

“Polaritonic crystals: from optical response to Casimir effect “

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● Introduction

● Polaritonic Crystals

- periodic structures: superradiance, polaritonic crystal limit
- quasicrystals: scaling and fractality

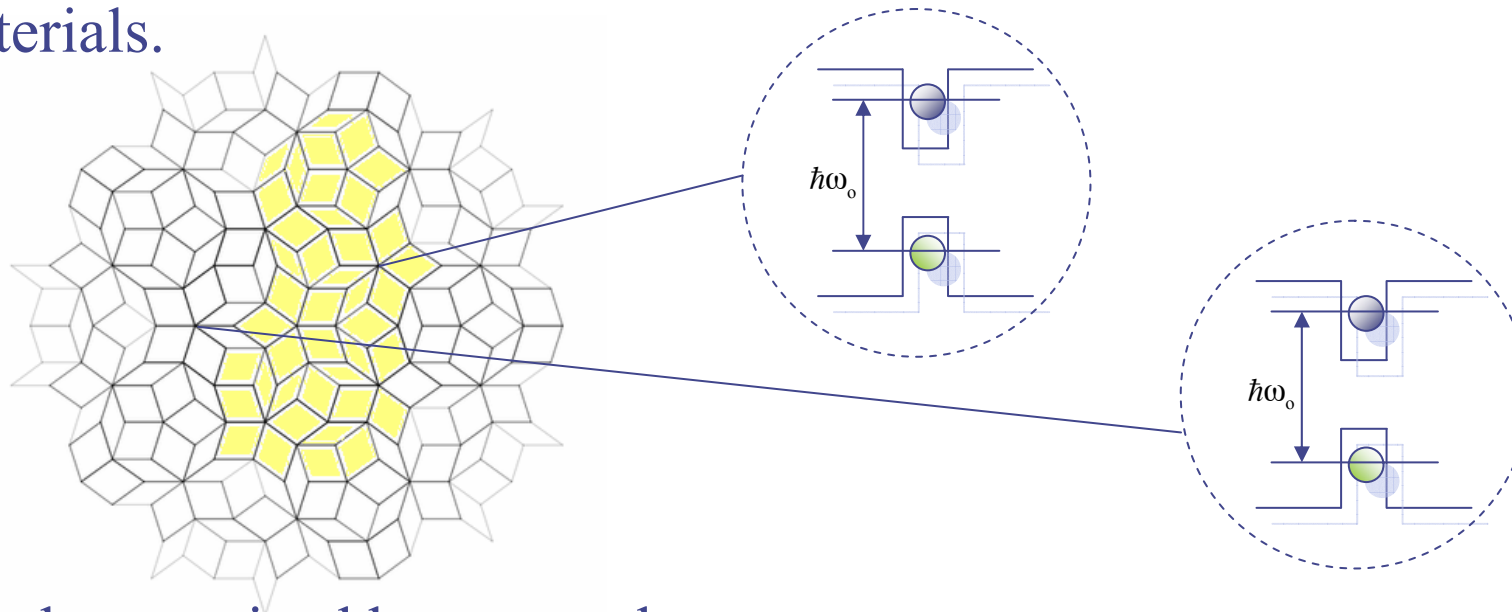
● Optical response

- e.m. Green function for a general shaped superstructure

● Casimir effect

- definition
- material design for force control
 - numerical approach
 - preliminary results

Polaritonic crystals are structured media (photonic crystals) with polariton poles in the dielectric susceptibility of some constituent materials.



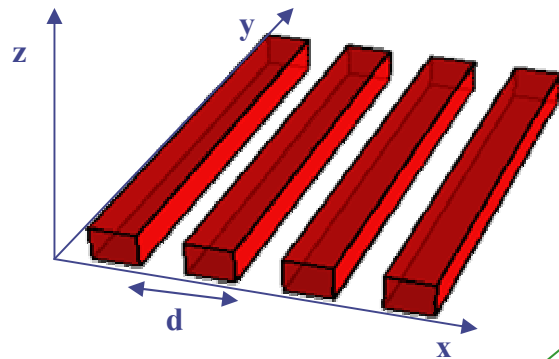
Are characterized by a complex dielectric tensor, function of spatial variables and frequency.

$$\varepsilon_{ij}(\vec{r}, \omega) = \varepsilon'_{ij}(\vec{r}, \omega) + i\varepsilon''_{ij}(\vec{r}, \omega)$$

In the photonic crystal limit \Rightarrow

$$\varepsilon(\vec{r}, \omega) = \varepsilon'(\vec{r})$$

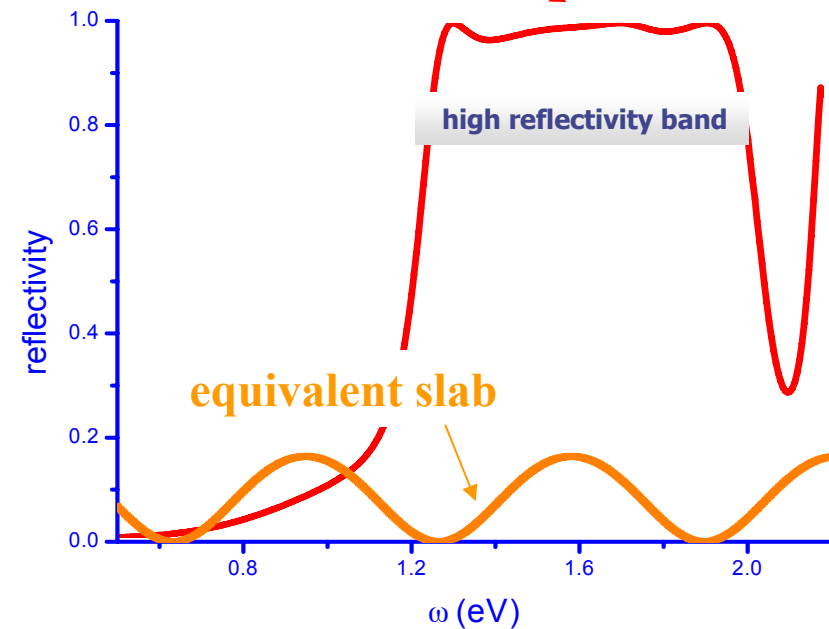
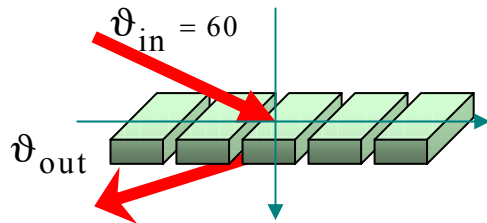
Self-sustained dielectric grating slab can show



