An Attribute-based Front-end for FlyFast *

Viencenzo Ciancia[1], Diego Latella[1], and Mieke Massink[1]

Istituto di Scienza e Tecnologie dell'Informazione 'A. Faedo', CNR, Italy

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Abstract. Typical Collective Adaptive Systems (CAS) consist of a large number of interacting objects that coordinate their activities in a decentralised and often implicit way. The design of such systems is challenging. It requires scalable analysis tools and techniques to check properties of proposed system designs before they are put into operation. A promising technique is Fast Mean-Field Approximated Model-checking. In particular, the FlyFast model-checker uses an on-the-fly algorithm for bounded PCTL model-checking of selected individuals in the context of very large populations whose behaviour is approximated using deterministic limit techniques. Recently, specific modelling languages have been proposed for CAS. A key feature of such languages is the *attribute-based* interaction paradigm. In this paper we present an attribute-based coordination language as a front-end for FlyFast. Its formal probabilistic semantics is provided and a translation to the original FlyFast language is presented and proved correct.

Keywords: Collective Adaptive Systems; Probabilistic Model-Checking; On-the-fly Model-Checking; Mean-Field Approximation; Discrete Time Markov Chains.

1 Introduction and Related Work

Collective Adaptive Systems (CAS) consist of a large number of entities with decentralised control and varying degrees of complex autonomous behaviour. CAS are at the core of the envisioned smart cities of the future and encompass systems like smart urban transport and smart grids. The pervasive nature of CAS and thus their impact on society implies that it is extremely important to develop reliable rigorous design models as well as a priori analysis techniques of such models—covering all relevant aspects of their behaviour, including quantitative and emergent ones—before they are put into operation[1].

Model-checking has been widely recognised as a powerful approach to the automatic verification of concurrent and distributed systems. It consists of an efficient procedure that, given an abstract model of the system, decides whether

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[1] See, e.g. www.focas.eu/adaptive-collective-systems and www.quanticol.eu

the model satisfies a logical formula, typically drawn from a temporal logic. Unfortunately, traditional Model-checking suffers from the so called *state-space explosion* problem which hampers scalability of the approach. In particular, its application to *very large* models, like those typical of CAS, is infeasible. In [15, 13] Latella et al. presented a scalable Mean-field model-checking procedure for verifing bounded PCTL (Probabilistic Computation Tree Logic) [9] properties of selected individuals in the context of systems consisting of a large number of similar, but independent, interacting objects; a limited form of global system properties can be treated as well. The procedure can be used with huge population sizes, as typical of analysis techniques based on mean-field approximation; the average behaviour of the population is approximated using a population DTMC convergence result [18] and is used for representing the context in which the selected individuals operate (see [18, 15, 13] for details). The model-checking procedure is implemented in the tool FlyFast as an instantiation of a probabilistic on-the-fly model-checker; the latter is parametric on (the semantic model of) the modelling language [14, 13].

FlyFast comes with simple modelling language. An agent[2] is a finite state process, a generic state C of which is specified by a *state defining equation* like $C := a_1.C_1 + \ldots + a_r.C_r$. Intuitively, the above notation defines state C of the agent and postulates that there are r outgoing transitions from C,with action a_j labelling a transition going from C to C_j. A probability value is assigned to each action a by means of a *probability function definition* $a :: E$, where the actual probability is given by the value of expression E evaluated in the current *occupancy measure vector* $\mathbf{m}$. Assume a system is composed on N instances of the agent and that the states of the agent are $C_1, \ldots C_S$. The occupancy measure vector at the current time is the vector $(m_1, \ldots, m_S)$ s.t. m_j yields the *fraction* of agents currently in state C_j over the total number N of agents. A *system specification* is a triple composed by an *agent specification*—given as a set of state defining equations—a set of probability function definitions, and an initial global state. Finally, FlyFast provides the user with *formula declarations* which allow for the interpretation of bounded PCTL atomic propositions in the model at hand. The computational model is *clock-synchronous*; at each step each agent must perform an independent step (which may be an idle self-loop) so that the global state probabilities are given as the product of agent step probabilities, and a new occupancy measure vector can be computed. The global system behaviour is thus a DTMC as well as the stochastic process given by the occupancy measure vector. Notably, for N sufficiently large, the latter can be approximated deterministically, i.e. by a *function* of time. This brings to a dramatic decrease in size of the global state space: at each step, the total number of potential next states drops from S^N to S, which makes bounded PCTL model-checking of very large population systems possible (the interested reader is referred to [15, 13, 18] for details).

Recently, modelling and programming languages have been proposed specifically for autonomic computing systems and CAS [7, 3, 10]. Typically, in such

[2] In the context of FlyFast we use the words *agent*, *process* and *object* as synonyms.

frameworks, a system is composed of a set of independent *components* where a component is a process equipped with a set of *attributes* describing features of the component. A classical example of attribute is the component *location*. An additional *environment* is often used for the specification of common or global features. The attributes of a component can be *updated* during its execution so that the association between attribute *names* and attribute *values* is mantained in the dynamic *store* of the component. Attributes can be used in *predicates* appearing in language constructs for component interaction. For instance a component may *broadcast* a message to *all those components satisfying* a given predicate; similarly a component may wait for a message from *any of those components satisfying* a given predicate.

In the present paper, we propose an extended version, inspired by CARMA [3], of the FlyFast front-end modelling language, based on components and predicate-based interaction. Components are expressed as pairs *process-store* and process actions are *predicate based multi-cast* output and input primitives. Associated to each action there is also an (atomic) probabilistic store update. For instance, assuming components have an attribute named loc which takes values in the set of points of a space type, the following action will send value v via channel α to all components in the same location as the sender making it change location randomly: $\alpha^*[\mathsf{loc} = \mathbf{my}.\mathsf{loc}]\langle v\rangle\{\mathsf{loc} \leftarrow \mathsf{randomLoc}\}$, where randomLoc is assumed to be a random generator of points in the space—*multi-cast* interaction is denoted using the $\_^*$ notation, as in CARMA. A component which is located in a point in a regular grid (patch) and is willing to receive a message $\alpha\langle v\rangle$ only if it is sent from a component located at north of its own location may execute the following action: $\alpha^*[\mathsf{loc} = \mathsf{N}(\mathbf{my}.\mathsf{loc})](x)\{\}$, where N is assumed to be a function that, given a point ℓ, returns the point north of ℓ. The computational model is again *clock-synchronous*, at the component level. In addition, each component is equipped with a local *outbox*. The effect of an output action $\alpha^*[\pi_r]\langle\rangle\sigma$ is to deliver output label $\alpha\langle\rangle$ to the local outbox, together with the predicate π_r, which receiver components will be required to satisfy, as well as the current store γ of the component executing the action; the current store is updated according to update σ. Note that output actions are *non-blocking* and that successive output actions of the same component rewrite its outbox. An input action $\alpha^*[\pi_s]()\sigma$ by a component will be executed with a probability which is proportional to the fraction of all those components whose outboxes currently contain the label $\alpha\langle\rangle$, a predicate π_r which is satisfied by the component, and a store γ which satisfies in turn predicate π_s. If such a fraction is zero, then the input action will not take place (input is blocking), otherwise the action takes place, the store of the component is updated via σ, and its outbox cleared. Thus, as in the original FlyFast language, component interaction is probabilistic but now the *fraction* of the components satisfying the relevant predicates plays a role in the computation of transition probabilities. We provide the formal probabilistic semantics of the extended language and a translation to the original FlyFast language which makes the model-checker support the extended language. The translation is proved correct.

