

Article

Kinetic Modeling of the Direct Dimethyl Ether (DME) Synthesis over Hybrid Multi-Site Catalysts

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Abstract: This paper deals with the proposition of a kinetic model for the direct synthesis of DME via CO₂ hydrogenation in view of the necessary optimization of the catalytic system, reactor design, and process strategy. Despite the fact that DME synthesis is typically treated as a mere combination of two separated catalytic steps (i.e., methanol synthesis and methanol dehydration), the model analysis is now proposed by taking into account the improvements related to the process running over a hybrid catalyst in a rational integration of the two catalytic steps, with boundary conditions properly assumed from the thermodynamics of direct DME synthesis. Specifically, the CO₂ activation step at the metal–oxide interface in the presence of ZrO₂ has been described for the first time through the introduction of an ad hoc mechanism based on solid assumptions from inherent studies in the literature. The kinetic modeling was investigated in a tubular fixed-bed reactor operating from 200 to 260 °C between 1 and 50 bar as a function of a gas hourly space velocity ranging from 2500 to 60,000 NL/kg_{cat}/h, in a stoichiometric CO₂/H₂ feed mixture of 1:3 *v/v*. A well-detailed elementary mechanism was used to predict the CO₂ conversion rate and identify the key reaction pathways, starting with the analysis of the implicated reactions and corresponding kinetic mechanisms and expressions, and finally estimating the main parameters based on an appropriate modeling of test conditions.

Keywords: dimethyl ether; CO₂ valorization; hydrogen conversion; hybrid catalysts; kinetic modeling; fixed-bed reactor



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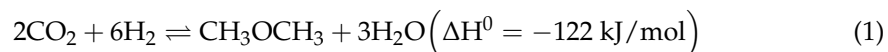


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1. Introduction

In consideration of the growing concern for climate change, the search for novel development models based on renewable energy sources with the potential to gradually replace fossil fuels is in the spotlight. On this road to progressive decarbonization, dimethyl ether (DME), the simplest of the ether compounds (chemical formula CH₃OCH₃), has been recently proposed as a very promising alternative fuel [1–3], being characterized by its high cetane number, low auto-ignition temperature, and near-zero emissions of particulate matter, carbon monoxide, and unburned hydrocarbons [4,5]. It also presents chemical and physical properties similar to LPG, enabling the use of existing storage and distribution infrastructure [6]. Additionally, DME can serve as a feedstock for the production of olefins via the DTO (DME-to-olefins) reaction pathway, which is characterized by high yields and conversions [7–9].

However, the real appeal of this compound lies in its potential synthesis reaction (Equation (1)), since recent research frontiers aim to employ CO₂ and H₂ as reactants [10,11]:



Using captured CO₂ and green H₂ produced via electrolysis, it is, therefore, possible to configure DME as an e-fuel directly produced in a sustainable CCUS (carbon capture utilization and storage) technology chain [12,13]. Furthermore, even though this aspect is still under investigation, DME can be steam-reformed, making it a potential circular hydrogen carrier [14]. For this to be feasible, however, it is necessary to make the synthesis process more efficient and cost-effective. Currently, DME is primarily produced from synthesis gas through an indirect method involving the use of two reactors in a typical cascade process [15]. In the first reactor, methanol synthesis is performed on a ternary metal–oxide catalyst (typically CZA, which is CuO–ZnO–Al₂O₃), while the second reactor allows the dehydration of the previously formed methanol over an acidic catalytic bed (usually γ -Al₂O₃ or a zeolite sample, such as HZSM-5) [10,16–18]. The real challenge is to achieve effective process intensification by implementing direct synthesis in a single reactor, using CO₂ as a feedstock, thus allowing the valorization of a thermodynamically “inert” compound, as well as reducing operating and investment costs [19,20]. Certainly, the main concern is the development of a multi-functional catalyst wherein a long-range inter-dispersion among metallic, oxide, and acidic functionalities must be achieved to catalyze both the methanol synthesis and DME production steps [14,21]. Indeed, by hybridizing the catalytic system for direct DME synthesis, the thermodynamic limitations behind the CO₂ hydrogenation process stopping the formation of methanol can be overcome, even if unavoidable water generation is a key contributor to possible catalyst deactivation mechanisms, such as copper sintering or migration of weak acidic sites [22–24].

Given the potential and critical issues associated with the direct synthesis of dimethyl ether from hydrogen and carbon dioxide, this work aims at investigating the performance, on a laboratory scale, of a previously optimized hybrid CuO–ZnO–ZrO₂/HZSM-5 catalytic system [22] by developing a kinetic model suitable to predict its experimental behavior in a wide set of operating conditions. Particularly, the exchange of alumina (a key component in the benchmark catalyst for methanol synthesis) with zirconia to enhance CO₂ adsorption, water tolerance, and heat management [25] has been limitedly studied from a microkinetic point of view [26,27]. Therefore, this knowledge gap inspired an ad hoc 18-parameter model based on three cascade steps (i.e., CO₂ hydrogenation to methanol, methanol dehydration, and rWGS) occurring in one pot over a hybrid catalytic system. This model aims at explaining a dual-site mechanism on two different adsorption sites by claiming that the CO₂ activation step at the metal–oxide interface is crucial prior to undergoing hydrogenation to methanol. Acidic functionality is then necessary for the subsequent dehydration into DME. As a consequence, the corresponding kinetic expressions were derived and appropriately integrated with all the reactions involved in the process. This resulted in a comprehensive model for the direct synthesis of DME, which led to the assessment of new mechanistic clues about the activity–selectivity pattern of the catalyst under investigation as necessary for further optimization of its design.