giant reflection bands

$R > 95\%$

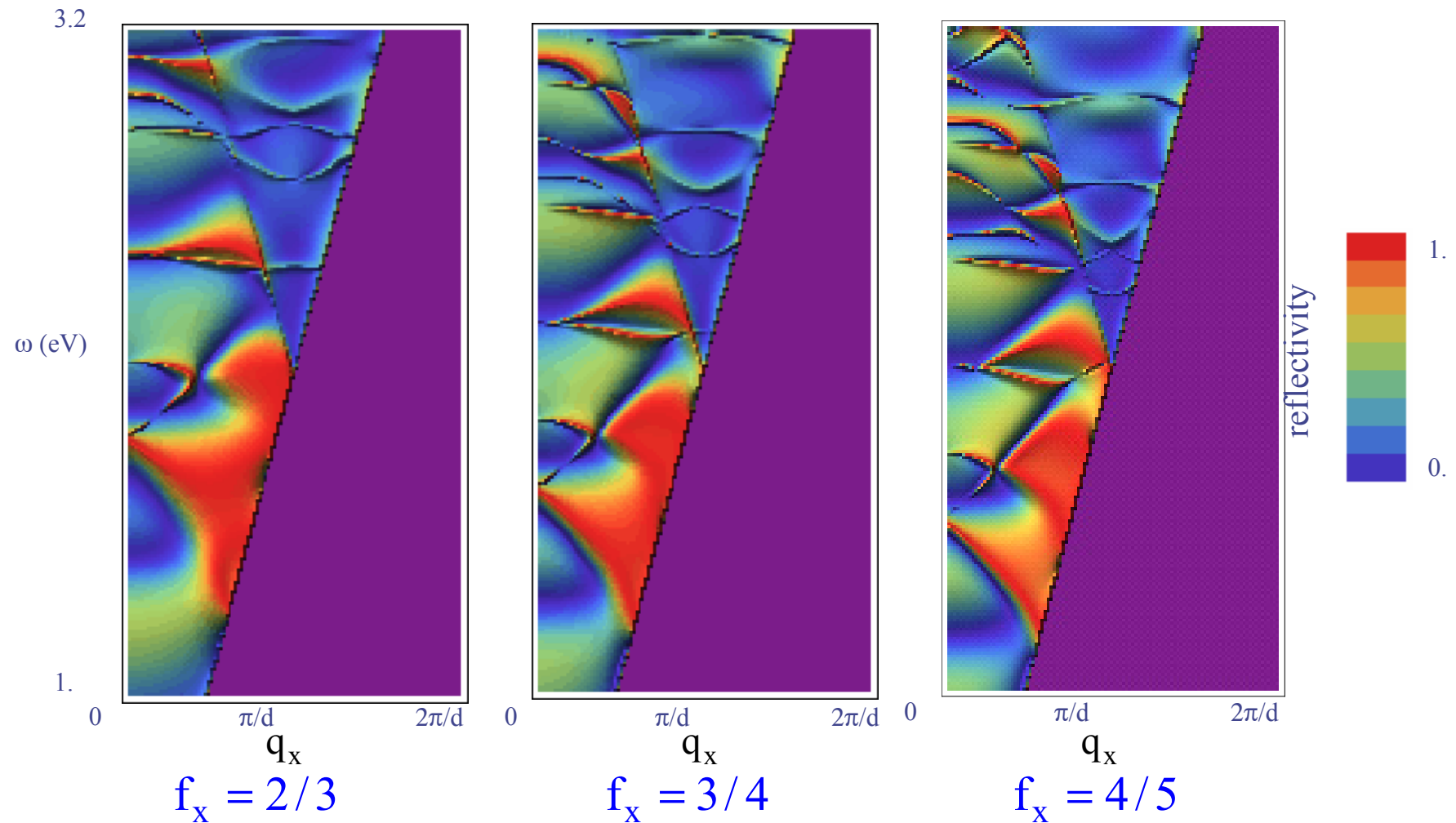
negative transmission



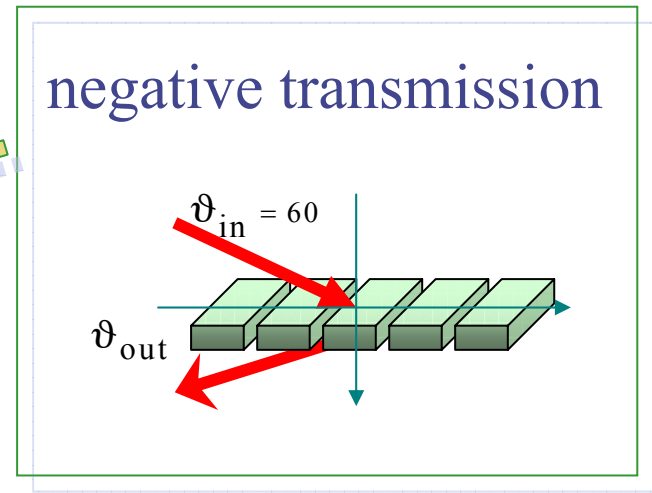
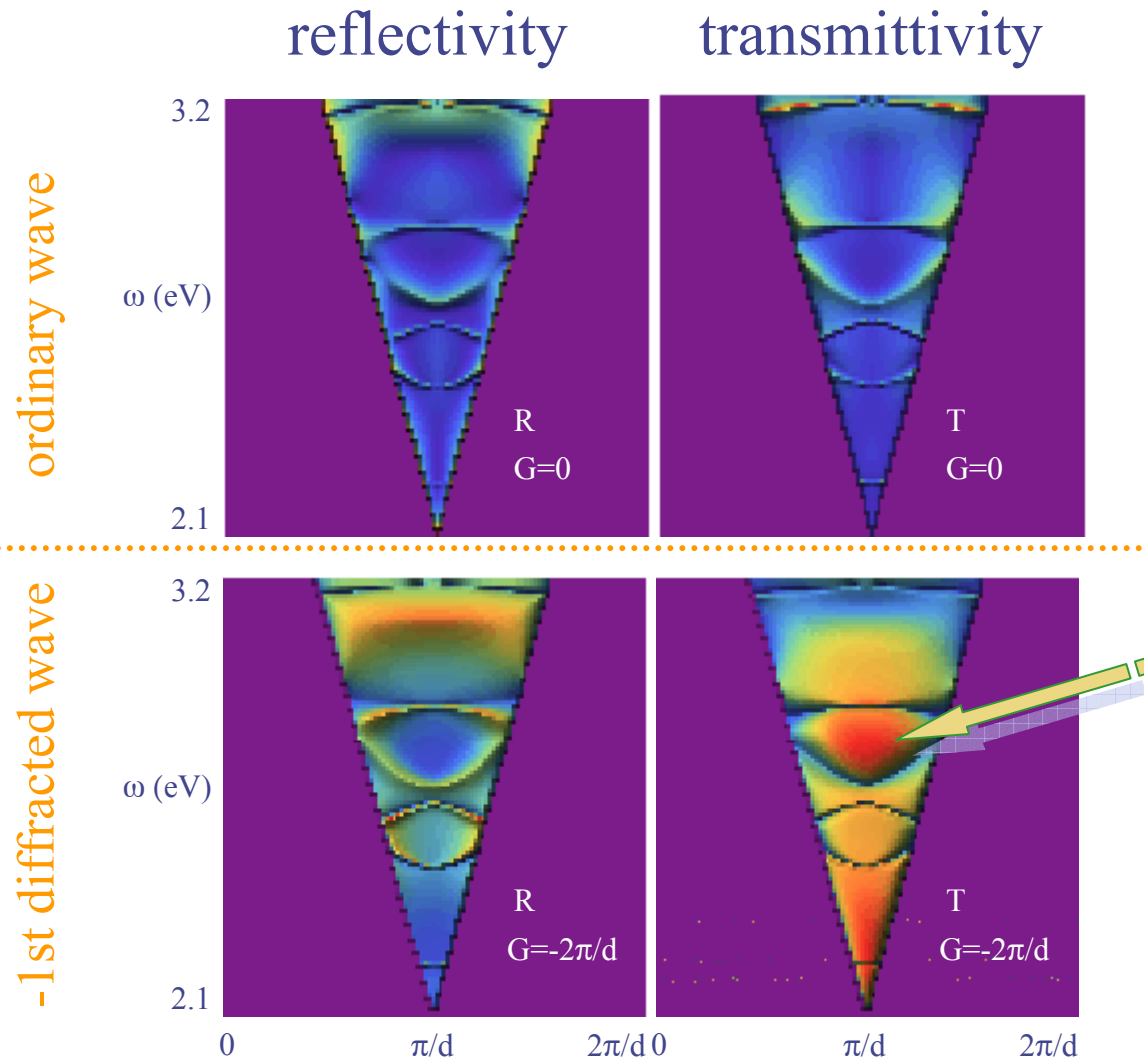
Mirror effect at the Brewster angle in semiconductor rectangular gratings

L.Pilozzi, A. D'Andrea, H. Fenniche, Phys.Rev. B **64**, 235319 (2001)

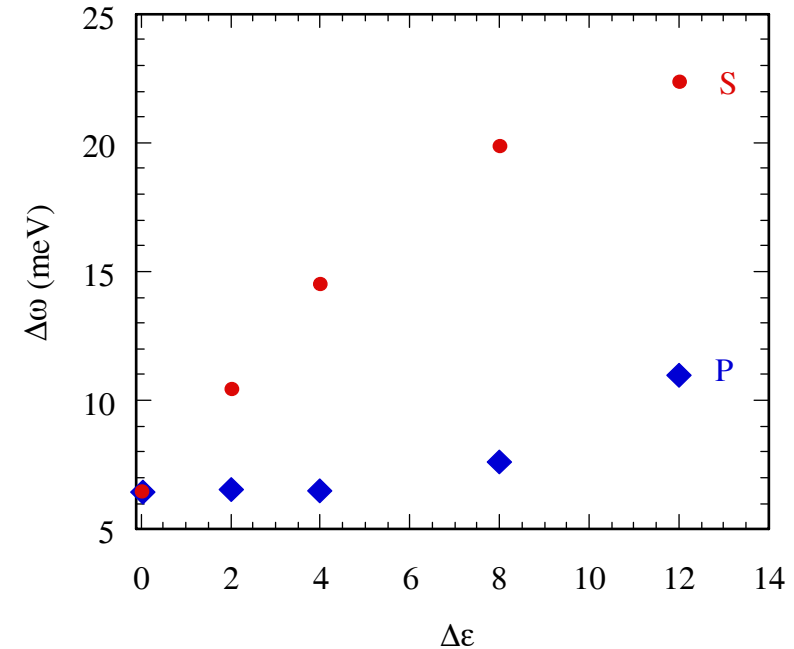
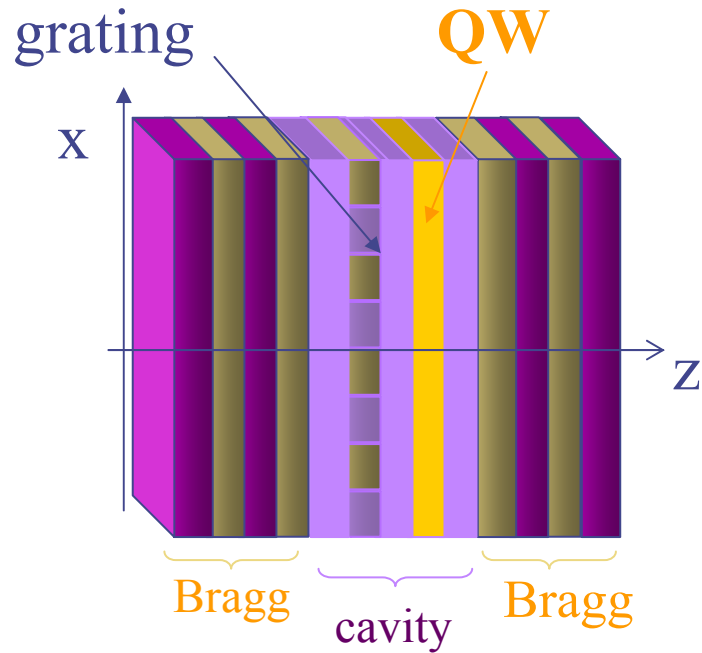
Reflectivity maps



