

1 **Quantitative interpretation of air radon progeny fluctuations in**  
2 **terms of stability conditions in the atmospheric boundary layer**

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10 **Abstract** Determining the mixing height using a tracer can improve the information obtained  
11 using traditional techniques. Here we provide an improved box model based on radon progeny  
12 measurements, which considers the vertical entrainment of residual layers and the variability in  
13 the soil radon exhalation rate. The potential issues in using progeny instead of radon have been  
14 solved from both a theoretical and experimental perspective; furthermore, the instrumental  
15 efficiency and the counting scheme have been included in the model. The applicability range  
16 of the box model has been defined by comparing radon-derived estimates with sodar and lidar  
17 data. Three intervals have been analyzed (“near-stable”, “transition” and “turbulent”), and  
18 different processes have been characterized. We describe a preliminary application case  
19 performed in Rome, Italy, while case studies will be required to determine the range limits that  
20 can be applied in any circumstances.

21

22 **Keywords** Aerosols and particles, Boundary-layer processes, Geochemical cycles, Model  
23 verification and validation, Modelling.

24

25 **1 Introduction**

26 The presence of air pollutants in the lower troposphere is highly influenced by meteorological

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27 conditions, which regulate turbulent mixing and the vertical and horizontal components of  
28 dispersion. Substances emitted into the atmospheric boundary layer (ABL) are gradually  
29 dispersed and eventually become completely mixed within this layer, given sufficient time and  
30 if there are no significant sinks (Seibert et al. 2000). The usual definition of the ABL involves  
31 considering the ABL to be the turbulent domain of the atmosphere adjacent to the ground. In  
32 this case, the ABL coincides with the mixing layer, i.e., a term commonly used in air pollution  
33 meteorology. The height of the mixing layer, the so-called “mixing height”, determines the  
34 available volume for the dispersion of pollutants, and this height is involved in many predictive  
35 and diagnostic methods and/or models used to assess pollutant concentrations. Furthermore,  
36 this variable is a critical parameter in atmospheric flow models (Lin 2012).

37 Traditionally, “profile-based” methods have been used to estimate the mixing height; these  
38 include direct measurement techniques obtained from remote sensing systems (radar, sodar or  
39 lidar) and sensors deployed on platforms (radiosondes, tethered balloons or masts) or aircraft.  
40 Furthermore, dynamical models provide fields relevant to the ABL, but the reliability of their  
41 performance needs to be better assessed; see, for instance, Nath and Patil (2006).

42 The scientific community considers the use of  $^{222}\text{Rn}$  (radon) to be a comparatively simple and  
43 economical approach for defining the stability conditions of the lower troposphere (Duenas et  
44 al. 1996, Perrino et al. 2001, Pasini et al. 2003, Sesana et al. 2003, Desideri et al. 2006, Zhang  
45 et al. 2006, Desideri et al. 2007, Perrino et al. 2008, Chambers et al. 2011, 2015) and for  
46 estimating the mixing height (Pasini and Ameli 2003, Sesana et al. 2003, 2006, Veleva et al.  
47 2010, Keller et al. 2011, Griffith et al. 2013). Earlier Guedalia et al. (1980) described this noble  
48 gas as a perfect tracer of ABL dilution features, and demonstrated that radon radioactivity  
49 represents a simple index of the stability state of the ABL. Once emitted by soil, radon leaves  
50 the surface by molecular diffusion or by convection, and enters the atmosphere where it is  
51 distributed by turbulent mixing (Porstendorfer 1994). The radon decay products are metallic  
52 elements that are easily fixed to existing aerosol particles in the atmosphere. The reduction of  
53 these particles in the atmosphere occurs either by radioactive decay or by removal processes  
54 (dry deposition, rainout, washout). The distribution of this aerosol component in the troposphere  
55 is controlled mainly by turbulent mixing.

56 Radioactivity measurements show a typical time variability reaching a maximum concentration  
57 during the night, in conditions of strong stability, and a minimum during the day when the  
58 mixed layer is well developed and vertical dilution occurs. Otherwise, low quasi-constant

59 values are found in advective situations characterized by mixing due to turbulence. From a  
 60 qualitative point of view, the activity of  $^{222}\text{Rn}$ , or its progeny counts, is proportional to the  
 61 stability conditions of the lower troposphere. If advection occurs, the contribution of different  
 62 air masses must be carefully assessed.

63 A better description of mixing and exchange processes in the ABL under different conditions  
 64 can be obtained by estimating the vertical radon profile. The vigour of atmospheric mixing, in  
 65 fact, regulates the vertical radon profile in the ABL (Williams et al. 2011). The structure of the  
 66 lower troposphere can be simplified with a one-dimensional ABL model, composed of a mixing  
 67 layer and a residual layer. While the mixing layer is characterized by vertical profiles controlled  
 68 by meteorological conditions, the residual layer can be described by constant concentrations of  
 69 radon and its progeny with altitude. The transition between the two layers is usually manifest  
 70 by a sharp gradient at the top of the mixing layer (Lopez et al. 1974, Vinod Kumar et al. 1999).  
 71 Therefore, if we assume a homogeneous exchange coefficient in the vertical profile within the  
 72 stable layer, the approximation of the nocturnal stable layer via a box model is supported  
 73 (Guedalia et al. 1980). The top of this box, defined as the “equivalent mixing height” ( $h_e$ ), is a  
 74 semi-quantitative index of the dispersion properties of this layer; low values of  $h_e$  are related to  
 75 low dispersion power and high concentrations of primary pollutants.

76 In Guedalia et al. (1980), the calculation of the top of the box was performed by means of

$$h_e = \frac{\phi \Delta t}{C_{[t]} - C_{[0]}}, \quad (1)$$

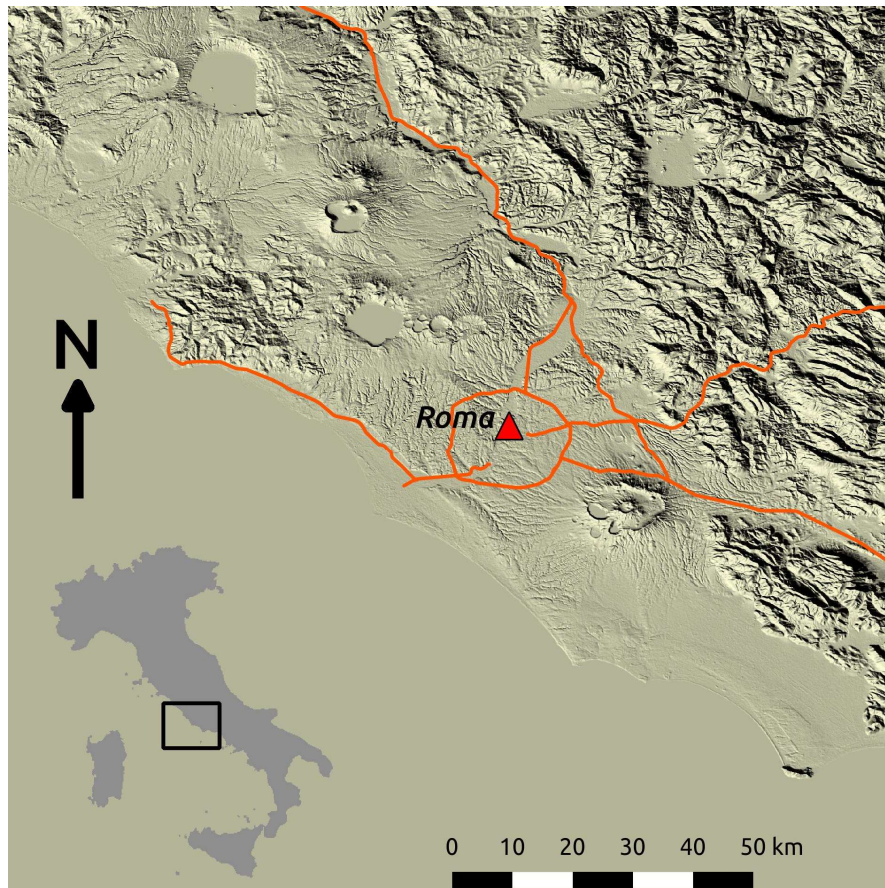
77 where  $\phi$  is the radon flux at the surface,  $\Delta t$  is the time interval from the start of accumulation,  
 78  $C_{[t]}$  is the radon concentration at time  $t$  ( $\text{Bq m}^{-3}$ ) and  $C_{[0]}$  is the radon concentration at the  
 79 beginning of accumulation ( $\text{Bq m}^{-3}$ ). Allegrini et al. (1994) showed that  $h_e$  can be properly  
 80 identified as the height at which a parcel of air emanating from the ground ceases to rise, at least  
 81 in nocturnal situations where advection is negligible. Furthermore, they quantified the layer  
 82 depth over a town by coupling a temperature profile from radiosonde ascents made in the  
 83 suburbs and the near-surface air temperature measured at the radon detection site within the  
 84 town. The high correlation between the estimated urban mixing height ( $h_u$ ) and  $h_e$  supports the  
 85 correct estimation of the urban mixing height by the box model, at least when turbulence in the  
 86 mixing layer is thermally driven. This approach has been tested only under stable and non-  
 87 advective conditions, with no rain, constant relative humidity, constant atmospheric pressure  
 88 and a limited space-time interval. This framework implies, first of all, that radon exhalation

89 from the ground is constant over time and spatially homogeneous. If our measurements are  
90 progeny-based the box model requires also that: i) the fraction of radon daughters attached to  
91 the particulate matter is constant, ii) the equilibrium factor between decay product is constant,  
92 iii) the vertical profile is constant through the mixing layer (in the case of single-height  
93 observations), and iv) the radon concentration is directly proportional to the number of  
94 detected  $\alpha$  or  $\beta$  counts. This last issue represents substantially an instrumental component that  
95 requires a calibration obtained using a reference material or independent techniques.

