Consiglio Nazionale delle Ricerche

ISTITUTO DI ELABORAZIONE DELLA INFORMAZIONE

PISA

On Deciding Observational Congruence of Finite State CCS Expressions by Rewriting

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Work partially supported by Progetto Finalizzato Sistemi Informatici e Calcolo Parallelo, Obiettivo LAMBRUSCO

Nota Interna B4-10

Marzo 1990

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DRAFT

1. Introduction

In this paper we propose a term rewriting approach [HO80] to verify the behavioural equivalence between recursive (finite-state) CCS specifications [Mil80]. Verifications are performed by executing the axiomatic presentation of behavioural equivalences by means of an associated term rewriting system [DIN90]. Till now, we have applied our approach to the axiomatic presentation of the observational congruence for finite CCS [HM85, Mil85]. In that case, when trying to execute the axiomatization by means of an equivalent term rewriting system, it results that the completion process diverges, i.e. the equivalent term rewriting system has an infinite number of rules. We have recovered this divergence by defining a particular rewriting strategy [IN89] that is able to compute the normal form of a finite CCS term and verify the observational congruence of two terms without performing any completion of its axiomatization. In doing that, we have been supported by a precise notion of *normal form* of a finite CCS term with respect to the observational congruence.

Now, we extend our term rewriting approach to (guarded) recursive CCS terms. In fact, a correct and complete axiomatization of the observational congruence for recursive (finite-state) CCS terms has been given by Milner [Mil89]. But, differently from finite CCS, the completeness of such axiomatization has not been proved by resorting to the notion of normal form. Thus, we do not have any information about the existence and the structure of the *normal form of a recursive CCS term*. Furthermore, because of the presence of the axioms for recursion, the term rewriting system associated to the axiomatization is not terminating.

We propose to study the axiomatic presentation of observational congruence in the framework of the *infinite normal forms* [DK89, DKP89]. It results that the term rewriting system associated to our axiomatization does not satisfy some requirements of the theory developed in [DK89, DKP89]. Thus, we extend the framework to our case, show the existence of normal forms for recursive CCS terms and define their structure.

Given this result, we can define a *finite representation* of the infinite normal forms and a rewriting strategy that permits to decide the observational congruence of (finite-state) CCS terms. Such strategy has to be proved correct and complete with respect to the axiomatization by showing that it computes a specific fair derivation.

2. Observational Congruence and CCS Finite-State Expressions

In this section we present the subclass of CCS expressions under consideration [Mil89]. Let E be the class of *recursive* CCS expressions generated by the following syntax:

E ::= nil | X | μ.E | recX.E | E + E

where μ ranges over a set Act ∪ {τ} of actions and τ is the so-called *internal action*.

A *free occurrence* of X in E is *guarded* if it occurs within some subexpression μ.F ($\mu \neq \tau$) of E. The *variable* X is *guarded* in E if every free occurrence of X in E is guarded, otherwise X is *unguarded* in E. A recursive expression recX.E is *guarded* if X is guarded in F. An expression E is *guarded* if every recursive subexpression of E is guarded.

In the following we deal with the class E_g of guarded (*closed*) CCS expressions (i.e. every variable is bound to a rec operator) and the well-known behavioural equivalence, *obseravtional congruence* [Mil80].

A correct and complete axiomatization of observational congruence for the subclass of finite CCS expressions, i.e. with no variables and rec operator, has been given in [HM85, Mil85].

In [IN89] we have defined a (correct and complete) rewriting strategy $\rightarrow_{\text{strat}}$ which is able to compute the OBS-normal form of a finite CCS expression and verify the observational congruence of two finite expressions.

Let us now consider the following (correct and complete) axiomatization OBSREC_g of observational congruence E_g [Mil89]:

S1.	E + F = F + E	T1.	μ.τ.E = μ.E
S2.	E + (F + G) = (E + F) + G	T2.	E + τ.E = τ.E
S3.	E + E = E	T3.	μ.(E + τ.F) + μ.F = μ.(E + τ.F)
S4.	E + NIL = E		
R1.	recX.E = E{recX.E/X}		
R2	If F = E{F/X} then F = recX.E, provided X is guarded in E		

The completeness of the axiomatization OBSEQ = OBSREC_g - {R1, R2} for finite CCS has been shown by resorting to a notion of *normal form* of a term with respect to observational congruence [HM85, Mil85]. Thus, E1 and E2 can be proved observationally congruent by reducing them to their normal forms (by applying the so-called Absorption Lemma) and then by checking for equivalence of the normal forms modulo the associative-commutative (AC) axioms S1, S2. The notion of normal form has driven us in defining the rewriting strategy $\rightarrow_{\text{strat}}$ for finite CCS.

This is not the case for the completeness of OBSREC$_g$ which does not resort to a definition of "recursive normal form" over E_g [Mil89]. In fact, it has been shown by relying on the notion of solution of suitable set of equations, where a set of equation is an equational characterization of a recursive expression. The main difference in proving completeness consists in dealing with "saturated" sets of equations, obtained by means of an operation which is opposite to the reductions applied by the Absorption Lemma in the finite case.

On the other hand, in defining a recursive normal form over E_g, we are supported by a notion of (unique) normal form on process graphs. In [BK88] a characterization of the kind of transformations necessary to obtain the unique normal graph w.r.t. observational congruence is defined. They show that, in order to obtain the recursive normal form, it is necessary to eliminate another source of redundance w.r.t. the finite case: bisimilar nested nodes. These transformations will drive our definition of a rewriting strategy on recursive expressions.

- The first problem is that we want to work at the syntactic level on terms, thus we need an injective correspondence between a term and its infinite tree semantics. This is not the case, in general it exists a number of different terms denoting the same tree. Thus, we use a result by [CKV74] to restrict our attention only to canonical terms, i.e. it is possible to reduc, algorithmically, each equivalent recursive term to the canonical one. In this way we assume to work only on canonical terms.

3. A rewriting relation on E_g

Let us consider R2, we want to replace it with a more convenient rule.

Collapsing Rule (CR)

Given E = recX.E', if F = recY.F' is a 'proper' subexpression of E' such that E{X/F} = F{X/Y} and FreeVar(E) = FreeVar(F), then E = E{X/F}.