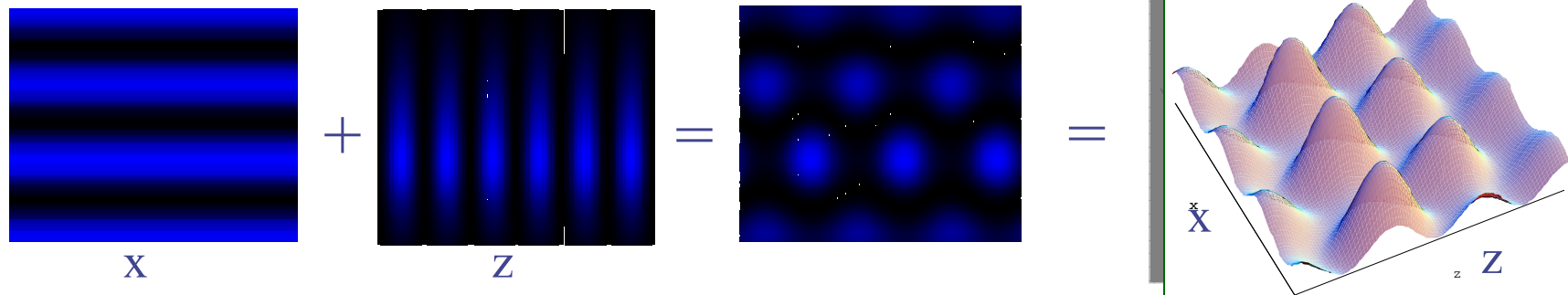
f_x = filling factor



Rabi splitting enhancement



Electric field in the cavity



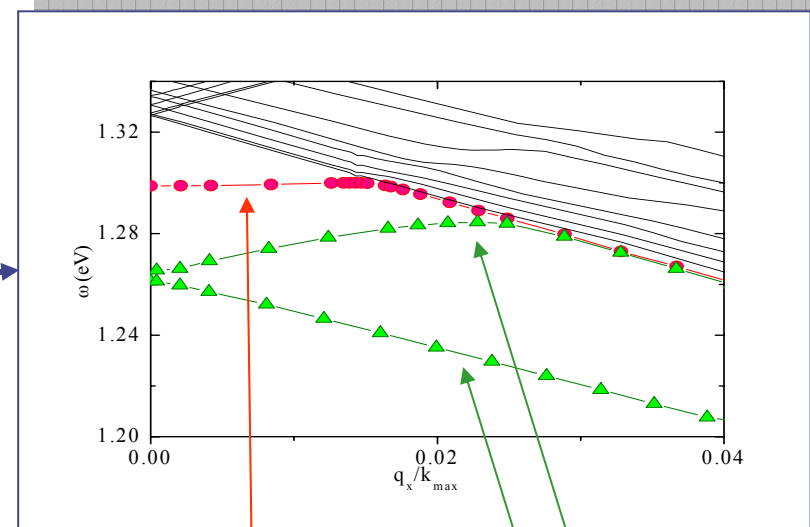
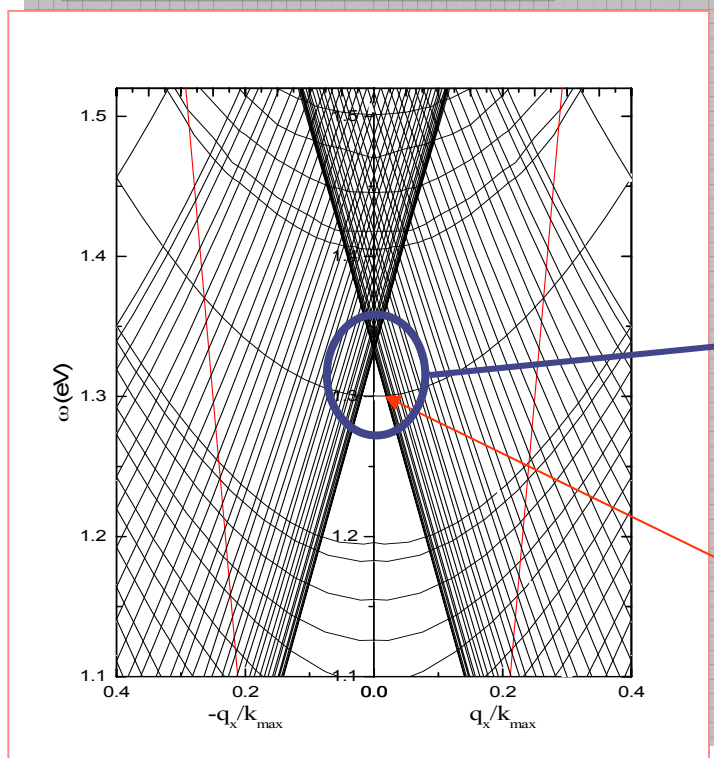
Grating-induced enhancement of exciton-polariton Rabi splitting in a planar microcavity

L.Pilozzi, A. D'Andrea, Phys.Rev. B **61**, 4771 (2000)

$$G_{cp}(z, z', \omega, k_{//}) = g(z, z', \omega, k_{//}) + \frac{h_{ox}(z, \omega) h_{xo}(z', \omega)}{E_x(k_{//}) + \Sigma_{xx}(k_{//}, \omega) - \hbar\omega - i\gamma}$$

Cavity polariton modes

Polariton self-energy



Cavity mode

grating modes

Effect of lateral periodicity on the optical response of a quantum well in a distributed Bragg reflector cavity: A simplified description via the Green function of a cavity polariton
 L.Pilozzi, A. D'Andrea, k. Cho, Phys.Rev. B **76**, 245312 (2007)

Microscopic nonlocal response theory

$$\epsilon(\vec{r}, \vec{r}', \omega) = \epsilon_b(\vec{r}, \omega) + 4\pi\chi(\vec{r}, \vec{r}', \omega)$$

- **Microscopic self-consistent approach**
- **Characteristic optical features dependent on system size and shape.**

the microscopic spatial
structure of the polarization

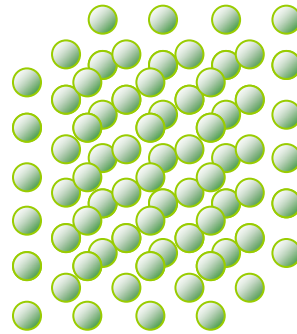


reflects on

the response field intensity

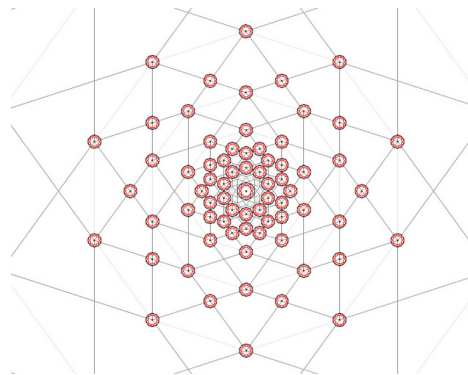
$$\vec{P}'(\vec{r}, \omega) = \int d^3r' \chi'(\vec{r}, \vec{r}', \omega) \vec{E}(\vec{r}', \omega)$$

