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VIBRATIONS OF SHAFTS ROTATING ON LUBRICATED BEARINGS

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SYNOPSIS We put in evidence certain properties of the forces generated by the lubricant in rotor bearings. The properties are relevant in the study of self-excited or forced vibrations of rotors.

1. INTRODUCTION

Analyses of shaft vibrations under the influence of lubricant action in the bearings are usually so complex that recourse must be made to numerical procedures. Thus specific results can be obtained; the overall qualitative appreciation of phenomena is often lost, however.

To reduce difficulties and push the direct analysis more deeply, we have already partially exploited the idea of studying separately certain dynamic properties of the oil film: precisely those which evidence the existence of paths of the journal centre along which the oil force has convenient properties. Separately one can then check the compatibility of such movements of the journal with other dynamic conditions deriving from hypotheses on shafts elastic behaviour.

Here the analysis of Ref. 1 is completed: in the first part a detailed study is made of properties of the linearized expressions of the oil forces in the neighbourhood of a steady state of the journal under load. The results are relevant for instance in an analysis of behaviour of a shaft rotating on two differently loaded bearings. In the second part large amplitude movements of the journal centre are considered: again extending results of Ref. 1 we consider the asymptotic expressions of lubricant forces when the eccentricity ratio is almost 1, covering cases when the lubrication is complete or when the film is ruptured by cavitation.

2. A property of the linearized expression of lubricant forces.

The radial and transverse components of the lubricant force acting on a rotating journal will be called F_e, F_n with

$$F_e = H\, f_e\, (a, a', \psi, \psi'; \underline{B}),$$
$$F_n = H\, f_n\, (a, a', \psi, \psi'; \underline{B}),$$

$$H = \frac{R\, b^3\, \omega\, \mu}{c^2},$$

where:

- R, b, c are radius, width, radial clearance of the bearing,
- ω is the rotor speed (radians per unit time) and
- μ is the viscosity of the lubricant;
- f_e, f_n are non-dimensional functions of eccentricity ratio a (true eccentricity, c a), of the derivative a' of a with respect to non-dimensional time $\tau = \omega t$, of the attitude angle ψ evaluated with reference to a preferred direction (the direction of the load, for instance) and of its derivative ψ', and finally of a vector $\underline{B}$ of parameters characterizing the bearing (for a cylindrical full bearing $\underline{B} = (c/R, b/R)$).

If $\bar{a}, \bar{\psi}$ are the values of a, ψ in a steady journal position under a load W, then

$$\bar{f}_e = f_e(\bar{a}, 0, \bar{\psi}, 0; \underline{B}) = -\frac{W}{H}\cos\bar{\psi},$$
$$\bar{f}_n = f_n(\bar{a}, 0, \bar{\psi}, 0; \underline{B}) = \frac{W}{H}\sin\bar{\psi},$$

It is convenient to introduce a system of reference $\mathcal{E} = (\bar{O}, x, y)$ with origin in the steady-state position of the journal centre and with x-axis oriented as the attitude vector $\Omega\bar{O}$ (Ω, bearings centre) and to introduce approximate components F_x, F_y of the lubricant force in the neighbourhood of the steady state:

$$F_x \simeq H(\bar{f}_e + a_{11}\xi + a_{12}\eta + b_{11}\xi' + b_{12}\eta'),$$

$$F_y \simeq H(\bar{f}_n + a_{21}\xi + a_{22}\eta + b_{21}\xi' + b_{22}\eta'),$$

where $\xi = x/c$, $\eta = y/c$.

We can explore now the possibility that special elliptic trajectories exist:

$$\xi = c_1 \cos\nu\tau + c_2 \sin\nu\tau,$$
$$\eta = c_3 \cos\nu\tau + c_4 \sin\nu\tau, \qquad (2.1)$$

(ν, ratio of whirl to running speed), such that the linearized expression of the excess force

$$(F_x - H\bar{f}_e,\ F_y - H\bar{f}_n)$$

coincides, along these trajectories, with the expression of the sum of an elastic restoring force (of components $-Kc\xi$, $-Kc\eta$; K, rigidity) and of a viscous damping force (of components $-Ec\xi'$, $-Ec\eta'$; E, damping constant):

$$a_{11}\xi + a_{12}\eta + b_{11}\xi' + b_{12}\eta' = -\varkappa\xi - \varepsilon\xi',$$
$$a_{21}\xi + a_{22}\eta + b_{21}\xi' + b_{22}\eta' = -\varkappa\eta - \varepsilon\eta',$$
$$\varkappa = \frac{cK}{H}, \quad \varepsilon = \frac{c\omega E}{H}. \qquad (2.2)$$