Correctness wrt R2

recX.recY.E = recX.E{X/Y}

recX.E = E if X $\notin$ FreeVar(E)

Ap : recX.τ.E = τ.recX.E{τ.X/X}

Problema che porta all'introduzione di Ap + l'idea di derivazione interessante

@@Simple terms like t = recX.τ.(a.X+P) or recX.τ.a.X, i.e. generic terms recX.τ.E, where E

contains directly prefixed occurrences of X, show that among all the derivations from t, fair derivations can be only obtained by applying $\rightarrow_{strat}$ infinitely. In fact, after the first unfolding step, any other rewriting by $\rightarrow_{R1}$ produces a redex for $\rightarrow_{strat}$, which has to be reduced in order to guarantee fairness. This is repeated infinitely and it follows that for such derivations it is not possible to single out an index, from which we can rewrite only by $\rightarrow_{R1}$.
Let us characterize the infinite applications of $\rightarrow_{strat}$ due to infinite rewritings by $\rightarrow_{R1}$.@@@

Proposition The axiom Ap is correct with respect to the axiomatixation $OBSREC_g$.

Proof $\tau.recX.E\{\tau.X/X\} \rightarrow_{R1} \tau.E\{(\tau.recX.E\{\tau.X/X\})/X\}$
By applying R2 we obtain $\tau.recX.E\{\tau.X/X\} = recX.\tau.E$. ♦

Discorso sull'estensione di $\rightarrow_{strat}$ al caso ricorsivo ottenendo $\rightarrow_{strat_e}$.

Now, the rewriting relation we use is the following:

$\rightarrow_{rec} = \rightarrow_{strat_e} \cup \rightarrow_{R1} \cup \rightarrow_{CR} \cup \rightarrow_{Ap}$ **modulo AC and TN**

where TN is the relation associated to the canonical transformation of a term.

It is immediate to verify that $\rightarrow_{rec}$ is a non terminating rewriting relation, thus we introduce the framework in which infinite rewritings can be studied.

3.1. Term Rewriting Systems and Infinite Normal Forms

We assume that the reader is familiar with the basic concepts of term rewriting systems. We summarize the most relevant definitions below, while we refer to [DK89, DKP89, HO80] for more details.

Let Σ be a set of operators, V be a set of variables and T denotes the set $T_\Sigma(V)$ of terms over Σ and V. An *equational theory* is any set $E = \{(s, t) \mid s, t \in T\}$. Elements (s, t) are called *equations* and written s = t. Let $\sim_E$ be the smallest symmetric relation that contains E and is closed under monotonicity and substitution. Let $=_E$ be the reflexive-transitive closure of $\sim_E$.

A *term rewriting system* (TRS) R is any set $\{(l_i, r_i) \mid l_i, r_i \in T, V(r_i) \subseteq V(l_i)\}$. The pairs (l_i, r_i) are called *rewriting rules* and written $l_i \rightarrow r_i$. The *rewriting relation* $\rightarrow_R$ on T is defined as the smallest relation containing R that is closed under monotonicity and substitution. A term t *rewrites* to a term s, written $t \rightarrow_R s$, if there exist $l_k \rightarrow r_k$ in R, a substitution σ and a subterm t/u, called *redex*, such that $t/u = \sigma(l_k)$ and $s = t[u \leftarrow \sigma(r_k)]$. An *equational* TRS is a tuple (R, E), written R/E, where R is a TRS and E an equational theory. The rewriting relation $\rightarrow_{R/E}$ is defined by $=_E \bullet \rightarrow_R \bullet =_E$, where $\bullet$ denotes composition of relations.
Let $\xrightarrow{+}$ and $\xrightarrow{*}$ denote the transitive and transitive-reflexive closure of $\rightarrow$, respectively.

Two terms s, t *converge modulo E*, written $s \downarrow_{R,E} t$, if there exist terms s', t' such that $s \xrightarrow{*}_R s' =_E t' \ {}_R\!\xleftarrow{*} t$. R is *confluent modulo E* if whenever $s \ {}_R\!\xleftarrow{*} t \xrightarrow{*}_R q$, then $s \downarrow_{R,E} q$.
A TRS R is *E-terminating* if there is no infinite sequence $t_1 \rightarrow_{R/E} t_2 \rightarrow_{R/E} t_3 \rightarrow_{R/E} \cdots$.
A term t is *in R-normal form* if there exists no term s such that $t \rightarrow_R s$. A term s is a *R-normal form* of t if $t \xrightarrow{*}_R s$ and s is in R-normal form; in this case we write $t \rightarrow^!_R s$.

Given a (possibly infinite) rewriting relation $\rightarrow$, let us recall the following definitions [DKP89]: **Tutte queste definizioni devono essere estese modulo AC!!!**

Definition 1 (ω-rewriting) $t \rightarrow^{\omega} t_{\infty}$ iff $t \xrightarrow{*} t_{\infty}$ or there exists a chain

$t = t_0 \rightarrow t_1 \rightarrow \ldots \rightarrow t_n \rightarrow \ldots$ such that $\lim_{n \rightarrow \infty} t_n = t_{\infty}$.

Definition 2 (ω-terminating) $\rightarrow$ is ω-terminating if for any infinite chain

$t = t_0 \rightarrow t_1 \rightarrow \ldots \rightarrow t_n \rightarrow \ldots$ of terms, the limit $\lim_{n \rightarrow \infty} t_n$ exists.

Definition 3 (top-terminating) $\rightarrow$ is top-terminating if there are no infinite chains

$t = t_0 \rightarrow t_1 \rightarrow \ldots \rightarrow t_n \rightarrow \ldots$ of terms with infinitely many rewrites at the topmost occurrence.

Definition 4 (ω-confluence) $\rightarrow$ is ω-confluent if ${}^{\omega}\!\leftarrow \bullet \rightarrow^{\omega}$ implies $\rightarrow^{\omega} \bullet {}^{\omega}\!\leftarrow$.
In other words, for any t, t_1, t_2 such that $t \rightarrow^{\omega} t_1$ and $t \rightarrow^{\omega} t_2$, there exists t' such that $t_1 \rightarrow^{\omega} t'$ and $t_2 \rightarrow^{\omega} t'$.

Definition 5 (ω-canonicity) $\rightarrow$ is ω-canonical if it is ω-terminating and ω-confluent.

The limit of an infinite chain from a term t may be seen as an "infinite normal form" of t. Thus, ω-termination implies the existence of an infinite normal form for any t, while ω-confluence implies the uniqueness of the infinite normal form of t, if it exists.

Definition 6 (ω-normal form) A term t_{∞} is an ω-normal form of t iff $t \rightarrow^{\omega} t_{\infty}$ and t_{∞} is minimal for $\rightarrow$, i.e. if $t_{\infty} \rightarrow t'$, then $t' = t_{\infty}$.

Thus, an ω-normal form need not be irreducible.