Related Work. As we mentioned before, this work has been inspired by CARMA [3], which in turn shares features with SCEL [7]. There are several aspects of either languages that are not present in our proposal. The main reason for the absence of most of them is the fact that this work is intended as a proof of concept rather than the realisation of a ready-to-use tool for reasoning about CAS. So we aim at keeping the language minimal and focussing only on attribute-based interaction in the context of stochastic and mean-field semantics/model-checking. A feature of CARMA we do not consider here is the notion of global environment, since it represents a singularity point that does not fit well with limit approximation techniques. Finally, we point out that the stochastic semantics of CARMA are based on *time inhomogeneous* CTMCs, due to the fact that action parameters may be time dependent, while we use DTMCs as semantic basis. The notion of the outbox is reminiscent of the notion of the ether in PALOMA [8] in the sense that the collection of all outboxes together can be though of as a kind of ether. On the other hand, such a collection is intrinsically distributed among the components so that it cannot represent a bottleneck in the execution of the system neither a singularity point in the deterministic approximation. Fluid model-checking for *continuous time* systems is addressed in [4] where a global model-checking procedure for the Continuous Stochastic Locic (CSL, [2]) is given which is based on a continuous limit approximated semantics model. Predicate-/attribute-based inter-process communication has been originally proposed in [16] where several variants of predicate-/attribute-based communication primitives—including blocking / non-blocking, bounded / unbounded—are discussed in the context of a study on high-level language constructs for distributed systems with decentralised control (see for instance [19]). The notion of predicate-/attribute-based interaction is central in the definition of SCEL [7] where its synchronous-communication variant has been given formal semantics. Asynchronous-communication variants have been defined for stochastic versions of SCEL [17]. An attribute-interaction based calculus is proposed in [1] where broadcast communication links among components are dynamically established on the basis of the interdependences determined by predicates over attributes. A reduction semantics approach is adopted where each transition involves the group composed of both sender and receivers. Attribute π-Calculus has been proposed in [12] and extended to Imperative π-Calculus in [11]; in both calculi, which inherit the classical point-to-point communication paradigm of the π-Calculus, as opposed to multi-cast, attributes are related to messages rather than to processes. None of the above mentioned works on predicate-/attribute-based languages addresses mean-field approximated model-checking so, to the best of our knowledge, the present paper is the first proposal on the subject.

2 Attribute-based Interaction Modelling Language

In this section we define a simple population description language. A *system* is defined as a population of N identical interacting components. At any point in time, each component can be in any of its finitely many states and the evolution

of the system proceeds in a *clock-synchronous* fashion: at each clock tick each member of the population must either execute one of the transitions that are enabled in its current state, or remain in such a state[3]. Finally, each component is equipped with a finite set of *attributes*; the current *store* $\gamma \in \Gamma$ of the component maps each attrribute *name* to an attribute *value*.

2.1 Syntax

A *component specification* is a pair (Δ, F) where Δ is a finite set of *state-defining* equations, one for each *state* of the component and F is a set of auxiliary function definitions[4]. We let $\mathcal{S}$, ranged over by $C, C', C_1, \ldots$ denote the (denumerable, non-empty) set of all states which can be used in equations. Each equation defines the transitions from the state to other states of the component; each transition is labelled by the *action* the component performs when the transition takes place. The general format of a state defining equation is:

$$C := [g_1]P_1 + \ldots + [g_r]P_r \tag{1}$$

where each *guard* $[g]$ is a *predicate* π defined according to the following grammar:

$$\pi ::= \top \mid \bot \mid e_1 \bowtie e_2 \mid \neg\pi \mid \pi_1 \ \wedge \ \pi_2$$

$\top$ ($\bot$, resp.) stands for the truth value *true* (*false* resp.), while $\bowtie \in \{\geq, >, \leq, <\}$; we let $\bowtie \in \{>, <\}$. An *expression* e can be an attribute name a, or $\mathbf{my}.a$ referring to the value of a in the component where it occurs, a value v in / variable x over given set $\mathcal{V}$. In defining equations as above, we abbreviate $[\top]P_j$ with P_j and we omit summands of the form $[\bot]P_j$.

Each P_j in a state defining equation as above is of the form $p_j :: act_j.C_j$, where p_j is a probability expression, i.e. an expression with value in $[0, 1]$, built from constants $v \in [0, 1]$ and the special operator $\mathsf{frc}\, C$ combined using standard arithmetics operators; for state C, $\mathsf{frc}\, C$ returns the *fraction* of the components which are currently in state C, over the total of N components. Clearly, the use of the frc operator allows action (and, ultimately, transition) probability to depend on the system global state. Actions act_j are defined according to the following grammar:

$$act ::= \alpha^*[\pi]\langle e_1, \ldots, e_n\rangle\sigma \mid \alpha^*[\pi](x_1, \ldots, x_n)\sigma$$

We assume a countable set of *action types* $\mathcal{A}$, with $\alpha \in \mathcal{A}$. The effect of an *output* action $\alpha^*[\pi]\langle e_1, \ldots, e_n\rangle\sigma$ is the broadcast of the message $\alpha\langle v_1, \ldots, v_n\rangle$—where v_j is the value of expression e_j in the current store—to all those components satisfying predicate π and which are willing to accept the communication. This

[3] In practice, the fact that the components are identical is not a strong limitation since each component may consist of several different sets of states, with each state in a given set being unreachable from states of other sets. Each such a set of states can be seen as a component with a different behaviour.

[4] The specific syntax of auxiliary function definitions is irrelevant and left out here.

is achieved by means of delivering the message, together with some additional information, to the outbox of the component executing the action, as we will discuss in detail in Section 2.2. In addition, the store of the component executing the action is updated according to the *update* σ, which is a function from Γ to the class of probability distributions over Γ—i.e., in the general case, the update may be probabilistic. Similarly, an *input* action $\alpha^*[\pi](x_1,\ldots,x_n)\sigma$ is used to receive a message $\alpha\langle v_1,\ldots,v_n\rangle$ sent by a component satisfying predicate $\pi[v_1/x_1,\ldots,v_n/x_n]$. More specifically, the probability of executing the input action will be proportional to the fraction of components which have sent the message, while satisfying predicate $\pi[v_1/x_1,\ldots,v_n/x_n]$ and requiring a predicate which is satisfied by the component executing the input action. Also input actions are provided with a store update σ whereas the component outbox is cleared as (a side) effect of their execution.

For example, assuming loc is the attribute yielding at any point in time the current location in space of the component, the following action will send value v to all components in the same location as the sender $\alpha^*[\mathsf{loc} = \mathbf{my}.\mathsf{loc}]\langle v\rangle\sigma_s$ while a component which is willing to receive the message $\alpha\langle v\rangle$ only if it is sent from a component located at the location at north of its own one and only if $v \leq 10$ may execute the following action: $\alpha^*[\mathsf{loc} = \mathsf{N}(\mathbf{my}.\mathsf{loc}) \ \wedge \ x \leq 10](x)\sigma_r$, where N is assumed to be a function that, given a location ℓ in a regular grid (patch), returns the location north of ℓ.

In the sequel, we shall call *address predicates* the pradicates $[\pi]$ used for identifying the partners in input/output actions. For updates, we use the following notation: $\{a_1 \leftarrow e_1^\gamma,\ldots,a_t \leftarrow e_t^\gamma\}$ where e_j^γ is an expression which may also include functions—the definition of which are to be provided in F—which may depend on the component store and produce random results, as we shall see below. All attributes not occurring in $\{a_1,\ldots,a_t\}$ are left unchenged by the update.

We require that any attribute name a occurring in a guard $[g]$, or in the expressions $\langle e_1,\ldots,e_n\rangle$ of the message in an output action, or in the expressions $e_1^\gamma,\ldots,e_t^\gamma$, must appear in the form $\mathbf{my}.a$ (thus referring to the value of the attribute in the local store of the component). An attribute name a may appear both with and without the $\mathbf{my}.$ prefix in the address predicate π.

Intuitively, equation (1) defines state C of the component at hand and postulates that there are r potential outgoing transitions from C, with action act_j labelling a transition going from C to C_j. The actual transitions will be detarmined by the value of the guards. Note that it may happen that the current cumulative probability value of the enabled transitions is less than 1; for this reason, the language provides the construct $\mathbf{rest} :: \alpha^*[\pi]\langle e_1,\ldots,e_n\rangle\sigma.C$, where **rest** is defined as the residual probability; it is required that there is at most one **rest**-branch (typically the last one) in every state defining equation. Only output actions are allowed in **rest**-branches; this ensures that the residual probability is not affected by the fraction of those component in the system satisfying the address predicate. Obviously, in a given component specification there is exactly one defining equation for each state of the component. We let $\mathcal{S}_\Delta$ denote the

finite set of states defined by Δ. Similarly, Γ_Δ, $\mathcal{A}_\Delta$ and Π_Δ denote the sets of stores associated to Δ, and the action types and predicates occurring in (the equations of) Δ. Finally, we let $\mathcal{V}_\Delta$ denote the set of values which a component specified by Δ is defined upon (by this we mean attribute values as well as values and tuples which can be sent by an instance of the component specification). Note that we assume $\mathcal{V}_\Delta$ is a finite set—thus also Γ_Δ is finite; model finiteness is a common assumption for modelling languages supported by automatic analysis and verification tools.