2. Experimental Section

This section is divided into two parts: synthesis of the hybrid catalyst and validation of its performance through a fixed-bed reactor.

2.1. Catalyst Synthesis

The hybrid catalyst was synthesized through the gel oxalate coprecipitation method, as elsewhere described [28]. Firstly, the nitrate precursors, namely Cu(NO₃)₂·2.5H₂O, Zn(NO₃)₂·6H₂O, and ZrO(NO₃)₂·nH₂O (in a relative Cu/Zn/Zr atomic ratio of 60/30/10), were dissolved in an ethanolic solution and added, at room temperature in a slow and

constant mode, to a vigorously stirred ethanolic solution containing oxalic acid (20 wt.% in excess with respect to the stoichiometric amount necessary to precipitate the metal precursors) and the finely dispersed HZSM-5 zeolite (Si/Al = 23 mol/mol) in a weight ratio of 1:1 with respect to the final oxide composition of the catalyst. The precipitate was stirred for 3 h, then aged overnight. The catalyst was filtered, dried at 90 °C for 16 h, and then calcined at 350 °C for 4 h.

2.2. Catalyst Testing

In order to assess the activity of the hybrid catalyst, CO₂ hydrogenation to DME was studied in a full kinetic regime, according to conditions designed to rule out the control of diffusional phenomena [11,17]. All the catalytic measurements were carried out in a fixed-bed stainless steel reactor with an internal diameter of 4 mm and a length of 21 mm, loaded with 250 mg of catalyst, and housed within a stainless-steel rod to minimize temperature gradients. The 40–70 mesh (210–420 μm) fraction was selected as a suitable compromise between reactor pressure drop and catalyst particle mechanical resistance. Catalytic tests were carried out at a total pressure of 1, 10, 30, or 50 bar in a temperature range between 200 and 260 °C and gas hourly space velocity of 2500, 8800, and 60,000 NL/kg_{cat}/h, employing a feed mixture of CO₂/H₂/N₂ in a 3:9:1 *v/v* ratio. Before each test, the catalyst underwent an in situ reduction at 300 °C for 1 h using pure hydrogen flow at atmospheric pressure. The reaction stream was analyzed via a gas chromatograph equipped with a three-column separation system connected to a flame ionization detector (FID) and a thermal conductivity detector (TCD). No formation of methane or higher hydrocarbons was detected under the applied conditions. Quantitative data were expressed in terms of CO₂ conversion (X_{CO_2}), DME yield (Y_{DME}) and DME and CO selectivity (S_{DME} and S_{CO}), as defined in Equations (2)–(5):

$$X_{CO_2} = \frac{F_{CO_2}^{in} - F_{CO_2}^{out}}{F_{CO_2}^{in}} \quad (2)$$

$$Y_{DME} = \frac{F_{DME}^{out}}{F_{CO_2}^{in}} \quad (3)$$

$$S_{DME} = \frac{F_{DME}^{out}}{F_{DME}^{out} + F_{Meth}^{out} + F_{CO}^{out}} \quad (4)$$

$$S_{CO} = \frac{F_{CO}^{out}}{F_{DME}^{out} + F_{Meth}^{out} + F_{CO}^{out}} \quad (5)$$

In Equations (2)–(5), F_i^{in} and F_i^{out} are the inlet and the outlet molar flow rates of component *i*, respectively. Conversion and selectivity data were determined by both internal standard and mass-balance methods, averaging three independent measurements with an accuracy of ±3%.

3. Kinetic Modeling

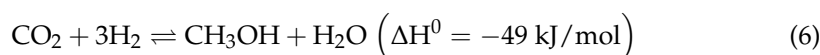
The development of a kinetic model for direct CO₂-to-DME hydrogenation was carried out in two steps:

1. Analysis of the implicated reactions and corresponding kinetic mechanisms and expressions.
2. Estimation of missing parameters based on appropriate modeling of test conditions.

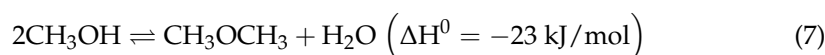
3.1. Reaction Kinetics Investigation

In direct DME synthesis from CO₂ and H₂, three reactions (Equations (6)–(8)) are involved [10,29–31]:

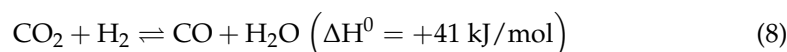
1. CO₂ hydrogenation to methanol (CHM):



2. Methanol dehydration to DME (MDD):



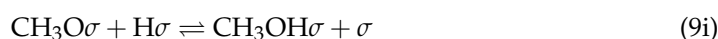
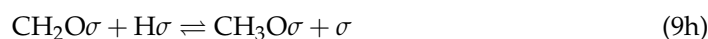
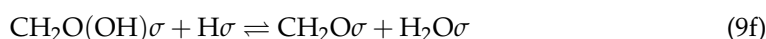
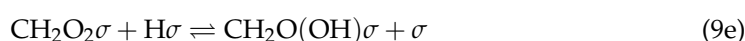
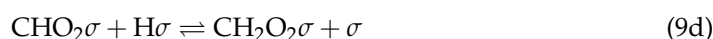
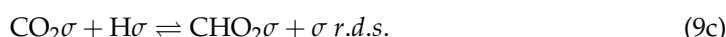
3. Reverse water–gas shift (rWGS):