96 Radon can be measured either as an  $\alpha$  or  $\beta$  particle emitter associated with the decay of its short-  
97 lived progeny. Several instrumental approaches support the determination of the radon  
98 concentration in air. Considering continuous techniques only, Frank et al. (2012) classified the  
99 available devices used in “one-filter” and “two-filters” systems. The first method is based on  
100 measuring the  $\alpha$  or  $\beta$  activity directly on the filter through which air is passed. These systems  
101 can be equipped with “selective” detectors, capable of discriminating different nuclides at  
102 specific energies, or by “gross” detectors, which estimate the total activity emitted by collected  
103 particles. The second group is based on a two-step collection that isolates the progeny associated  
104 directly with  $^{222}\text{Rn}$  present in air. This latter approach represents the reference method in the  
105 framework of the Global Atmospheric Watch Programme of the World Meteorological  
106 Organization. The operational conditions can be perfectly controlled and the  $^{222}\text{Rn}$   
107 measurements in the air are more reliable and sensitive compared to the first group. These  
108 systems are more complex than “one-filter” devices and they require more resources in terms  
109 of maintenance and logistics. The cost-effectiveness of “one-filter” systems is an important  
110 feature that favours the diffusion of this type instead of “research” instruments. These simplified  
111 systems introduce, unfortunately, interference due to the contemporary presence of several  
112 radon daughters, with different half-lives, and to the variability of disequilibrium between those  
113 nuclides and radon in the atmosphere.

114 From this perspective, the model must include a conversion coefficient that can support the  
115 estimation of single-height radon activity based of total  $\beta$  counts, which is affected both by  
116 disequilibrium effects and by instrumental efficiency. The aim of the present study is to  
117 investigate the use of mixing-height modelling by using radon progeny instead of direct  $^{222}\text{Rn}$   
118 measurements. The approach is, firstly, based on a general modification of the box model  
119 considering not only nocturnal stable conditions and a variable radon soil exhalation. Secondly,

120 we present a calibration protocol for use in converting progeny observations into radon air  
121 activity. Finally, we check the validity of our progeny-based model with ground-based  
122 techniques (sodar and lidar).



123  
124 **Fig 1** Location map of the study area (Rome, Italy). Red lines represent the major roadways.  
125

## 126 **2 Methods**

127 The adaptation of a simplified ABL box model to radon-progeny observations required the  
128 development of appropriate equations designed to support the comparison between radon-  
129 progeny derived mixing height and independent estimates obtained by traditional techniques  
130 such as sodar and lidar. Firstly, we present in this section the experimental set-up useful for  
131 validating our model. Then we describe from a general point of view our box model and we  
132 define the adaptation required to the specific site and instrumentation included in this study  
133 case.

134

## 135 2.1 Experimental

136 We describe the experimental set-up at first, because constraints to the modelling are induced  
137 by the study site and by the radon-progeny instrument.

138

### 139 2.1.1 *The study site*

140 The field survey was conducted in the city of Rome, Italy, at the Sapienza University Campus  
141 (latitude 41°54'05"N, longitude 12°30'57"E). The sampling site (Fig. 1) was on the roof of the  
142 Physics Department, approximately 75 m above the sea level, 20 m above the ground. The  
143 rooftop facility features standard meteorological sensors that provide air temperature and  
144 relative humidity. The observing period commenced on 20 June and concluded on 12 July  
145 2011.

146

### 147 2.1.2 *Radon progeny detector*

148 The natural radioactivity was measured using an automatic stability monitor (A PBL Mixing  
149 Monitor, FAI Instruments, Fontenuova, Rome, Italy), comprising a sampler for the collection  
150 of particulate matter on filter membranes and a Geiger-Muller counter for determining the total  
151  $\beta$ -activity of the short-lived radon progeny attached to the particles. The instrument operates on  
152 two filters at the same time: while the sampling phase is acting on one filter for 1 h, the  $\beta$   
153 detection is performed on the other filter. These instrumental features ensure that the short-lived  
154  $\beta$  activity of the particles is continuously determined over an integration time of 1 h and that  
155 the  $\beta$  measurement period is long enough to guarantee highly accurate results. The residual  
156 radioactivity is taken into account using a software procedure. The accuracy of the  
157 determination is improved by the automatic subtraction of the background radiation (Perrino et  
158 al. 2000), while the lower limit of detection of the stability monitor has been estimated at 0.15  
159 Bq m<sup>-3</sup>. This value is affected by the conversion factor ( $c_f$ ) defined in Section 3.2 ( $c_f = 0.77 \text{ s}^{-1}$   
160  $\text{Bq}^{-1}$ ) but reference materials are necessary for a more detailed definition of the lower limit of  
161 detection. The maximum instrumental error at the lowest counting level was about 3%.

162

### 163 2.1.3 *Sodar*

164 The instrument considered in the present study was a fully automated monostatic triaxial  
165 Doppler sodar, which allows a continuous display of the thermal turbulent structure of the

166 atmosphere, the vertical velocity and its standard deviation, and the wind speed and direction.  
167 It features three different 1.5-m diameter antennae/channels: one is oriented vertically, and the  
168 other two are oriented north-south and east-west and are tilted at 20 degrees from the zenith.  
169 The system radiates three short tone bursts, one for each antenna, at the three different  
170 frequencies (1750, 2000, and 2250 Hz) with a temporal resolution of 6 sec. This pattern results  
171 in an operative range of approximately  $10^3$  m, starting from a first useful range gate of  
172 approximately 25 m. The vertical resolution is 7 m for the echoed signal, and the horizontal  
173 resolution is 28 m. An extensive description of the instrument, including the electronic and data  
174 processing system, is given by Mastrantonio et al. (1994) and references therein.

175 Because the emitted acoustic waves are scattered by small-scale temperature fluctuations, i.e.,  
176 the thermal turbulence, the mixing height can be estimated from sodar measurements using  
177 objective or subjective methods applied to the digitized range-corrected vertical profiles of  
178 signal intensity (range corrected signal). In the present study, a very reliable technique  
179 originally proposed by Beyrich (1993) and Beyrich and Weill (1993) has been used. Under  
180 convective conditions, the mixing height is defined as the height of an elevated secondary  
181 maximum that corresponds with the zone of strong turbulence at the capping inversion. Under  
182 stable conditions, the mixing height is determined from the minimum of the first derivative or  
183 from the maximum curvature of the range corrected signal, depending on the stage of the ABL  
184 evolution and on the shape of the range-corrected signal profile (Beyrich 1997, Casasanta et al.  
185 2014).

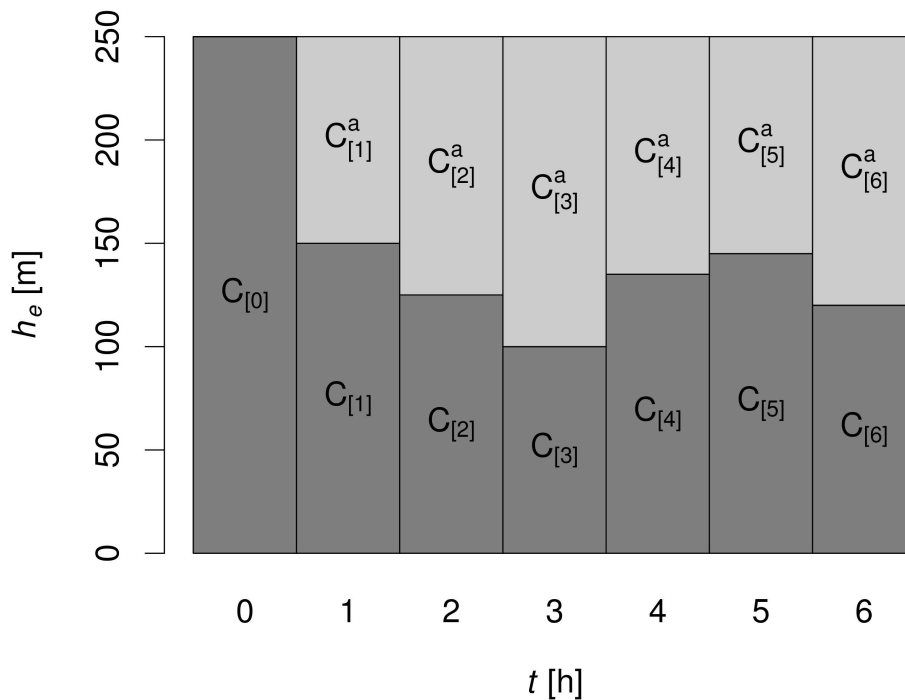
186

#### 187 *2.1.4 Lidar*

188 The lidar instrument deployed for this experiment was a custom-made, fully automated  
189 monostatic elastic backscatter lidar device, specially designed to observe the atmospheric  
190 aerosol vertical profile in the ABL through the entire troposphere. The radiation source is a  
191 Handy HYL 102 (Quanta System S.p.A.) Q-switched Nd:YAG laser with a second harmonic at  
192 532 nm and a repetition rate of 20 Hz. The backscattered radiation is collected by a 100-mm  
193 Cassegrain telescope and by a 50-mm large field-of-view refractor telescope to observe the  
194 strong echo from the lowest atmosphere. In both the collectors, narrow-band interference filters  
195 are used to filter the collimated signals. This feature reduces the sky light, making it possible to  
196 obtain measurements in full daylight. The incoming radiation is detected by photomultipliers.  
197 The signals from both telescopes are matched in the overlapping altitude ranges to produce a

198 continuous profile between approximately 100 m and 10 km, with a vertical resolution of 7.5  
 199 m. The instrument can also measure the linear depolarization ratio, but because such  
 200 measurements have not been used in this work, the relevant data will not be described here. The  
 201 acquisition system has been set to perform an integration of the backscattered signals over 15  
 202 s, corresponding to 300 laser shots, but all of the following analyses were performed on profiles  
 203 averaged over 5 min.

204 The custom-made software controls the system handling, the quality assurance, and the time  
 205 scheduling. For the whole measurement campaign, the lidar was programmed to perform  
 206 measurements for 5 min before and after every hour, thereby creating two vertical profiles  
 207 around each hour. The hourly mixing height was then retrieved by applying the well-known  
 208 wavelet covariance transform method to these two profiles (Cohn and Angevine 2000, Davis et  
 209 al. 2000, Brooks 2003, Pal et al. 2010) and taking the average value.