Example 1 (A spatially distributed SEIR Model). As a model for space we use the *bi-dimensional Regular GRID* [6], where for each point ℓ the following specific operators are defined, with the usual North, South, East, West meaning: $\mathsf{N}(\ell), \mathsf{S}(\ell), \mathsf{E}(\ell), \mathsf{W}(\ell)$. Each component is equipped with a *position* attribute, named loc, which is always yielding the current position (i.e. point) in space of the component and is the only attribute of the component. Note that, given the abstract nature of the bi-dimensional Regular GRID, such a "point" could be a physical point is space, but also a specific region (or patch) in a pateched representation of spaces. We will implicitly refer to the second interpretation in the sequel.

The purpose of auxiliary function `Jump` declared in Figure 1 is twofold: (i) it defines a function from positions to discrete probability distributions which, given position ℓ, characterizes a probability distribution which assignes probability $p_{\mathsf{N}}(\ell)$ to $\mathsf{N}(\ell)$, probability $p_{\mathsf{S}}(\ell)$ to $\mathsf{S}(\ell)$, and so on and (ii) defines a random position generator which, given position ℓ, randomly returns a new position according to the specified probabilities. Note that the probabilities are themselves functions of the position and they are assumed being declared as additional auxiliary functions. For messages, we use the null data value $\bullet$ only; consequently, no variable is needed and we use the notation $\_$ in input actions.

In the equation for $\mathtt{S}$ in Figure 1, probability constants $h, m_N, \ldots, m_W$ are factors in $[0, 1]$ with cumulative value less then of equal to 1, each to be multiplied by the actual probability of the associated (input) action. The latter will be computed as the fraction of the local states which satisfy the required predicate. The resulting values, when taken all together, will characterize a probability *sub*-distribution; the residual probability will be associated to a **rest**-self-loop. Similar considerations apply to the probability constants in the definition of other states (e.g. i in the figure). We assume $h > m_N \approx m_S \approx m_E \approx m_W$. In other words, an agent has higher probability to get the infection from agents in the same place than from agents in adjacent places; the probability drops to zero in all other cases.

A system is modelled as a population of N instances of a component, so a *system specification* is a triple $(\Delta, F, \boldsymbol{\Sigma_0})^{(N)}$ where (Δ, F) is a component specification and $\boldsymbol{\Sigma_0}$ is the *initial (system) global state*, which will be discussed below. In the sequel we will often write Δ instead of (Δ, F).

$$\texttt{Jump}(\ell) := [p_{\mathsf{H}}(\ell) :: \ell;\ p_{\mathsf{N}}(\ell) :: \mathsf{N}(\ell);\ p_{\mathsf{S}}(\ell) :: \mathsf{S}(\ell);\ p_{\mathsf{E}}(\ell) :: \mathsf{E}(\ell);\ p_{\mathsf{W}}(\ell) :: \mathsf{W}(\ell)]$$

$$\begin{aligned}
\mathtt{S} := {} & h :: \mathtt{inf}^*[\mathsf{loc} = \mathbf{my}.\mathsf{loc}](_)\{\mathbf{my}.\mathsf{loc} \leftarrow \mathtt{Jump}(\mathbf{my}.\mathsf{loc})\}.\mathtt{E} + \\
& m_N :: \mathtt{inf}^*[\mathsf{loc} = \mathsf{N}(\mathbf{my}.\mathsf{loc})](_)\{\mathbf{my}.\mathsf{loc} \leftarrow \mathtt{Jump}(\mathbf{my}.\mathsf{loc})\}.\mathtt{E} + \\
& m_S :: \mathtt{inf}^*[\mathsf{loc} = \mathsf{S}(\mathbf{my}.\mathsf{loc})](_)\{\mathbf{my}.\mathsf{loc} \leftarrow \mathtt{Jump}(\mathbf{my}.\mathsf{loc})\}.\mathtt{E} + \\
& m_E :: \mathtt{inf}^*[\mathsf{loc} = \mathsf{E}(\mathbf{my}.\mathsf{loc})](_)\{\mathbf{my}.\mathsf{loc} \leftarrow \mathtt{Jump}(\mathbf{my}.\mathsf{loc})\}.\mathtt{E} + \\
& m_W :: \mathtt{inf}^*[\mathsf{loc} = \mathsf{W}(\mathbf{my}.\mathsf{loc})](_)\{\mathbf{my}.\mathsf{loc} \leftarrow \mathtt{Jump}(\mathbf{my}.\mathsf{loc})\}.\mathtt{E} + \\
& ext :: \mathtt{ext}^*[\bot]\langle\bullet\rangle\{\mathbf{my}.\mathsf{loc} \leftarrow \mathtt{Jump}(\mathbf{my}.\mathsf{loc})\}.\mathtt{E} + \\
& sr :: \mathtt{rec}^*[\bot]\langle\bullet\rangle\{\mathbf{my}.\mathsf{loc} \leftarrow \mathtt{Jump}(\mathbf{my}.\mathsf{loc})\}.\mathtt{R} + \\
& \mathbf{rest} :: \mathtt{nsc}^*[\bot]\langle\bullet\rangle\{\mathbf{my}.\mathsf{loc} \leftarrow \mathtt{Jump}(\mathbf{my}.\mathsf{loc})\}.\mathtt{S}
\end{aligned}$$

$$\begin{aligned}
\mathtt{E} := {} & ei :: \mathtt{act}^*[\bot]\langle\bullet\rangle\{\mathbf{my}.\mathsf{loc} \leftarrow \mathtt{Jump}(\mathbf{my}.\mathsf{loc})\}.\mathtt{I} + \\
& er :: \mathtt{rec}^*[\bot]\langle\bullet\rangle\{\mathbf{my}.\mathsf{loc} \leftarrow \mathtt{Jump}(\mathbf{my}.\mathsf{loc})\}.\mathtt{R} + \\
& \mathbf{rest} :: \mathtt{nsc}^*[\bot]\langle\bullet\rangle\{\mathbf{my}.\mathsf{loc} \leftarrow \mathtt{Jump}(\mathbf{my}.\mathsf{loc})\}.\mathtt{E}
\end{aligned}$$

$$\begin{aligned}
\mathtt{I} := {} & ii :: \mathtt{inf}^*[\top]\langle\bullet\rangle\{\mathbf{my}.\mathsf{loc} \leftarrow \mathtt{Jump}(\mathbf{my}.\mathsf{loc})\}.\mathtt{I} + \\
& ir :: \mathtt{rec}^*[\bot]\langle\bullet\rangle\{\mathbf{my}.\mathsf{loc} \leftarrow \mathtt{Jump}(\mathbf{my}.\mathsf{loc})\}.\mathtt{R} + \\
& \mathbf{rest} :: \mathtt{nsc}^*[\bot]\langle\bullet\rangle\{\mathbf{my}.\mathsf{loc} \leftarrow \mathtt{Jump}(\mathbf{my}.\mathsf{loc})\}.\mathtt{I}
\end{aligned}$$

$$\begin{aligned}
\mathtt{R} := {} & rs :: \mathtt{loss}^*[\bot]\langle\bullet\rangle\{\mathbf{my}.\mathsf{loc} \leftarrow \mathtt{Jump}(\mathbf{my}.\mathsf{loc})\}.\mathtt{S} \\
& \mathbf{rest} :: \mathtt{nsc}^*[\bot]\langle\bullet\rangle\{\mathbf{my}.\mathsf{loc} \leftarrow \mathtt{Jump}(\mathbf{my}.\mathsf{loc})\}.\mathtt{R}
\end{aligned}$$