4. Infinite Normal Forms for Finite-State CCS Expressions

We aim at proving that the rewriting relation $\rightarrow_{rec} = \rightarrow_{strat_e} \cup \rightarrow_{R1} \cup \rightarrow_{CR} \cup \rightarrow_{Ap}$ is canonical over E_g.

Let us first state the following facts on the rewriting rules in $\rightarrow_{rec}$:

1. $\rightarrow_{strat_e}$ deletes internal actions and/or subterms working on rec bodies and on the external context of a rec term separately, i.e. its redexes do not involve rec bodies *and* the external context (except the situations in which rec terms are seen as constants(?), for example

recX.a.X + τ.recY.a.Y $\rightarrow_{strat_e}$ τ.recY.a.Y);

2. $\rightarrow_{CR}$ reduces a rec term recX.E[recY.F] by replacing the internal rec term with X. To be applied, $\rightarrow_{CR}$ checks subterms for equivalence by using $\rightarrow_{strat_e} \cup \rightarrow_{CR} \cup \rightarrow_{Ap}$ in the test E{X/F} = F{X/Y} (see def.), but $\rightarrow_{CR}$ does not apply such possible reductions. Thus, the term resulting from the application of $\rightarrow_{CR}$ is reducible according to $\rightarrow_{strat_e} \cup \rightarrow_{CR} \cup \rightarrow_{Ap}$;

3. differently from $\rightarrow_{strat_e}$ and $\rightarrow_{CR}$, $\rightarrow_{Ap}$ does not reduce a term by deleting subterms and/or internal actions or by replacing rec terms with variables, but transforms a rec term into a prefix term;

4. once $\rightarrow_{Ap}$ has been applied obtaining a term τ.recX.E{τ.X/X}, redexes can exist for $\rightarrow_{strat_e}$ in E{τ.X/X}. This is the case when there exist directly prefixed occurrences of the variable X in the body E;

5. $\rightarrow_{R1}$ unfolds recursive expressions by replacing variable occurrences with recursive expressions;

6. if $t \rightarrow^{\omega} t_{\infty}$ and $t_{\infty} \rightarrow_{strat_e} \cup \rightarrow_{CR} \cup \rightarrow_{Ap} t'$, then $t' \neq t_{\infty}$.

Therefore, infinite expressions can be derived from recursive expressions *only* by using $\rightarrow_{R1}$. On the other hand, 6. implies that $\rightarrow_{strat_e} \cup \rightarrow_{CR} \cup \rightarrow_{Ap}$ does not preserve the limit.

We would like to prove that $\rightarrow_{rec}$ is ω-canonical. To do that, we have to prove that $\rightarrow_{rec}$ is:

1. top-terminating;
2. ω-terminating;
3. ω-confluent.

4.1. ω-termination of $\rightarrow_{rec}$

Proposition 1 The rewriting relation $\rightarrow_{strat_e} \cup \rightarrow_{CR} \cup \rightarrow_{Ap}$ is finitely terminating and top-terminating over E_g.

Proof It follows from the fact that, given any term $t \in E_g$, there exists a finite number of rewriting steps by $\rightarrow_{strat_e} \cup \rightarrow_{CR} \cup \rightarrow_{Ap}$ for any occurrence, included the topmost one. ♦

Proposition 2 The rewriting relation $\rightarrow_{rec}$ is top-terminating over E_g.

Proof For Proposition 1 the relation $\rightarrow_{strat_e} \cup \rightarrow_{CR} \cup \rightarrow_{Ap}$ is finitely terminating and top-terminating over E_g. An infinite derivation can be obtained by only applying $\rightarrow_{R1}$ (see facts above). This relation rewrites every guarded term t = recX.E into a term t' which contains t at occurrences prefixed (at least) by a guard. If t = recX.E with E = recY.E', the axiom recX.recY.E = recX.E{X/Y} can be applied, thus avoiding the rewriting of the term by $\rightarrow_{R1}$ at the topmost occurrence. If t = recX.E with recY.E' as a top-level summand in E, it is obvious that after the first rewriting step by $\rightarrow_{R1}$ obtaining t', recY.E' is not a topmost occurrence of t' anymore. Every term in E_g contains a finite number of recursive guarded subterms. Thus, the guardness condition and Proposition 1 imply that $\rightarrow_{rec}$ cannot apply infinite rewriting steps at the topmost occurrence. ♦

Proposition 3 The rewriting relation $\rightarrow_{rec}$ is ω-terminating over E_g.

Proof It follows from Theorem 11 in [DKP89], since $\rightarrow_{rec}$ is top-terminating over E_g for Proposition 2. ♦

Now, we can single out "interesting" infinite derivations.

Definition 7 (fair derivation [DKP89]) A derivation is fair if no redex persists forever.

Proposition 4 ($\rightarrow_{R1}$ does not destroy redexes for $\rightarrow_{strat_e} \cup \rightarrow_{CR} \cup \rightarrow_{Ap}$)
Given E ∈ E_g, let the subexpression E/u be a redex for $\rightarrow_{strat_e} \cup \rightarrow_{CR} \cup \rightarrow_{Ap}$.
If E $\rightarrow_{R1}$ E' on the subexpression recX.F of E, then:
i. if E/u does not occur in F, then E/u still occurs in E';
ii. otherwise (E/u occurs in F), not only E/u occurs in E', but $\rightarrow_{R1}$ produces as many new redexes E/u in E' as the number of the occurrences of X in F.

Proof It follows from the definition of the rewriting rule $\rightarrow_{R1}$. ♦

Proposition 5 Every fair derivation over E_g has a finite number of rewritings by $\rightarrow_{strat_e} \cup \rightarrow_{CR} \cup \rightarrow_{Ap}$.

Proof Let us consider the possibility of infinite rewritings by $\rightarrow_{strat_e} \cup \rightarrow_{CR} \cup \rightarrow_{Ap}$ over E_g when rewriting by $\rightarrow_{R1}$. Any fair derivation has a limit (Proposition 3) and $\rightarrow_{R1}$ does not destroy the redexes for $\rightarrow_{strat_e} \cup \rightarrow_{CR} \cup \rightarrow_{Ap}$ (Proposition 4). Now, we show that rewriting by $\rightarrow_{R1}$ does not produce new redexes for $\rightarrow_{strat_e} \cup \rightarrow_{CR} \cup \rightarrow_{Ap}$ infinitely.