For existence the necessary and sufficient condition is the vanishing of the determinant of a Voigt matrix. As the determinant is the sum of two squares two conditions ensue:

$$\nu^2\left[b_{12}b_{21} - (b_{11}+\varepsilon)(b_{22}+\varepsilon)\right] = a_{12}a_{21} - (a_{11}+\varkappa)(a_{22}+\varkappa),$$
$$\nu\left[(b_{11}+\varepsilon)(a_{22}+\varkappa) + (b_{22}+\varepsilon)(a_{11}+\varkappa) - b_{12}a_{21} - b_{21}a_{12}\right] = 0.$$

This system has two solutions in ε, $\varkappa$, which can be given explicitly

$$\varepsilon = -\frac{1}{2}(b_{11}+b_{22}) \pm \sqrt{2}\,g\left((h^2+4\nu^2g^2)^{1/2} - h\right)^{1/2}, \qquad (2.3)$$
$$\varkappa = -\frac{1}{2}(a_{11}+a_{22}) \pm \frac{1}{\sqrt{2}}\left((h^2+4\nu^2g^2)^{1/2} - h\right)^{1/2};$$

here

$$h = \frac{\nu^2}{4}(b_{11}-b_{22})^2 - \frac{1}{4}(a_{11}-a_{22})^2 - a_{12}a_{21} + \nu^2 b_{12}b_{21}, \qquad (2.4)$$
$$g = \frac{1}{4}(b_{11}-b_{22})(a_{11}-a_{22}) + \frac{1}{2}(b_{21}a_{12} + a_{21}b_{12}).$$

Formulae (2.3) can be put to many uses. For instance elliptic whirls of the type (2.1) with $\nu = 1$ can be caused in a shafts journal by lack of balance in the shaft; then, the sharpness of the response can be judged directly trough the q-factor of the journal

$$q = \left(\frac{\varkappa}{\varepsilon}\right)_{\nu=1},$$

if the rotor is rigid or, otherwise, through a q-factor of the system rotor-bearing easily deduced in terms of formulae (2.3).

Elliptic whirls of type (2.1) occur also in the journals of a balanced shaft under transition conditions before oil whirl instability sets in. If the rotor is symmetric with respect to the mid-plane and the two bearings are equal, then, at transition, $\varepsilon = 0$, and the analysis of Ref. 1 applies. Another interesting case is when the bearings are different or at least differently loaded (See Ref. 2). Then, at transition, ε cannot be zero in both bearings, but rather the signs are different and the energy lost to the lubricant in one bearing is gained from the lubricant in the other. A detailed analysis will appear in a separate paper.

3. Properties of lubricant forces during large amplitude whirls: full lubrication.

The lubricant forces in a plain cylindrical bearing with full lubrication have expressions which can be approximated, when the eccentricity ratio a tends to 1, as follows

$$F_e = -H h_e \frac{a'}{(1-a)^\alpha}\left[1 + o(1-a)\right], \qquad (3.1)$$
$$F_n = H h_n \frac{1-2\psi'}{(1-a)^{\alpha-1}}\left[1 + o(1-a)\right],$$

where h_e, h_n are positive constants and α is larger than 1. One can then inquire about the existence of paths for the journal centre, such that, along those paths,

$$F_e \simeq -Kca - W\cos\psi, \qquad (3.2)$$
$$F_n \simeq W\sin\psi, \text{ when } a \to 1.$$

The idea here is obviously similar to that which has led to the developments of Sect. 2; we imagine that the excess lubricant force $F_e + W\cos\psi$, $F_n - W\sin\psi$ behaves as an elastic restoring force and exploit asymptotic techniques rather than a process of linearization.

One finds (Ref. 3) that (3.1), (3.2) require that a and ψ develop in time as follows

$$a = 1 - \left[\frac{h_e}{(\alpha-1)h\tau}\right]^{\frac{1}{\alpha-1}}\left[1 - \frac{2p}{(\alpha-1)h\tau}\sin\frac{\tau}{2}\right] + o\left(\tau^{\frac{\alpha}{\alpha-1}}\right), \qquad (3.3)$$

$$\psi = \frac{\tau}{2} + \frac{h_e}{h_n}\frac{p}{(\alpha-1)h\tau}\cos\frac{\tau}{2} + o(\tau^{-1}),$$

where

$$h = \frac{Kc}{H}, \quad p = \frac{W}{H}.$$

It is possible now to check the compatibility of these trajectories of the journal with whirling modes of the shaft (as implied by the appropriate dynamic equations) in some important cases: an elastic shaft of uniform cross-section or a massless elastic shaft carrying a massive flywheel. Compatibility can be achieved (see Ref. 3) if the running speed ω is less than $2\,\omega_*^{(\infty)}$, where $\omega_*^{(\infty)}$ is the lowest critical speed of the shaft on rigid supports. The mode of vibration approximates, as $\tau \to \infty$, the mode of vibration of the shaft when resting on elastic supports of rigidity K such that $2\,\omega_*^{(K)} = \omega$ (here $\omega_*^{(K)}$ is the lowest critical speed on supports of rigidity K).