Fig. 1. A spacially distributed SEIR model

2.2 Probabilistic Semantics

In order to model component interactions within a system, each component is equipped with a local outbox. Let Λ^O_Δ be the set $\Lambda^O_\Delta = \{\alpha\langle\mathbf{v}\rangle | \alpha \in \mathcal{A}_\Delta, \mathbf{v} \in \mathcal{V}_\Delta\}$. An *outbox-state* $O \in \mathcal{O}_\Delta = \{\langle\rangle\} \cup (\Gamma_\Delta \times \Pi_\Delta \times \Lambda^O_\Delta)$ is either empty or a triple $(\gamma, \pi, \alpha\langle\mathbf{v}\rangle)$. A *component-state* Σ is a triple $\Sigma = (C, \gamma, O) \in \mathcal{S}_\Delta \times \Gamma_\Delta \times \mathcal{O}_\Delta = \Omega_\Delta$, where C, γ, O are the current state, store and outbox-state of the component, respectively. If the component-state is the target of a transition modelling the execution of an *output* action, then $O = (\gamma', \pi, \alpha\langle\mathbf{v}\rangle)$, where γ' is the store of the (component-state) source of the transition, π is the predicate used in the action—actualized with γ'—and $\alpha\langle\mathbf{v}\rangle$ the actual message sent by the action. If, instead, the component-state is the target of a transition for an *input* action, then $O = \langle\rangle$, i.e. the empty outbox. The idea is that, whenever a component executes an output action, the related information will be available in the component's outbox *only during the next clock tick*; in the next state, (other) components will be able to get the message by means of corresponding input actions. After such a tick, the outbox will be empty or filled with the information generated by a subsequent output action of the component. It is worth noting that *the collection* of the outboxes of all the component instances in a system state is conceptually similar to the notion of the Ether (state), where communication takes place, as in PALOMA [8]. Note anyway that the collection is intrinsecally distributed and represents in no way a bottleneck for component interaction. A *global state* is

a tuple $\boldsymbol{\Sigma} = ((C_1, \gamma_1, O_1), \ldots, (C_N, \gamma_N, O_N)) \in \Omega_\Delta^N$ where $\boldsymbol{\Sigma}_{[j]} = (C_j, \gamma_j, O_j)$ is the component-state of the j-th instance in the population[5], for $j = 1 \ldots N$. We say that N is the *population size* of the system. In the sequel, we will omit the explicit indication of the size N in $(\Delta, F, \boldsymbol{\Sigma_0})^{(N)}$, and elements thereof or related functions, writing simply $(\Delta, F, \boldsymbol{\Sigma_0})$, when this cannot cause confusion. In summary, a system specification can be thought of as process algebraic clock-synchronous parallel composition of N processes.

The probabilistic behaviour of a system can be derived from its specification $(\Delta, F, \boldsymbol{\Sigma_0})^{(N)}$. We remind that Ω_Δ is finite, since so are sets $\mathcal{S}_\Delta, \Gamma_\Delta$ and $\mathcal{O}_\Delta$. Assume $\Omega_\Delta = \{\Sigma_1, \ldots, \Sigma_S\}$ and let $\mathcal{U}^S$be the set $\{\mathbf{m} \in [0,1]^S | \sum_{i=1}^S \mathbf{m}_{[i]} = 1\}$, i.e. the unit simplex of dimension S; we can assume, w.l.o.g. that there is a total ordering on Ω_Δ so that we can unambiguously associate each component m_j of a vector $\mathbf{m} = (m_1, \ldots, m_S) \in \mathcal{U}^S$ with a distinct element Σ_j of $\{\Sigma_1, \ldots, \Sigma_S\}$. With each global state $\boldsymbol{\Sigma}^{(N)}$ an *occupancy measure* vector $\mathbf{M}^{(N)}(\boldsymbol{\Sigma}^{(N)}) \in \mathcal{U}^S$ is associated where $\mathbf{M}^{(N)}(\boldsymbol{\Sigma}^{(N)}) = (M_1^{(N)}, \ldots, M_S^{(N)})$ where $M_i^{(N)} = \frac{1}{N} \sum_{n=1}^N \mathbf{1}_{\{\boldsymbol{\Sigma}_{[n]}^{(N)} = \Sigma_i\}}$ for $i = 1, \ldots, S$ and the value of $\mathbf{1}_{\{\alpha=\beta\}}$ is 1, if $\alpha = \beta$, and 0 otherwise. So, for $\Sigma_i = (C_i, \gamma_i, O_i)$, $M_i^{(N)}$ is the *fraction*, in the current global state $\boldsymbol{\Sigma}^{(N)}$, of the component instances which are in state C_i, have store γ_i and outbox O_i, over the total number N. We assume semantic intepretation functions $\mathbf{E_L}[\![\cdot]\!]$ and $\mathbf{E_R}[\![\cdot]\!]$ for the *local*, *remote* respectively, intepretation of expressions and predicates and a function $\mathbf{E_P}[\![\cdot]\!]$ for the interpretation of probability expressions. $\mathbf{E_L}[\![e]\!]$ ($\mathbf{E_R}[\![e]\!]$, respectively) takes a local (remote, respectively) store γ as an argument, whereas $\mathbf{E_P}[\![p]\!]$ takes an occupancy measure vector $\mathbf{m}$ as an argument. We note that $\mathbf{E_L}[\![a]\!]_\gamma = a, \mathbf{E_L}[\![\mathbf{my}.a]\!]_\gamma = \gamma(a)$, $\mathbf{E_R}[\![a]\!]_\gamma = \gamma(a)$, and $\mathbf{E_P}[\![\mathsf{frc}\, C]\!]_\mathbf{m} = \sum_{i=1}^S \{\mathbf{m}_{[i]} | \Sigma_i = (C, \gamma_i, O_i)\}$; moreover, $\mathbf{E_R}[\![\mathbf{my}.a]\!]_\gamma$, $\mathbf{E_P}[\![\mathrm{tt}]\!]_\mathbf{m}$, $\mathbf{E_P}[\![\mathrm{ff}]\!]_\mathbf{m}$, $\mathbf{E_L}[\![\mathsf{frc}\, C]\!]_\gamma$, and $\mathbf{E_R}[\![\mathsf{frc}\, C]\!]_\gamma$ are undefined as are, for the sake of simplicity, $\mathbf{E_P}[\![a]\!]_\mathbf{m}$, $\mathbf{E_P}[\![\mathbf{my}.a]\!]_\mathbf{m}$. The definition of the above semantic interpretation functions on composition terms can be given recursively on the structure of the terms and is left out here. In particular, we assume them extended to tuples. Similarly, we assume standard techinques and machinery for the definition of auxiliary functions in store updates and their sematic interpretation: the semantic interpretation of update σ in the current store γ will denoted by $\mathbf{E_U}[\![\sigma]\!]_\gamma$—in this paper, updates do not depend on the current occupancy measure vector, i.e. the frc operator cannot occur in their specification. $\mathbf{E_U}[\![\sigma]\!]_\gamma$ is a function from stores to probability values.

Let Λ_Δ be defined as $\Lambda_\Delta = \Lambda_\Delta^O \cup \Lambda_\Delta^I$, with Λ_Δ^O as above, and $\Lambda_\Delta^I = \{\alpha(\mathbf{v}) | \alpha \in \mathcal{A}_\Delta, \mathbf{v} \in \mathcal{V}_\Delta\}$. A component specification (Δ, F) characterises the (component) transition probability matrix as a function of occupancy measure vectors $\mathbf{m}$, $\mathbf{K}^{(N)} : \mathcal{U}^S \times \Omega_\Delta \times \Omega_\Delta \to [0,1]$ such that $\mathbf{K}^{(N)}(\mathbf{m})_{\Sigma,\Sigma'}$ is the probability of one step jump from component-state Σ to component-state Σ', given (that the global

[5] Appropriate syntactical shorthands can be introduced for describing the initial state, e.g. $\langle(\mathtt{S}, \gamma_\mathtt{s})[2000], (\mathtt{E}, \gamma_\mathtt{e})[100], (\mathtt{I}, \gamma_\mathtt{i})[200], (\mathtt{R}, \gamma_\mathtt{r})[0]\rangle$ for 2000 objects initially in state `S` etc. In addition specific syntax can be used for specifying the initial stores like, e.g. $\{a_1 \leftarrow v_1, \ldots, a_n \leftarrow v_n\}$.