crystals



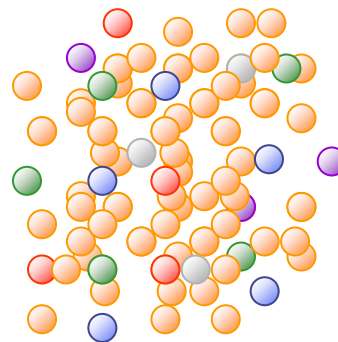
long range order
and
periodicity

quasicrystals

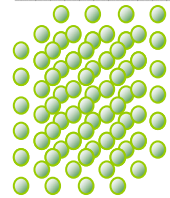


long range order
no periodicity

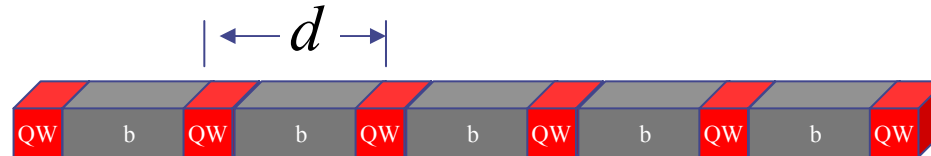
disordered systems



no long range order
no periodicity



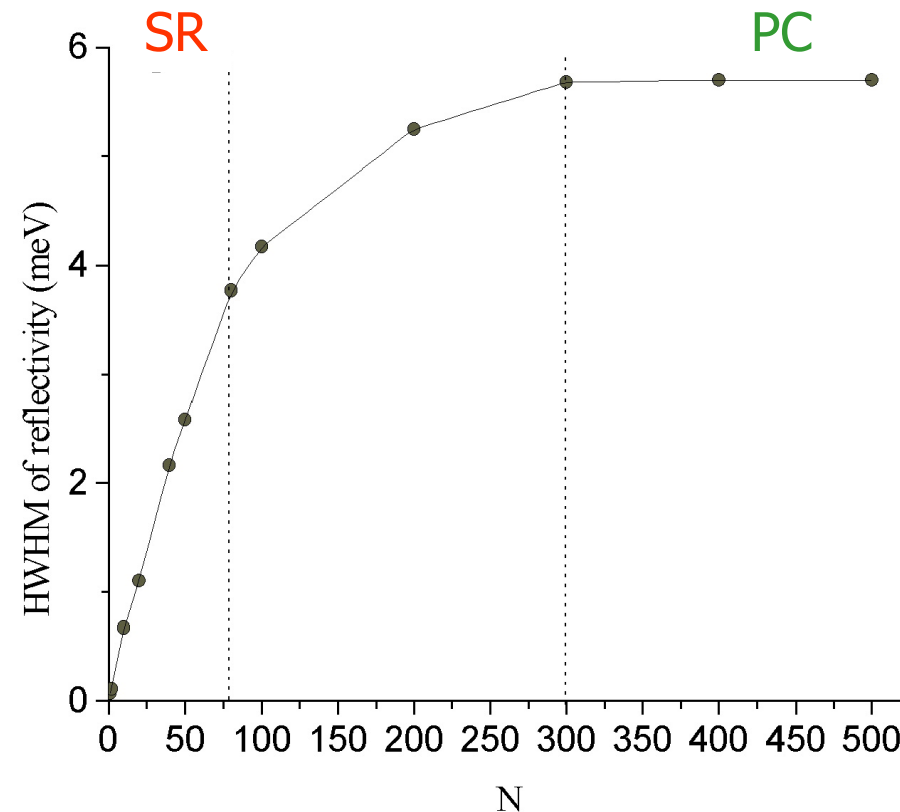
MQW: superradiance (SR) and polaritonic crystal (PC) limit



$$d = \frac{\lambda(\omega_o, \epsilon_b)}{2}$$

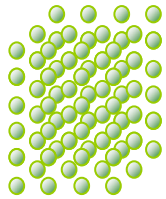
ω_o = exciton resonance

N=quantum well number



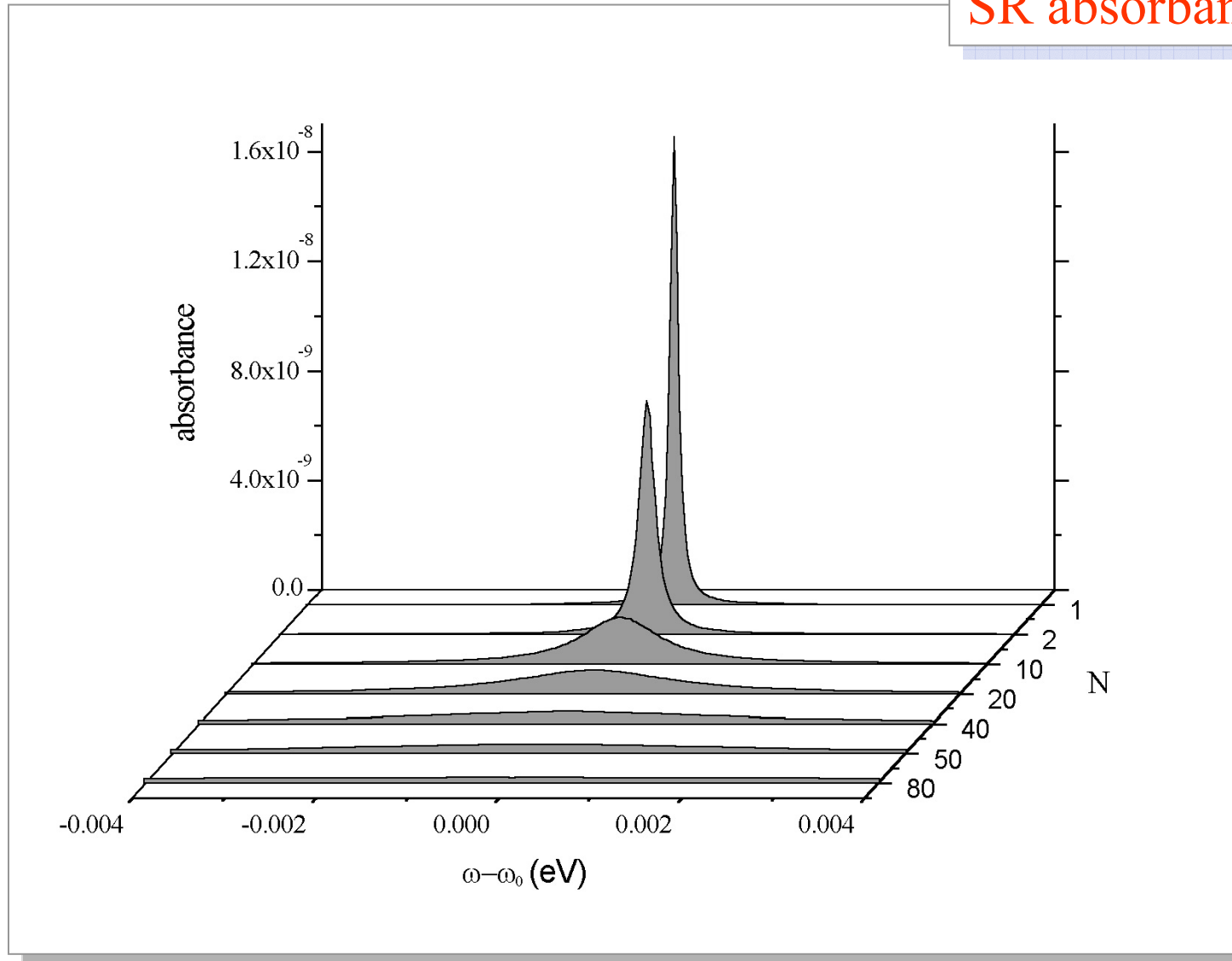
Spatial dispersion effects on the optical properties of a resonant Bragg reflector

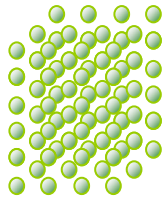
L.Pilozzi, A. D'Andrea, K. Cho, Phys.Rev. B **69**, 205311 (2004)



