GIMC-SIMAI YOUNG 2022

Retrieval of surface emissivity from FORUM-EE9 simulated measurements: optimization of constraints

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University of Pavia, September 30, 2022

Forum mission

FORUM (Far-infrared Outgoing Radiation Understanding and Monitoring):

- Fourier Transform Spectrometer (FTS);
- End-to-end (E2E) simulator;
- 9th ESA's Earth Explorer mission (EE9);
- Complete emission spectrum at the top of the atmosphere (TOA) $\rightarrow$ unique picture of the Earth's radiative budget;
- $100 - 1600$ cm^{-1} region of the atmosphere (FIR and part of MIR) $\rightarrow$ more than 95% outgoing longwave flux lost by our planet.

Targets:

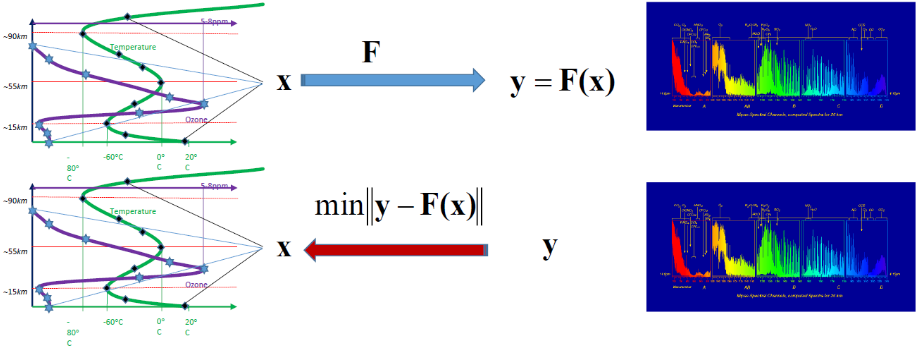
- Upper Troposphere and Lower Stratosphere Water Vapor;
- **Surface emissivity in polar and dry regions**;
- Cirrus clouds characteristics.

Final aim: improving the accuracy of climate models

Direct and inverse problem

Direct problem: from the atmospheric status vector x find the simulated spectrum $y = F(x)$, with F known as **forward model**.

Inverse problem: from the measured spectrum y find the parameter vector x (**retrieval vector**) which minimizes $||y - F(x)||$.



Direct problem

Radiative transfer equation (homogeneous mean, fixed frequency ω):

Lambert-Beer law + Planck law

$$\frac{d}{dx}I(x) = -\rho(x)\alpha(x)I(x) + \rho(x)\alpha(x)B\big(T(x)\big),$$

where:

- I is the intensity of light ray;
- x is the coordinate along the direction of the light beam;
- $\rho\alpha$ is the attenuation coefficient;
- $B\big(T(x)\big)$ is the Planck function depending on the temperature T.

Continuous solution for the altitude layer from 0 to x_N:

$$I(x_N) = I(x_0)e^{-\int_0^{x_N}\rho(x)\alpha(x)dx} + \int_0^{x_N}\rho(x)\alpha(x)B\big(T(x)\big)e^{\int_x^{x_N}\rho(x')\alpha(x')dx'}dx$$

Inverse problem and Optimal Estimation method

Inverse problem: determine an estimate of x from the measurements y.

The inverse problem is very **ill-conditioned**.

Given a Gaussian measurement error $\varepsilon = y - F(x)$, with $S_y = \mathbb{E}[\varepsilon\varepsilon^t]$.
Suppose there is an apriori estimate x_a of x with error $\varepsilon_a = x - x_a$ and $S_a = \mathbb{E}[\varepsilon_a\varepsilon_a^t]$. We can compute:

$$P(y, x_a) = \frac{1}{(2\pi)^{\frac{n}{2}}|S_a|}\, e^{\dfrac{(x_a - x)^t S_a^{-1}(x_a - x)}{2}}\, \frac{1}{(2\pi)^{\frac{m}{2}}|S_y|}\, e^{\dfrac{(y - F(x))^t S_y^{-1}(y - F(x))}{2}},$$

where:

- $|\ldots|$ is the determinant of A;
- n is the dimension of x;
- m is the dimension of y.