system state induces) occupancy measure vector $\mathbf{m}$. $\mathbf{K}^{(N)}(\mathbf{m})_{\Sigma,\Sigma'}$ is computed by making use of a transition relation $(C,\gamma,O) \xrightarrow{\lambda,p} (C',\gamma',O')$ over the space of component-states Ω_Δ, with transition labels drawn from $\Theta_\Delta \subset \Lambda_\Delta \times [0,1]$. More specifically, The transition relation is the relation $\rightarrow \subseteq \Omega_\Delta \times \Theta_\Delta \times \Omega_\Delta$ such that $(C,\gamma,O) \xrightarrow{\lambda,p} (C',\gamma',O')$ iff $C := \sum_{j\in J}[g_j]p_j :: act_j.C_j$ is the defining equation for C and $p = \sum_{k\in J}\{\bar{p}_k | (C,\gamma,O) \xrightarrow{\lambda,\bar{p}_k}_k (C',\gamma',O')\}$, where $\xrightarrow{\lambda,\bar{p}_k}_k$ is the least relation induced by the rules in Figure 2.

$$\frac{\begin{array}{c} C := \sum_{j\in J}[g_j]p_j :: act_j.C_j \quad k \in J \quad \mathbf{E_L}[\![g_k]\!]_\gamma = \mathrm{tt} \\ act_k = \alpha^*[\pi]\langle \mathbf{e}\rangle\sigma \quad p = \mathbf{E_U}[\![\sigma]\!]_\gamma(\gamma') \end{array}}{(C,\gamma,O) \xrightarrow{\alpha\langle \mathbf{E_L}[\![\mathbf{e}]\!]_\gamma\rangle, p\cdot \mathbf{E_P}[\![p_k]\!]_\mathbf{m}}_k (C_k,\gamma',(\gamma,\mathbf{E_L}[\![\pi]\!]_\gamma,\alpha\langle \mathbf{E_L}[\![\mathbf{e}]\!]_\gamma\rangle))} \tag{2}$$

$$\frac{\begin{array}{c} C := \sum_{j\in J}[g_j]p_j :: act_j.C_j \quad k \in J \quad [g_k]p_k = \mathbf{rest} \\ act_k = \alpha^*[\pi]\langle \mathbf{e}\rangle\sigma \quad p = \mathbf{E_U}[\![\sigma]\!]_\gamma(\gamma') \end{array}}{(C,\gamma,O) \xrightarrow{\alpha\langle \mathbf{E_L}[\![\mathbf{e}]\!]_\gamma\rangle, p\cdot(1-\sum_{j\in(J\setminus\{k\})}\mathbf{E_P}[\![p_j]\!]_\mathbf{m})}_k (C_k,\gamma',(\gamma,\mathbf{E_L}[\![\pi]\!]_\gamma,\alpha\langle \mathbf{E_L}[\![\mathbf{e}]\!]_\gamma\rangle))} \tag{3}$$

$$\frac{\begin{array}{c} C := \sum_{j\in J}[g_j]p_j :: act_j.C_j \quad k \in J \quad \mathbf{E_L}[\![g_k]\!] = \mathrm{tt} \\ act_k = \alpha^*[\pi](\mathbf{x})\sigma \quad \mathbf{v} \in \mathcal{V}_\Delta \quad p = \mathbf{E_U}[\![\sigma[\mathbf{v}/\mathbf{x}]]\!]_\gamma(\gamma') \\ f = \sum_{i=1}^{S}\{\mathbf{m}_{[i]} | \Sigma_i = (C_i',\gamma_i',(\gamma_i'',\pi_i',\alpha\langle \mathbf{v}\rangle) \ \wedge \\ \qquad \mathbf{E_R}[\![\pi_i']\!]_\gamma = \mathbf{E_R}[\![\mathbf{E_L}[\![\pi[\mathbf{v}/\mathbf{x}]]\!]_\gamma]\!]_{\gamma_i''} = \mathrm{tt}\} \end{array}}{(C,\gamma,O) \xrightarrow{\alpha(\mathbf{v}), p\cdot \mathbf{E_P}[\![p_k]\!]_\mathbf{m}\cdot f}_k (C_k,\gamma',\langle\rangle)} \tag{4}$$

Fig. 2. Probabilistic Semantics Rules

The component transition matrix function $\mathbf{K}^{(N)}(\mathbf{m})_{\Sigma,\Sigma'}$ is readely defined as follows: $\mathbf{K}^{(N)}(\mathbf{m})_{\Sigma,\Sigma'} = \sum_{(\lambda,p)\in\Theta_\Delta}\{p | \Sigma \xrightarrow{\lambda,p} \Sigma'\}$. Note that all the above summations are finite under our assumption that such is $\mathcal{V}_\Delta$.

The behaviour of the system is the result of the parallel-synchronous execution of the N instances of the component. Thus, the probabilistic behaviour of the system is characterised by the DTMC $\mathbf{X}^{(N)}(t)$ with initial probability distribution $\boldsymbol{\delta}_{\boldsymbol{\Sigma_0}}$ and one step probability matrix $\mathbf{P}^{(N)}$ defined by the following product:

$$\mathbf{P}^{(N)}_{\boldsymbol{\Sigma},\boldsymbol{\Sigma'}} = \Pi_{n=1}^{N}\mathbf{K}^{(N)}(\mathbf{M}^{(N)}(\boldsymbol{\Sigma}))_{\boldsymbol{\Sigma}_{[n]},\boldsymbol{\Sigma'}_{[n]}}. \tag{5}$$

Of course, the 'occupancy measure' view of the evolution in time of stochastic process $\mathbf{X}^{(N)}(t)$ is again a DTMC, namely the *occupancy measure DTMC*, which is defined as expected: $\mathbf{M}^{(N)}(t) = \mathbf{M}^{(N)}(\mathbf{X}^{(N)}(t))$.

2.3 Bounded PCTL

We recall that, given a set $\mathscr{P}$ of atomic propositions, the syntax of PCTL *state formulas* Φ and *path formulas* φ is defined as follows, where $\mathsf{ap} \in \mathscr{P}$ and $k \geq 0$:

$$\Phi ::= \mathsf{ap} \mid \neg\Phi \mid \Phi \wedge \Phi \mid \mathcal{P}_{\bowtie p}(\varphi) \qquad \text{where } \varphi ::= \mathcal{X}\,\Phi \mid \Phi\,\mathcal{U}^{\leq k}\,\Phi.$$

PCTL formulas are interpreted over *state labelled* DTMCs, which are pairs $(\mathcal{M}, L)$ where $\mathcal{M}$ is a DTMC and L is a mapping from the set of states of $\mathcal{M}$ to $2^{\mathscr{P}}$; for each state s, $L(s)$ is the set of atomic propositions true in s[6].

For the purposes of bounded PCTL model-checking, system specifications are enriched with the declaration of three different kinds of atomic propositions. A declaration of the form $ap\,\mathsf{at}\,C$ associates atomic proposition ap to state $C \in \mathcal{S}_{\Delta}$. Thus ap must be included in the set $L(\boldsymbol{\Sigma})$ for each global state $\boldsymbol{\Sigma} = ((C_1, \gamma_1, O_1), \ldots, (C_N, \gamma_N, O_N))$ such that $C_1 = C$ (recall here that FlyFast performs model-checking of *the first* object in the context of the global system). A declaration of the form $ap\,\mathsf{def}\,(\mathbf{my}.a \bowtie v)$ associates atomic proposition ap to all component-states (C, γ, O) s.t. attribute a is $\bowtie v$. So, ap must be included in the set $L(\boldsymbol{\Sigma})$ for each global state $\boldsymbol{\Sigma} = ((C_1, \gamma_1, O_1), \ldots, (C_N, \gamma_N, O_N))$ such that $\mathbf{E_L}[\![\mathbf{my}.a \bowtie v]\!]_{\gamma_1} = \mathrm{tt}$. Finally, a limited form of *global* atomic predicate is provided by means of a declaration of the form $ap\ \mathsf{def}\ (\mathsf{frc}\,C \bowtie v)$; in this case, ap must be included in the set $L(\boldsymbol{\Sigma})$ for each global state $\boldsymbol{\Sigma}$ s.t. the fraction in $\boldsymbol{\Sigma}$ of the component states (C, γ, O), for any γ and O, is $\bowtie$ than $v \in [0, 1]$.