MQW: super-radiant regime

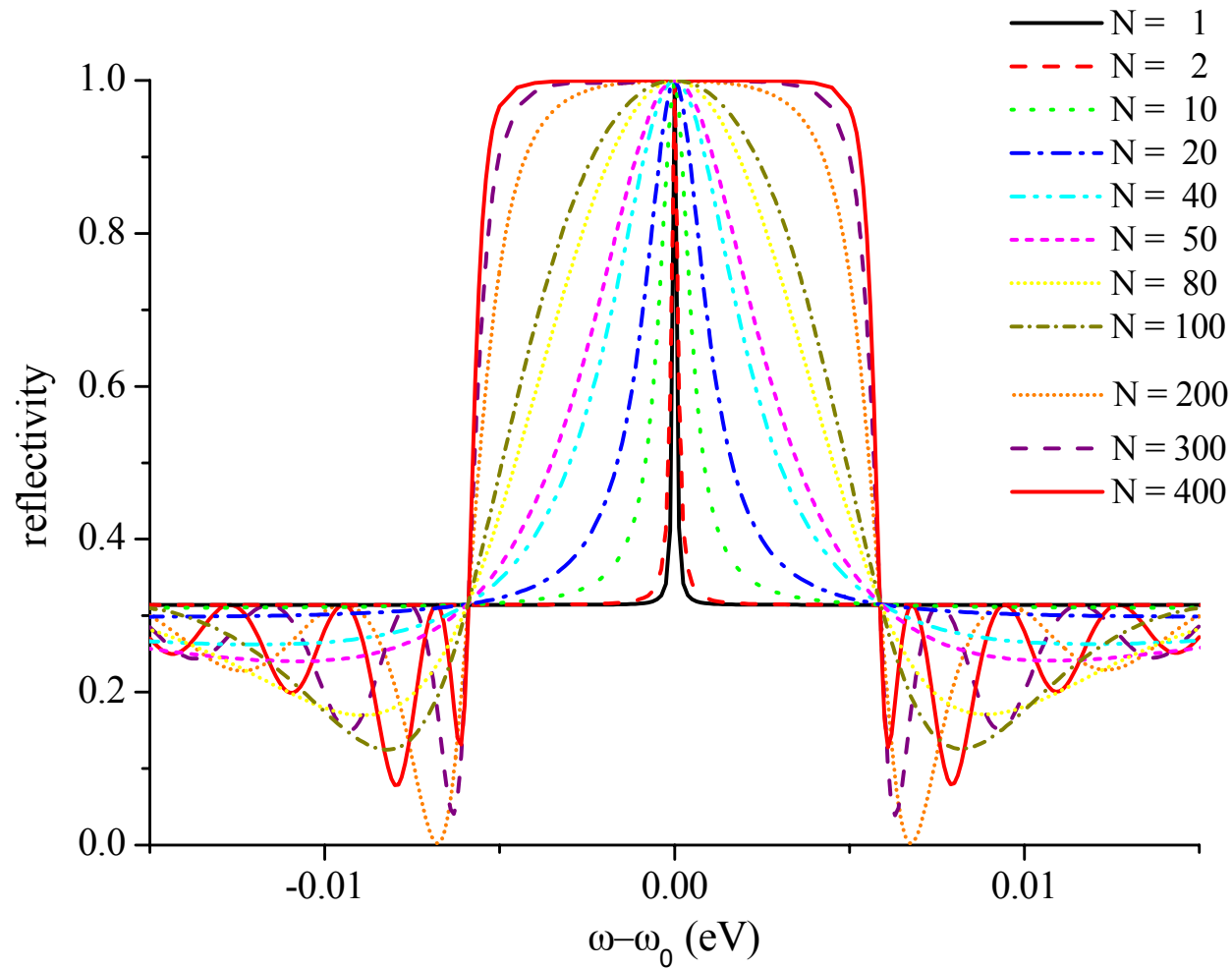
SR absorbance

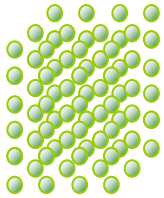




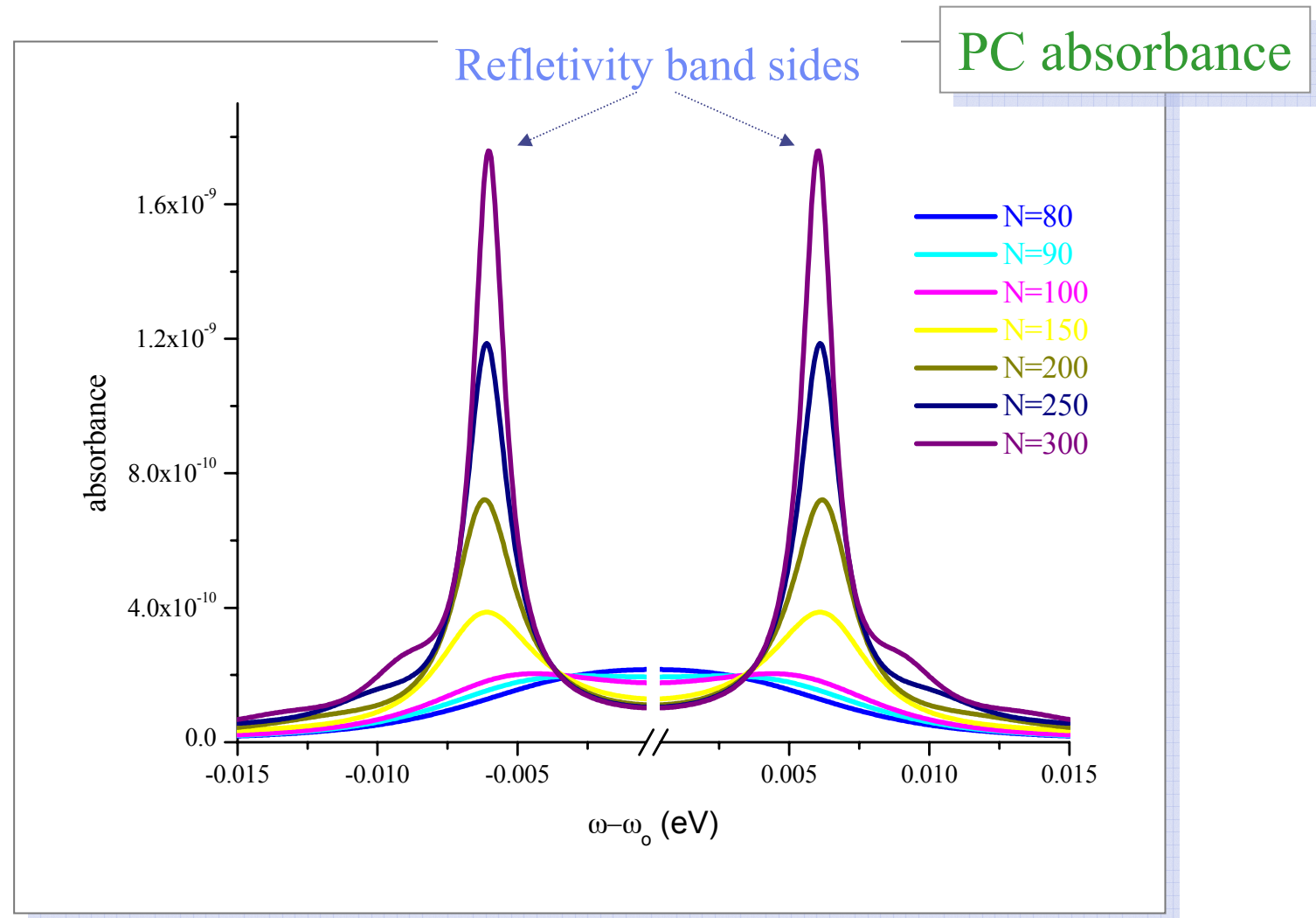
MQW: polaritonic crystal regime

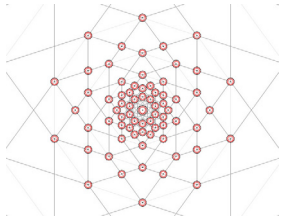
PC reflectivity



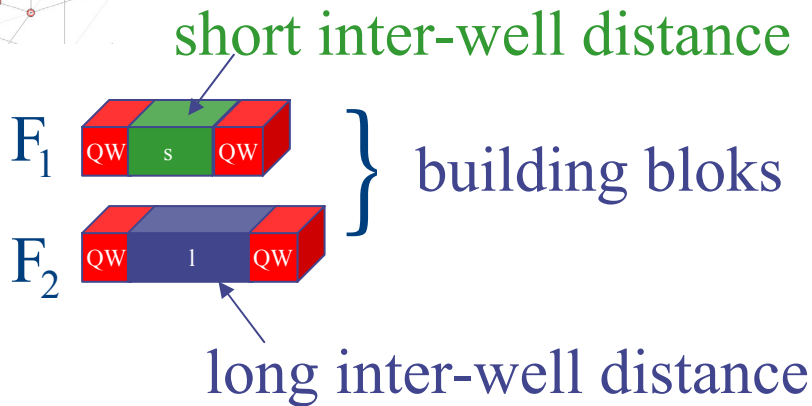


MQW: polaritonic crystal regime





QW Fibonacci chains: scaling and fractals



golden ratio

$$l/s = \frac{\sqrt{5} + 1}{2}$$

$$F_{j+1} = F_j \cup F_{j-1}$$



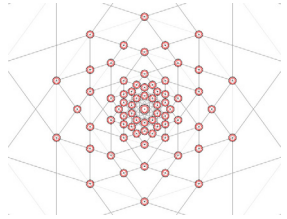
QW Fibonacci chain

Resonant Fibonacci quantum well structures in one dimension

A.N. Poddubny, L.Pilozzi, M.M.Voronov, E.L.Ivchenko, Phys.Rev. B **77**, 113306 (2008)