As an alternative to the asymptotic condition (3.2) one can explore the case when (q, a constant)

$$F_e \simeq -Kca - W\cos\psi, \qquad (3.4)$$
$$F_n \simeq W\sin\psi + Hq\psi, \text{ when } a \to 1.$$

As a consequence one finds (Ref. 3) instead of (3.3) the asymptotic expressions

$$a = 1 - \left[\frac{h_e}{(\alpha-1)h\tau}\right]^{\frac{1}{\alpha-1}}\left[1 - \frac{2p}{(\alpha-1)h\tau}\sin\gamma\tau\right] + o\left(\tau^{-\frac{\alpha}{\alpha-1}}\right),$$

$$\psi = \gamma\tau + \frac{h_e}{2\gamma h_n}\frac{p}{(\alpha-1)h\tau}\cos\gamma\tau + o(\tau^{-1}),$$

γ, a constant.

Compatibility with the equations of motion for a shaft is now achieved only for $\gamma = (\omega_*^{(\infty)}/\omega)$

and $\omega > 2\omega_*^{(\infty)}$. The mode of vibration is approximately the fundamental mode on rigid supports (at all speeds $\omega > 2\omega_*^{(\infty)}$); the amplitude increases linearly with time.

4. The case of partial lubrication.

The analysis of Sect.3 has essentially a theoretical interest, because, in practice, during vibrations at high eccentricity ratio the lubricant film is usually ruptured. Then, explicit expressions for the oil forces are available only for the case of short bearings (Ref. 4); therefore we restrict our attention here to that case. Even so the analytical situation is very complex, because a new variable is involved: the angle α which specifies the attitude of the oil film

$$\alpha = \arctan \frac{2a'}{(1-2\psi')a} ; \qquad (4.1)$$

precisely one has

$$F_e = \tfrac{1}{2} H \{(1-2\psi')ag_1(a,\alpha)-2a'g_2(a,\alpha)\} ,$$
$$(4.2)$$
$$F_n = \tfrac{1}{2} H \{(1-2\psi')ag_3(a,\alpha)-2a'g_1(a,\alpha)\} ,$$

where g_1, g_2, g_3 have complicated expressions in terms of the variables a and α (Ref.4)

$$g_1 = \frac{-2a \cos^3\alpha}{(1-a^2 \cos^2\alpha)^2}$$

$$g_2 = \frac{[3+(2-5a^2)\cos^2\alpha]\, a \,\mathrm{sen}\,\alpha}{(1-a^2)^2 (1-a^2\cos^2\alpha)^2} + \qquad (4.3)$$

$$+ \frac{1+2a^2}{(1-a^2)^{5/2}} \left\{ \frac{\pi}{2} + \arctan\left[\frac{a\,\mathrm{sen}\,\alpha}{(1-a^2)^{1/2}}\right]\right\} ,$$

$$g_3 = \frac{(1-2\cos^2\alpha + a^2\cos^2\alpha)\, a\,\mathrm{sen}\,\alpha}{(1-a^2)(1-a^2\cos^2\alpha)^2} -$$

$$+ (1-a^2)^{-3/2}\left[\frac{\pi}{2} + \arctan \frac{a\,\mathrm{sen}\,\alpha}{(1-a^2)^{1/2}}\right] .$$

A case in which the analysis is still relatively simple is when

$$\alpha = o\,[(1-a)^{\gamma}] , \qquad \gamma > 0 ,$$

and the oil force is required to behave as follows

$$F_e \simeq - Kca , \qquad F_n \simeq 0 , \quad \text{when } a \to 1 .$$

It corresponds to circumstances where the region of cavitation rotates with the frequency of whirl without appreciable rocking and it is compatible only with the behaviour of a vertical shaft (or a light shaft: no appreciable load on the bearings).

Then one finds the condition $\gamma = 1/2$ and the following asymptotic expressions

$$\alpha \simeq - m\,\tau^{-1/3}, \quad a \simeq 1 - n\,\tau^{-2/3} ,$$
$$(4.4)$$
$$1 - 2\psi' \simeq \frac{4n}{3m}\,\tau^{-4/3} ,$$

where m and n are positive constants.

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