CO hydrogenation to DME is not considered since it can be obtained from the linear combination of Equations (7) and (8), and is also significantly inhibited by the production of formate, an intermediate that strongly binds copper active sites [32,33]. For each reaction, the kinetic mechanism was reconstructed, identifying the rate-determining step (r.d.s.) [34]. As for methanol dehydration and rWGS, previously elaborated mechanisms from the literature were adopted due to the conventional nature of the employed catalyst (HZSM-5 and Cu-based systems, respectively) [35,36], while an ad hoc kinetic mechanism was developed for CO₂ hydrogenation, based on experimental evidence [37]. Finally, the kinetic expression of the reaction rate was derived for each mechanism, assuming that all elementary steps, except the r.d.s., were at thermodynamic equilibrium and adopting the Langmuir–Hinshelwood–Hougen–Watson (LHHW) model [38]. In light of the moderate operating conditions in terms of pressure and temperature, the gas phase was assumed to be ideal, with a unitary fugacity coefficient (see Appendix A). A “dual site” model is thus proposed, distinguishing two different active sites on the hybrid catalyst [37,39]: the metal–oxide site (σ) and the zeolitic acid site ($\tilde{\sigma}$).

3.1.1. CO₂ Hydrogenation Mechanism

The introduction of zirconia in the catalyst composition, instead of the alumina present in the benchmark catalyst, has a limited history in the literature related to kinetic modeling [26]. Thus, an ad hoc mechanism was developed to further explore its specific behavior. More in detail, the CO₂ activation step is a highly debated aspect, with two generally proposed alternatives [40,41]: the formate route, based on the formation of HCOO intermediate, and the rWGS route, which involves the conversion of CO₂ into CO. Although an unambiguous agreement on the exact mechanism does not exist yet, most of the recent experimental findings suggest a leaning towards the formate route, as described by Equations (9a)–(9j):



According to this route, one molecule of CO₂ activated on a partially oxidized Cu(δ+) site (9b) initially reacts with one atom of hydrogen previously activated on a metallic site (9a), giving rise to a formate species (9c). The latter then reacts with other activated hydrogen atoms in a cascade sequence to produce dioxomethylene species (9d), formaldehyde (9f), methoxy species (9f), and finally methanol (9i) in the last hydrogenation step.

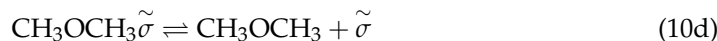
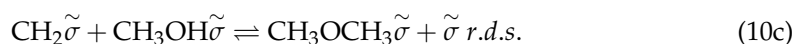
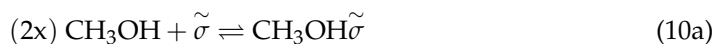
3.1.2. Methanol Dehydration Mechanism

The methanol dehydration reaction strongly relies on the zeolitic component of the catalyst, considering that density, strength, accessibility, and nature of the acid sites (Brønsted or Lewis) are all features capable of influencing the DME productivity [22,37,42]. Several kinetic models have been proposed in the literature, primarily differing from the zeolite topology [26]. However, in terms of reaction mechanism, most studies have converged on identifying two main alternative pathways [42,43]:

Associative pathway, which involves the formation of DME from two adsorbed methanol molecules.

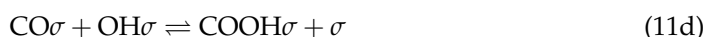
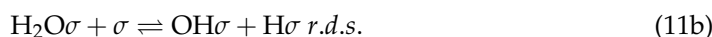
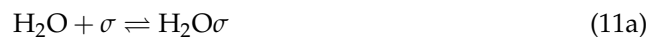
The dpathway is characterized by the adsorption of a methanol molecule followed by its dissociation into a methoxyl group, which then reacts with another methanol molecule to produce DME.

Overall, due to the relatively low operating temperatures (far below 400 °C) and the mesoporous structure of the hybrid catalyst utilized [25], it can be concluded that the predominant pathway is the associative one [44]. Concerning the mechanism over a HZSM-5 zeolite, Lu et al. [35] proposed the following sequence of elementary reactions (Equations (10a)–(10d)):



3.1.3. rWGS Mechanism

The rWGS reaction mechanism can be reconstructed based on its counterpart reaction. More specifically, WGS predominantly occurs on copper sites (at the interface with other metal oxides), meaning that the introduction of zirconia can act as a promoter, enhancing the activity of the existing sites without inducing any further reaction [45,46]. It is therefore possible to refer to some mechanisms already found in the literature that identify the carboxyl route as the predominant pathway for the WGS reaction [36,47], as exemplified by the set of Equations (11a)–(11g):



3.2. Dual-Site Adsorption on the Hybrid Catalyst

While the final dehydration step of methanol to DME is known to be mediated by the zeolitic acidic site [15,16,18], an adequate concentration of both metallic and metal–oxide sites is claimed to be somehow necessary for the primary methanol formation [32,33]. In

particular, it should be noted that the adsorption of CO₂ can even occur on bare oxides, while the dissociation of H₂ more efficiently takes place on metallic surfaces; however, a possible activation over metal–oxide surfaces is not ruled out [40,41]. Therefore, this model proposes the use of a unique metal–oxide site for both CO₂ and H₂ adsorption, assumed to be the active center in CO₂ hydrogenation to methanol and in rWGS reaction. Accordingly, the rate equations proposed for this dual-site LHHW mechanism are summarized in Equations (12)–(14):

$$r_{CHM} = \frac{K_{cin_{CHM}} \left(P_{CO_2} P_{H_2}^{1/2} - \frac{P_{CH_3OH} P_{H_2O}}{K_{eq_{CHM}} P_{H_2}^{5/2}} \right)}{\left(1 + K_{CO} P_{CO} + K_{CO_2} P_{CO_2} + K_{CH_3OH} P_{CH_3OH} + K_{H_2O} P_{H_2O} + \sqrt{K_{H_2} P_{H_2}} \right)^2} \quad (12)$$