If a redex for $\rightarrow_{strat_e} \cup \rightarrow_{CR} \cup \rightarrow_{Ap}$ occurs in the body of a rec expression, every rewriting step by $\rightarrow_{R1}$ produces new similar redexes. For the fairness hypothesis, such redexes will be eventually reduced. Thus, infinitely rewriting by $\to_{strat_e} \cup \to_{CR} \cup \to_{Ap}$ can arise from the 'combination' between a rec body and its external context.

As $\to_{Ap}$ is concerned, its introduction as rewriting rule is motivated (see section 3) to prevent a situation of infinite reductions by $\to_{strat_e}$, where every reduction is produced when rewriting by $\to_{R1}$. In fact, when directly prefixed occurrences $\mu.X$ of X occur in the expression E in recX.τ.E, replacing recX.τ.E in μ.recX.τ.E with τ.E{recX.τ.E/X} gives rise to a redex for $\to_{strat_e}$, which is produced by the 'combination' between the external context represented by μ.[] and a piece of the rec body τ.E{recX.τ.E/X}.

Further infinitely reducible combinations between a rewc body and its external context do not exist, because the guardness hypothesis implies that a redex for $\to_{strat_e} \cup \to_{CR} \cup \to_{Ap}$ can arise, due to such a combination, only after a finite number of rewriting by $\to_{R1}$. ♦

This means that for any derivation it is possible to single out an index N, such that for every $n \geq N$ t_n is rewritten only by $\to_{R1}$ (independently from the fairness of the derivation).

Corollary 1 For any derivation $t = t_0 \to_{rec} t_1 \to_{rec} \dots \to_{rec} t_n \to_{rec} \dots$ over E_g, there exists an index N such that $t_n \to_{R1} t_{n+1}$ for $n \geq N$.

Actually, we prove a stronger result, a "structuring" condition, on fair derivations.

Corollary 2 For any fair derivation $t = t_0 \to_{rec} t_1 \to_{rec} \dots \to_{rec} t_n \to_{rec} \dots$ over E_g, there exists an index N such that $t_n \to_{R1} t_{n+1}$ and not $t_n \to_{strat_e} \cup \to_{CR} \cup \to_{Ap} t_{n+1}$ for $n \geq N$.

Proof Any derivation has a limit for ω-termination property and the limit is reached by only applying $\to_{R1}$ from some index on. Therefore, any fair derivation over E_g can be "structured" in two parts: in the former $\to_{rec}$ is applied, while in the latter *only* $\to_{R1}$ can be applied, thus reaching the limit.

4.2. ω-confluence of $\to_{rec}$

If a derivation is not fair, there exist "hanging" redexes for $\to_{strat_e} \cup \to_{CR} \cup \to_{Ap}$ along the derivation, which are not destroyed by $\to_{R1}$ (Proposition 4). It follows that the limit of the derivation contains such redexes and it is not an ω-normal form.

Now, we show that the limit of a fair derivation is an ω-normal form. In other words, we guarantee that all the possible reductions by $\to_{strat_e} \cup \to_{CR} \cup \to_{Ap}$ are applied in the first part of the derivation and in the second one *only* $\to_{R1}$ can be applied.

Structured fairness implies that there do not exist reductions by $\rightarrow_{strat_e} \cup \rightarrow_{CR} \cup \rightarrow_{Ap}$ "hanging" on t_n, $n \geq N$, for some N. To prove that the limit is an ω-normal form, we show that no reductions by $\rightarrow_{strat_e} \cup \rightarrow_{CR} \cup \rightarrow_{Ap}$, which are not applicable on the finite terms along the derivation, are applicable on the limit. Vice versa, we show that if t has an ω-normal form t', there exists a fair derivation with limit t'.

Proposition 7 Given $\rightarrow_{rec}$ and any term $t \in E_g$, then:

i.) if t admits an ω-normal form t', then it exists a fair derivation $t = t_0 \rightarrow_{rec} t_1 \rightarrow_{rec} \dots \rightarrow_{rec} t_n \rightarrow_{rec} \dots$ with $\lim_{n\rightarrow\infty} t_n = t'$;

ii.) for any fair derivation $t = t_0 \rightarrow_{rec} t_1 \rightarrow_{rec} \dots \rightarrow_{rec} t_n \rightarrow_{rec} \dots$ with $\lim_{n\rightarrow\infty} t_n = t'$, t' is an ω-normal form of t.

Proof

i.) The term t admits an ω-normal form t', hence (Definition 6) $t \rightarrow^{\omega}_{rec} t'$ and t' cannot be reduced by $\rightarrow_{strat_e} \cup \rightarrow_{CR} \cup \rightarrow_{Ap}$. By contradiction, let us suppose that it does not exist a fair derivation which computes an ω-normal form of t. Let D be a derivation $t = t_0 \rightarrow_{rec} t_1 \rightarrow_{rec} \dots \rightarrow_{rec} t_n \rightarrow_{rec} \dots$ such that its limit $t' = \lim_{n\rightarrow\infty} t_n$ is an ω-normal form of t. Let us suppose that D is not fair. For Definition 7, there exists a "hanging" reduction by $\rightarrow_{strat_e} \cup \rightarrow_{CR} \cup \rightarrow_{Ap}$ along the derivation and, for Proposition 4, it can be applied to the limit t' as well. This contradicts the hypothesis that t' is an ω-normal form.

ii.) Let D be a fair derivation $t = t_0 \rightarrow_{rec} t_1 \rightarrow_{rec} \dots \rightarrow_{rec} t_n \rightarrow_{rec} \dots$ with $\lim_{n\rightarrow\infty} t_n = t'$. By contradiction, let us suppose that t' is not an ω-normal form of t.

We have to consider two situations:

1. t' can be reduced on a (finite/infinite) redex, which is the infinite form of a redex, which already occurs from a finite t_n on and is preserved till the limit by $\rightarrow_{R1}$ for Proposition 4;

2. t' can be reduced on an infinite redex, the finite form of which never occurs in the finite terms t_n along D.