210

211 **Fig 2** Temporal evolution of the nocturnal stable layer (dark grey box). The light grey area above represents the  
 212 residual layer.

## 213 2.2 Mixing-height modelling

214 The evolution of the preferred box model should clearly include the introduction of the radon  
 215 decay contribution and the description of a multi-layer structure. The contribution of the



216 residual layer was introduced by Sesana et al (2003) and Pasini (2009), who approached the  
 217 problem from a theoretical point of view. Griffith et al. (2013) defined a formulation of the so-  
 218 called “dilution” term based on the formation of the residual layer every time the mixing height  
 219 is lowered. Pasini et al. (2014) proposed a different approach based on a permanent residual  
 220 layer that develops after the first compression of the day. This difference is coupled to the  
 221 discrimination between compression an expansion based on the activity derivative instead of  
 222 the height differential. We believe that these differences and the soil flux modelling are the most  
 223 important innovation introduced herein. We preferred this approach because the removal of the  
 224 residual layer is not a high-frequency process, so that our hypothesis consists in considering the  
 225 residual term persisting over at least one night.

226 In Fig. 2, we define compression conditions when the stable layer depth decreases ( $i = 1, 2, 3,$   
 227 6) and expansion situations when  $h_e$  increases ( $i = 4, 5$ ). In the following discussion,  $\lambda$  is the  
 228  $^{222}\text{Rn}$  decay constant ( $\text{s}^{-1}$ ),  $t$  is our sampling interval (s), and  $C^a$  is the calculated concentration  
 229 in the residual layer ( $\text{Bq m}^{-3}$ ). Additionally, we adopt the symbolic form  $h_{e[n,m]} = h_{e[n]} - h_{e[m]}$  for  
 230 the difference between equivalent mixing heights at times  $n$  and  $m$ , respectively.

231 In compression cases, the generalization of Eq. 1 reads:

$$h_{e[i]} = \frac{\phi}{\lambda} \frac{1 - e^{-\lambda\Delta t}}{C_{[i]} - C_{[i-1]}e^{-\lambda\Delta t}}, \quad (2)$$

232 and the concentration in the residual layer after the  $i^{\text{th}}$  compression is,

$$C_{[i]}^a = \frac{C_{[i-1]}^a e^{-\lambda\Delta t} \Delta h_{e[0,i-1]} + C_{[i-1]} e^{-\lambda\Delta t} \Delta h_{e[i-1,i]}}{\Delta h_{e[0,i]}}. \quad (3)$$

233 If the stable layer depth increases and the overlying air is included in the box, i.e., in cases of  
 234 expansion, the equivalent mixing height can be calculated as,

$$h_{e[i]} = \frac{\frac{\phi}{\lambda} (1 - e^{-\lambda\Delta t}) + h_{e[i-1]} (C_{[i-1]} - C_{[i-1]}^a) e^{-\lambda\Delta t}}{C_{[i]} - C_{[i-1]}^a e^{-\lambda\Delta t}}, \quad (4)$$

235 and the concentration above the top of the box (in the residual layer) is

$$C_{[i]}^a = C_{[i-1]}^a e^{-\lambda\Delta t}. \quad (5)$$

236

### 237 2.2.1 Soil radon-flux submodel

238 A second main aim of the present study is to test the introduction of a variable emanation rate  
 239 instead of a constant radon flux. The Rn source term can be modelled or derived by inter-

240 comparison with other techniques (Griffith et al. 2013). However, the radon flux originating  
 241 from the surface is a complex process influenced by many factors (Sun et al. 2004, Voltaggio  
 242 et al. 2006, Zhuo et al. 2008), and although pedology and geology are disciplines that are not  
 243 commonly involved in atmospheric modelling, the support provided by a multidisciplinary  
 244 approach focused on radon emanations from the soil is important. The definition of the radon-  
 245 emitting source can, in fact, improve atmospheric models (Szegvary et al. 2007).  
 246 The simplest way to predict the exhalation rate is the application of idealized models based on  
 247 the porous media transport theory. This sub-model follows the direction of Zhuo et al. (2008),  
 248 who proposed a combined model in which the soil radon emanation power and the soil water  
 249 saturation are the main parameters that control the radon flux, viz.

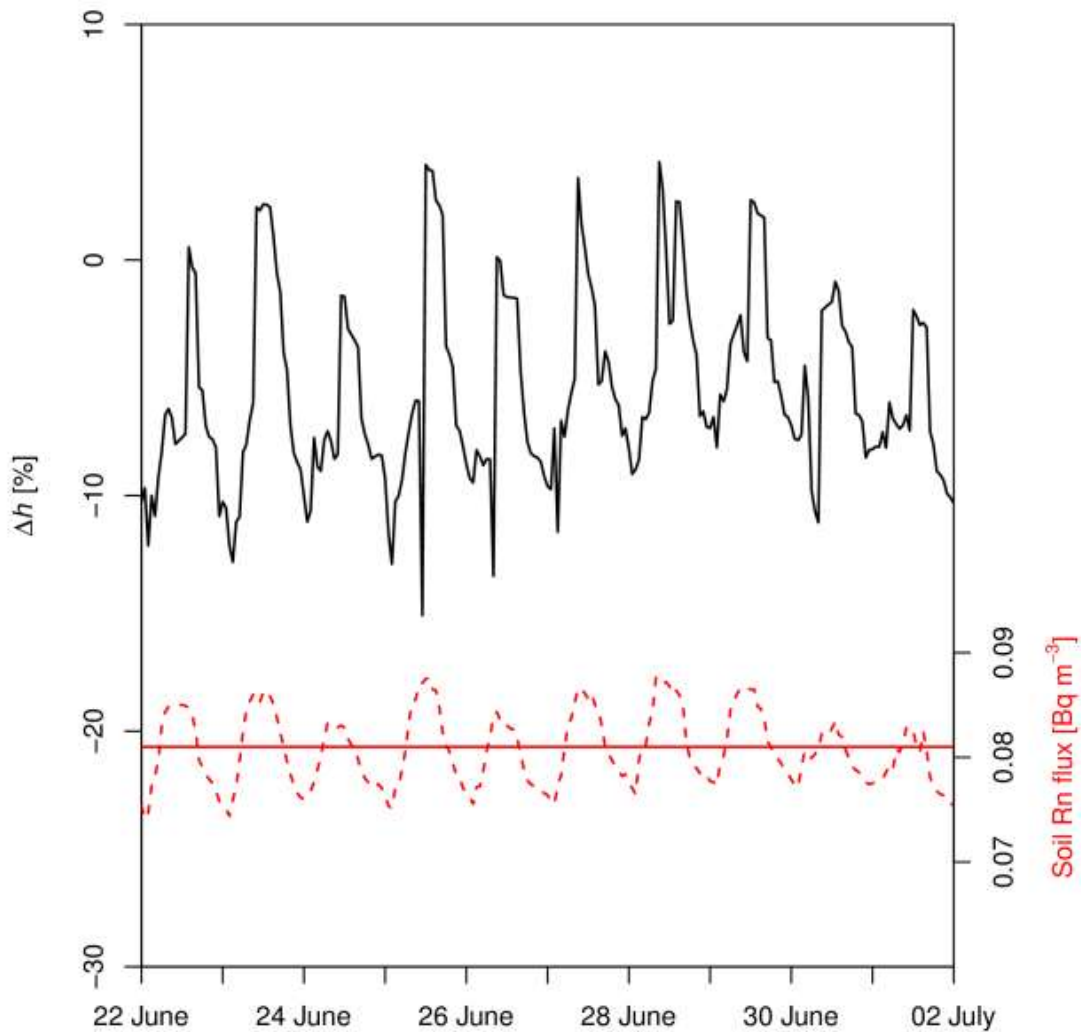
$$\phi = R\rho_b\varepsilon\left(\frac{T_s}{273}\right)^{0.75}\sqrt{\lambda D_0 p \exp(-6Sp - 6S^{14}p)}. \quad (6)$$

250 Here  $\phi$  is the radon flux ( $\text{Bq m}^{-2}$ ),  $R$  is the  $^{226}\text{Ra}$  soil content ( $\text{Bq kg}^{-1}$ ),  $\rho_b$  is the soil bulk density  
 251 ( $\text{kg m}^{-3}$ ),  $T_s$  is the soil temperature (K),  $D_0$  is the  $^{222}\text{Rn}$  diffusion coefficient in air,  $S$  is the soil  
 252 water saturation,  $p$  is the soil total porosity and  $\varepsilon$  is the radon emanation power. This kind of  
 253 model is based on a steady-state condition that considers the dominant contribution of diffusion  
 254 and neglects the forced flow due to horizontal atmospheric pressure gradients. The altitude  
 255 above the ground of the sampling site supported the assumption of steady-state soil exhalation.  
 256 This approach can produce, of course, an underprediction of the soil exhalation rate especially  
 257 in terms of high frequency variations and we focus our attention on this issue in later studies.  
 258 Once the soil flux is parametrized by a sub-model based on Eq. 6 (the details of which are  
 259 presented in Appendix 1), we are able to compare the results from standard and improved  
 260 models (Fig. 3). The difference between the two models is defined as

$$\Delta h = \frac{(h_e^{\text{variable}} - h_e^{\text{constant}})}{h_e^{\text{constant}}}. \quad (7)$$

261 Although small discrepancies (less than 5%) were frequently observed in correspondence with  
 262 expansion conditions, slight but significant differences (approximately -10%) were detected  
 263 during the nighttime.

264



265

266 **Fig 3** Percent difference in mixing height (black line) between equivalent mixing heights estimated using  
 267 constant (continuous red line) and variable (dotted red line) soil radon fluxes.

268

269 The latter deviation is negligible if we consider the absolute amount of discrepancy (only 5-10  
 270 m, with a mixing height of approximately 50-100 m during the nighttime). These negative  
 271 anomalies are often associated with major variations in soil humidity, and they may be  
 272 consistent when strong advection occurs. Further validations are required to confirm the  
 273 performance of the improved model. The accordance between the two considered models could  
 274 be, in fact, influenced by the under-prediction in unsteady conditions and by the absence of  
 275 sharp variations in terms of meteorological conditions (precipitation, humidity, etc.) during the

276 survey.