Inverse problem and Optimal Estimation method

We can rewrite P as:

$$P(y, x_a) = \frac{1}{(2\pi)^{\frac{n+m}{2}} |S_a| \, |S_y|} \, e^{-\frac{1}{2}\left[(x_a - x)^t S_a^{-1}(x_a - x) + (y - F(x))^t S_y^{-1}(y - F(x))\right]}$$

Optimal estimation method for the inversion:

The maximization of the probability that a given parameter vector is compatible with the measurements is equivalent to the minimization of the quantity:

$$\chi^2(x) = (x_a - x)^t S_a^{-1}(x_a - x) + \big(y - F(x)\big)^t S_y^{-1}\big(y - F(x)\big).$$

The minimization

Gauss-Newton method (GN) + Levenberg-Marquardt technique (LM):

$$x_{k+1} = x_k + \left(K_k^t S_y^{-1} K_k + S_a^{-1} + \alpha_k \, \mathrm{diag}(\mathrm{K_k^t S_y^{-1} K_k}) \right)^{-1} \cdot$$
$$\cdot \left[K_k^t S_y^{-1} (y - F(x_k)) + S_a^{-1}(x_a - x_k) \right],$$

where k is the iteration index, α_k is the Marquardt parameter at iteration k and $K_k = \nabla F(x_k)$.

Why LM?

- the damping term α_k helps in the inversion of the matrix to be computed;
- for large values of α_k, $x_{k+1} - x_k$ goes to $-\frac{\nabla \chi^2(x)}{\alpha(x)}$, which is a descend direction for the cost function.

Drawback: premature convergence.

Surface emissivity

Emissivity of the surface of a material:
effectiveness in emitting energy as thermal radiation (visible radiation and infrared radiation).
Each body re-emits part of the energy that reaches it in the form of thermal energy, and reflects the rest.

For the energy conservation law:

$$E_{absorbed} + E_{reflected} = E_{incident}.$$

Emissivity (ϵ) and **Reflectivity** (r) are defined as:

$$\epsilon = \frac{E_{absorbed}}{E_{incident}},$$

$$r = \frac{E_{reflected}}{E_{incident}},$$

$$\epsilon + r = 1.$$

Iterative Variable Strength regularization (IVS)

Aim: Regularization of the retrieved surface emissivity profile.

Why?
No correlations in the a-priori VCM to avoid cross-talks between spectral ranges with different sensitivity to surface emissivity $\rightarrow$ better reconstruction in the transition intervals $\rightarrow$ oscillations in the retrieved profile.

Method:
It is an a-posteriori regularization consisting in adding a Tikhonov term to χ^2 :

$$\chi^2(x) = (x_a - x)^t S_a^{-1}(x_a - x) + \left(y - F(x)\right)^t S_y^{-1}\left(y - F(x)\right) + (x_s - x)^t R_\Lambda (x_s - x),$$

where

- x_s is an estimate of the solution,
- $R_\Lambda = L_i^t \Lambda L_i$ is such that:
  - L_i is a linear operator approximating the i-th derivative: $(L_i x_k)_j \simeq \frac{d^i}{d\omega^i} x_k(\omega_j)$,
  - Λ is a positive diagonal matrix such that $\Lambda_{jj} = \lambda(\omega_j)$.

IVS regularization

Let

- k be the iteration count at convergence for the minimization of χ^2 without the regularization term,
- $x_{\mathrm{OE}} \equiv x_{k+1}$,
- $x_s = 0$,

then the G-N minimizer x_Λ has the form:

$$x_\Lambda = x_k + \left(K_k^t S_y^{-1} K_k + S_a^{-1} + \alpha_k \, \mathrm{diag}(\mathrm{K_k^t S_y^{-1} K_k}) + \mathrm{R}_\Lambda \right)^{-1} \cdot$$
$$\cdot \left[K_k^t S_y^{-1} (y - F(x_k)) + S_a^{-1} (x_a - x_k) - R_\Lambda x_k \right].$$

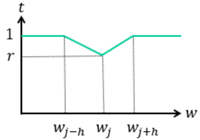
IVS regularization

The procedure:
Starting with a large $\Lambda^{(0)} = \lambda^{(0)} I$, we decrease the profile until both the following conditions are fulfilled:

- $\left| x_\Lambda^q(\omega) - x_{\mathrm{OE}}^q(\omega) \right| \leq w_e(\omega) \sqrt{S_y(\omega,\omega)}$,
- $\nu_\Lambda^q(\omega) \leq w_r(\omega) \nu_{\mathrm{OE}}^q(\omega)$,

where q is the regularization step, $\nu_\Lambda(\omega)$ and $\nu_{\mathrm{OE}}(\omega)$ are the spectral resolutions of the x_Λ and x_{OE} profiles respectively.

Decreasing the λ profile:
for each point ω_j such that the conditions are not satisfied we multiply by a triangular function $t_j(\omega)$:



with $r = 0.99$, $h = 1$ or $h = 2$.