The situation **1.** can concern redexes of any rule in $\rightarrow_{strat_e} \cup \rightarrow_{CR} \cup \rightarrow_{Ap}$, but contradicts the hypothesis of fairness of D. On the other hand, **2.** can concern $\rightarrow_{strat_e}$ and $\rightarrow_{CR}$, but not $\rightarrow_{Ap}$, because, in order to be applied, only $\rightarrow_{strat_e}$ and $\rightarrow_{CR}$ require equivalence of subexpressions, i.e. equivalence of infinite trees which can be represented by means of syntactically different expressions. Thus, it could happen that a reduction by $\rightarrow_{strat_e}$ or $\rightarrow_{CR}$ is never detected on the finite terms in D, because it involves subexpressions which are semantically equivalent, but syntactically different. Such subexpressions become syntactically equivalent at the limit and the reduction can be applied. The situation described can never occur because the equivalence between subexpressions in $\rightarrow_{strat_e}$ and $\rightarrow_{CR}$ is checked modulo TN. The transformation TN recognizes as equivalent those subexpressions which are syntactically different but have the same tree semantics, and the possible reduction by $\rightarrow_{strat_e}$ or $\rightarrow_{CR}$ can be applied on the finite terms in D. ♦

Proposition 7 allows us to restrict our attention to fair derivations, as they compute ω-normal forms at the limit. To show the ω-confluence of $\rightarrow_{rec} = \rightarrow_{strat_e} \cup \rightarrow_{R1} \cup \rightarrow_{CR} \cup \rightarrow_{Ap}$, we have to prove the uniqueness of ω-normal forms, i.e. every fair derivation from a term t computes the *same* ω-normal form (Rewriting by $\rightarrow_{rec}$ is modulo **AC** and **TN**).

This means that given any term t and any two fair derivations from t,

D_1: $t = t_0 \rightarrow_{rec} t_1 \rightarrow_{rec} \dots \rightarrow_{rec} t_n \rightarrow_{rec} \dots$ with $\lim_{n\rightarrow\infty} t_n = t_\infty$ and

D_2: $t = t'_0 \rightarrow_{rec} t'_1 \rightarrow_{rec} \dots \rightarrow_{rec} t'_n \rightarrow_{rec} \dots$ with $\lim_{n\rightarrow\infty} t'_n = t'_\infty$

we have to prove $t_\infty =_{AC} t'_\infty$.

D_1 and D_2 are fair derivations, then (Corollary 2) there exist (finite) indexes N_1, N_2 such that:

- $t_k \rightarrow_{R1} t_{k+1}$ and not $t_k \rightarrow_{strat_e} \cup \rightarrow_{CR} \cup \rightarrow_{Ap} t_{k+1}$ for every $k \geq N_1$
- $t'_j \rightarrow_{R1} t'_{j+1}$ and not $t'_j \rightarrow_{strat_e} \cup \rightarrow_{CR} \cup \rightarrow_{Ap} t'_{j+1}$ for every $j \geq N_2$.

Moreover, t_∞ and t'_∞ are ω-normal forms for Proposition 7. Thus, every possible reduction by $\rightarrow_{strat_e} \cup \rightarrow_{CR} \cup \rightarrow_{Ap}$ has been applied before N_1 in D_1 and before N_2 in D_2, and *only* rewritings by $\rightarrow_{R1}$ are applied after N_1 in D_1 and after N_2 in D_2.

It follows that to prove $t_\infty =_{AC} t'_\infty$ means to prove $t_{N1} =_{TN} t'_{N2}$, i.e.

Proposition $t_\infty =_{AC} t'_\infty$ if and only if $t_{N1} =_{TN} t'_{N2}$.

It obviously(?) follows from rewriting t_{N1} and t'_{N2} only by $\rightarrow_{R1}$.

Therefore, in order to prove the uniqueness of ω-normal forms, i.e. the ω-confluence of the non terminating relation $\rightarrow_{rec}$, it is sufficient to prove the confluence of the same relation by restricting to the subderivations from any term t to t_{N1} and t'_{N2}, respectively, for any two fair derivations D_1, D_2 from t.

N.B. This is not possible in [DKP89] because there fairness does not imply a structuring condition on fair derivations into two parts, where the former is 'finite' and in the latter only non terminating rules are applied.

Hence, we consider the subderivations

D'_1: $t = t_0 \rightarrow_{rec} t_1 \rightarrow_{rec} \cdots \rightarrow_{rec} t_{N1}$

D'_2: $t = t'_0 \rightarrow_{rec} t'_1 \rightarrow_{rec} \cdots \rightarrow_{rec} t'_{N2}$

and here the relation $\rightarrow_{rec}$ is treated as a terminating relation. As terminating rewriting relations are concerned, confluence is equivalent to local confluence [HO80]. Local confluence can be decided by means of the critical pairs generated by superposing the rewriting rules, representing the situation in which a term can be rewritten by two (or more) rules on non independent subterms.

Let us now consider the following facts:

1. $\rightarrow_{strat_e}$ cannot superpose with itself (it follows from its definition: $\rightarrow_{strat_e}$ applies all the possible reductions in OBSEQ, thus returning an OBS-normal form).
2. $\rightarrow_{R1}$ does not destroy the redexes for $\rightarrow_{strat_e} \cup \rightarrow_{CR} \cup \rightarrow_{Ap}$ (see Proposition 5). It follows that, in analysing the possible superpositions between the rules in $\rightarrow_{rec}$, we can omit superpositions of every rule with $\rightarrow_{R1}$. Note that, since $\rightarrow_{CR}$ and $\rightarrow_{Ap}$ always work on rec terms, they always superpose with $\rightarrow_{R1}$ (at least in two occurrences as regards $\rightarrow_{CR}$). (RIV. quella tra CR e R1)

Let us prove the ω-confluence of the relation $\rightarrow_{rec}$ by showing the local confluence of $\rightarrow_{rec}$ in the subderivations D'_1 and D'_2. This is obtained by cases on the possible superpositions between the rules in $\rightarrow_{rec}$.

AC-unification nella superposition dà problemi, come nel caso terminante???

Proposition 8 Given $\rightarrow_{rec} = \rightarrow_{strat_e} \cup \rightarrow_{R1} \cup \rightarrow_{CR} \cup \rightarrow_{Ap}$, any term t and the finite subderivations D'_1 and D'_2 of any two fair derivations from t,

D'_1: $t = t_0 \rightarrow_{rec} t_1 \rightarrow_{rec} \cdots \rightarrow_{rec} t_{N1}$ and

D'_2: $t = t'_0 \rightarrow_{rec} t'_1 \rightarrow_{rec} \cdots \rightarrow_{rec} t'_{N2}$

then $t_{N1} = t'_{N2}$ (modulo TN).

Proof

Let $t_i = t'_i$, $0 \leq i < \min(N1,N2)$, be the first term in D'_1 and D'_2 such that t_i can be rewritten by means of different rules in $\rightarrow_{rec}$ ($t_j = t'_j$, $j \leq i$). Let us first examine the possible superpositions between $\rightarrow_{strat_e}$ and the other rules $\rightarrow_{CR}$ and $\rightarrow_{Ap}$.