3 A simple Translation to FlyFast

Given system specification $\Upsilon = (\Delta_{\Upsilon}, F_{\Upsilon}, \boldsymbol{\Sigma_0})^{(N)}$, let $\mathcal{I}_{\mathcal{S}}$ be a total injection from $\Omega_{\Delta_{\Upsilon}}$ to $\mathcal{S}$ and $\mathcal{I}_{\mathcal{A}}$ be a total injection from $(\mathcal{S}_{\Delta_{\Upsilon}} \times \Gamma_{\Delta_{\Upsilon}}) \times \Lambda_{\Delta_{\Upsilon}} \times \Omega_{\Delta_{\Upsilon}}$ to $\mathcal{A}$. In the sequel we show how to build from Υ a FlyFast [13, 15] system specification $\mathcal{I}(\Upsilon) = \langle \Delta, A, \mathbf{C}_0 \rangle$.

For action $act = \alpha^*[\pi]\langle e_1, \ldots, e_n\rangle\sigma$ or $act = \alpha^*[\pi](x_1, \ldots, x_n)\sigma$, we let $T(act) = \alpha$, $P(act) = \pi$, $V(act) = \langle e_1, \ldots, e_n\rangle$ and $U(act) = \sigma$. $\mathsf{SUM}\{t|\mathsf{cond}(t)\}$ denotes the *syntactical* term representing the sum of terms $t \in \{t|\mathsf{cond}(t) = \mathrm{tt}\}$, i.e. $t_1 + \ldots + t_n$, if $\{t|\mathsf{cond}(t) = \mathrm{tt}\} = \{t_1, \ldots, t_n\} \neq \emptyset$ and 0 if $\{t|\mathsf{cond}(t) = \mathrm{tt}\}$ is the empty set. Finally, by $t * t'$ we mean the *syntactical* term representing the product of terms t and t'.

For each state equation $C := \sum_{j \in J}[g_j]p_j :: act_j.C_j$ in Δ_{Υ}:

1. For each *output* action $\alpha^*[\pi]\langle \mathbf{e}\rangle\sigma = act_k$ with $k \in J$, and $\gamma \in \Gamma_{\Delta_{\Upsilon}}$, let $J_{\alpha,\sigma,\gamma}$ be the largest subset of J s.t. there is $C' \in \mathcal{S}_{\Delta}$ s.t. for all $j \in J_{\alpha,\sigma,\gamma}$ $C_j = C'$, $T(act_j) = \alpha$, $\mathbf{E_L}[\![P(act_j)]\!]_{\gamma} = \mathbf{E_L}[\![\pi]\!]_{\gamma}$, $\mathbf{E_L}[\![V(act_j)]\!]_{\gamma} = \mathbf{E_L}[\![\mathbf{e}]\!]_{\gamma}$ and $\mathbf{E_U}[\![U(act_j)]\!]_{\gamma} = \mathbf{E_U}[\![\sigma]\!]_{\gamma}$.

[6] We refer to [9] for the formal definition of PCTL and to [15, 13] for the details of its instantiation in FlyFast.

For each $\gamma' \in \Gamma_{\Delta_\Upsilon}$, with $\xi = \mathcal{I}_{\mathcal{A}}((C,\gamma),\alpha\langle \mathbf{E_L}[\![\mathbf{e}]\!]_\gamma\rangle,(C',\gamma',(\gamma,\mathbf{E_L}[\![\pi]\!]_\gamma,\alpha\langle \mathbf{E_L}[\![\mathbf{e}]\!]_\gamma\rangle)))$, the following action probability function definition is included in A:

$$\xi :: \mathbf{E_U}[\![\sigma]\!]_\gamma(\gamma') * \mathsf{SUM}\{p_j | j \in \hat{J}_{\alpha,\sigma,\gamma}\}$$

where $\hat{J}_{\alpha,\sigma,\gamma} = \{j \in J_{\alpha,\sigma,\gamma} | [g_j]p_j \neq \mathbf{rest} \ \wedge \ \mathbf{E_L}[\![g_j]\!]_\gamma = \mathrm{tt}\}$;
Moreover, for each outbox $O \in \mathcal{O}_{\Delta_\Upsilon}$, the following summand is added to the equation in Δ for state $\mathcal{I}_{\mathcal{S}}(C,\gamma,O)$:

$$\xi.\mathcal{I}_{\mathcal{S}}(C',\gamma',(\gamma,\mathbf{E_L}[\![\pi]\!]_\gamma,\alpha\langle \mathbf{E_L}[\![\mathbf{e}]\!]_\gamma\rangle));$$

2. For each $\mathbf{v} \in \mathcal{V}$, *input* action $\alpha^*[\pi](\mathbf{x})\sigma = act_k$, with $k \in J$, and $\gamma \in \Gamma_{\Delta_\Upsilon}$, let $J_{\alpha,\sigma,\gamma}$ be the largest subset of J s.t. there is $C' \in \mathcal{S}_\Delta$ s.t. for all $j \in J_{\alpha,\sigma,\gamma}$ $C_j = C'$, $T(act_j) = \alpha$ and $\mathbf{E_U}[\![U(act_j[\mathbf{v}/\mathbf{x}])]\!]_\gamma = \mathbf{E_U}[\![\sigma[\mathbf{v}/\mathbf{x}]]\!]_\gamma$. For each $\gamma' \in \Gamma_{\Delta_\Upsilon}$, with $\xi = \mathcal{I}_{\mathcal{A}}((C,\gamma),\alpha(\mathbf{v}),(C',\gamma',\langle\rangle))$, the following action probability function definition is included in A:

$$\xi :: \mathbf{E_U}[\![\sigma[\mathbf{v}/\mathbf{x}]]\!]_\gamma(\gamma') * \mathsf{SUM}\{(p_j * \Phi_j) | j \in \hat{J}_{\alpha,\sigma,\gamma}\}$$

where $\hat{J}_{\alpha,\sigma,\gamma} = \{j \in J_{\alpha,\sigma,\gamma} | [g_j]p_j \neq \mathbf{rest} \ \wedge \ \mathbf{E_L}[\![g_j]\!]_\gamma = \mathrm{tt}\}$ and term Φ_j is: $\mathsf{SUM}\{\mathsf{frc}\,\mathcal{I}_{\mathcal{S}}((\tilde{C},\tilde{\gamma},(\bar{\gamma},\bar{\pi},\alpha\langle\mathbf{v}\rangle))) | \mathbf{E_R}[\![\bar{\pi}]\!]_\gamma = \mathbf{E_R}[\![\mathbf{E_L}[\![\pi_j]\!]_\gamma]\!]_{\bar{\gamma}} = \mathrm{tt}\}$.
Moreover, for each outbox $O \in \mathcal{O}_{\Delta_\Upsilon}$, the following summand is added to the equation in Δ for state $\mathcal{I}_{\mathcal{S}}(C,\gamma,O)$:

$$\xi.\mathcal{I}_{\mathcal{S}}(C',\gamma',\langle\rangle);$$

3. If there exists $k \in J$ s.t. $[g_k]p_k = \mathbf{rest}$, and $act_k = \alpha^*[\pi]\langle\mathbf{e}\rangle\sigma$, let $\bar{A}$ be the set of probability function definitions which has been constructed in steps (1) and (2) above; note that for every $\zeta :: p * q \in \bar{A}$, q is either a probability constant p_j occurring in a branch $[g_j]p_j :: act_j.C_j$ of the defining equation for C, or it is a term of the form $\mathsf{SUM}\{(p_h * \Phi_h) | h \in H\}$, for some index set H. We define $\bar{p} = (1 - \mathsf{SUM}\{q | \zeta :: p * q \in \bar{A}\})$ and, for all $\gamma' \in \Gamma_{\Delta_\Upsilon}$, with $\xi = \mathcal{I}_{\mathcal{A}}((C,\gamma),\alpha\langle \mathbf{E_L}[\![\mathbf{e}]\!]_\gamma\rangle,(C_k,\gamma',(\gamma,\mathbf{E_L}[\![\pi]\!]_\gamma,\alpha\langle \mathbf{E_L}[\![\mathbf{e}]\!]_\gamma\rangle)))$, the following action probability function definition is included in A:

$$\xi :: \mathbf{E_U}[\![\sigma]\!]_\gamma(\gamma') * \bar{p}.$$

Moreover, for each outbox $O \in \mathcal{O}_{\Delta_\Upsilon}$, the following summand is added to the equation in Δ for state $\mathcal{I}_{\mathcal{S}}(C,\gamma,O)$:

$$\xi.\mathcal{I}_{\mathcal{S}}(C_k,\gamma',(\gamma,\mathbf{E_L}[\![\pi]\!]_\gamma,\alpha\langle \mathbf{E_L}[\![\mathbf{e}]\!]_\gamma\rangle));$$

4. no other action probability function definition and transition is included.