277

### 278 2.2.3 Gross $\beta$ counts versus air radon activity -- Theoretical solution

279 The critical step in using radon progeny as a tool for modelling the mixing height of the  
 280 boundary layer is the conversion between gross  $\beta$  counts and air radon activity. This issue can  
 281 be approached in two complementary ways. The first one is based on finding a theoretical  
 282 solution considering the gross  $\beta$  counts emitted by filters where radon and thoron progeny are  
 283 collected. Essentially, four nuclides contribute to the total  $\beta$  emissions because the  $\beta$  branching  
 284 rate of  $^{218}\text{Po}$  is negligible,

$$\beta_{tot} = \epsilon_s \{ \epsilon_{1024} [^{214}\text{Pb}]_{\beta} + \epsilon_{3272} [^{214}\text{Bi}]_{\beta} + \epsilon_{2252} [^{212}\text{Pb}]_{\beta} + \epsilon_{4999} [^{212}\text{Bi}]_{\beta} \}, \quad (8)$$

285 where  $\beta_{tot}$  is the total counts,  $\epsilon_s$  is the sampling efficiency,  $\epsilon_{keV}$  is the detector efficiency at a  
 286 specific energy,  $[^{xxx}\text{C}]_{\beta}$  is the  $\beta$  activity of a specific nuclide. While the sampling efficiency,  
 287 generally considered to be approximately 100% (Islam and Haque 1994) and homogeneous for  
 288 all the considered nuclides, is a negligible term of the Eq. 8 ( $\epsilon_s = 1$ ), the detector efficiency is  
 289 a key parameter that rules the total  $\beta$  counts. The first two members of Eq. 8 are related to the  
 290  $^{222}\text{Rn}$  decay series having half-lives of 26.8 and 19.9 min respectively. The remaining decay  
 291 component are associated with the  $^{220}\text{Rn}$  decay series with 10.2 and 1.0 h half-lives. The  
 292 instrument presented in section 2.1.2 discriminates between the contributions of short-lived  
 293 products ( $^{222}\text{Rn}$  progeny) and long-lived products ( $^{220}\text{Rn}$  progeny). Considering also the lower  
 294 presence of thoron associated with the altitude of the sampling point, Eq. 8 can be limited to the  
 295 first two members.

296 The decay of those isotopes during the sampling and the counting phases regulate the final  
 297 measurement. The first phase can be described by the following,

$$\frac{d[^{218}\text{Po}]_{filt}}{dt_s} = \nu[^{218}\text{Po}]_{air} - \lambda_{218\text{Po}}[^{218}\text{Po}]_{filt}, \quad (9a)$$

$$\begin{aligned} \frac{d[^{214}\text{Pb}]_{filt}}{dt_s} &= \nu[^{214}\text{Pb}]_{air} + \lambda_{218\text{Po}}[^{218}\text{Po}]_{filt} \\ &- \lambda_{214\text{Pb}}[^{214}\text{Pb}]_{filt}, \end{aligned} \quad (9b)$$

$$\frac{d[^{214}\text{Bi}]_{filt}}{dt_s} = \nu[^{214}\text{Bi}]_{air} + \lambda_{214\text{Pb}}[^{214}\text{Pb}]_{filt} - \lambda_{214\text{Bi}}[^{214}\text{Bi}]_{filt}, \quad (9c)$$

298 where the solutions estimate the  $\beta$  pre-counting activity of each nuclide, whereas other

299 equations regulate the following counting phase,

$$\frac{d[{}^{218}\text{Po}]_{\beta}}{dt} = -\lambda_{218\text{Po}}[{}^{218}\text{Po}]_{\text{filt}}, \quad (10a)$$

$$\frac{d[{}^{214}\text{Pb}]_{\beta}}{dt} = \lambda_{218\text{Po}}[{}^{218}\text{Po}]_{\text{filt}} - \lambda_{214\text{Pb}}[{}^{214}\text{Pb}]_{\text{filt}}, \quad (10b)$$

$$\frac{d[{}^{214}\text{Bi}]_{\beta}}{dt} = \lambda_{214\text{Pb}}[{}^{214}\text{Pb}]_{\text{filt}} - \lambda_{214\text{Bi}}[{}^{214}\text{Bi}]_{\text{filt}}. \quad (10c)$$

300 Considering the low  $\beta$  decay branch ratio of  ${}^{218}\text{Po}$ , the solution can be simplified as  
301 demonstrated by Islam and Haque (1994),

$$\begin{aligned} \beta_{\text{tot}} = & \epsilon_{1024}\nu f_{214\text{Pb}}[{}^{222}\text{Rn}]_{\text{air}}[4.28 \times 10^5(1 - e^{-\lambda_{214\text{Pb}}t_s})(e^{-\lambda_{214\text{Pb}}t_c^i} - e^{-\lambda_{214\text{Pb}}t_c^f})] + \\ & -\epsilon_{1024}\nu f_{214\text{Pb}}[{}^{222}\text{Rn}]_{\text{air}}[1.81 \times 10^5(1 - e^{-\lambda_{214\text{Bi}}t_s})(e^{-\lambda_{214\text{Bi}}t_c^i} - e^{-\lambda_{214\text{Bi}}t_c^f})] + \\ & +\epsilon_{3272}\nu f_{214\text{Bi}}[{}^{222}\text{Rn}]_{\text{air}}\left[4.84 \times 10^4(1 - e^{-\lambda_{214\text{Bi}}t_s})\left(e^{-\lambda_{214\text{Bi}}t_c^i} - e^{-\lambda_{214\text{Bi}}t_c^f}\right)\right], \quad (11) \end{aligned}$$

302 where  $\nu$  is the sampling rate ( $\text{m}^3 \text{min}^{-1}$ ),  $t_s$  is the filter sampling time (min),  $t_c$  is the initial (i)  
303 and the final (f) counting time elapsed after the sampling period,  $[{}^{222}\text{Rn}]_{\text{air}}$  is the radon air  
304 activity,  $\epsilon_{(\text{keV})}$  is the detector efficiency at a specific energy,  $f_{214\text{Pb}}$  is the equilibrium factor  
305 between  ${}^{214}\text{Pb}$  and radon in the atmosphere,  $f_{214\text{Bi}}$  is the equilibrium factor between  ${}^{214}\text{Bi}$  and  
306 radon in the atmosphere. The counting strategy of an instrument regulates the relationship  
307 between the conversion factor ( $c_f$ ) and the remaining input variables,

$$\begin{aligned} \beta_{\text{cpm}} = & \frac{\epsilon_{1024}\nu f_{214\text{Pb}}[{}^{222}\text{Rn}]_{\text{air}}[F_1 - F_2] + \epsilon_{3272}\nu f_{214\text{Bi}}[{}^{222}\text{Rn}]_{\text{air}}F_3}{t_c^f - t_c^i} = \\ & = 60c_f\nu t_s[{}^{222}\text{Rn}]_{\text{air}}, \quad (12) \end{aligned}$$

308 where  $F_1 = 4.28 \times 10^5(1 - e^{-\lambda_{214\text{Pb}}t_s})(e^{-\lambda_{214\text{Pb}}t_c^i} - e^{-\lambda_{214\text{Pb}}t_c^f})$ ,  $F_2 = 1.81 \times 10^5(1 -$   
309  $e^{-\lambda_{214\text{Bi}}t_s})(e^{-\lambda_{214\text{Bi}}t_c^i} - e^{-\lambda_{214\text{Bi}}t_c^f})$ ,  $F_3 = 4.84 \times 10^4(1 - e^{-\lambda_{214\text{Bi}}t_s})(e^{-\lambda_{214\text{Bi}}t_c^i} -$   
310  $e^{-\lambda_{214\text{Bi}}t_c^f})$ , which implies

$$c_f = \frac{\epsilon_{1024}\nu f_{214\text{Pb}}[F_1 - F_2] + \epsilon_{3272}\nu f_{214\text{Bi}}F_3}{60(t_c^f - t_c^i)t_s}. \quad (13)$$

311 This mathematical treatment allows us to reduce the number of input variables in the model to  
312 just the detector efficiency and the radon-progeny equilibrium factor, though these two  
313 variables cannot be easily estimated with a detector that measures the gross  $\beta$  activity. For this

314 reason, Eq. 13 cannot be solved analytically. Nevertheless, this theoretical treatment represents  
 315 a starting point for understanding the relationship between gross  $\beta$  counts and air radon activity.  
 316 The definition of the equilibrium factor between radon and its progeny also cannot be  
 317 determined with our instrumentation; therefore we assume here that the equilibrium is complete  
 318 between radon progeny ( $f_{214Pb} \approx f_{214Bi} \approx f$ ). This assumption is consistent with Jacobi and  
 319 Andre (1963), who found, at 20 m above the ground, a negligible disequilibrium between the  
 320 two considered nuclides under different mixing conditions. The conversion equation (Eq. 13)  
 321 can be consequently simplified as

$$c_f \approx \bar{\epsilon} f \nabla, \quad (14)$$

322 where  $\bar{\epsilon}$  is the overall detector efficiency term,  $\nabla$  is a term depending on the counting scheme  
 323 of the instrument and  $f$  is the degree of disequilibrium between radon and its progeny. While  
 324 the first two terms are specific to the used instrument, the latter can vary between 0.8 and 0.95  
 325 during the day depending on the mixing state of the atmosphere and the altitude (Vinod Kumar  
 326 et al. 1999). We were not able to exactly determine these parameters during the survey, so we  
 327 preferred to use experimental sodar observations and fix a constant conversion factor.

