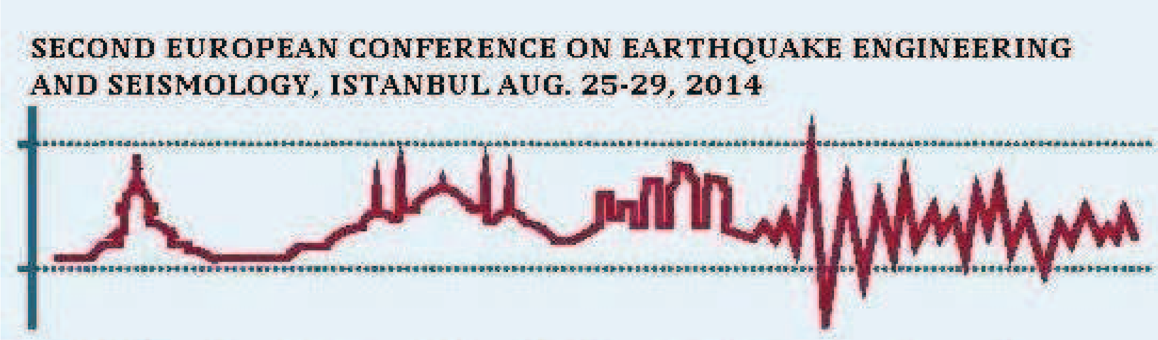

STATISTICAL MODELLING FOR IDENTIFICATION OF EARTHQUAKE CLUSTERS

Elisa VARINI[1], Yosihiko OGATA[2]

It is widely recognized that earthquake clustering is a main feature of the seismicity and a seismic area can be affected by different types of earthquake clusters, such as aftershock sequences and swarms, due to its peculiar tectonic and volcanic environments. Different occurrence rates are expected to be observed in a sufficiently long period, each corresponding to and characterizing a different type of earthquake clusters.

In this study we propose a probabilistic approach to model different types of earthquake clusters, also named states of the system, in order to identify and quantify them. To this end, we assume a state-space model (*X*,*Y*) in which the states of the hidden (unobserved) process *X* drive different realizations of the observed process *Y*.

The earthquakes (observations) are first associated with a state and, conditioned on that state, follow an ETAS (Epidemic-Type Afetrshock-Sequence) point process. The hidden state process *X* is assumed to be a pure jump Markov process and the hazard function $\lambda(t|H_t)$ of the observed process Y is given by the following relation:

$$\lambda(t|H_t) = \sum_{s\in S} \lambda_s(t|H_t)\delta_s(X_t) \ . \qquad (1)$$

where $\delta_s(X_t) = 1$ if $X_t = s$ and $\delta_s(X_t) = 0$ otherwise.

The problem of the likelihood approximation is solved by particle filtering technique and parameter estimation is dealt with by Markov Chain Monte Carlo method in the Bayesian framework.

We analyse two earthquake sequences: the former occurred off the east coast of Izu Peninsula (Japan) in 1998 and the latter started in 2011, a week after the Tohoku-Oki earthquake, Northwest of Lake Inawashiro.

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Statistical modelling for identification of earthquake clusters

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INTRODUCTION

It is widely recognized that earthquake clustering is a main feature of the seismicity. A seismic area can be affected by different types of earthquake clusters, such as aftershock sequences and swarms, due to its peculiar tectonic and volcanic environments. Different occurrence rates are expected to be observed in a sufficiently long period, each corresponding to and characterizing a different type of earthquake clusters. For this purpose we model seismic sequences by doubly stochastic Poisson processes. These processes belong to the family of the state-space models and are such that the observed process of earthquake occurrence times is a point process whose conditional intensity function is assumed to be dependent on both the past history and the current state. In particular we consider two point processes drawn from the literature on statistical seismology: the simple Poisson model and the epidemic-type aftershock-sequence model. The hidden state process is assumed to be a stationary Markov process so that the current state depends only on the last visited state. Bayesian analysis of the 1998 Izu swarm is carried out: a sequential Monte Carlo method is applied to approximate the likelihood function and Markov Chain Monte Carlo methods are used for the parameter estimation.