$$r_{MDD} = \frac{K_{cin_{MDD}} \left(P_{CH_3OH}^2 - \frac{P_{CH_3OCH_3} P_{H_2O}}{K_{eq_{MDD}}} \right)}{\left(1 + K_{H_2O} P_{H_2O} + K_{CH_3OH} P_{CH_3OH} + K_{CH_3OCH_3} P_{CH_3OCH_3} \right)^2} \quad (13)$$

$$r_{rWGS} = - \frac{K_{cin_{rWGS}} \left(P_{H_2O} P_{CO} - \frac{P_{CO_2} P_{H_2}}{K_{eq_{rWGS}}} \right)}{\left(1 + K_{CO} P_{CO} + K_{CO_2} P_{CO_2} + K_{CH_3OH} P_{CH_3OH} + K_{H_2O} P_{H_2O} + \sqrt{K_{H_2} P_{H_2}} \right)^2} \quad (14)$$

Remarkably, since both the steps of CO₂ hydrogenation to methanol and rWGS are characterized by a competition for the same adsorption site (σ), the denominators of Equations (12) and (14) include the terms for CO adsorption ($K_{CO} P_{CO}$) and CH₃OH adsorption ($K_{CH_3OH} P_{CH_3OH}$), respectively, as inhibition contributions for the corresponding reactions.

3.3. Parameter Estimation Method

In order to validate the kinetic expressions, it is necessary to describe the laboratory setup via a predictive model, which is crucial for parameter estimation. In this light, the fixed-bed reactor was assimilated to an isothermal and isobaric PFR (plug flow reactor). The experimental setup was designed to minimize the impact of internal and external diffusional resistances, which were consequently neglected. Therefore, one-dimensional steady-state mass balances (Equations (15) and (16)) were written for each compound along the axial coordinate:

$$\frac{dG_i}{dz} = \frac{1}{L} \sum_{j=1}^3 v_{ij} r_j \quad (15)$$

$$G_i(z=0) = y_i^{in} G_{TOT}^{in} \quad (16)$$

In Equation (15), L represents the reactor length, r_j is the rate of reaction j (Equations (12)–(14)) and v_{ij} is the stoichiometric coefficient of compound i in reaction j . In Equation (16), y_i^{in} is the inlet molar fraction of component i and G_{TOT}^{in} is the total inlet gas hourly space velocity. The balances are, indeed, expressed in terms of space velocity of each compound, noted as G_i , which is here defined as the ratio between the molar flow rate F_i and the total weight of the catalyst M_c (Equation (17)):

$$G_i = \frac{F_i}{M_c} \quad (17)$$

It is important to note that the kinetic expressions of reaction rates (present in Equation (15)) incorporate temperature-dependent equilibrium, kinetic, and adsorption constants. While the former can be derived from the existing literature, being only subject to the thermodynamics of the reactions involved (see Table 1) [48], the latter two constants are specific to the developed catalyst and require experimental data regression for determination. More precisely, the Arrhenius and Van 't Hoff equations express their temperature

dependence and were reparametrized (see Equations (18) and (19)) to improve numerical stability in fitting and decrease numerical correlation between estimated parameters [49,50]:

$$K_{cin_j} = k_{cin_j} \exp \left[-E_{A_j} \left(\frac{T_{rif}}{T_R} - 1 \right) \right] \quad (18)$$

$$K_i = k_i \exp \left[-\Delta H_{ads_i} \left(\frac{T_{rif}}{T_R} - 1 \right) \right] \quad (19)$$

Table 1. Expressions of equilibrium constants' dependence on temperature [48].

Parameter	Temperature Dependence
$K_{eq_{CHM}}$	$10^{\frac{3066}{T} - 10.592} \text{ [bar}^{-2}\text{]}$
$K_{eq_{MDD}}$	$\exp \left(\frac{4019}{T} + 3.707 \cdot \log(T) - 2.783 \cdot 10^{-3} \cdot T + \right.$ $\left. + 3.8 \cdot 10^{-7} \cdot T^2 - 6.561 \cdot 10^4 \cdot T^{-3} - 26.64 \right)$
$K_{eq_{rWGS}}$	$10^{-\frac{2073}{T} + 2.029}$

In Equations (18) and (19), T_{rif} is a reference temperature, assumed to be equal to the experimental mean value of 503.15 K; T_R denotes the reactor temperature; k_{cin_j} and k_i are pre-exponential factors; and E_{A_j} and ΔH_{ads_i} represent energetic terms. The latter were made dimensionless, according to Equations (20) and (21) (where R is the universal gas constant), to decrease differences in magnitude orders and reduce computational costs [51]:

$$E_{A_j} = \frac{E_{A_j}^*}{T_{rif} R} \quad (20)$$

$$\Delta H_{ads_i} = \frac{\Delta H_{ads_i}^*}{T_{rif} R} \quad (21)$$

Both pre-exponential factor and energetic terms were estimated by minimizing the weighted sum ϕ of square relative errors between experimental molar fractions Y_i^k and model outputs \hat{Y}_i^k (Equation (22)) [52]:

$$\phi = \sum_{k=1}^{N_p} \sum_i \left[w_i \cdot \left(\frac{Y_i^k - \hat{Y}_i^k}{Y_i^k} \right)^2 \right] \quad (22)$$

In Equation (22), N_p represents the number of experimental tests, while w_i is the weight assigned to compound i . Optimization was carried out by applying a Matlab algorithm based on the *fmincon* function to find the minimum value, providing a set of 100 experimental data points as input. The starting point was varied in different trials, looking at achieving a global minimum, while an additional set of 60 data points was used to validate the outcomes obtained.