328

### 329 **3 Results**

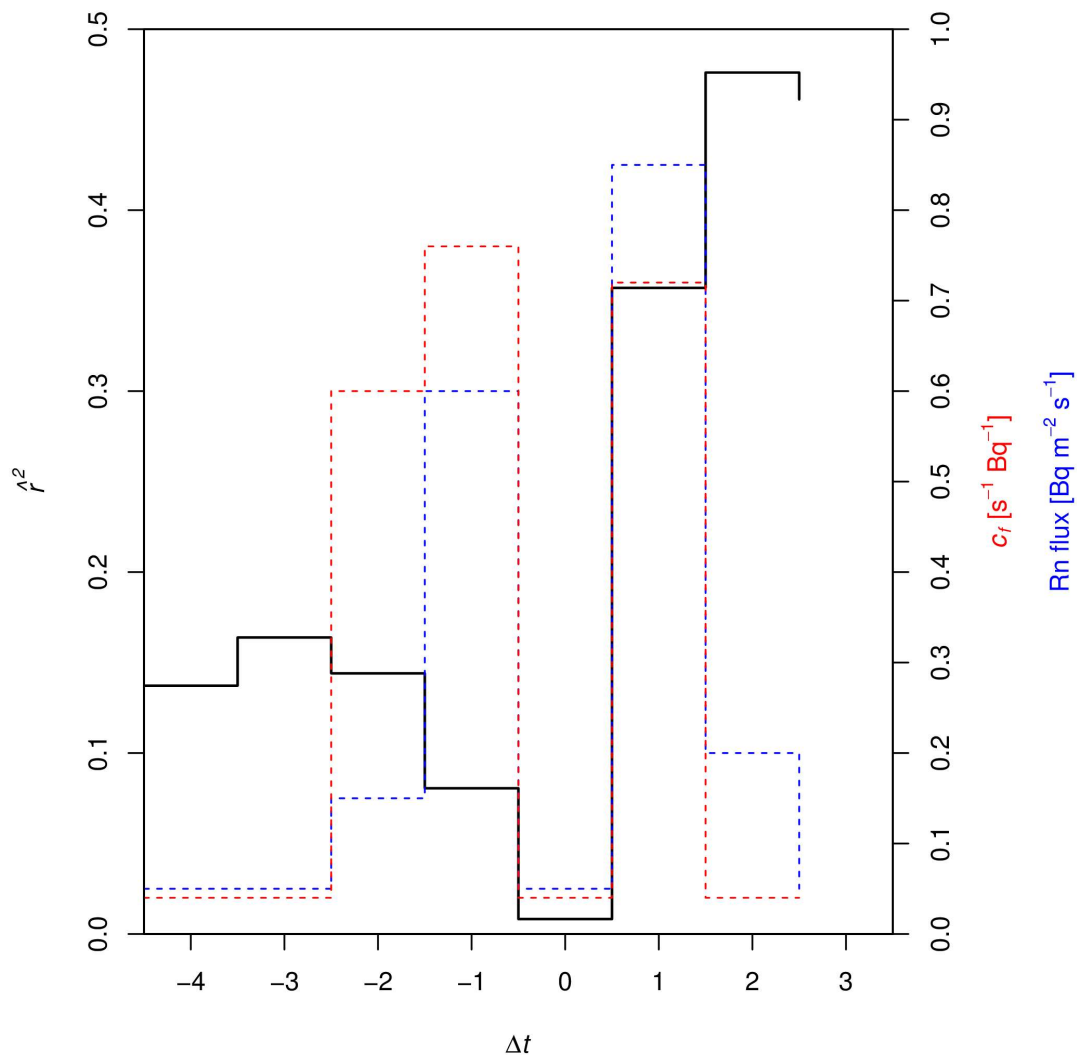
330 The comparison between the selected techniques provided the opportunity to optimize the  
 331 model in terms of input variables and time synchronization. The optimization was first  
 332 conducted considering only the sodar observations during the night when the near-stable  
 333 conditions are predominant. The lidar observations were later used to validate the model output  
 334 and to estimate the model performance under turbulent conditions.

335

#### 336 **3.1 Time synchronization**

337 The first issue addressed was the time synchronization between radon progeny dynamics and  
 338 the sodar estimates of the mixing height. Assuming that the sodar observations are based on the  
 339 turbulent thermal structure of the lower troposphere, the diffusion of radon progeny through the  
 340 mixing layer is assumed to produce a delay.

341 Considering the definition of the mixing height offered by Seibert et al. (2000), “The mixing  
 342 height is the height of the layer adjacent to the ground over which pollutants or any constituents  
 343 emitted within this layer or entrained into it become vertically dispersed by convection or  
 344 mechanical turbulence within a time scale of about an hour”, we first checked whether our  
 345 observations were consistent with this constraint.



346  
 347 **Fig 4** Statistical estimation of radon diffusion delay. The continuous black line shows the maximum calculated  
 348 linear regression coefficient ( $\hat{r}^2$ ) obtained for sodar observations and radon-derived estimates. The dotted lines  
 349 show the input values required to obtain the best correlation. The variables are the radon exhalation rate (blue) and  
 350 the conversion factor  $c_f$  from gross  $\beta$  counts to air radon content (red). The number of hours between radon  
 351 observations and sodar measurements is described by  $\Delta t$ .  
 352

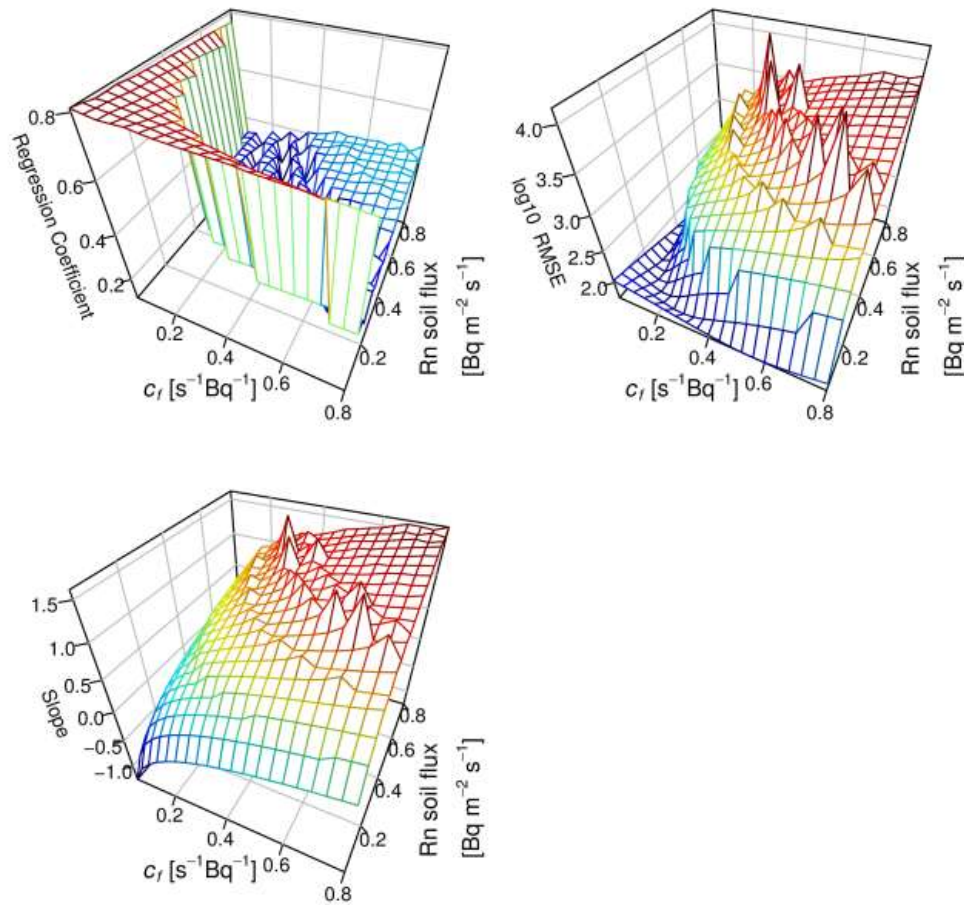
353 The analysis was performed by looking for the best fit in both types of estimates under nocturnal  
354 conditions (Fig. 4), while the statistical relationship was calculated considering a narrow time  
355 window from 2300 to 0400 UTC. The model simulations used 400 combinations of values for  
356 the soil radon flux and conversion factor, with a selected dataset of 66 observations. The former  
357 parameter varied between 0.01 and 0.8 Bq m<sup>-2</sup> s<sup>-1</sup>, and the latter ranged between 0.01 and 0.8.  
358 The radon flux interval was selected based on values available in the literature (Tuccimei and  
359 Soligo 2008), and the conversion factor maximum was determined based on an average detector  
360 efficiency of 50-60%. The best fit was obtained using a 2-h shift between radon-derived mixing  
361 height and sodar estimates. One hour can be ascribed to the start/finish of the sampling phase  
362 (1-h long), and the residual one hour suggests that the diffusion of radon in the atmospheric  
363 layer under nocturnal weak stable conditions is consistent with the mixing-height definition.

364

### 365 3.2 Gross $\beta$ counts versus air radon activity -- Experimental solution

366 Model runs with different input variables defined the conversion factor between gross  $\beta$  counts  
367 and Rn air activity (required by the model). The conversion factor is controlled by an  
368 instrumental component, which is dependent on the detector efficiency, and by the soil  
369 exhalation rate of radon. The best combination of both variables was selected to achieve the  
370 best linear regression coefficient ( $r^2$ ), the lowest root-mean-square error (*RMSE*) and the closest  
371 regression coefficient to 1. In this case, the statistical relationships was calculated considering  
372 the whole sodar dataset between 15 and 8. We selected 126 observations, and the model ran  
373 using 400 combinations of values for soil radon flux and conversion factor. In this phase, the  
374 soil radon flux also varied between 0.01 and 0.8 Bq m<sup>-2</sup> s<sup>-1</sup>, and the conversion factor ranged  
375 between 0.01 and 0.8. An inverse relationship between the two variables was observed (Fig. 5).  
376 A sharp decrease in the regression coefficient occurred along a hyperbolic-shaped limit that  
377 corresponded to an increase in the *RMSE*. The combinations of input variables that produced a  
378 slope between the sodar observations and the radon-derived estimates closer to 1 corresponded  
379 to the lowest *RMSE* values. Furthermore, the optimal combination of the parameters was found  
380 when the mean soil radon flux is 0.08 Bq m<sup>-2</sup> s<sup>-1</sup>) and the conversion factor ( $c_f$ ) is 0.77.