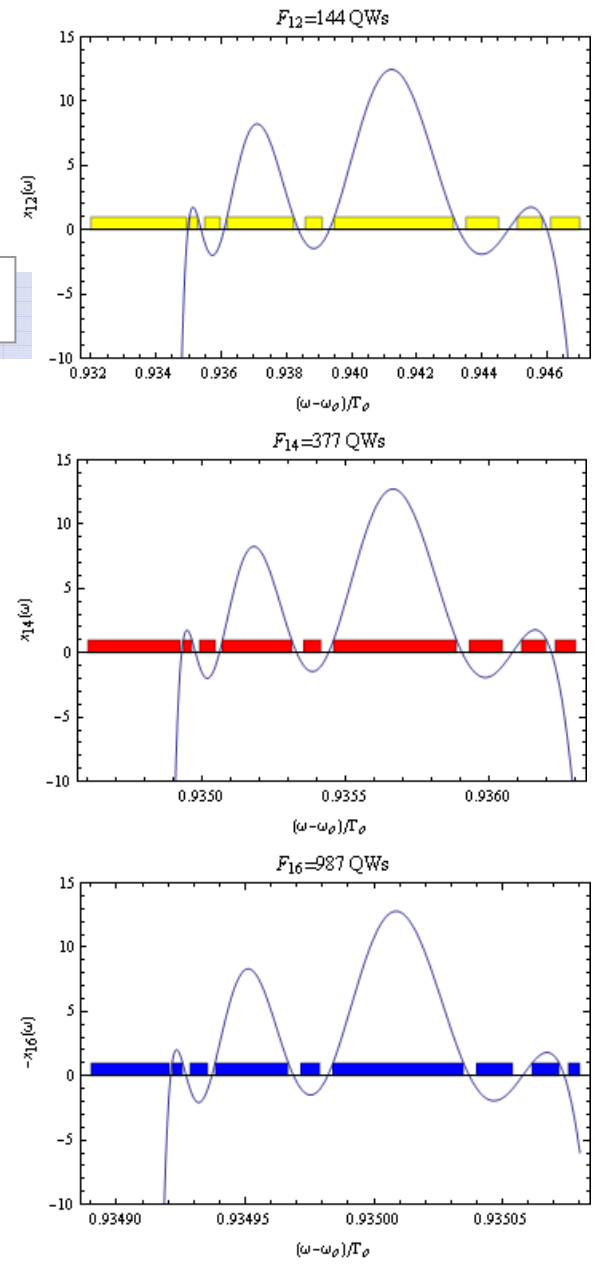
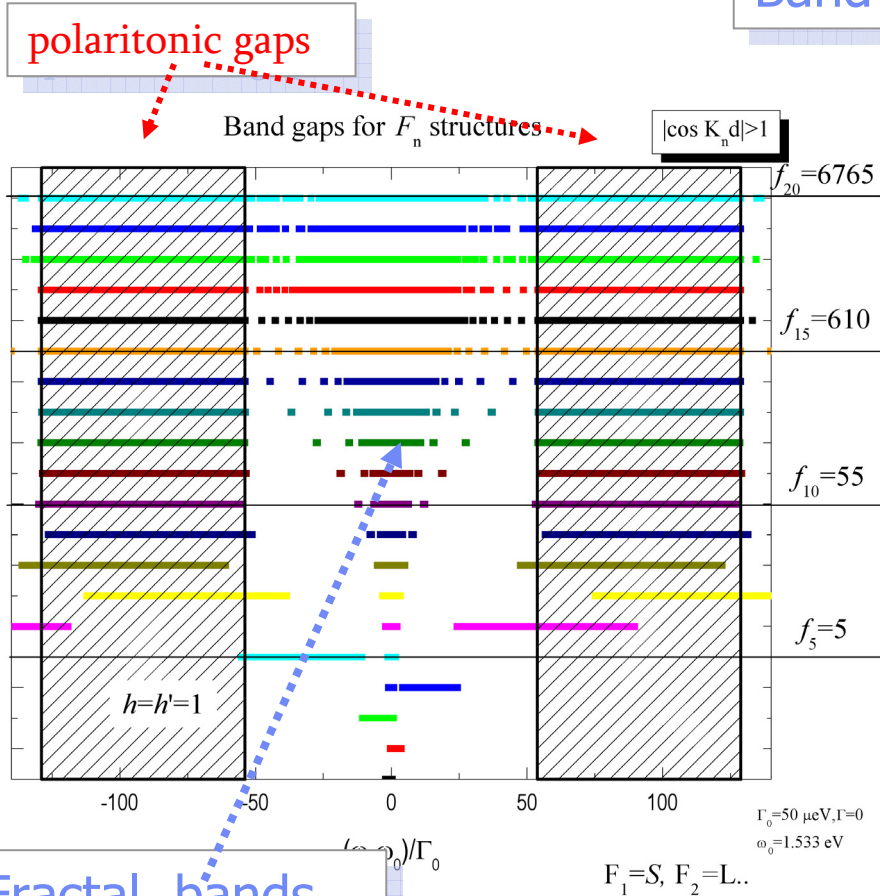
Exciton-polaritonic quasicrystalline and aperiodic structures

A.N. Poddubny, L.Pilozzi, M.M.Voronov, E.L.Ivchenko, Phys.Rev. B **80**, 115314 (2009)



QW Fibonacci chain

Band scaling



Maxwell eqs.

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E}(\vec{r}, \omega) - q^2 \vec{E}(\vec{r}, \omega) = 4\pi q^2 \left[\vec{P}_b(\vec{r}, \omega) + \vec{P}_x^1(\vec{r}, \omega) + \vec{P}'(\vec{r}, \omega) \right]$$

$$q = \frac{\omega}{c}$$

macroscopic r-dependence

$$\vec{P}_b(\vec{r}, \omega) = \chi_b(\vec{r}, \omega) \vec{E}(\vec{r}, \omega)$$

microscopic r-dependence

$$\vec{P}_x^1(\vec{r}, \omega) = \sum_{\mu} \frac{P_{o\mu}(\vec{r})}{E_{\mu o} - \hbar\omega - i\gamma} \int d\vec{r}' P_{o\mu}(\vec{r}') E(\vec{r}', \omega)$$

P' remaining part of induced polarization

Kikuo Cho

“*Reconstruction of Macroscopic Maxwell Equations*” Springer Tracts. 237 (2010)

“*Optical Response of Nanostructures: Microscopic Nonlocal Theory*” Springer (2003)

Different ways to renormalize the Green function:

- 1) Green function for vacuum

$$\left[\vec{\nabla} \times \vec{\nabla} \times - q^2 \vec{\mathbb{I}} \right] \vec{G}_{\text{vac}}(\vec{r}, \vec{r}', \omega) = 4\pi \vec{\mathbb{I}} \delta(\vec{r} - \vec{r}')$$

No polarization is normalized into EM field

$$\mathbf{E}(\vec{r}, \omega) = \mathbf{E}_o(\vec{r}, \omega) + \int \vec{G}_{\text{vac}}(\vec{r}, \vec{r}', \omega) \mathbf{P}(\vec{r}', \omega) d\vec{r}'$$

incident field in vacuum

polarization induced field

2) Cavity Green function

Only background polarization is renormalized

$$\vec{\nabla} \times \vec{\nabla} \times \vec{g}(\vec{r}, \vec{r}', \omega) - q^2 \{1 + 4\pi\chi_b(\vec{r}, \omega)\} \vec{g}(\vec{r}, \vec{r}', \omega) = 4\pi q^2 \vec{I} \delta(\vec{r} - \vec{r}')$$

$$E(\vec{r}, \omega) = E_o^{\text{cav}}(\vec{r}, \omega) + \int \vec{g}(\vec{r}, \vec{r}', \omega) \left[\vec{P}_x^1(\vec{r}, \omega) + \vec{P}'(\vec{r}, \omega) \right] d\vec{r}'$$

General shaped superstructure

EM Green function with nonlocal renormalization of background dielectrics

$$\vec{\nabla} \times \vec{\nabla} \times \vec{g}(\vec{r}, \vec{r}', \omega) - q^2 \{1 + 4\pi\chi_b(\vec{r}, \omega)\} \vec{g}(\vec{r}, \vec{r}', \omega) = 4\pi q^2 \vec{I} \delta(\vec{r} - \vec{r}')$$

a) Non-local form of the background local polarization

$$\vec{P}_b(\vec{r}, \omega) = \chi_b(\vec{r}, \omega) \int d^3 r' \delta(\vec{r} - \vec{r}') \vec{E}(\vec{r}', \omega)$$

b) Degenerate kernel

$$\delta(\vec{r} - \vec{r}') = \sum_v \phi_v^*(\vec{r}) \phi_v(\vec{r}')$$

$$\vec{P}_b(\vec{r}, \omega) = \sum_v \chi_{bv}(\vec{r}, \omega) \phi_v^*(\vec{r}) \int d^3 r' \phi_v(\vec{r}') \vec{E}(\vec{r}', \omega)$$

3) Green function of cavity polariton

we include the linear polarization

$$\left[\vec{\nabla} \times \vec{\nabla} \times -q^2 \epsilon_b(\vec{r}, \omega) I \right] \vec{G}_{cp}(\vec{r}, \vec{r}', \omega) - \sum_{\mu} \frac{4\pi q^2 P_{o\mu}(\vec{r})}{E_{\mu o} - \hbar\omega - i\gamma} H_{\mu o}(\vec{r}', \omega) = 4\pi q^2 \vec{I} \delta(\vec{r} - \vec{r}')$$