Let $\mathbf{K}^{(N)}_{\mathcal{I}(\Upsilon)} : \mathcal{U}^S \times \mathcal{I}_{\mathcal{S}}(\Omega_\Delta) \times \mathcal{I}_{\mathcal{S}}(\Omega_\Delta) \to [0,1]$ be the step probability function associated to $\mathcal{I}(\Upsilon)$ by the FlyFast language probabilistic semantics definition (see [13, 15] for details) and $\mathbf{K}^{(N)}_{\Upsilon} : \mathcal{U}^S \times \Omega_\Delta \times \Omega_\Delta \to [0,1]$ be the step probability function for Υ as defined in Section 2.2. It is easy to see that:

Theorem 1. *For all $N > 0$, occupancy measure vector $\mathbf{m} \in \mathcal{U}^S$ and $\Sigma, \Sigma' \in \Omega_\Delta$ the following holds:* $\mathbf{K}^{(N)}_\Upsilon(\mathbf{m})_{\Sigma,\Sigma'} = \mathbf{K}^{(N)}_{\mathcal{I}(\Upsilon)}(\mathbf{m})_{\mathcal{I}_\mathcal{S}(\Sigma),\mathcal{I}_\mathcal{S}(\Sigma')}$.

Proof (scketch).

$$
\begin{aligned}
& \mathbf{K}^{(N)}_\Upsilon(\mathbf{m})_{(C,\gamma,O),(C',\gamma',O')} \\
= \quad & \{\text{Def. } \mathbf{K}^{(N)}_\Upsilon(\mathbf{m})\} \\
& \textstyle\sum_{(\lambda,p)\in\Theta_\Delta}\{p|(C,\gamma,O) \xrightarrow{\lambda,p} (C',\gamma',O')\} \\
= \quad & \{\text{Def. } \rightarrow\} \\
& \textstyle\sum_{(\lambda,p)\in\Theta_\Delta}\{p|p = \sum_{k\in J}\{\bar{p}_k|(C,\gamma,O) \xrightarrow{\lambda,\bar{p}_k}_k (C',\gamma',O')\} \ \wedge \\
& \qquad\qquad C := \textstyle\sum_{j\in J}[g_j]p_j :: act_j.C_j \text{ is the def. eq. of } C \text{ in } \Upsilon\}
\end{aligned}
$$

Consider the outer summation and suppose $(\alpha\langle\mathbf{v}\rangle, p)$ be the index of a summand. This means that $\mathbf{v} = \alpha\langle\mathbf{E_L}[\![\mathbf{e}]\!]_\gamma\rangle$ for some $\mathbf{e}$. For the sake of simplicity, assume there is only one instance of such a summand and there is only one $k \in J$ such that the following transition is derived using the rules of Section 2: $(C,\gamma,O) \xrightarrow{\alpha\langle\mathbf{v}\rangle,\bar{p}_k}_k (C',\gamma',O')$[7]. So, we have $\mathbf{K}^{(N)}_\Upsilon(\mathbf{m})_{(C,\gamma,O),(C',\gamma',O')} = \bar{p}_k$ such that $(C,\gamma,O) \xrightarrow{\alpha\langle\mathbf{v}\rangle,\bar{p}_k}_k (C',\gamma',O')$, where $C := \sum_{j\in J}[g_j]p_j :: act_j.C_j$ is the defining equation for C. Suppose $[g_k]p_k \neq \mathbf{rest}$, so that Rule (2) of Figure 2 has been used for generating the transition. This implies that $\mathbf{E_L}[\![g_k]\!]_\gamma = \mathrm{tt}$, $\bar{p}_k = \mathbf{E_U}[\![\sigma]\!]_\gamma(\gamma') \cdot \mathbf{E_P}[\![p_k]\!]_\mathbf{m}$, $C' = C_k$, and $O' = (\gamma, \mathbf{E_L}[\![\pi]\!]_\gamma, \alpha\langle\mathbf{E_L}[\![\mathbf{e}]\!]_\gamma\rangle)$. Under the above conditions, by definition of the translation algorithm, the action $\xi = \mathcal{I}_\mathcal{A}((C,\gamma),\alpha\langle\mathbf{v}\rangle,(C',\gamma',O'))$ and related action probability function definition $\xi :: \mathbf{E_U}[\![\sigma]\!]_\gamma(\gamma') * \mathsf{SUM}\{p_k\}$ are included in the FlyFast model. Moreover, the summand $\xi.\mathcal{I}_\mathcal{S}(C',\gamma',O')$ is added in the equation for state $\mathcal{I}_\mathcal{S}(C,\gamma,O)$ in the FlyFast model. Using the semantics definition of the FlyFast language [13, 15], we get that the probability assigned to ξ is $\mathbf{E_U}[\![\sigma]\!]_\gamma(\gamma') \cdot \mathbf{E_P}[\![p_k]\!]_\mathbf{m}$, that is, exactly $\bar{p_k}$. Thus $\mathbf{K}^{(N)}_\Upsilon(\mathbf{m})_{\Sigma,\Sigma'} = \mathbf{K}^{(N)}_{\mathcal{I}(\Upsilon)}(\mathbf{m})_{\mathcal{I}_\mathcal{S}(\Sigma),\mathcal{I}_\mathcal{S}(\Sigma')}$. The proof for all the other cases is similar. •

The translation of atomic proposition declarations into FlyFast formula declarations is the obvious one and is shown in Figure 3 where $\mathsf{OR}\{e|\mathsf{cond}(e)\}$ denotes the *syntactical* term representing the disjunction of expressions $e \in \{e|\mathsf{cond}(e) = \mathrm{tt}\}$, i.e. $e_1|\ldots|e_n$, if $\{e|\mathsf{cond}(e) = \mathrm{tt}\} = \{e_1,\ldots,e_n\} \neq \emptyset$ and ff, if $\{e|\mathsf{cond}(e) = \mathrm{tt}\}$ is the empty set.

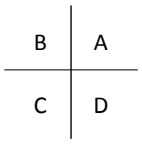
[7] The more general case where there are more instances of the same action only complicates notation and is not different, conceptually, from the simple case we are proving here.

atomic prop. decl.	FlyFast formula declaration
$\mathsf{ap}\,\mathsf{at}\,C$	$\mathsf{ap} : \mathsf{OR}\{\mathcal{I}_{\mathcal{S}}((C',\gamma,O))\mid(C',\gamma,O)\in\Omega_{\Delta_\Upsilon}\ \wedge\ C'=C\}$
$\mathsf{ap}\,\mathsf{def}\,(\mathbf{my}.a \bowtie v)$	$\mathsf{ap} : \mathsf{OR}\{\mathcal{I}_{\mathcal{S}}((C,\gamma,O))\mid(C,\gamma,O)\in\Omega_{\Delta_\Upsilon}\ \wedge\ \gamma(a)\bowtie v\}$
$\mathsf{ap}\,\mathsf{def}\,(\mathsf{frc}\,C \bowtie v)$	$\mathsf{ap} : \mathsf{SUM}\{\mathsf{frc}\,\mathcal{I}_{\mathcal{S}}((C',\gamma,O))\mid(C',\gamma,O)\in\Omega_{\Delta_\Upsilon}\ \wedge\ C'=C\}\bowtie v$

Fig. 3. Translation of atomic proposition declarations. The translation is not defined whenever $\mathsf{OR}\{e|\mathsf{cond}(e)\} = \mathrm{ff}$ or $\mathsf{SUM}\{e|\mathsf{cond}(e)\} = 0$ and suitable error messages are generated.

4 Epidemic Example Revisited

We return to the distributed Epidemic example of Fig. 1 where, for the sake of simplicity, we consider a simple patched space, consisting of the usual four quadrants A, B, C, D in the Cartesian Plane, as in Fig. 4 (left). We model a 'flow' from quadrant C to quadrant A by defining the jump probabilities as in the table in Fig. 4 (right)[8], where $l = 0.6$ and $s = 1 - l$, so that $l > s$.