THE PROPOSED STATE-SPACE MODEL

Let $(\mathbf{X}, \mathbf{Y})$ follow a continuous-time state-space model [4], in particular a doubly stochastic Poisson process [3] defined as follows:

- the STATE PROCESS $\mathbf{X} = (X_t)_{t \geq 0} = (S_n, J_n)_{n \in \mathcal{N}}$ is a hidden (unobserved) pure jump Markov process such that it visits the state J_n at the jump time S_n:

$$X_t = J_n \quad \text{for all} \quad t \in [S_n, S_{n+1})$$

The state process $\mathbf{X}$ is completely defined by the initial probabilities $\{\phi_i(0)\}_{i \in \mathcal{X}}$, where $\phi_i(0) = P(X_0 = i)$, and by the stationary transition kernel $Q = (q_{ij})_{i,j \in \mathcal{X}}$, where $q_{ij} \geq 0$ for $i \neq j$ and $q_{ii} = -\sum_{j: j \neq i} q_{ij}$. This implies that:

  - the transition probability from state i to state j is $p_{ij} = P(J_n = j \mid J_{n-1} = i) = -\frac{q_{ij}}{q_{ii}}$,
  - the holding time $Z_n = S_n - S_{n-1}$ in state $J_{n-1} = i$ is exponentially distributed with rate $-q_{ii}$.

In this application we assume three possible states, that is $J_n \in \{1, 2, 3\}$ for all n.

- the OBSERVED PROCESS $\mathbf{Y} = (Y_t)_{t \geq 0} = (t_j, m_j)_{j \in \mathcal{N}}$ is a marked point process, where t_j denotes the occurrence time of the j^{th} earthquake in the region and m_j the corresponding magnitude.
The observed process $\mathbf{Y}$ is completely defined by its hazard function conditionally on both the state X_t and the observed history $Y_{0:t}$ up to time t:

$$\lambda(t \mid X_t, Y_{0:t}) = \begin{cases} \mu & \text{if } X_t = 1 \\ \mu + \displaystyle\sum_{j: t^*_{S_n} \leq t_j < t} \frac{k e^{\gamma_2 (m_j - M_0)}}{(t - t_j + c)^p} & \text{if } X_t = 2 \\ \mu + \displaystyle\sum_{j: t^*_{S_n} \leq t_j < t} \frac{k e^{\gamma_3 (m_j - M_0)}}{(t - t_j + c)^p} & \text{if } X_t = 3 \end{cases}$$

The first expression above corresponds to a stationary Poisson process, the others are versions of the well-known Epidemic-Type Aftershock-Sequence (ETAS) models [1, 2]. We assume:

  - $\gamma_2 < \gamma_3$, which implies that the efficiency of a shock in generating its aftershocks relative to its magnitude is different when the system is in states 2 or 3;
  - $(t^*_{S_n}, m^*_{S_n})$ denotes the last earthquake occurred by the most recent state transition X_{S_n} with respect to time t. This earthquake (red bar in Figure 1) is thought as the event triggering the subsequent cluster (green time interval in Figure 1).

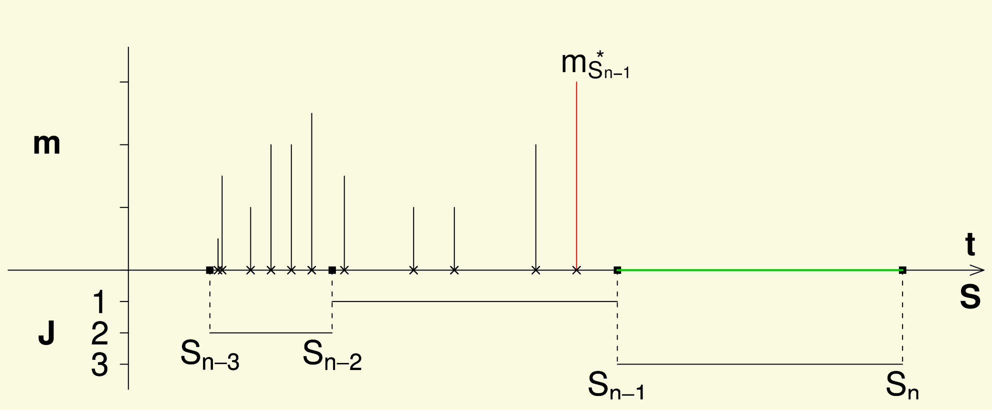

Figure 1: Outline of the proposed state-space process: observed process (upper quadrant) and hidden state process (lower quadrant).The event (red bar) triggers the upcoming earthquake cluster covering the time interval between the jump times S_{n-1} and S_n (green line).