381

382 **Fig 5** Statistical output of the performed model runs with different combinations of input parameters. Estimation  
 383 of the linear regression coefficient ( $r^2$ ) (a), of the root-mean-square error ( $RMSE$ ) (b), and of the slope (c), between  
 384 the sodar observations and the radon-derived mixing height.

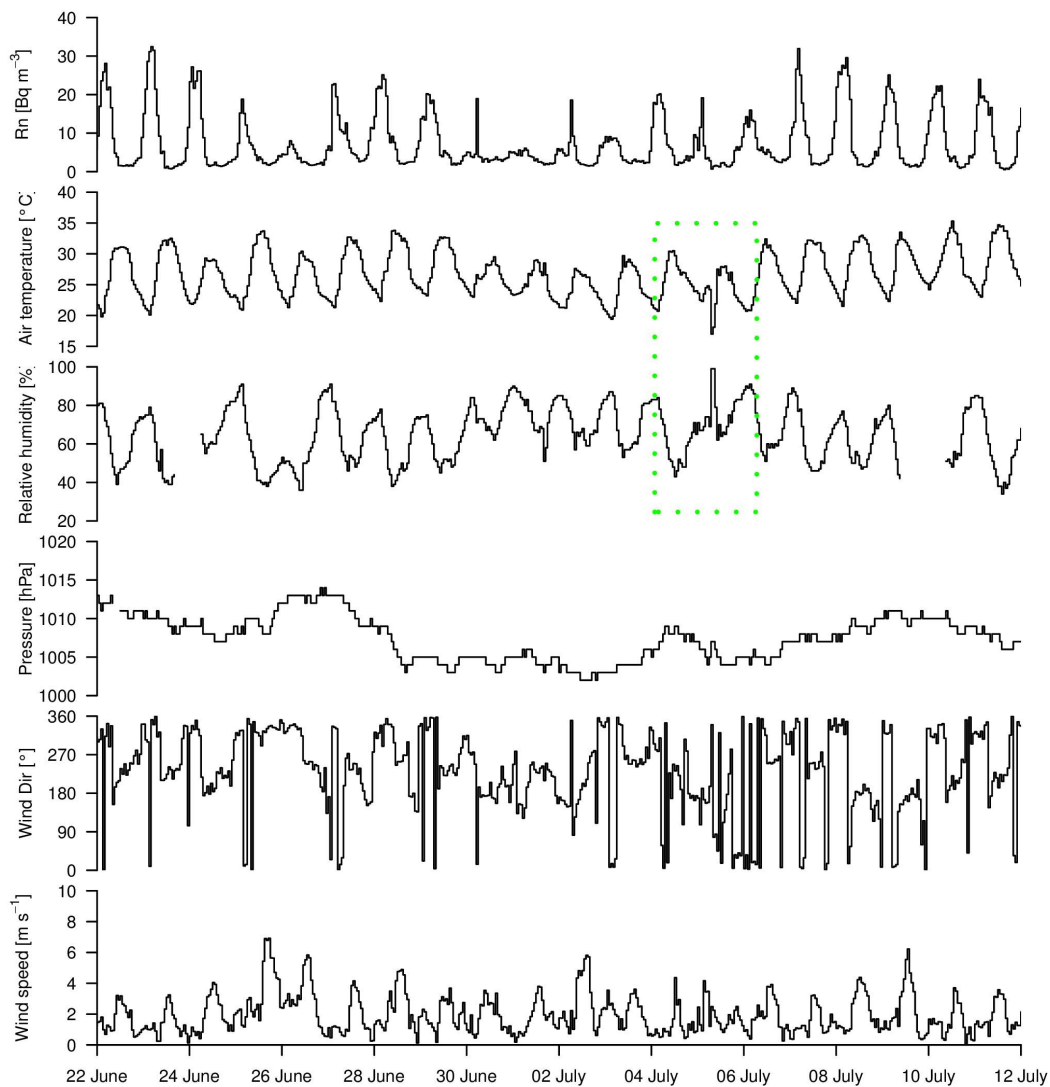
385

### 386 3.3 Model validation

387 The performance of our model was validated using different independent techniques, such as  
 388 sodar and lidar. While sodar estimates of mixing height are more reliable under nocturnal  
 389 conditions, lidar observations are more consistent during the day. We defined a “stability limit”  
 390 in terms of equivalent radon activity where the agreement between sodar and radon-derived  
 391 estimates is consistent. On the other hand, we fixed a “turbulent limit” where advection is a  
 392 major component and box-modelling assumptions are not respected. Finally, we discriminated  
 393 different transition phases considering the time gradient of equivalent radon activity.

#### 394 3.3.1 Meteorological framework

395 The meteorological conditions during the survey can be summarized in Fig. 6. The major  
 396 meteorological parameters were reported in combination to the equivalent radon activity. We  
 397 observed only one precipitation event during July 5 with about 1 mm of rain in the early  
 398 morning.

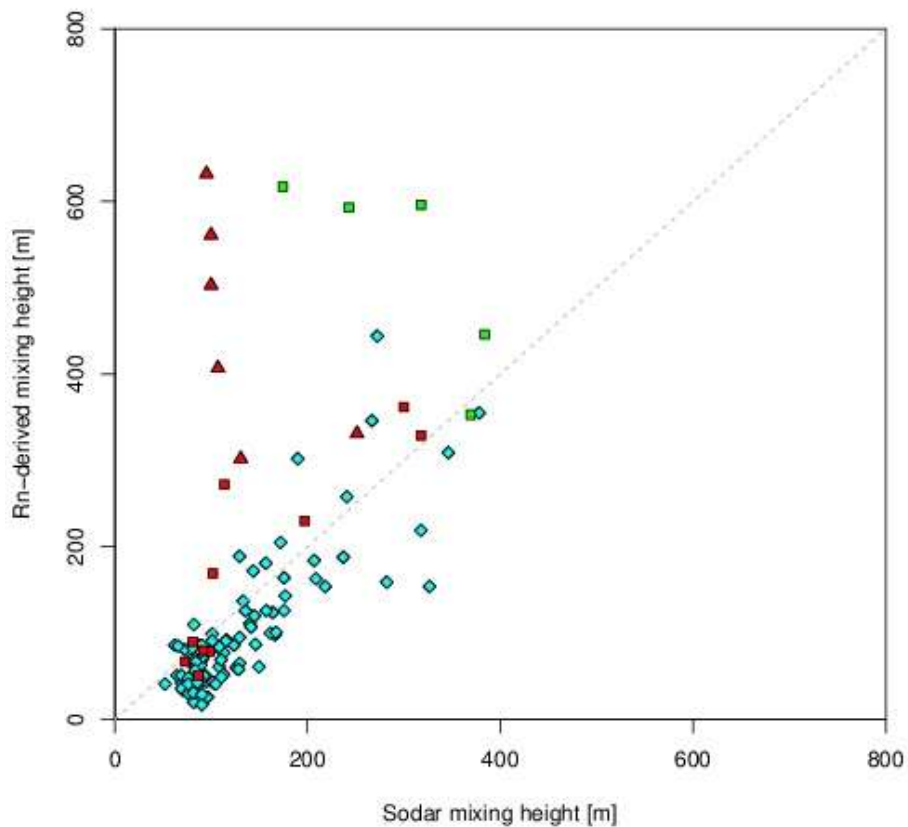


399

400 **Fig 6** Summary plot of the major meteorological parameters. The dotted green rectangle highlighted the  
 401 precipitation event occurred during the survey. Wind parameters are derived from sodar observations at an altitude  
 402 of 48 m above the ground.

403

404 The most important feature was the airflow that showed the typical sea-breeze pattern for this  
 405 area (Caballero and Lavagnini 2002). The flow was dominated by the sea breeze (from the  
 406 south-western sector) from 1000 to 2100 UTC and was influenced mainly by topography  
 407 (mainly from the northern sector) during the night and early morning. The sea-breeze  
 408 component was active during the day and the front cross the city on late afternoon. It is possible  
 409 that during the evening (after 1700 UTC) the sea breeze was coupled to the up-slope flows  
 410 directed to the geomorphological elements (the Sabina mountains and Tiber valley) located  
 411 north eastern respect to the investigated site. While the switch from land to sea winds occurred  
 412 at 0900-1000 UTC, the transition sea-to-land occurred at 1700-2100 UTC.



413

414 **Fig 7** Comparison between the sodar observations and the radon-derived mixing heights. Observations are  
 415 classified as near-stable (cyan diamond), heavy expansion (green square), soft compression (red triangle) and  
 416 heavy compression (red square).

417

418

419 3.3.2 Near-stable conditions (radon vs. sodar)

420 The estimated values defined in Section 3.2 were used for the entire investigated period as input  
 421 parameters, and the comparison between all of the available observations yielded a good  
 422 agreement (Fig. 7). Few measurements were outliers, and the outliers that did exist were related  
 423 to situations out of the near-stable conditions required by our model. Based on a 21-day survey,  
 424 sodar provided 125 1h-averaged observations over 504 h that can be considered optimal for  
 425 mixing height estimation, and only 17 observations ( $\approx 14\%$ ) appeared to be outliers (Table 1).  
 426 In detail, 92 observations ( $\approx 73\%$  of the total) were characterized by radon values greater than  
 427  $9 \text{ Bq m}^{-3}$  in the air, and no outliers were detected. The slope between the two independent  
 428 estimates is 0.82, and the linear regression coefficient was approximately 0.88. Consequently,  
 429 above this limit (now defined as “stability limit”), we had stability conditions or transition  
 430 phases (compressions or expansions) consistent with the box model definition. The radon-  
 431 derived mixing heights under the stability limit were all below 400 m, with an average deviation  
 432 from sodar estimates of approximately  $-28 \text{ m}$ . The negative deviation of radon-derived mixing  
 433 heights with respect to the sodar values indicated an underestimation that could reflect the  
 434 different nature of the mixing height associated with the considered techniques or some  
 435 limitations to our model associated to disequilibrium variations (not considered by the average  
 436 estimation of the conversion factor).