Case 1 $t_i \rightarrow_{strat_e} t_{i+1}$ and $t_i \rightarrow_{CR} t'_{i+1}$.

Case 1.1 Redex($\rightarrow_{CR}$) is a subterm of redex($\rightarrow_{strat_e}$). We distinguish two situations:

1.1.1 $\rightarrow_{strat_e}$ deletes a summand.

Given $t_i = E_1 + ... + E_n$, let us suppose that $\rightarrow_{strat_e}$ deletes a summand E_i which is also reducible by $\rightarrow_{CR}$. From the definition of $\rightarrow_{strat_e}$ it follows that there exists a summand E_k in t_i which contains some derivative E', which is equivalent to E_i according to $\rightarrow_{strat_e}$. Since $\rightarrow_{strat_e}$ does not make use of $\rightarrow_{CR}$, also E' can be rewritten by $\rightarrow_{CR}$ and the resulting summand E'_k can be then rewritten by $\rightarrow_{strat_e}$ (Fact 3) obtaining a summand E''_k. On the other side, once E_i has been rewritten by $\rightarrow_{CR}$ obtaining E'_i, the previous applicability of $\rightarrow_{strat_e}$ on t_i deleting E_i means that there exists E' in some summand E_k such that E' is equivalent to E_i according to $\rightarrow_{strat_e}$. Hence, E_k can be reduced by $\rightarrow_{CR}$ to a summand E'_k which, together with E'_i, can be rewritten by $\rightarrow_{strat_e}$ into E''_k thus deleting E'_i. (Note that $\rightarrow_{strat_e}$ is also applicable on E'_i and E'_k, resulting from the two applications of $\rightarrow_{CR}$, see Fact 3).

$t_i = E_1 + ... + E_n$

$\downarrow_{strat_e}$ $\qquad\qquad$ $\downarrow_{CR}$

$E_1 +...+ E_{i-1} + E_{i+1} +...+ E_k[E'] +...+ E_n$ $\qquad$ $E_1 +...+ E'_i +...+ E_k[E'] +...+ E_n$

$\downarrow_{CR}$ *on E'* $\qquad\qquad$ $\downarrow_{CR}$ *on E'*

$E_1 +...+ E_{i-1} + E_{i+1} +...+ E'_k +...+ E_n$ $\qquad$ $E_1 +...+ E'_i +...+ E'_k +...+ E_n$

$\downarrow_{strat_e}$ $\qquad\qquad$ $\downarrow_{strat_e}$ *on E'_i, E'_k and the sum of the resulting terms*

$E_1 +...+ E_{i-1} + E_{i+1} +...+ E''_k +...+ E_n$

1.1.2 $\rightarrow_{strat_e}$ deletes an internal action.

This situation can be simply depicted by the following figure:

$t_i\ [\mu.\tau.E]$ $\qquad\qquad$ *E contains a redex for $\rightarrow_{CR}$*

$\downarrow_{strat_e}$ $\qquad\qquad$ $\downarrow_{CR}$

$t_i\ [\mu.E]$ $\qquad\qquad$ $t_i\ [\mu.\tau.E']$

$\downarrow_{CR}$

$t_i\ [\mu.E']$ $\qquad\qquad$ $\downarrow_{strat_e}$ *(it reduces E' and deletes the internal action)*

$\downarrow_{strat_e}$

Fact 3 $\qquad$ $t_i\ [\mu.E'']$

Case 1.2 Redex($\rightarrow_{strat_e}$) is subterm of redex($\rightarrow_{CR}$).

This situation is independent from the kind of reduction performed by $\rightarrow_{strat_e}$.

$\rightarrow_{CR}$ applies on a suitable subterm recX.E[recY.F]. If $\rightarrow_{strat_e}$ applies on E in the external context of recY.F and/or in F, $\rightarrow_{CR}$ recognizes such reductions but it does not apply them (Fact 3). The term t_i [recX.E{X/recY.F}] resulting from the application of $\rightarrow_{CR}$ can be reduced by $\rightarrow_{strat_e}$ which applies all the possible reductions according to OBSEQ, the initial redexes for $\rightarrow_{strat_e}$ included. On the other side, $\rightarrow_{strat_e}$ on t_i reduces F to F' and the external context of recY.F in E to E". The resulting term is still reducible by $\rightarrow_{CR}$ (otherwise t_i should not be reducible by $\rightarrow_{CR}$ on recX.E[recY.F]). The confluence to the same term is obtained by applying $\rightarrow_{strat_e}$ again.

t_i [recX.E[recY.F]]

$\downarrow_{strat_e}$ $\downarrow_{CR}$

t_i [recX.E"[recY.F']] t_i [recX.E{X/recY.F}]

$\downarrow_{CR}$

t_i [recX.E"{X/recY.F'}] $\downarrow_{strat_e}$

$\downarrow_{strat_e}$

t_i [recX.E']

Case 2 $t_i \rightarrow_{strat_e} t_{i+1}$ and $t_i \rightarrow_{Ap} t'_{i+1}$.

Case 2.1 Redex($\rightarrow_{Ap}$) is a subterm of redex($\rightarrow_{strat_e}$).

The same reasoning applied in **Case 1.1** can be followed. In particular, the **Case 2.1.2** can be depicted by means of the same figure as for **Case 1.1.2** with $\rightarrow_{CR}$ replaced by $\rightarrow_{Ap}$.
(Rivedere il caso 2.1.1)

Case 2.2 Redex($\rightarrow_{strat_e}$) is a subterm of redex($\rightarrow_{Ap}$).

This situation can be depicted by the following figure:

t_i [recX.τ.E] *E contains a redex for $\rightarrow_{strate}$*

$\downarrow_{strat_e}$ $\downarrow_{Ap}$

t_i [recX.τ.E'] t_i [τ.recX.E{τ.X/X}]

$\downarrow_{Ap}$ *(E{τ.X/X} is still*

t_i [τ.recX.E'{τ.X/X}] $\downarrow_{strat_e}$ *reducible by $\rightarrow_{strat_e}$)*

(possible application of Fact 5) $\downarrow_{strat_e}$

t_i [τ.recX.E"]

Note that in t_i [τ.recX.E"] the internal action τ can be deleted by $\rightarrow_{strat_e}$ dependently on the context.

Let us now consider the possible superpositions between $\rightarrow_{Ap}$ and $\rightarrow_{CR}$.

Case 3 $t_i \rightarrow_{Ap} t_{i+1}$ and $t_i \rightarrow_{CR} t'_{i+1}$.

