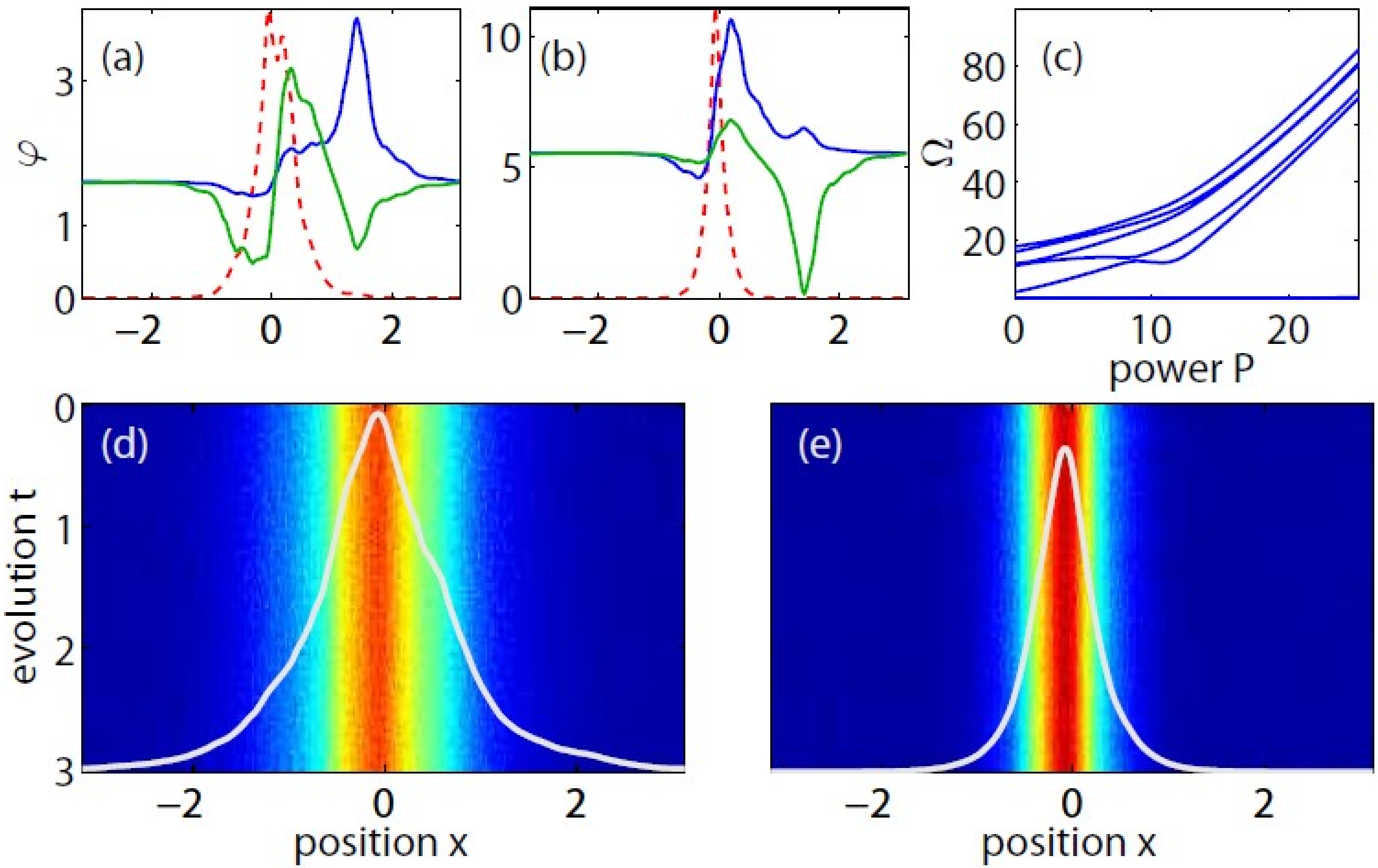
$$\delta\psi = [u(x) + iv(x)]\exp(\Omega t)$$

$$-\Omega^2 u = L_1 L_0 u$$

$$L_0 = -\partial_x^2 - E + V(x) - \varphi(x)^2 \text{ and } L_1 = -\partial_x^2 - E + V(x) - 3\varphi(x)^2$$

Numerical calculated eigenvalues and eigenstates

- The nonlinear Anderson states are stable



FOCUSING

Rose and Weinstein, 1988