BAYESIAN INFERENCE

Let $\theta = (\mu, k, \gamma_2, \gamma_3, c, p, p_{12}, p_{21}, p_{31}, q_{11}, q_{22}, q_{33})$ be the vector of parameters to be estimated. The aim of the Bayesian analysis is to estimate the POSTERIOR DISTRIBUTION of θ, that is expressed as follows:

$$p(\theta \mid Y_{0:T}) \propto \mathcal{L}(Y_{0:T} \mid \theta) \pi_0(\theta) ,$$

where $\mathcal{L}(Y_{0:T} \mid \theta)$ and $\pi_0(\theta)$ denote the likelihood function and the prior distribution of the parameters, respectively. By applying a Markov chain Monte Carlo method based on the Metropolis-Hastings algorithm, a Markov chain of parameter vectors $\{\theta_r : r = 1, \ldots, R\}$ is generated. For large sample size R, this Markov chain turns out to approximate the posterior distribution of θ. A crucial issue in this procedure is how the likelihood function is evaluated.
The LIKELIHOOD ESTIMATION is performed by a sequential Monte Carlo method, known as Particle Filtering, which works as follows. For every partition $\{\tau_h\}_{h=0}^{H}$ of the time interval $[0, T]$, it is proved that:

$$\mathcal{L}(Y_{0:T} \mid \theta) = \prod_{h=1}^{H} \sum_{j \in \mathcal{X}} \varphi_j(\tau_h) ,$$

where $\varphi_j(\tau_h) = E_{\mathbf{X}}\left[\mathcal{L}(Y_{\tau_{h-1}:\tau_h} \mid X_{\tau_{h-1}:\tau_h}, \theta) \delta_j(X_{\tau_h})\right]$ is proportional to the filtering probabilities $\phi_j(\tau_h) = P(X_{\tau_h} = j \mid Y_{0:\tau_h}, \theta)$. For all h, the distribution $p(X_{0:\tau_h} \mid Y_{0:\tau_h}, \theta)$ is approximated by a weighted particle set $\{(x^{(i)}_{0:\tau_h}, \omega^{(i)}_h)\}_{i=1}^{P}$ and the unnormalised filtering probabilities by:

$$\varphi_j(\tau_h) \approx \sum_{i=1}^{P} \omega^{(i)}_h \delta_j(x^{(i)}_{\tau_h}) .$$

Each particle is essentially composed by a possible state history $x^{(i)}_{0:\tau_h}$ on the interval $[0, \tau_h]$ and by a probability weight $\omega^{(i)}_h$. The particle set at time τ_{h-1}, updated through the observations in $(\tau_{h-1}, \tau_h]$, produces the particle set at time τ_h; this sequential procedure is called Particle Filtering and enables us to learn sequentially from the upcoming data.

APPLICATION TO THE 1998 IZU SWARM

The Izu area is located at north of the Philippine and Pacific tectonic plates, the so-called Izu-Bonin-Mariana arc. Its seismicity is typically associated with dike intrusion (shallow magmatic activity) around the active volcanos along the plate tectonic convergent boundary.

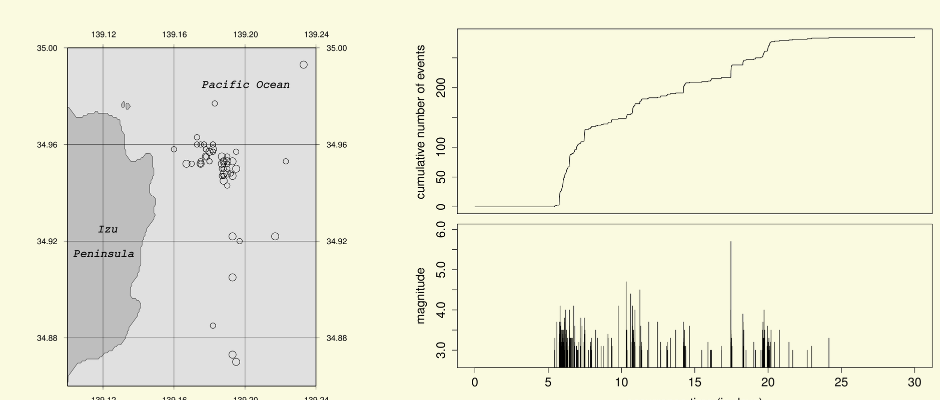

Figure 2: [Left] Map of the epicentre locations of the 1998 Izu swarm. [Right] Cumulative number of events (stepwise line) and magnitude (vertical lines) versus occurrence times.