4. Results and Discussion

4.1. Experimental Results

A sensitivity analysis was carried out to evaluate the influence of temperature and pressure on product distribution, as depicted in Figure 1.

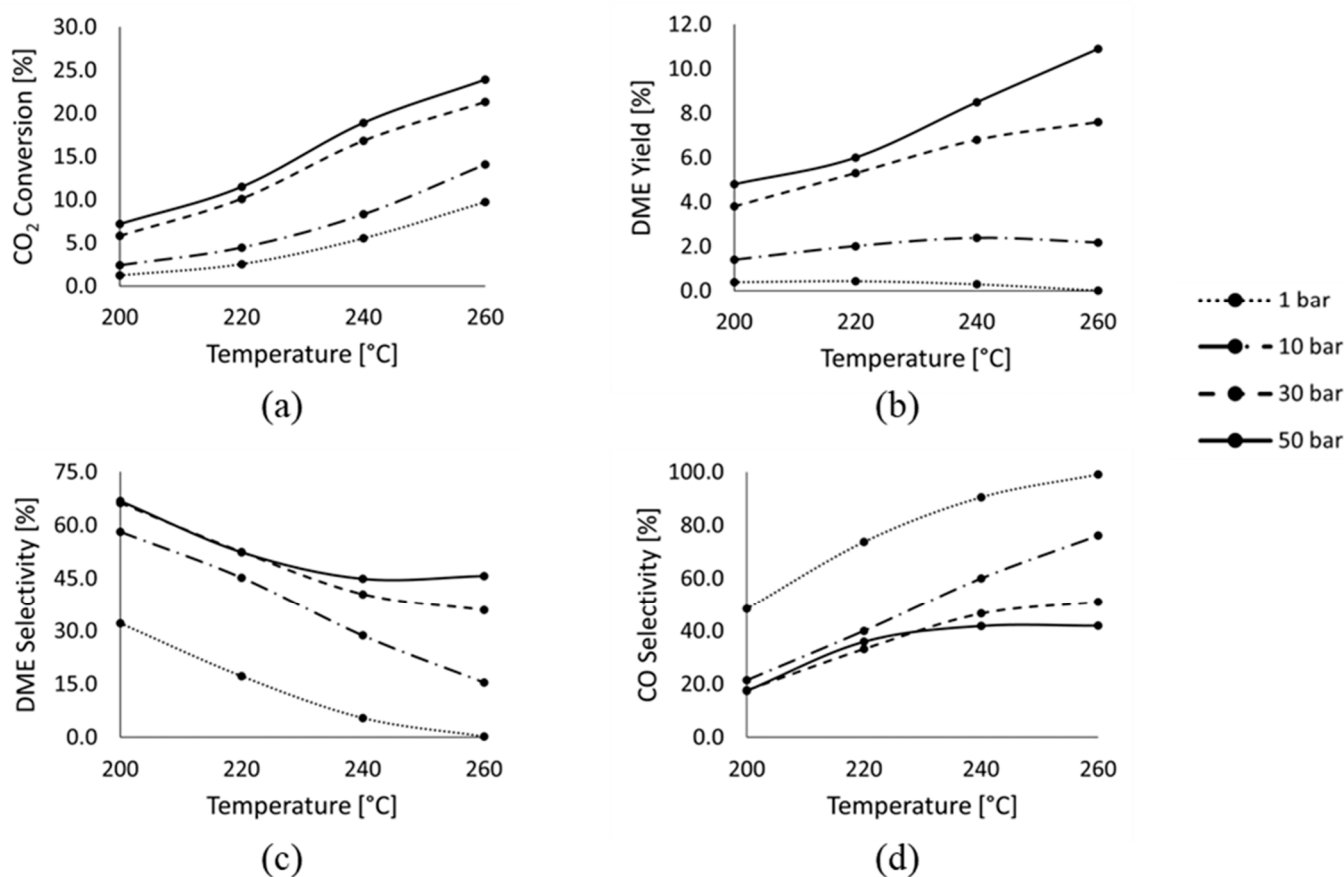


Figure 1. Influence of temperature and pressure on CO₂ conversion (a), DME yield (b), DME selectivity (c), and CO selectivity (d), evaluated with an inlet flow rate of 8800 NL/(kg_{cat} h).

On the whole, the results are consistent with the thermodynamic analysis of the system [10,31,53]. High pressure is preferable during the DME synthesis reaction as it leads to a decrease in the number of moles. On the other hand, an increase in temperature favors CO₂ conversion due to its positive kinetic effect on the reaction rate. However, it simultaneously decreases DME selectivity while raising CO selectivity, in agreement with the thermal tonality both of DME synthesis (exo-) and rWGS (endo-). Although a plateau in DME and CO selectivity was observed above 240 °C for runs performed both at 30 and 50 bar, a further increase in reaction temperature unfortunately triggers severe catalyst deactivation. Indeed, a larger amount of water formed above those conditions dramatically determines morphological changes in the catalyst surface, prompting a drastic activity decay (ca. 20% with respect to the initial activity) in a few hours of time on stream (<20 h) [17,22]. Therefore, these findings not only indicate that the process performance is governed by kinetics [15], but also suggest an optimal compromise between CO₂ conversion and DME selectivity at moderately high pressures (30 or 50 bar) and intermediate temperatures (<260 °C), so as to limit any issue related to catalyst lifetime.