Condition	Radon		Sodar		Lidar	
	Activity	Gradient	Obs.	$\Delta$	Obs.	$\Delta$
	$\text{Bq m}^{-3}$	$\text{Bq m}^{-3}$		m		m
Near-stable	$\geq 9$		92	-28	123	-120
Soft compression	3 - 9	$\leq +0.7$	6	+310	24	+110
Heavy compression		$> +0.7$	10	+26	35	-250
Soft expansion		$\geq -0.8$	0		18	+300
Heavy expansion		$< -0.8$	6	+400	47	+610
Turbulent	$\leq 3$		11	+1400	155	+2000

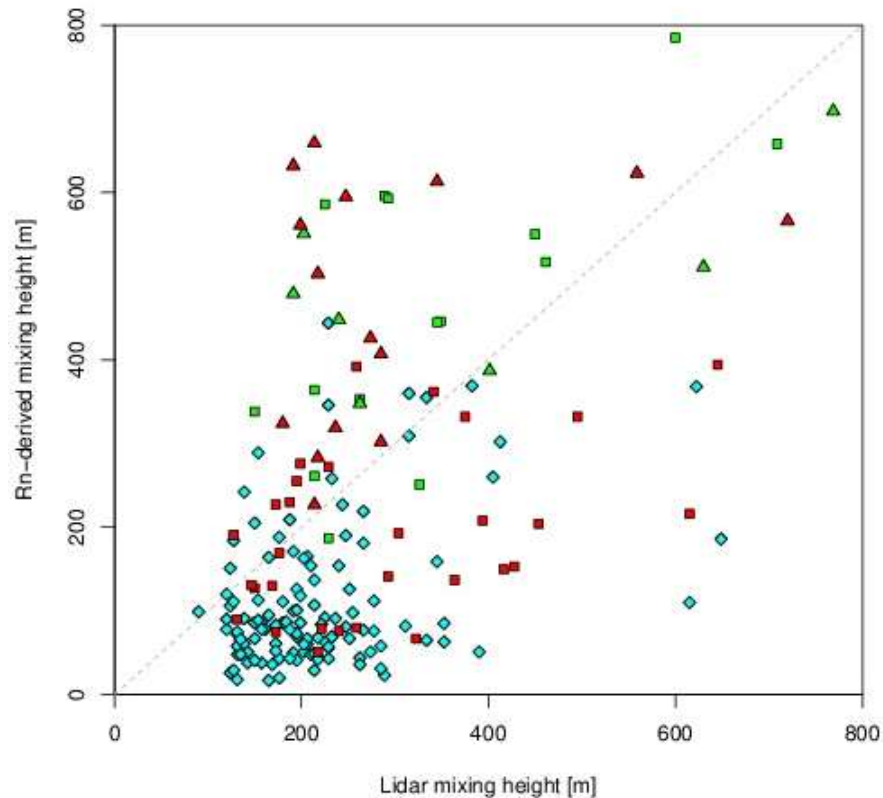
437 **Table 1** Summary of the observed conditions with the different techniques.  $\Delta$  represents the deviation of radon-  
 438 derived mixing height from sodar or lidar estimates.

439

440 In opposition to this accordance between the two approaches, we observed a net deviation in  
441 case of air radon activities below  $3 \text{ Bq m}^{-3}$ . Below this limit, which we define as the “turbulent  
442 limit”, we observed 11 events ( $\approx 10\%$  of the total) with an important overestimation of  
443 approximately 1,400 m above sodar mixing-height values ( $\approx 300 \text{ m}$ ). Furthermore, intermediate  
444 situations can be defined if we consider the air radon activities included between the above-  
445 mentioned limits. If positive increases in air radon activity ( $> 0.7 \text{ Bq m}^{-3}$ ) occur, we can identify  
446 “heavy compression” situations suitable for box modelling. We defined 10 events of this type,  
447 with an average deviation from sodar estimates of approximately +26 m and a linear regression  
448 coefficient of 0.65. These events occurred generally on evening, just before the near-stable  
449 situations, indicating the switch between sea to land breeze. We can infer a weak  
450 underestimation of the box model associated with sharp variations in terms of radon activities  
451 and disequilibrium between radon and its progeny. The other intermediate conditions (“soft  
452 compression”, “soft expansion” and “heavy expansion”) were characterized by sodar mixing-  
453 height values of approximately 130 m, 370 m and 290 m, respectively, with an average  
454 deviation of +400 m from the sodar estimates. Situations of heavy expansion (with air radon  
455 activity between 3 and  $9 \text{ Bq m}^{-3}$  and decreases greater than  $-0.8 \text{ Bq m}^{-3}$ ) occurred six times  
456 (five days) and they occurred generally in the morning (0700-1000 UTC). These events were  
457 detected at the transition between the near-stable and the turbulent situations, indicating the  
458 land-to-sea air masses switch. The box model overestimation can be related to significant  
459 variations in terms of sources. The radon flux under this situations is strongly overestimated  
460 because sea water has a lower radon exhalation rate compared to rocks. The number of soft-  
461 compression situations (with air radon activity between 3 and  $9 \text{ Bq m}^{-3}$  and increases of less  
462 than  $+0.7 \text{ Bq m}^{-3}$ ) and soft-expansion events (with air radon activity between 3 and  $9 \text{ Bq m}^{-3}$   
463 and decreases of less than  $+0.7 \text{ Bq m}^{-3}$ ) combined to sodar observations were limited. The  
464 possible interpretation of such conditions will be discussed in the comparison with lidar  
465 observations. The above-described definition of the box model constraints requires a larger  
466 dataset to rigorously define the model’s applicability limits, and should be tested for a longer  
467 period and applied in different geological condition. The presented limits refer to a situation  
468 where local outcropping rocks have a natural content of  $^{226}\text{Ra}$  of approximately  $100 \text{ Bq kg}^{-1}$ .  
469 Therefore, lower contents will consequently be associated with lower reference values.

470 *3.3.3 Turbulent conditions (radon vs. lidar)*

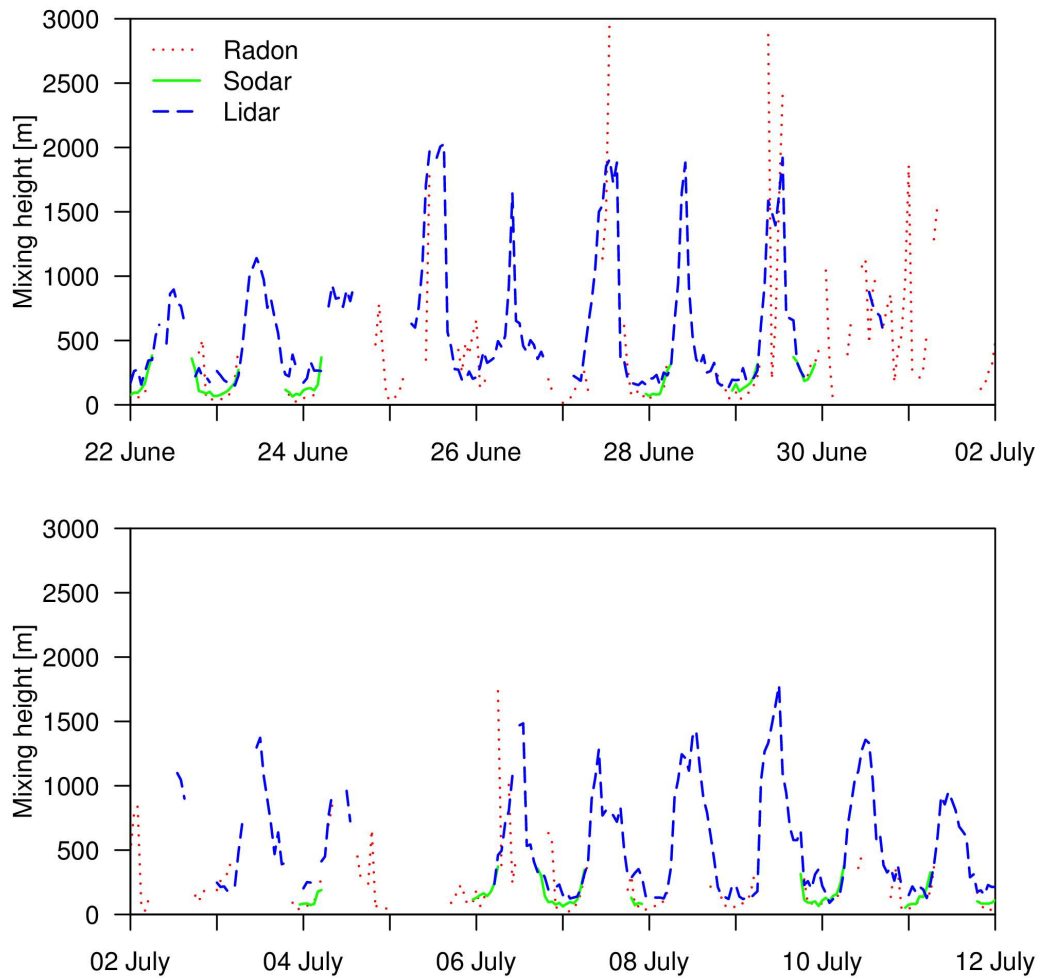
471 Using the considered parameters, the evolution of the mixing height during the period showed  
 472 good agreement between sodar mixing-heights and the radon-modelled estimates. The  
 473 comparison between radon estimates and lidar mixing-heights (Fig. 8) exhibits a different  
 474 behaviour.



475

476 **Fig 8** Radon vs. lidar observations. Observations are classified as near-stable (cyan diamond), heavy expansion  
 477 (green square), soft expansion (green triangle), soft compression (red triangle) and heavy compression (red square).  
 478

479 Lidar performed during the survey produced 402 1-h averaged observations, and considering  
 480 the stability limit, 123 observations ( $\approx 30\%$  of the total) were characterized by stable conditions  
 481 consistent with the box model definition (Table 1). The radon-derived mixing heights above the  
 482 stability limit were all below 400 m, with an average deviation from lidar estimates of  
 483 approximately  $-120$  m. The radon-derived mixing heights in this case were also underestimated,  
 484 and the lowest level of detection for lidar was approximately 100 m.