A series of earthquake swarms off the east coast of Izu Peninsula, central Japan, has occurred intermittently in the last few decades [1]. Looking through the 1998 swarms (Figure 2), the events seem prone to occur in clusters: sometimes thick clusters with low/moderate magnitude, sometimes more scattered clusters with moderate/high magnitude. Nevertheless the strongest events are frequently isolated in time.

Table 1 shows the chosen prior and proposal distributions used to estimate the parameters of the state-space model by the MCMC method. At each time t, the hazard function of Figure 3 is evaluated by replacing the parameters with their ergodic means $\hat{\theta}$ and it is given by combining the three hazard functions of the considered Poisson and ETAS models, weighted by the estimated filtering probabilities $\hat{\Phi}_j(t)$ in each state:

$$\hat{\lambda}(t \mid Y_{0:t}) = \sum_{j=1}^{3} \hat{\Phi}_j(t) \hat{\lambda}(t \mid X_t = j, Y_{0:t})$$

Figure 4 shows which is the most probable state (1 in red, 2 in green and 3 in blue) at each time t according to the estimated filtering probabilities.

parameter	prior distr.	proposal distr.	erg. mean	(st.dev.)
μ	$\Gamma(0.05, 0.0016)$	$logN(\mu^{(r-1)}, 0.0004)$	0.0165	(0.0096)
k	$\Gamma(1, 0.81)$	$logN(k^{(r-1)}, 0.0015)$	0.1915	(0.0344)
γ_2	$\Gamma(0.5, 0.16)$	$logN(\gamma_2^{(r-1)}, 0.1)$	0.2498	(0.1652)
γ_3	$\Gamma(2, 2.25)$	$logN(\gamma_3^{(r-1)}, 1.2)$	1.9778	(1.6255)
c	$\Gamma(0.5, 0.16)$	$logN(c^{(r-1)}, 0.0025)$	0.1705	(0.0518)
p	$\Gamma(1, 0.49)$	$logN(p^{(r-1)}, 0.021)$	1.4490	(0.0957)
p_{12}	$U(0,1)$	$B(p_{12}^{(r-1)}, \frac{p_{12}^{(r-1)}(1-p_{12}^{(r-1)})}{2})$	0.5195	(0.2911)
p_{21}	$U(0,1)$	$B(p_{21}^{(r-1)}, \frac{p_{21}^{(r-1)}(1-p_{21}^{(r-1)})}{2})$	0.5054	(0.2929)
p_{31}	$U(0,1)$	$B(p_{31}^{(r-1)}, \frac{p_{31}^{(r-1)}(1-p_{31}^{(r-1)})}{2})$	0.4529	(0.2794)
$-q_{11}$	$\Gamma(200, 30000)$	$logN(-q_{11}^{(r-1)}, 28000)$	170.43	(164.60)
$-q_{22}$	$\Gamma(200, 30000)$	$logN(-q_{22}^{(r-1)}, 28000)$	342.36	(191.19)
$-q_{33}$	$\Gamma(200, 30000)$	$logN(-q_{33}^{(r-1)}, 28000)$	181.28	(157.78)

Table 1: MCMC input-output for the state-space model on Izu data.

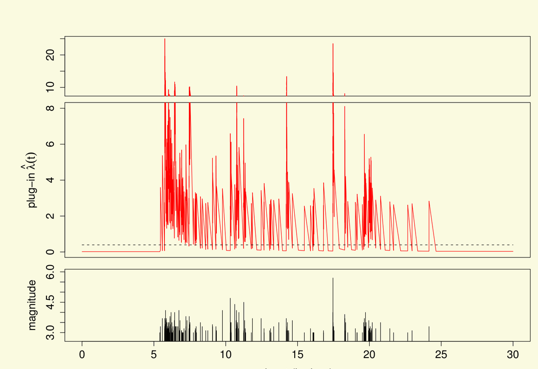

Figure 3: Hazard function evaluated in the ergodic means of the parameters.