4.2. Reaction Kinetics Investigation Results

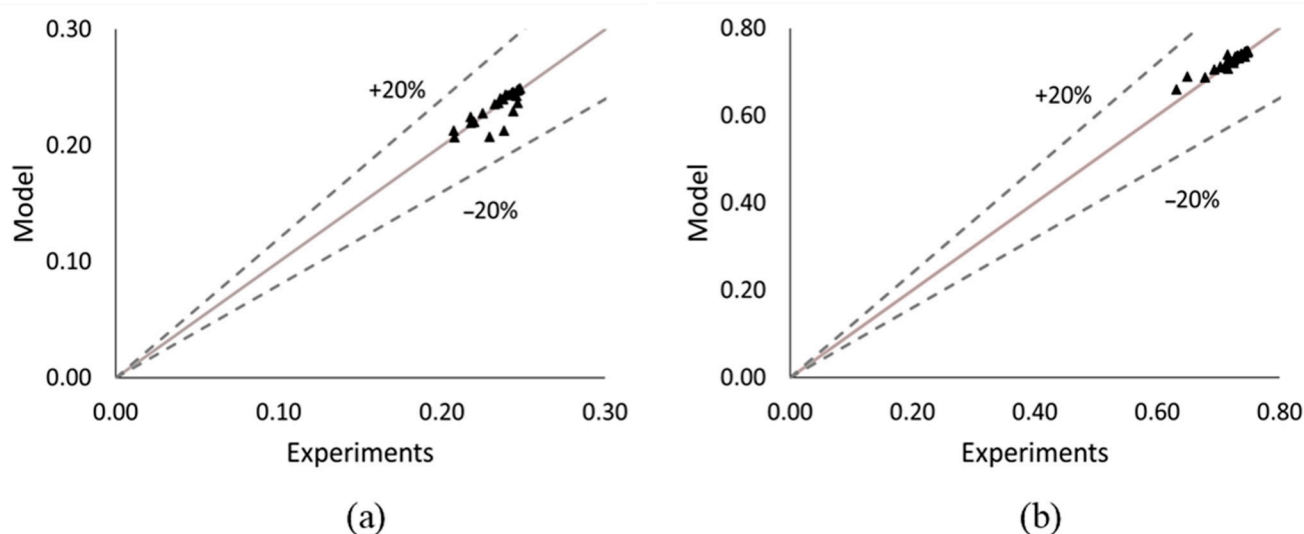
Falling within the typical range of values reported for similar computational models [26], the parameters resulting from the estimation procedure are listed in Table 2, with Figure 2 illustrating the comparison between experimental data and model outputs.

Table 2. Estimated parameters for the proposed kinetic model and corresponding confidence interval with a level of significance equal to 0.05.

Parameter	Unit	Pre-Exponential Factor (k_{cin_j} or k_i)	Energetic Term (E_{A_j} or $-\Delta H_{ads_i}$)
$K_{cin_{CHM}}$	$[mol \cdot g_{cat}^{-1} \cdot s^{-1} \cdot bar^{-3/2}]$	$(2.56 \pm 0.05) \cdot 10^{-4}$	44.95 ± 1.75
$K_{cin_{MDD}}$	$[mol \cdot g_{cat}^{-1} \cdot s^{-1} \cdot bar^{-2}]$	0.26 ± 0.01	99.73 ± 1.21
$K_{cin_{rWGS}}$	$[mol \cdot g_{cat}^{-1} \cdot s^{-1} \cdot bar^{-2}]$	$(6.16 \pm 0.12) \cdot 10^{-3}$	39.52 ± 1.71
K_{H_2}	$[bar^{-1}]$	$(8.54 \pm 0.32) \cdot 10^{-2}$	33.57 ± 1.38
K_{CO_2}	$[bar^{-1}]$	9.56 ± 0.10	14.23 ± 0.93
K_{CO}	$[bar^{-1}]$	8.97 ± 0.49	10.41 ± 0.40
K_{H_2O}	$[bar^{-1}]$	8.02 ± 0.43	22.95 ± 0.86
K_{CH_3OH}	$[bar^{-1}]$	5.70 ± 0.29	7.07 ± 0.11
$K_{CH_3OCH_3}$	$[bar^{-1}]$	5.20 ± 0.10	60.53 ± 0.66

The model provided satisfactory results in calculating the molar fractions of CO₂ and H₂, with an average relative error of 1.5%, which is comparable to the chromatographic tolerance. Moreover, despite an average relative error of around 40%, the model suitably also works in the prediction of the molar fractions of DME and CO, considering a minimal absolute error in the order of 0.001. Only in the case of the other compounds (i.e., CH₃OH or H₂O), the large number of estimated parameters (which is equal to 18) cannot determine a lower discrepancy in the model fitting, so that a relatively small error in absolute terms has a strong impact in relative terms.

In any case, even the highest deviation recorded for large methanol concentrations (see Figure 2f) cannot be considered a relevant issue since the simulation model targets the one-pot synthesis of DME and the methanol selectivity always attains relatively low values (below 10%) under all the experimental conditions applied. More importantly, these results evidently confirmed previous mechanistic clues on the catalytic behavior, with positive effects on the process performance played by an increase in space velocity or reaction pressure. However, further optimization of the hybrid catalyst design is required for higher CO₂ conversion rates below 220 °C, where CO formation can be minimized. Finally, not only the catalyst formulation but also the reactor configuration should be properly designed in order to tackle the issues related to water formed during the process, dramatically affecting catalyst lifetime and process performance.