	$\mathtt{p}_H$	$\mathtt{p}_N$	$\mathtt{p}_S$	$\mathtt{p}_E$	$\mathtt{p}_W$
A	l	0	$s/2$	0	$s/2$
B	$s/2$	0	$s/2$	l	0
C	s	$l/2$	0	$l/2$	0
D	$s/2$	l	0	0	$s/2$

Fig. 4. The Cartesian quadrants (left) and the jump probabilities (right).

We consider a model in which initially there are 10.000 components in state `S` in quadrant C and 100 in state `S` in quadrant A. The non-zero values of the parameters are the same for each quadrant, defined as follows: $h = 0.2, ext = 0.1, ei = 0.4, ii = 0.8, ir = 0.2, rs = 0.1, m_N = m_S = m_E = m_W = 0.05$.

Fig. 5 shows the fast-simulation results[9] for the model for each of the four quadrants. This functionality is built in in the FlyFast tool. In the figure, the fractions of numbers of the componets in each of the four states at each of the four locations are shown. Note that these fractions correspond to appropriate predicates on standard atomic propositions; for instance the fraction of components in state `S` at quandrant A is captured by $\mathtt{s} \wedge \mathtt{a}$, assuming the following declarations: $\mathtt{s}\,\mathsf{at}\,\mathtt{S}$ and $\mathtt{a}\,\mathsf{def}\,(\mathbf{my}.\mathsf{loc} = A)$. The simulation of single elements, taken as the average over 10 runs shows a very good correspondence with the fast-simulation results. The results also show good correspondence to the original SEIR model [5] when the probability to move between quadrants is set to zero and in the initial state the total population is in state `S` and in one specific quadrant in the former model. Besides fast simulation, that gives an idea of the

[8] We assume: `N(A) = N(B) = S(C) = S(D) = E(A) = E(D) = W(B) = W(C) = undefined`, with `[loc = undefined] =` $\perp$ for all `loc` .

[9] Experiments have been performed using the FlyFast on-the-fly mean field model checker on a PC with an Intel Core i7 1.7GHz, RAM 8Gb.

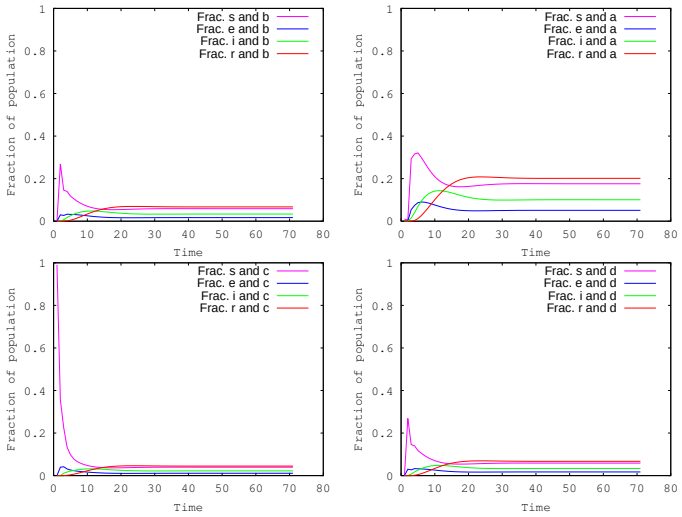


Fig. 5. Fast simulation for each of the four quadrants.

average global behaviour of the system, we can also analyse the behaviour of a single component in the context of the overall behaviour. We consider two example properties as illustration. Let us first consider a component initially in state `S` and located in A and let atomic propositions `i` and `c` be declared as follows: `i` at `I` and `c` def $(\mathbf{my}.\mathsf{loc} = C)$. The following formula (P1) states that the probability is greater than p that that the component ends up infected in quadrant C by time t: $\mathcal{P}_{>p}(\mathrm{tt}\,\mathcal{U}^{\leq t}\,(i \wedge c))$. FlyFast allows to study the dynamics of the actual probability as a function of t, by means of the notation $\mathcal{P}_{=?}(\mathrm{tt}\,\mathcal{U}^{\leq t}\,(i \wedge c))$ and the resulting graph, for the above initial conditions and for the first 70 time units is shown in Fig. 6 (left). For comparison, the formula for an agent starting in C and ending up in A and being infected is shown as well in the same figure. The results for a more complicated, nested, formula (P2) are shown in Fig. 6 (right). P2 expresses the probability, over time, of a component reaching a situation in which it is neither infected and located in A nor exposed and located in A, and from which it can reach a state in which it has a probability higher than 0.25 to be infected and located in `C` within the next 5 time units, with `e` at `E`: $\mathcal{P}_{=?}(\mathrm{tt}\ \mathcal{U}^{\leq t}\ ((\neg(i \wedge a) \wedge \neg(e \wedge a)) \wedge \mathcal{P}_{>0.25}(\mathrm{tt}\ \mathcal{U}^{\leq 5}\,(i \wedge c))))$. The formula has been considered for a component which is initially in A and in state `S`; the figure shows also a similar formula, where the role of A and C is exchanged.

For both types of properties a considerable difference in the probabilities can be observed for an agent that is initially located in A or in C, due to the flow of movement that has been introduced. This illustrates a clear dependence of the results on the dynamically changing spatial distribution of components. The total number of states, actions and transitions for the resulting FlyFast *object specification* is 52, 114 and 468 respectively, while the number of states of the

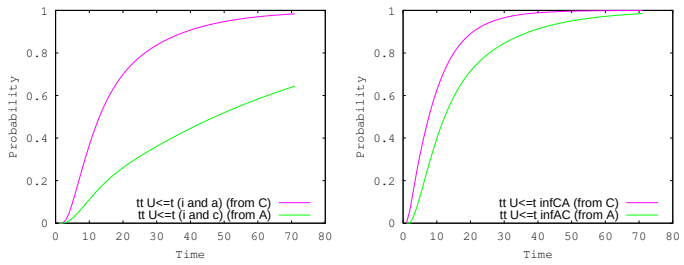


Fig. 6. Model checking results for properties P1 (left) and P2 (right).

global *approximated* model which have been generated for the analysis of formula P2 is 2.323 (2.185 when A and C are swapped). The model checking time for the more complicated nested formula P2 and for all values of t (70) is 10.343 (9.921 when A and C are swapped) $ms \approx 148\,(141)\,ms$ per checking session, for a model with a total population of 10.100 objects. A well-known feature of mean-field model checking is that the model checking time is independent of the size of the population, however, further experimentation with more extended spatial models and more attributes, that do effect this time, is planned as future work.

5 Conclusions

The attribute-based interaction paradigm is deemed fundamental for agent interaction in the context of Autonomic or Collective Adaptive Systems [7, 17, 3, 1, 10]. In this paper we have presented a attribute-based coordination modelling language as a front-end for FlyFast, an on-the-fly mean-field model-checker for bounded PCTL. The language extends the original FlyFast modelling language by replacing its actions with input (output, respectively) actions where senders (receivers, respectively) are specified by means of predicates on dynamic attributes on system components, where a component is a process/attribute-store pair. A translation to the standard FlyFast language has been presented, its correctness has been showed as well as an example of its application to a simple case study.

It should be noted that the introduction of attributes in a process model is an *intrinsic* source of complexity in terms of component state-space size. Such an increase, in the worst case, goes with $|\mathcal{V}_\Delta|^{|Att_\Delta|} \cdot (|\mathcal{A}^O_\Delta| + 1)$, where Att_Δ is the set of attributes of the component, $\mathcal{V}_\Delta$ is the set of values they can take, and $\mathcal{A}^O_\Delta$ is the set of output actions occurring in the component specification (which may appear in its outbox). The obvious consequence of this is that one has to carefully ponder the importance and necessity of each and every new attribute used in a model, although, it must be kept in mind that the real source of state-space explosion is the size of the system, and this issue is addressed by Mean-Field approximation. A first optimisation consists in considering only reachable

component states as well as eliminating actions with constant zero probability. A possible additional line of investigation is the study of techniques for DTMC minimization to Mean-field analysis, so that the number of difference equations can decrease as a consequence, in a similar way as for CTMCs and the number of differential equations in fluid flow analysis [20].

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