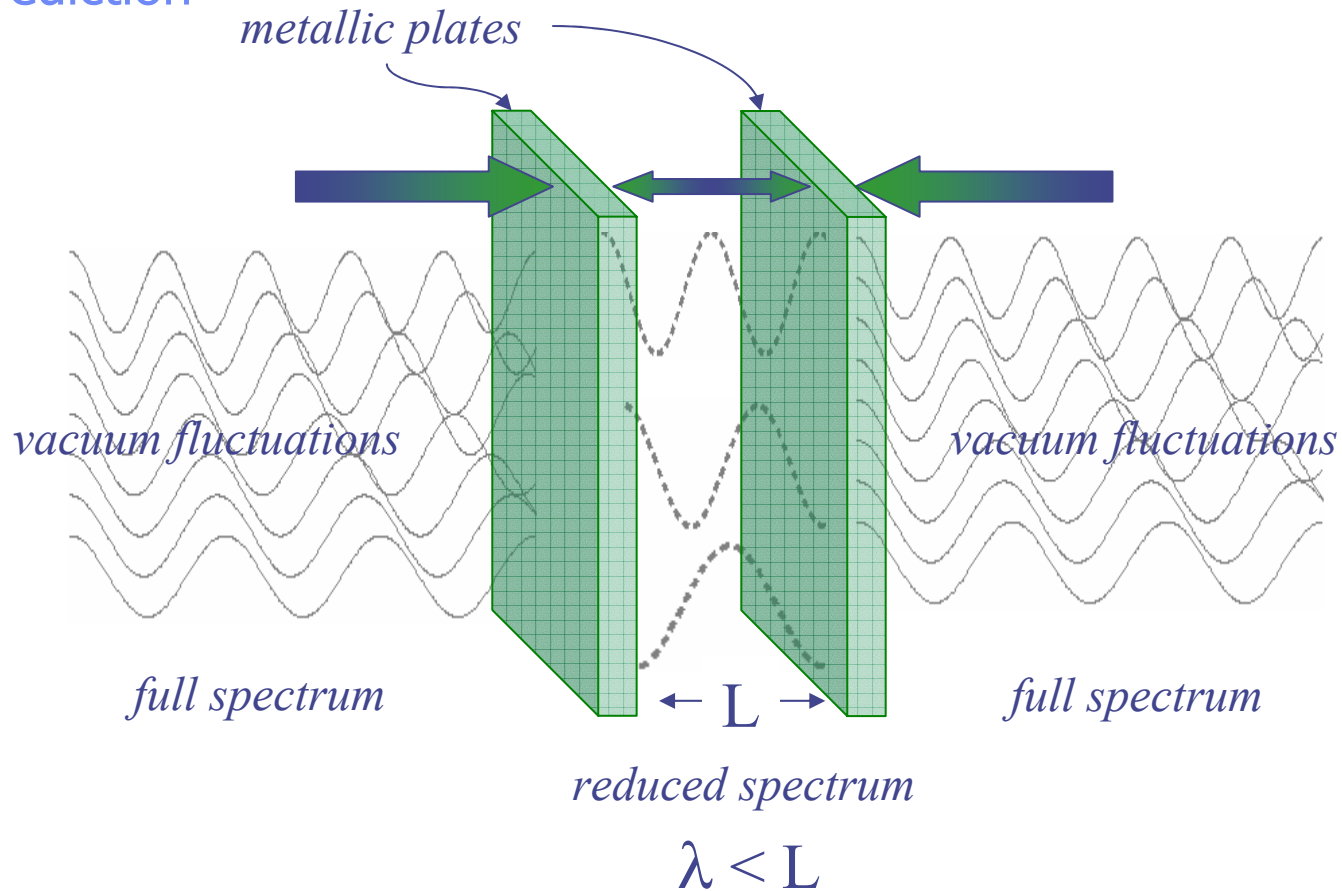
$$H_{\mu o}(\vec{r}, \omega) = \int d\vec{r}'' P_{\mu o}(\vec{r}'', \omega) G_{cp}(\vec{r}, \vec{r}'', \omega)$$

$$\vec{G}_{cp}(\vec{r}, \vec{r}', \omega) = g(\vec{r}, \vec{r}', \omega) + \sum_{\mu} \frac{h_{o\mu}(\vec{r}, \omega)}{E_{\mu o} - \hbar\omega - i\gamma} H_{\mu o}(\vec{r}', \omega)$$

field induced by $P_{o\mu}(\vec{r})$
in the empty cavity

$$h_{o\mu}(\vec{r}, \omega) = \int d\vec{r}' g(\vec{r}, \vec{r}', \omega) P_{o\mu}(\vec{r}', \omega)$$

Prediction



H. B. G. Casimir, *Proc. K. Ned. Akad. Wet.* **51**(1948) 793

Zero point energy modification of the radiation field can produce an attractive force between neutral conductors.

$$F(L)/A = -\frac{\partial U(L)}{\partial L} \frac{1}{A} = \frac{\pi^2 \hbar c}{240L^4}$$

1 atmosphere for $L=10$ nm

- Change in e.m. field boundary conditions induces change in mode frequencies and then in the zero point energy:

$$E = \sum_n \frac{1}{2} \hbar \omega_n \quad \omega_n = c \sqrt{k_{//}^2 + \frac{\pi^2}{L^2} n^2}$$

$$E(L) = \frac{\hbar c A}{\pi} \sum_{n=0}^{\infty} 2\pi \int_0^{\infty} k_{//} dk_{//} \sqrt{k_{//}^2 + \frac{\pi^2}{L^2} n^2} \quad U(L) = E(L) - E(\infty) = -\frac{\pi^2 \hbar c}{720 L^3} A$$

- Casimir Forces as e.m. **stress energy tensor** can be obtained via Fluctuation-Dissipation theorem from the imaginary-frequency ($\omega = i\omega$) **Green's function**.

$$\langle E_j(\vec{x}) E_k(\vec{x}') \rangle = \frac{\hbar}{\pi} \omega^2 G_{jk}(i\omega, \vec{x} - \vec{x}')$$

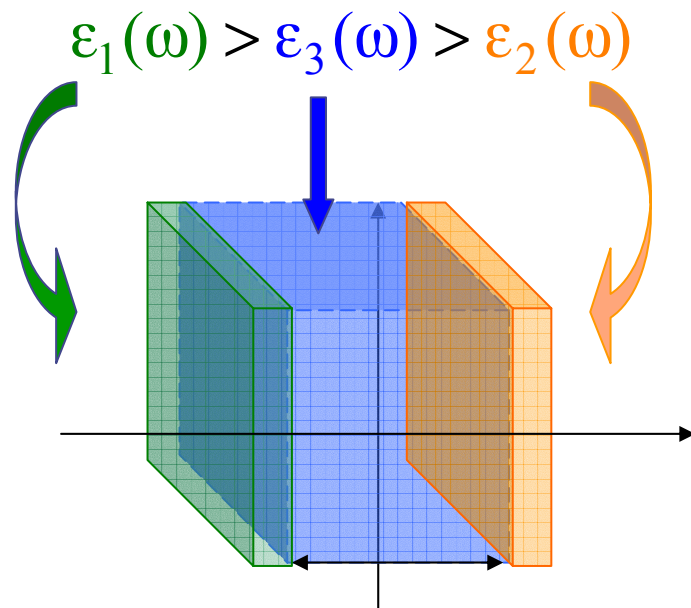


Measurements

- Marcus Sparnaay at Philips in Eindhoven, in 1958 confirmed the Casimir force existence but was not able to reach sufficient accuracy
- **1996 Steve K. Lamoreaux**
Los Alamos National Laboratory
"Demonstration of the Casimir Force in the 0.6 to 6 μm Range"
Phys. Rev. Lett. **78**, 5–8 (1997)
measurement agreement with theory **10%**
- **1997 Umar Mohideen and Anushree Roy**
University of California at Riverside
"Precision Measurement of the Casimir Force from 0.1 to 0.9 μm "
Phys. Rev. Lett. **81**, 004549 (1997)
measurement agreement with theory **1%**

Material design for force control:
crossover from attraction to repulsion

As shown theoretically in a seminal paper by [Evgeny Lifshitz](#)
in certain circumstances repulsive forces can arise



Casimir repulsion
experimentally demonstrated in
Gold-Bromobenzene-Silicon

nature

Vol 457|8 January 2009|doi:10.1038/nature07610

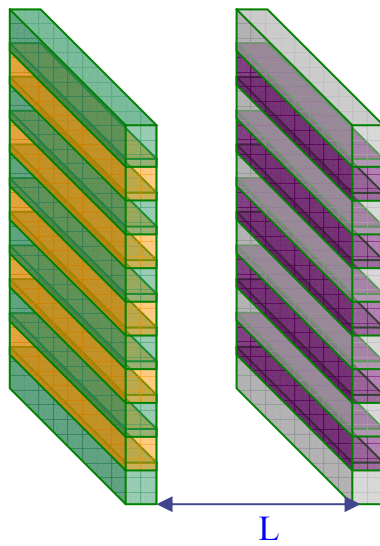
LETTERS

Measured long-range repulsive Casimir-Lifshitz forces

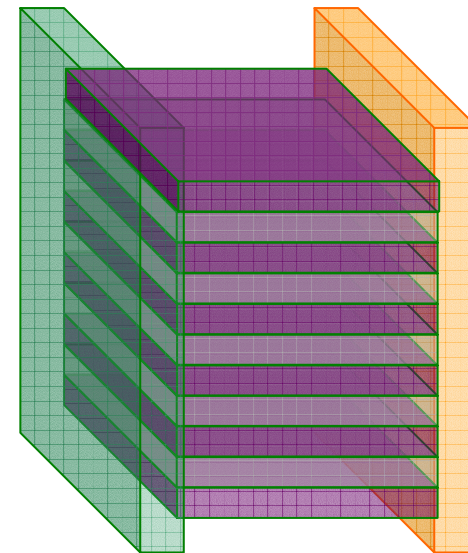
J. N. Munday¹, Federico Capasso² & V. Adrian Parsegian³

Can geometry alone
give rise to Casimir repulsion?

