Stability of network indexes defined trough matrix functions

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One of the major goals of network analysis is to identify important components in a network by exploiting the topological structure of connections between its nodes. To this end, recent years have seen the introduction of many new measures of importance of a node or a set of nodes, defined in terms of suitable entries of functions of matrices $f(A)$, for different choices of f and A [2]. Measures based on matrix functions are particularly relevant as they are able to capture the global structure of connections involving a node u, whereas most of other models rely on the local behavior around u. On the other hand, the matrix function approach requires a significant computational effort to address the entries of $f(A)$. This is particularly prohibitive when the network changes frequently and the important components have to be updated.

Given the adjacency matrix A of a graph $G = (V, E)$, in this work we address the problem of estimating the changes in the entries of $f(A)$ with respect to changes in the edge set E. Intuition suggests that, if the topology of connections in the new graph $\tilde{G} = (V, \tilde{E})$ is not significantly distorted, relevant components in G maintain their leading role in $\tilde{G}$. By exploiting the literature on functions of banded matrices [4, 1], we propose a bound showing that the magnitude of the variation of the entry $f(A)_{u,v}$ decays exponentially with the distance in G that separates either u or v from the set of nodes touched by the edges that are perturbed.

The interests of this bounds relies on the fact that often one needs to know only "who are" the first few most important nodes in the graph, whereas the nodes whose edge connection tends to change more often are those having a marginal role in the graph [3], and typically the distance in G from important nodes and nodes having a marginal role is large.

References

[1] M. Benzi and V. Simoncini, *Decay bounds for functions of Hermitian matrices with banded or Kronecker structure*, SIAM J. Matrix Anal. Appl., 36 (2015), pp. 1263–1282.

[2] E. Estrada and D. J. Higham, *Network properties revealed through matrix functions*, SIAM review, 52 (2010), pp. 696–714.

[3] A. N. Langville and C. D. Meyer, *Who's# 1?: the science of rating and ranking*, Princeton University Press, 2012.

[4] S. Pozza and V. Simoncini, *Decay bounds for non-Hermitian matrix functions*, arXiv:1605.01595v3 [math.NA], (2016).

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