Case 3.1 Redex($\rightarrow_{CR}$) is a subterm of redex($\rightarrow_{Ap}$).

This situation can be simply depicted by the following figure:

$$\begin{array}{ccc} & t_i\,[\text{recZ}.\tau.(\text{E'}[\text{recX.E}[\text{recY.F}]])] & \\ & \downarrow_{Ap} \qquad\qquad \downarrow_{CR} & \\ t_i\,[\tau.\text{recZ.E'}[\text{recX.E}[\text{recY.F}]]\{\tau.\text{Z/Z}\}] & & t_i\,[\text{recZ}.\tau.(\text{E'}[\text{recX.E}\{\text{X/recY.F}\}])] \\ & \downarrow_{CR} \qquad\qquad \downarrow_{Ap} & \\ & t_i\,[\tau.\text{recZ.E'}[\text{recX.E}\{\text{X/recY.F}\}\{\tau.\text{Z/Z}\}]] & \end{array}$$

Case 3.2 Redex($\rightarrow_{Ap}$) is a subterm of redex($\rightarrow_{CR}$).

Let t_i[recX.E[recY.F]] be given, where recX.E[recY.F] is redex for $\rightarrow_{CR}$ and the external context of recY.F in E and/or F contain redexes for $\rightarrow_{Ap}$. If $\rightarrow_{Ap}$ is first applied on any redex in the external context of recY.F or in F, the resulting term is still reducible by $\rightarrow_{CR}$ (otherwise recX.E[recY.F] could not be a redex for $\rightarrow_{CR}$). On the other side, once $\rightarrow_{CR}$ has been applied, the resulting term can be still reduced by $\rightarrow_{Ap}$ if a redex($\rightarrow_{Ap}$) is in the external context of recY.F, while it is not reducible if a redex($\rightarrow_{Ap}$) is only in F.

Let $\rightarrow_{Ap}$ be applicable on the external context of recY.F in E, we have the following picture:

$$\begin{array}{ccc} & t_i[\text{recX.E}[\text{recY.F}]] & \\ & \downarrow_{Ap} \qquad\qquad \downarrow_{CR} & \\ t_i[\text{recX.E'}[\text{recY.F}]] & & t_i[\text{recX.E}\{\text{X/recY.F}\}] \\ & \downarrow_{CR} \qquad\qquad \downarrow_{Ap} & \\ & t_i[\text{recX.E'}\{\text{X/recY.F}\}] & \end{array}$$

Let $\rightarrow_{Ap}$ be applicable on F, we have the following picture:

$$\begin{array}{ccc} & t_i[\text{recX.E}[\text{recY.F}]] & \\ & \downarrow_{Ap} \qquad\qquad \downarrow_{CR} & \\ t_i[\text{recX.E}[\text{recY.F'}]] & & t_i[\text{recX.E}\{\text{X/recY.F}\}] \\ & \downarrow_{CR} & \\ & t_i[\text{recX.E}\{\text{X/recY.F'}\}] & \end{array}$$

and t_i[recX.E{X/recY.F'}] = t_i[recX.E{X/recY.F}].

Let $\rightarrow_{Ap}$ be applicable on recX.E, i.e. $E = \tau.E'$. This implies that also recY.F is a redex for $\rightarrow_{Ap}$, i.e. $F = \tau.F'$, otherwise $\rightarrow_{CR}$ is not yet applicable on recX.E[recY.F].
We have the following picture:

$$t_i[recX.\tau.(E'[recY.\tau.F'])]$$

$$\downarrow_{Ap} \qquad\qquad\qquad \downarrow_{CR}$$

$$t_i[\tau.recX.E'[recY.\tau.F']\{\tau.X/X\}] \qquad t_i[recX.\tau.(E'\{X/recY.\tau.F'\})]$$

(now $\rightarrow_{CR}$ is not applicable) $\qquad \downarrow_{Ap} \qquad\qquad\qquad \downarrow_{Ap}$

$$t_i[\tau.recX.E'[\tau.recY.F'\{\tau.Y/Y\}]\{\tau.X/X\}] \qquad t_i[\tau.recX.E'\{X/recY.\tau.F'\}\{\tau.X/X\}]$$

(now $\rightarrow_{CR}$ is again applicable) $\qquad \downarrow_{CR}$

$$t_i[\tau.recX.E'[\tau.\{X/recY.F'\{\tau.Y/Y\}\}]\{\tau.X/X\}]$$

and $t_i[\tau.recX.E'[\tau.\{X/recY.F'\{\tau.Y/Y\}\}]\{\tau.X/X\}] = t_i[\tau.recX.E'\{X/recY.\tau.F'\}\{\tau.X/X\}]$.

Let us now consider the possible superpositions between $\rightarrow_{CR}$ and itself.

Case 4 $\; t_i \rightarrow_{CR} t_{i+1}$ and $t_i \rightarrow_{CR} t'_{i+1}$.

This is the case when a redex for $\rightarrow_{CR}$ is contained in another redex for $\rightarrow_{CR}$, as in t_i[recX.E[recY.F[recZ.G]]] such that both recX.E[recY.F[recZ.G]] and recY.F[recZ.G] are redexes for $\rightarrow_{CR}$. Applying $\rightarrow_{CR}$ on the external redex results in the replacement of recY.F[recZ.G] with X, thus loosing the previously possible reduction on it. On the other side, applying $\rightarrow_{CR}$ on the internal redex results in a term $t_i[recX.E[recY.F\{Y/recZ.G\}]]$, which is still reducible by $\rightarrow_{CR}$ (using $\rightarrow_{strat_c}$ for its application).

$$t_i[recX.E[recY.F[recZ.G]]]$$

(on the external rec redex) $\qquad \downarrow_{CR} \qquad\qquad\qquad \downarrow_{CR} \qquad$ *(on the internal rec redex)*

$$t_i[recX.E\{X/recY.F[recZ.G]\}] \qquad t_i[recX.E[recY.F\{Y/recZ.G\}]]$$

$$\downarrow_{CR}$$

$$t_i[recX.E\{X/recY.F\{Y/recZ.G\}\}]$$

and $t_i[recX.E\{X/recY.F[recZ.G]\}] = t_i[recX.E\{X/recY.F\{Y/recZ.G\}\}]$.

Let us now consider the possible superpositions between $\rightarrow_{Ap}$ and itself.

Case 5 $\; t_i \rightarrow_{Ap} t_{i+1}$ and $t_i \rightarrow_{Ap} t'_{i+1}$.