The amplitude is set either with the independent variable, or in number of points (**ztri**).

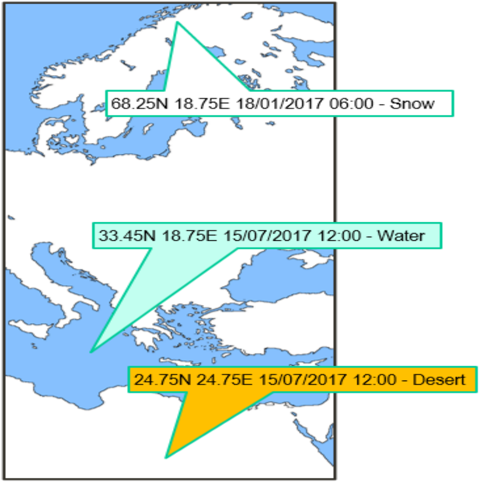
Retrieval Qualifiers

- χ^2;
- **DOF**: number of degrees of freedom of the solution;
- **POQ** (Profile oscillations quantifier, Ω_1): given a profile $x_i = x(\omega_i)$ it measures its oscillations:

$$\Omega_1 = \frac{1}{n-2} \sum_{i=2}^{n-1} \frac{\left| x_i - x_{i-1} - \frac{x_{i+1} - x_{i-1}}{\omega_{i+1} - \omega_{i-1}} (\omega_i - \omega_{i-1}) \right|}{\sqrt{(x_{i+1} - x_{i-1})^2 + (\omega_{i+1} - \omega_{i-1})^2}}$$

Test scenarios

Sensitivity to emissivity in the FIR depends on the **PWV** of the atmosphere: **total atmospheric water vapour contained in a vertical air column of unit area from the Earth's surface to the top of the atmosphere**

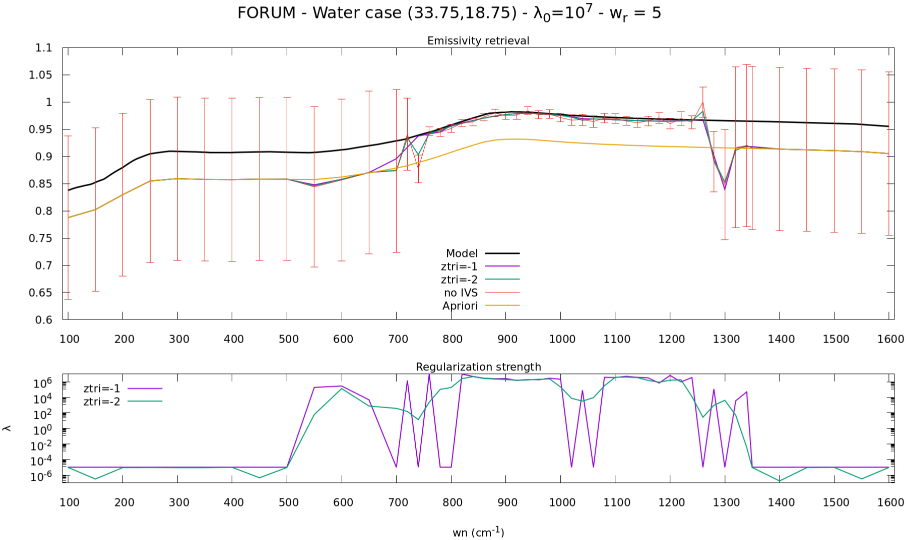


- Water case: $PWV = 36.33$ mm $\rightarrow$ wet atmosphere $\rightarrow$ **no sensitivity**.
- Snow case: $PWV = 3.31$ mm $\rightarrow$ dry atmosphere $\rightarrow$ **good sensitivity**.
- Desert case: $PWV = 23.14$ mm $\rightarrow$ fairly dry atmosphere $\rightarrow$ **some sensitivity**.

Results - water case

WATER CASE PVW: 36.33 mm			
	BEFORE IVS	AFTER IVS ($\lambda_0 = 10^7$, $w_e = 1$, $w_r = 5$)	
		h = 1	h = 2
DOF	29.167	9.524	11.048
POQ	271E-6	97E-6	165E-6
χ^2	1.0287	1.0365	1.0356