**Figure 2.** Cont.

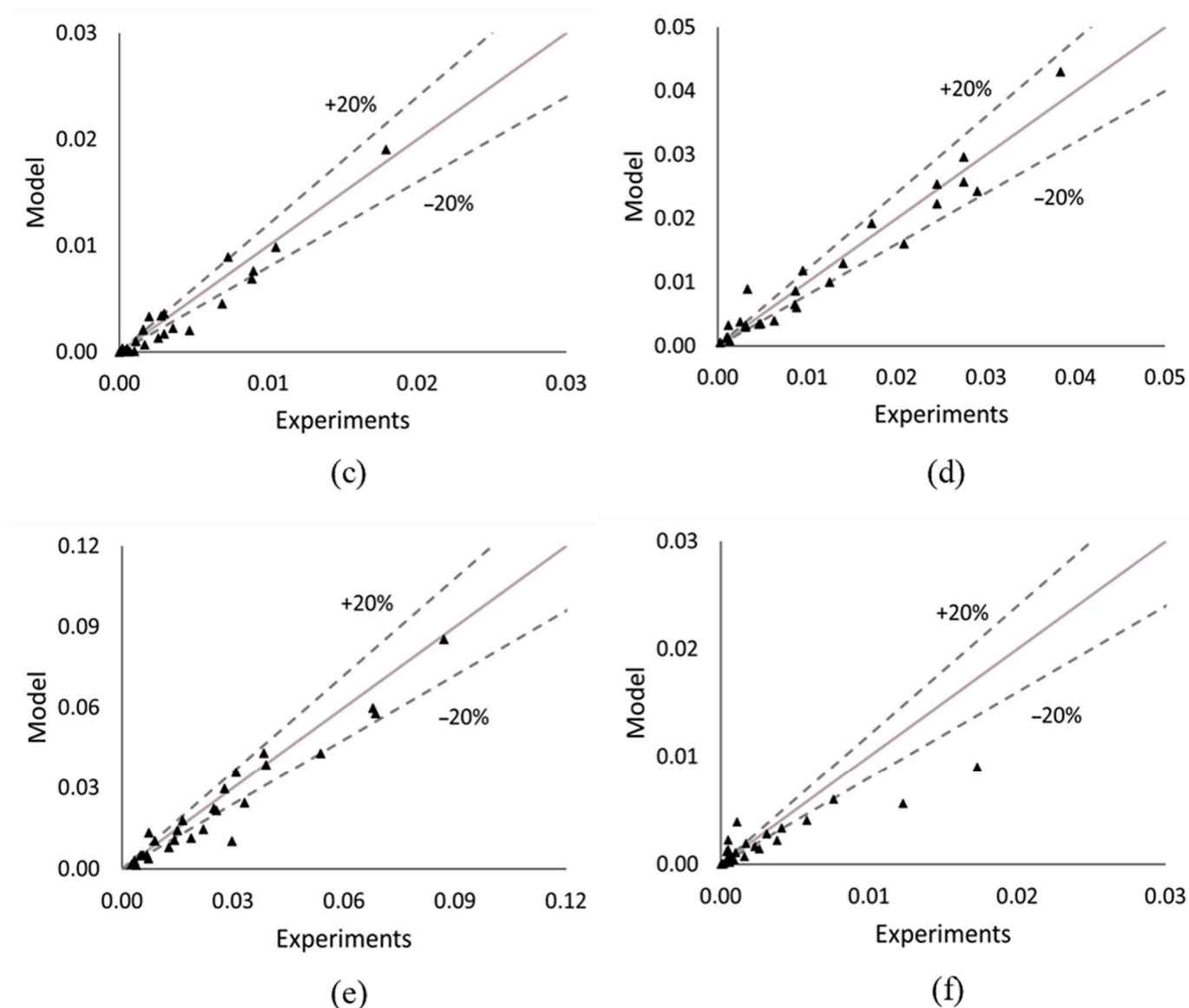


Figure 2. Comparison of experimental data (along the x-axis) and model results (along the y-axis) for the mole fractions of CO₂ (a), H₂ (b), DME (c), CO (d), H₂O (e), and methanol (f).

5. Conclusions

A comprehensive 18-parameter kinetic model was developed for the direct hydrogenation of CO₂ to DME over a hybrid CuO-ZnO-ZrO₂/HZSM-5 catalytic system, previously optimized in terms of composition, preparation method, and tuning of physico-chemical properties. The model was validated in a wide set of reaction pressure, temperature, and space velocity, clearly showing a reliable fitting of the estimated parameters, within an average absolute error of 10^{-4} , both for the process reactants (i.e., CO₂, H₂) and the target product (i.e., DME), with the correspondent experimental values. The accuracy of the model confirmed previous findings on the multi-site nature of the studied process, mainly following the formate route, according to which carbon dioxide is activated at the metal-oxide interface, then reacting with activated hydrogen migrating via spillover from metallic copper sites. Afterwards, formate species are hydrogenated to methanol, which, in line with an associative pathway, quickly turns into DME by condensation with another adsorbed methanol molecule over the acidic sites of the zeolite. Overall, the adequate accuracy in the prediction of the compositions related to the most abundant process compounds makes the model promising for further implementation within a plant simulation, prompting an

estimation of the costs associated with the direct DME synthesis process for an assessment of its actual convenience on an industrial scale.