Casimir force tailoring by complex asymmetric cavity geometries



Patterned mirrors



Patterned cavity

- numerical approach
Imaginary-frequency scattering-matrix techniques

$$P(d) = 2k_B T \sum_{n=0}^{\infty} \int_0^{\infty} \frac{d^2 \vec{q}_{//}}{2\pi} q_z(iw_n) \sum_{\alpha=S,P} \frac{r_{\alpha}^{(1)}(iw_n) r_{\alpha}^{(2)}(iw_n) e^{-2q_z d}}{1 - r_{\alpha}^{(1)}(iw_n) r_{\alpha}^{(2)}(iw_n) e^{-2q_z d}}$$

$$r_S^{\mu}(iw_n) = \frac{q_z - k_z^{\mu}}{q_z + k_z^{\mu}}$$

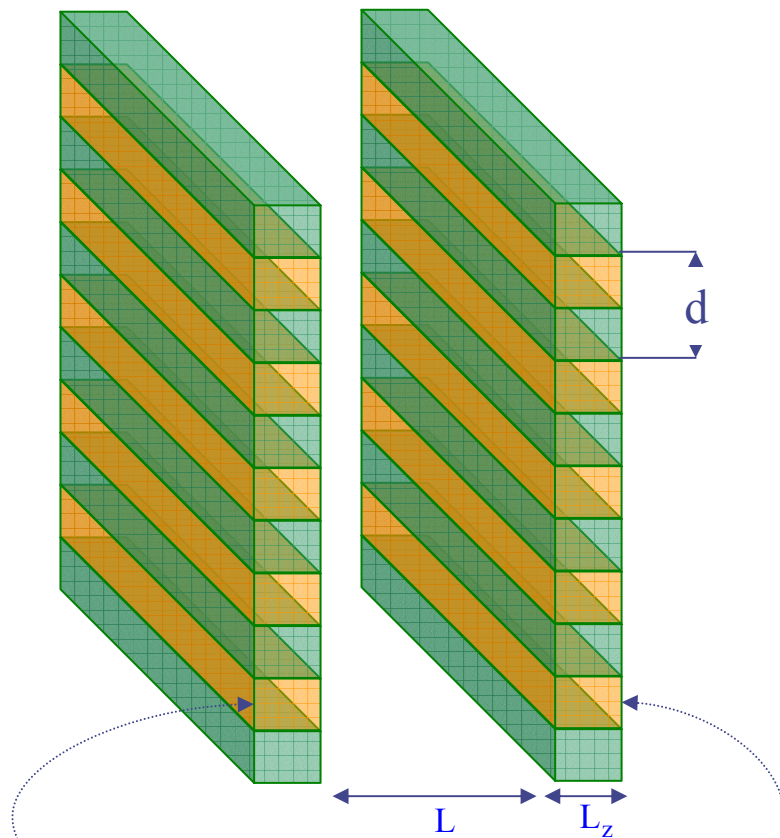
$$r_P^{\mu}(iw_n) = \frac{\epsilon_0 k_z^{\mu} - q_z \epsilon^{\mu}(iw_n)}{\epsilon_0 k_z^{\mu} + q_z \epsilon^{\mu}(iw_n)}$$

$$w_n = 2\pi \frac{k_B T}{\hbar} n$$

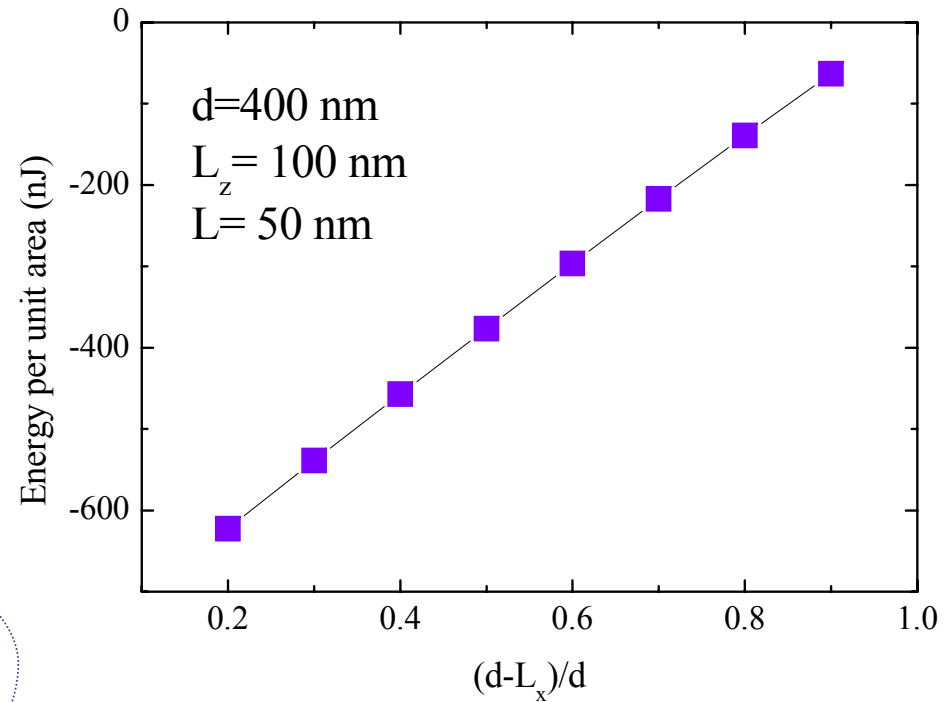
For patterned systems

$$P(L, T) = -k_B T \frac{\partial}{\partial L} \sum_{n=0}^{\infty} \int_0^{\infty} \frac{d^2 \vec{q}_{//}}{2\pi} \ln \det \left[\vec{I} - \vec{r}_1(iw_n) \vec{\chi}^>(L) \vec{r}_2(iw_n) \vec{\chi}^>(L) \right]$$

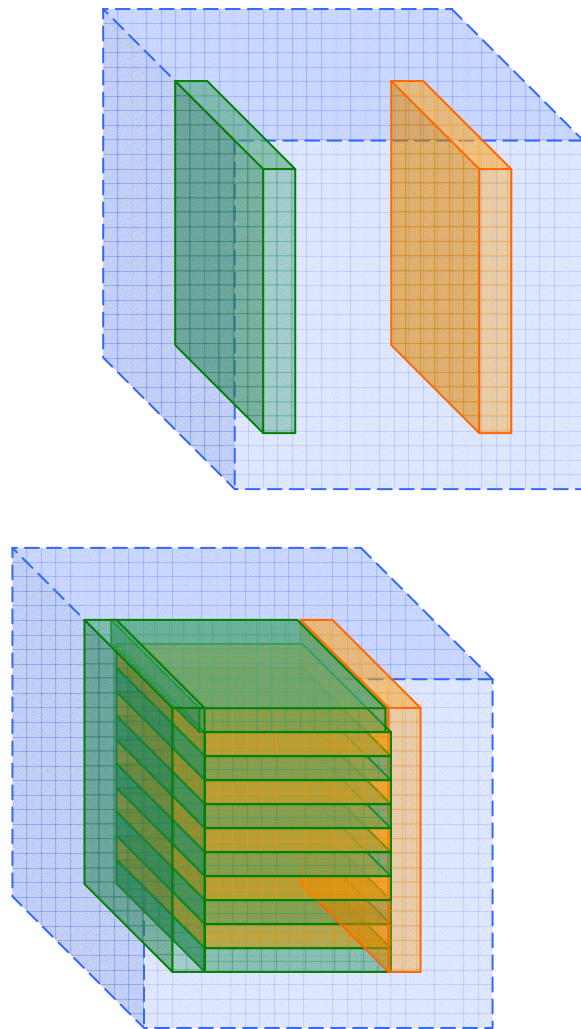
Casimir force tailoring by complex asymmetric cavity geometries



Patterned mirrors

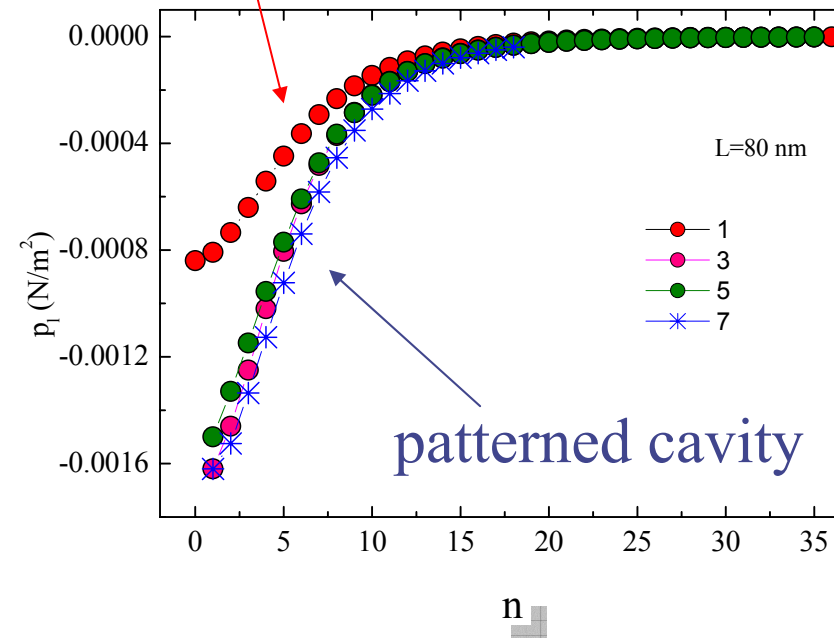


Casimir force tailoring by complex asymmetric cavity geometries single Matsubara frequency contributions



patterned cavity

homogeneous cavity



patterned cavity

Matsubara frequency

$$w_n = 2\pi \frac{k_B T}{\hbar} n$$

- We have discussed the optical properties of periodic and aperiodic polaritonic crystals and have shown how a particular renormalization of the Green function allows to consider more complex geometries.
- This renormalization is the required tool to consider complex geometries for the Casimir force tailoring

since
- this force can be computed as an integral of the mean electromagnetic stress tensor over all the frequencies, determined from the classical Green function as a consequence of the fluctuation-dissipation theorem.