485

486 **Fig 9** Temporal evolution of the mixing height estimated by sodar, lidar and radon progeny techniques. Radon  
 487 mixing heights are limited to conditions above the “turbulent limit”.

488

489 A total of 155 observations occurred under turbulent conditions ( $\approx 38\%$  of the total), and the  
 490 overestimation in this case was greater than 2 km over the mean lidar values (approximately  
 491 800 m). All the turbulent limit events were observed between 1000 and 2000 UTC and winds  
 492 were a combination of sea breeze (from the south-west sector) and other terrestrial directions  
 493 (north-north-west sector and south-south-east sector). An overestimation under these conditions  
 494 was expected, since advection is a dominant component and box modelling is not appropriate.

495 The important information that can be derived, in this case, is the start time and the duration of  
496 the turbulent period. Intermediate situations suitable for box modelling were detected in 35  
497 heavy compression events, with an average deviation from lidar estimates of approximately  
498  $-250$  m. The interpretation presented above is confirmed by the underestimation of the mixing  
499 height. Under these conditions, the performance of sodar was superior to lidar, and in this case  
500 radon modelling can support lidar in order to improve its results. The other intermediate  
501 conditions (“soft compression”, “soft expansion” and “heavy expansion”) were obtained with  
502 lidar mixing heights ranging from 600 to 900 m and with an average deviation of 200 m from  
503 lidar estimates. While this is consistent with the interpretation of heavy expansion events  
504 discussed above (Section 3.3.2), more indications can be obtained for soft compression and soft  
505 expansion. Some of these events ( $\approx 30\%$ ) were characterized by wind speeds  $> 2 \text{ m s}^{-1}$  occurring  
506 between 1000 - 1500 UTC, indicating variations in terms of air masses when the sea-breeze  
507 regime is not completely dominant. The remaining situations occurred in the late evening  
508 probably when the sea-breeze component was decaying. The variations of the radioactive  
509 features (radon activity and progeny disequilibrium), in these cases, were significant and our  
510 model failed to include these fluctuations.

511 Considering only the near-stable and weak-convection conditions (below 600 m), the  
512 comparison between the lidar and radon-derived mixing heights highlights the good  
513 performance of the radon-based estimates. Lidar yielded the best output during diurnal  
514 convection when the box model is out of the stated applicability range, in agreement with all of  
515 the available literature (Griffiths et al. 2013). The change in air mass fetch, under turbulent  
516 conditions, is dominant and box modelling is not appropriate. In this case, radon modelling can  
517 only outline the turbulent conditions and cannot make an exact prediction of the mixing height.  
518 Further implementations of the model are necessary to also include turbulent conditions.

519 A summary of these results is reported in Fig. 9, where the agreement between the sodar  
520 estimates and the radon-derived mixing height is consistent, especially under nocturnal near-  
521 stable conditions. The lidar observations are confirmed to be consistent, especially during daily  
522 conditions, but the extension of the operating conditions of our improved box model enhances  
523 the capacity to integrate both techniques.

524



#### 525 **4 Conclusions**

526 The estimation of the radon-derived mixing height can be improved by including the vertical  
527 entrainment of residual layers and variability in the soil radon exhalation rate in a box model.  
528 Our objective was to provide an improved box model and its validation; the efficiency in  
529 interpreting pollutant dynamics with this type of modelling has already been accomplished  
530 previously (Perrino et al. 2001, Chambers et al. 2015). The conversion of radon progeny gross  
531  $\beta$  counts into air radon activity is presented from both theoretical and experimental perspectives.  
532 This approach supported the definition of a conversion factor controlled by instrumental  
533 efficiency and counting scheme. The presence of complete equilibrium in the radon progeny is,  
534 at the moment, an important approximation assumed by our model. The comparison between  
535 radon-derived estimates, sodar mixing heights and lidar values supported the definition of the  
536 applicability range of this box model. It was possible to identify count limits that describe “near-  
537 stable” conditions, occurring especially in late afternoon, at night, and early morning periods,  
538 during which good agreement between radon-derived and sodar estimates was observed.  
539 Additional limits were identified for a “transition” range, occurring during early afternoon and  
540 late morning periods, during which different processes occur. However, only certain types of  
541 compressions can be included in the conditions where the box model hypothesis are satisfied.  
542 Specific limits and situations outside of the validity range of the box model for turbulent  
543 conditions can easily be identified. Further studies are of course necessary for the definition of  
544 more general limits between the different intervals, but the improved model provides an  
545 enhanced application range for a simple detection of mixing height. However, the contribution  
546 of our model can already help improve the description of diffusion processes involving air  
547 pollutants.

548

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550 The data used herein are available by contacting the corresponding author (salzano@iia.cnr.it).  
551 The sodar and lidar activities were performed with the support offered by Anna Maria  
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554 “Sapienza”. We are grateful to Scott D. Chambers, who contributed to improving and clarifying  
555 the manuscript.

556

## 557 6 Appendix 1

558 The Eq. 6 included the following values for Rome: 100 Bq kg<sup>-1</sup> for the <sup>226</sup>Ra soil content  
 559 (Voltaggio et al. 2006), 1.5 × 10<sup>3</sup> kg m<sup>-3</sup> for the soil bulk density (Voltaggio et al. 2006) and  
 560 0.45 for the soil total porosity (Voltaggio et al. 2006).  $\lambda$  and  $D_0$  are constants with values of 2.1  
 561 × 10<sup>-6</sup> s<sup>-1</sup> and 1.1 × 10<sup>-5</sup> m<sup>2</sup> s<sup>-1</sup>, respectively. Soil temperature, water saturation and emanation  
 562 power are time dependent variables that are influenced by meteorological conditions. The latter  
 563 can be estimated using Zhuo et al. 2008,

$$\varepsilon = \varepsilon_0[1 + a(1 - e^{-bS})]. \quad (15)$$

564 The emanation power at 25 °C ( $\varepsilon_0$ ) and the two constants  $a$  and  $b$  are specific for silty soils,  
 565 such as the those present in the Rome area.

566 Considering these equations, the estimation of hourly fluxes requires the solution of heat and  
 567 hydraulic balances to predict variations in terms of soil temperature and water saturation. The  
 568 definition of a box model with a single layer (1 m height), representing the superficial soil,  
 569 represents a preliminary approach. At this stage, several constraints are necessary to ensure a  
 570 simple solution, but further development of the model is required to improve the prediction.  
 571 Assuming that water infiltrates only vertically (no run-off and horizontal fluxes) and that no  
 572 temperature and water gradients are present in this layer, the hydraulic balance can be defined  
 573 as follow,

$$S = \frac{V_{water}}{V_{pores}} = \frac{V_{water}}{p V_{soil}} = \frac{10^{-3}(P_h - ET_0)}{p V_{soil}}. \quad (16)$$

574 The soil water saturation is defined as the ratio between water volume and pore volume, and  
 575 can be rewritten as a function of soil porosity and is consequently related to  $P_h$  (hourly  
 576 precipitation in mm),  $ET_0$  (hourly evapotranspiration in mm),  $V_{soil}$  (the considered soil volume  
 577 in m<sup>3</sup>) and  $p$  as soil porosity.

578 Moreover, the thermal balance can be expressed,

$$\Delta T = \frac{G}{C_v h}, \quad (17)$$

579 where  $\Delta T$  is the soil temperature variation (K),  $G$  is the soil heat flux (J m<sup>-2</sup>),  $C_v$  is the volumetric  
 580 heat capacity in J m<sup>-3</sup> K<sup>-1</sup> and  $h$  is the layer thickness (m). In detail, the volumetric heat capacity  
 581 can be computed as

$$C_v = c_{soil}\rho_b + c_w\rho_w S, \quad (18)$$

582 where  $c_{soil}$  is the average heat capacity of solid constituents in soil ( $J\ kg^{-1}\ K^{-1}$ ),  $\rho_b$  is the soil bulk  
 583 density ( $kg\ m^{-3}$ ),  $\rho_w$  is the water density ( $kg\ m^{-3}$ ),  $c_w$  is the water heat capacity and  $S$  is the soil  
 584 water saturation.

585 The calculation of  $ET_0$  and  $G$  was made following the approach of Allen et al (1998),

$$G = 0.1 R_n \text{ (day)}, \quad (19a)$$

$$G = 0.5 R_n \text{ (night)}. \quad (19b)$$

586 The soil heat flux ( $G$ ) in this case is related to  $R_n$ , which is the net radiation ( $J\ m^{-2}\ h^{-1}$ ),

$$ET_0 = \frac{0.408\Delta(R_n - G) + \gamma \frac{37}{T_a + 273.16} uVPD}{\Delta + \gamma(1 + 0.34u)}. \quad (20)$$

587 The Eq. 20 equation follows the Penman-Monteith method, where  $ET_0$  is the hourly  
 588 evapotranspiration ( $mm\ h^{-1}$ ),  $u$  is the wind speed ( $m\ s^{-1}$ ),  $R_n$  is the net radiation in  $J\ m^{-2}\ h^{-1}$ ,  $\gamma$  is  
 589 the psychrometric constant ( $kPa\ ^\circ C^{-1}$ ),  $\Delta$  is the slope of the saturation vapour pressure curve  
 590 ( $kPa\ ^\circ C^{-1}$ ),  $T_a$  is the air temperature ( $^\circ C$ ) and  $VPD$  is the vapour pressure deficit ( $kPa$ ). The  
 591 values of  $\gamma$ ,  $VPD$  and  $\Delta$  depend on air temperature, pressure and humidity, and they were  
 592 calculated using functions described in Alexandris and Kerkides (2003). The net radiation was  
 593 estimated using astronomical functions and cloud attenuation values obtained from cloudiness  
 594 observations, following Kasten and Czeplak (1980).

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