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Appendix A Fugacity Coefficient Evaluation

When working with real mixtures, it is crucial to determine the fugacity coefficient v_i of each component to measure its deviation from the ideal state. This can be performed by taking advantage of the fact that $\ln v_i$ is the partial molar property of $\ln v_m$, where v_m is the fugacity coefficient of the mixture, as described in Equation (A1) [54]:

$$\ln v_i = \ln v_m - \sum_{j \neq i} y_j \left(\frac{\partial \ln v_m}{\partial y_j} \right)_{P, T, y_{k \neq j \neq i}} \quad (\text{A1})$$

In Equation (A1), y_i denotes the molar fraction of compound i , while v_m was computed through the two-term virial equation of state (Equations (A2)) [55]:

$$\ln v_m = \frac{B_m P}{RT} \quad (\text{A2})$$

In Equation (A2), P is the reaction pressure, T is the reaction temperature, and R is the universal gas constant, whereas B_m denotes the second virial coefficient of the mixture, which can be evaluated from binary coefficients B_{ij} , as in Equation (A3):

$$B_m = \sum_{i=1}^7 \sum_{j=1}^7 y_i y_j B_{ij} \quad (\text{A3})$$

The binary coefficients B_{ij} can be calculated through Tsonopoulos’s correlation (Equations (A4)–(A7)), which extends the virial equation to polar fluids [56]:

$$\frac{B_{ij} P_{C_{ij}}}{RT_{C_{ij}}} = f^{(0)}(T_{R_{ij}}) + \omega_{ij} f^{(1)}(T_{R_{ij}}) + f^{(2)}(T_{R_{ij}}) \quad \text{where } T_{R_{ij}} = \frac{T}{T_{C_{ij}}} \quad (\text{A4})$$

$$f^{(0)}(T_{R_{ij}}) = 0.1445 - \frac{0.33}{T_{R_{ij}}} - \frac{0.1385}{T_{R_{ij}}^2} - \frac{0.0121}{T_{R_{ij}}^3} - \frac{0.000607}{T_{R_{ij}}^8} \quad (\text{A5})$$

$$f^{(1)}(T_{R_{ij}}) = 0.0637 + \frac{0.331}{T_{R_{ij}}^2} - \frac{0.423}{T_{R_{ij}}^3} - \frac{0.008}{T_{R_{ij}}^8} \quad (\text{A6})$$

$$f^{(2)}(T_{R_{ij}}) = \frac{a_{ij}}{T_{R_{ij}}^6} - \frac{b_{ij}}{T_{R_{ij}}^8} \quad (\text{A7})$$

The terms $f^{(0)}$, $f^{(1)}$ and $f^{(2)}$ represent the impact of intermolecular forces from spherically symmetrical molecules, molecular orientation, and polarity, respectively. All the

critical binary properties figuring in Equation (A4) are evaluated from the critical properties (i.e., critical volume v_C , critical temperature T_C , and critical pressure P_C) and acentric factors ω of pure components, as detailed in Equations (A8)–(A11) [57]:

$$v_{C_{ij}} = \frac{\left(v_{C_i}^{\frac{1}{3}} + v_{C_j}^{\frac{1}{3}}\right)^3}{8} \quad (\text{A8})$$

$$T_{C_{ij}} = \sqrt{T_{C_i} T_{C_j}} \quad (\text{A9})$$

$$P_{C_{ij}} = \frac{T_{C_{ij}}}{2v_{C_{ij}}} \left(\frac{P_{C_i} v_{C_i}}{T_{C_i}} + \frac{P_{C_j} v_{C_j}}{T_{C_j}} \right) \quad (\text{A10})$$

$$\omega_{ij} = \frac{\omega_i + \omega_j}{2} \quad (\text{A11})$$

The a_{ij} and b_{ij} parameters in Equation (A7) measure polar interactions and are exclusively non-zero for pairs of polar fluids. In such instances, they are assumed to be equal to the average of the constants of pure components, as outlined in Equations (A12) and (A13) [55]:

$$a_{ij} = \frac{a_i + a_j}{2} \quad (\text{A12})$$

$$b_{ij} = \frac{b_i + b_j}{2} \quad (\text{A13})$$

In the specific case analyzed, the only polar fluids considered are water, methanol, and dimethyl ether, whose parameters are listed in Table A1.

Table A1. a and b parameters for Equations (A12) and (A13) [55].

Compound	a	b
Water	0.0279	0.0229
Methanol	0.0878	0.0560
Dimethyl Ether	−0.01513	0

Based on the above, in Table A2 are reported the fugacity coefficients as determined under the most severe operating conditions (i.e., 200 °C and 50 bar). The analysis indicates that, in nearly all cases, these coefficients are very close to one, resulting in a negligible correction to the partial pressure. Moreover, although nitrogen shows a measurable deviation from its ideal behavior, its fugacity is not present in the kinetic expressions of the reaction rates (Equations (12)–(14)). Therefore, it was preferred to maintain the validity of the ideal gas hypothesis against a relatively modest enhancement in the model accuracy only achievable behind an increased computational complexity.

Table A2. Fugacity coefficients for different compositions, evaluated at 200 °C and 50 bar.

y_{CO_2}	y_{H_2}	y_{DME}	y_{CO}	y_{Meth}	y_{H_2O}	y_{N_2}	ϕ_{CO_2}	ϕ_{H_2}	ϕ_{DME}	ϕ_{CO}	ϕ_{Meth}	ϕ_{H_2O}	ϕ_{N_2}
0.23	0.69	-	-	-	-	0.08	0.97	1.03	-	-	-	-	0.24
0.21	0.67	-	0.02	-	0.02	0.08	0.97	1.03	-	1.02	-	0.96	0.24
0.21	0.67	0.005	0.007	0.002	0.016	0.09	0.97	1.03	0.98	1.01	0.95	0.96	0.24
0.20	0.63	0.01	0.03	0.01	0.03	0.09	0.97	1.03	0.97	1.01	0.94	0.95	0.24
0.19	0.62	0.02	0.02	0.01	0.06	0.08	0.96	1.04	0.97	1.02	0.93	0.94	0.24

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