This is the case when a redex for $\rightarrow_{Ap}$ is contained in another redex for $\rightarrow_{Ap}$ and it can be simply depicted by the following figure:

$$t_i[\text{recX}.\tau.(E[\text{recY}.\tau.F])]$$

(on the external redex) $\downarrow_{Ap}$ $\downarrow_{Ap}$ *(on the internal redex)*

$$t_i[\tau.\text{recX}.E[\text{recY}.\tau.F]\{\tau.X/X\}] \qquad t_i[\text{recX}.\tau.(E[\tau.\text{recY}.F\{\tau.Y/Y\}])]$$

$\downarrow_{Ap}$ $\downarrow_{Ap}$

$$t_i[\tau.\text{recX}.E[\tau.\text{recY}.F\{\tau.Y/Y\}]\{\tau.X/X\}]$$

♦

Thus, we have shown that the rewriting relation $\rightarrow_{rec}$ is ω-canonical, i.e. the ω-normal form of any term $t \in E_g$ exists and it is unique.
Let us define a "finite" representation of an ω-normal form:

Definition 8 (rec-normal-form)
A term $t \in E_g$ is a *rec-normal-form* if every term t' such that $t \overset{*}{\Rightarrow}_{R1} t'$ cannot be reduced by (or is in normal form w.r.t.) $\rightarrow_{strat_e} \cup \rightarrow_{CR} \cup \rightarrow_{Ap}$.

Corollary For any term $t \in E_g$ there exists a rec-normal-form and it is unique modulo TN.

Proof It follows from the fairness property and Propositions 7 and 8. ♦

Now, we show the completeness of $\rightarrow_{rec}$ with respect to the notion of rτ-normal process graph [BK88].

Proposition 9 Given any $t \in E_g$, let D: $t = t_0 \rightarrow_{rec} t_1 \rightarrow_{rec} \cdots \rightarrow_{rec} t_N \rightarrow_{R1} \cdots$ be a fair derivation from t, such that $t'_N = TN(t_N)$ is a rec-normal-form of t.
Then t'_N denotes a rτ-normal process graph.

Proof A rτ-normal process graph is a graph which is (see section on results by [BK88]):
i. rτ-rigid, i.e. there do not exist rτ-bisimilar nodes;
ii. minimal, i.e. there do not exist arcs, double edges and τ-loops.
Let us suppose to have defined a transformation $E_g \rightarrow$ Process-Graphs [Mil84, BK88].
(Note that the transformation in [Mil84] obtains root-unwound graphs).
By contradiction: let t'_N denote a graph $g(t'_N)$ which is not rτ-normal. This means that at least one of the two conditions above is not satisfied.
Let us consider i).: $g(t'_N)$ is not rτ-rigid and this means that it has not only the trivial rτ-

autobisimulation. Therefore, there exists a rτ-autobisimulation R of $g(t'_N)$ such that there exist two nodes s_i, $s_j \neq root(g(t_N))$, $s_i \neq s_j$ and $(s_i, s_j) \in R$. In other words, there exists a pair (s_i, s_j) of τ-bisimilar (internal and distinct) nodes in $g(t'_N)$. It follows that the subgraph $(g)_{s_i}$, $(g)_{s_j}$ with root s_i, s_j respectively, are τ-bisimilar. We have to consider different cases.

Case 1. $(g)_{s_i}$ and $(g)_{s_j}$ are different subgraphs and one is not a subgraph of the other one. They are τ-bisimilar and this implies that $(g)_{s_i}$ and/or $(g)_{s_j}$ are not τ-normal subgraphs. At syntactic level, this means that in t'_N there occur subterms which are not in rec-normal-form, thus contradicting the hypothesis that t'_N is a rec-normal-form.

Case 2. One of the two subgraphs, let us say $(g)_{s_j}$, is a subgraph of the other one, $(g)_{s_i}$:

$$(g)_{s_i} \supset (g)_{s_j}$$

2.1. The subgraphs denote the same infinite tree: $tree((g)_{s_i}) = tree((g)_{s_j})$.

This means that $(g)_{s_j}$ has been obtained as an unfolding by $\rightarrow_{R1}$ from s_j. This is not possible because $g(t'_N)$ corresponds to a canonical term t'_N, i.e. reduced w.r.t. TN.

2.2. The subgraphs denote different infinite trees, $tree((g)_{s_i}) \neq tree((g)_{s_j})$, such that $tree((g)_{s_i})$ contains the subtree $tree((g)_{s_j})$ which is τ-bisimilar to itself. There can be different situations:

2.2.1. The τ-bisimilar nodes s_i and s_j are connected by a unique path labelled with a τ-action **(figura)**. At syntactic level, this means that in t'_N there occur subterms $\mu.\tau.P$ for some P or $recX.\tau.E$ for some E containing directly prefixed occurrences $\mu.X$ of X, contradicting the hypothesis that t'_N is a rec-normal-form and therefore reduced with respect to $\rightarrow_{strat_e}$ and $\rightarrow_{Ap}$. (Note that $\mu.\tau.X = \mu.X$ is the only rule in $\rightarrow_{strat_e}$ with a prefix left hand side. It reduces vertically, while the other rules, i.e. the Absorption Lemma, have a summation left hand side and reduce horizontally).

2.2.2. The τ-bisimilar nodes s_i and s_j are placed in a more general context (figura). At syntactic level, this means that there is a subterm $recX.E$ of t'_N which contains a subterm $recX.F$ observationally equivalent to $recX.E\{X/F\}$. This contradicts the hypothesis that t'_N is a rec-normal-form and therefore reduced with respect to $\rightarrow_{CR}$.

Let us now consider the condition ii). Let t'_N denote a graph $g(t'_N)$ which is not minimal. This means that in $g(t'_N)$ there occur:

1. τ-loops;
2. arcs with primary edge labelled μ;
3. double edges.

While rewriting by $\rightarrow_{rec}$ over E_g, the case 1. cannot occur because the τ-loops generated during the collapsing operation on graphs, are removed at syntactic level by $\rightarrow_{strat_e}$ while executing the collapse on terms. The case 3. is a particular instance of 2., the only case to considered. The occurrence of an arc with primary edge labelled μ in a graph corresponds, at syntactic level, to the occurrence (in t'_N) of a subexpression as follows:

$\tau.(\ldots \mu.(\ldots\tau. \ldots (P + \ldots) \ldots) \ldots) + \mu.P$.

This subexpression is an instance of the Absorption Lemma and can be reduced by $\rightarrow_{strat_e}$ thus deleting $\mu.P$: this contradicts the hypothesis that t'_N is a rec-normal-form. ♦

5. Conclusions and Future Work

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