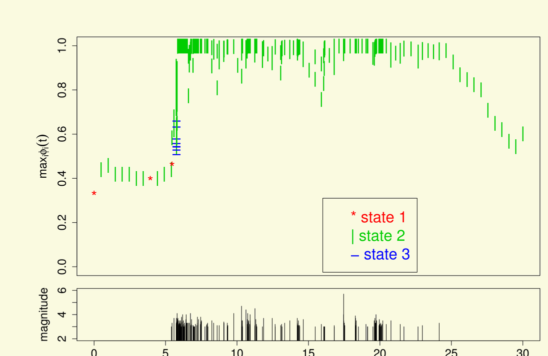

Figure 4: Estimated maximum filtering probabilities versus time and associated states.

COMPARISON OF THE STATE-SPACE MODEL AND THE POISSON/ETAS MODELS

Both Poisson model and ETAS model are fitted to Izu data in order to compare their perfomance with respect to the proposed state-space model.
Figure 5 and Tables 2-3 show input and output of the corresponding Bayesian analyses.

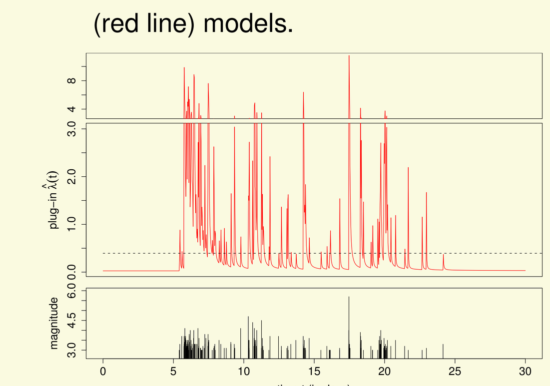

Figure 5: Hazard function given the ergodic means of the parameters for Poisson (dashed black line) and ETAS (red line) models.

Table 2: MCMC input-output for the Poisson model on Izu data.

parameter	prior distr.	proposal distr.	erg. mean	(st.dev.)
$\log \mu$	$\Gamma(-5.5, 6.25)$	$logN(\mu^{(i-1)}, 2)$	-0.9350	(0.0591)
μ			0.3933	(0.0232)

Table 3: MCMC input-output for the ETAS model on Izu data.

parameter	prior distr.	proposal distr.	erg. mean	(st.dev.)
μ	$\Gamma(0.5, 0.16)$	$logN(\mu^{(i-1)}, 0.0009)$	0.0261	(0.0121)
k	$\Gamma(5, 16)$	$logN(k^{(i-1)}, 0.002)$	0.2011	(0.0462)
γ	$\Gamma(10, 81)$	$logN(\gamma^{(i-1)}, 0.1)$	0.2946	(0.1929)
c	$\Gamma(1, 0.49)$	$logN(c^{(i-1)}, 0.005)$	0.2103	(0.0705)
p	$\Gamma(1, 0.49)$	$logN(p^{(i-1)}, 0.04)$	1.5360	(0.1258)

Table 4: Natural logarithm of the Bayes factors and marginal likelihoods.

A \ B	$Poisson$	$ETAS$	$\ln \hat{\mathcal{L}}(Y_{0:t} \mid A)$
$Poisson$	-	-	-549.16
$ETAS$	387.98	-	-161.18
$State-space$	400.31	12.33	-148.85

A criterion for Bayesian model selection is given by the Bayes factor

$$\mathcal{B}_{AB} = \frac{\hat{\mathcal{L}}(Y_{0:t} \mid A)}{\hat{\mathcal{L}}(Y_{0:t} \mid B)}$$

that is the ratio between the marginal likelihoods of two competing models A and B. According to the Jeffrey's scale, there is a decisive evidence in favour of M_A if $\ln \mathcal{B}_{AB}$ is greater than 4.6. Therefore, as shown in Table 4, the state-space model decisively has the best fit.

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