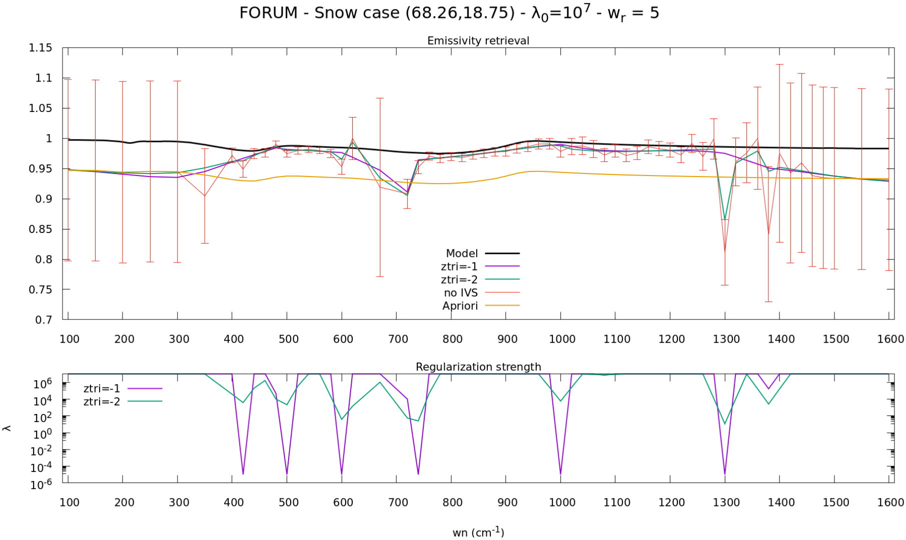
Results - water case



Results - snow case

SNOW CASE PVW: 3.31 mm			
	BEFORE IVS	AFTER IVS ($\lambda_0 = 10^7$, $w_e = 1$, $w_r = 5$) h = 1	h = 2
DOF	45.343	10.920	14.809
POQ	473E-6	34E-6	166E-6
χ^2	1.0262	1.0376	1.0341

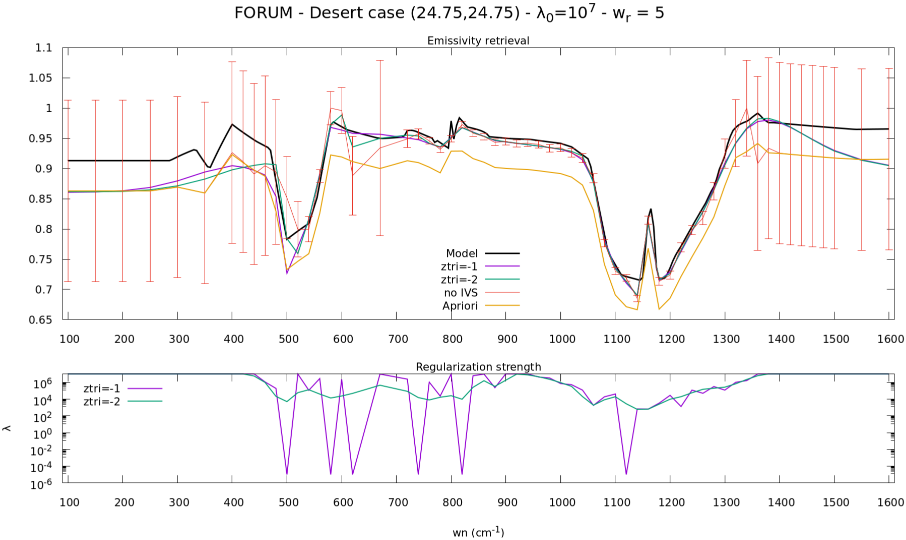
Results - snow case



Results - desert case

DESERT CASE PVW: 23.14 mm			
	BEFORE IVS	AFTER IVS ($\lambda_0 = 10^7$, $w_e = 1$, $w_r = 5$)	
		h = 1	h = 2
DOF	38.403	17.605	19.535
POQ	422E-6	254E-6	283E-6
χ^2	1.2046	1.2506	1.2409

Results - desert case



Conclusions, ongoing and future works

- the IVS for surface emissivity profile has been recently added to the official algorithm;
- in the optimal IVS setting it turned out to be crucial to start from a strong regularization and then softly reduce the λ profile in a quite large range of wave numbers;

Future work

- Optimization of the retrieval grid.
- Global map for the sensitivity to emissivity in the FIR and MIR.

Thank you!

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Retrieval grid: fine vs coarse

- **Fine grid**
  - Minimization of the smoothing error. Sharp features of the emissivity model are reproduced.
  - Reduced precision, possible biases.
- **Coarse grid**
  - If the retrieval grid step is larger than the retrieval feature, the feature cannot be reproduced.
  - Good precision. Each retrieval point is the average of a large number of measurements. Thus, the random error is smaller and there are no biases.

Retrieval grid: fine vs coarse