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Title: Another step in the path from macroseismic fields to probabilistic modeling of attenuation

It is beyond doubt that information provided by historical seismology can provide a valuable aid to understanding the evolution of the seismic phenomenon; we cannot therefore renounce to analyze macroseismic intensity data in order to assign the seismic hazard of an area; well aware that this requires the development of specific models and techniques different from those adopted in the study of instrumental data.
In this context the issue of macroseismic attenuation is also included. From the beginning of the 2000s, a path has been undertaken in the direction of probabilistic modeling of macroseismic attenuation, from the perspective of treating the whole process as random and not only of adding of a Gaussian error to the empirical relationships between magnitude, distance and intensity at site. In fact, the irregularities in the shape and extent of macroseismic fields depend on various factors and their potential interactions, like topography, pattern of population density, characteristics of regional geology, that cannot be reduced to a measurement error. Among the cornerstones of this approach there is the respect, as far as possible, of the ordinal nature of macroseismic intensity. According to this assumption, the intensity at any site is considered as an integer random variable that varies between 1 and the epicentral intensity I_0.

Beta-binomial model
As for any model, we started by simplifying reality and assuming that the attenuation trend is circular. Conditioned on the epicentral intensity I_0 and on a fixed epicentral distance, the intensity I_s at a given site was assumed to have a binomial distribution with parameter *p*. Given a point source and circular isoseismal lines bounding the points of equal intensity, one draws *J* circular bins around the epicenter and supposes that in all of the sites within each *j*-th bin, I_s, so as ΔI , has the same binomial distribution with parameter p_j, i.e.:

$$Pr(I_s = i|I_0 = i_0, p_j = \binom{i_0}{i} p_j^i (1 - p_j)^{(i_0 - i)}. \quad (1)$$

In its turn, each p_j has a beta distribution with hyperparameters α_j and β_j that, according to the Bayesian approach, are assigned by exploiting the information drawn from previous databases; then the posterior mean of each p_j provides the estimate of these paramaters. To extend the value of p at any epicentral-site distance one approximates the estimates $\hat{p}_j$ by the smoothing inverse power function *g(d)* = $\left[\frac{c_1}{c_1+d}\right]^{c_2}$, whose coefficients c_1, c_2 are estimated by the method of least squares. In this way one is able to forecast in terms of macroseismic intensity I_s at site the damage scenario that a future earthquake of given intensity I_0 could cause by the *smoothed* binomial probability distribution obtained by replacing p_j with *g(d)* in Eq. (1) and by using the mode i_{smooth} of this distribution as forecast value of the intensity I_s at any site distant *d* from the epicenter (Rotondi, Zonno (2004)). Three criteria were used to validate the results: the logarithmic scoring rule, the ratio between the probability that the fitted model assigns to an observation and the probability of the forecast value, and the absolute discrepancy between observed and estimated intensities at site.

Construction of learning sets
Another crucial point is the continuous updating of the model parameters in the light of the most recent databases and on the basis of the identification of sets of macroseismic fields homogeneous from the attenuation viewpoint. As regards the first point, we refer to the results obtained from the analysis of the Italian Databases DBMI15; about the second point, each of 538 macroseimic fields of DBMI15 has been characterized through summaries of the spatial distribution of the intensity decay; in particular, location and dispersion measures (mean and median values and 3rd quantile) have been computed for each set of distances from the epicenter to the sites where the same intensity I_s is recorded. All this information was collected in a matrix to which a hierarchical agglomerative method for cluster analysis was then applied obtaining 4 classes A, B, C, and D with similar attenuation trends decreasing in steepness. So, given an earthquake of intensity I_0 belonging, for instance, to the class B, the parameters $p_j$are estimated on the basis of the intensity records related to all the other earthquakes of the same class and with the same epicentral intensity (Zonno, Rotondi, Brambilla (2009)).

Anisotropic attenuation trend
Actually, it is well-known that, drawing the isoseismal lines of many earthquakes, the attenuation trend appears quite complex and not simply circular. Since it was observed that more rapid decay can be visibly recognizable along the direction perpendicular to that of the fault, it can be appropriate to use an elliptical shape for the isoseismal lines when information on the fault rupture that caused an earthquake, in particular on the direction and length of the rupture, is available (Agostinelli and Rotondi (2016)). The solution we found to do that, consists in a plane transformation that turns the ellipse of major axis equal to the fault rupture into the circle of radius equal to the width of the first bin; then one repeats the estimation procedure in the transformed plane and then one associates the estimated probability

distribution of the intensity I_s that will be felt at a site to the original position of that site (Rotondi *et al.* (2016)).
The problem arises when we do not have information on the causative fault, e.g. when the fault does not appear on the surface, being completely hidden underneath surface rock layers (blind fault). Taking into account that the shape of the area of highest intensity is generally elongate along the direction of the active fault plane, we propose to deduce the fault dimensions from those of the ellipsoid hull (or spanning ellipsoid) that includes all the sites with I_0 - I_s 1, i.e. the ellipsoid of minimal area such that all given points lie just inside or on the boundary of the ellipsoid. The idea is not known because already in 1973 Shebalin proposed to estimate the dimension and orientation of a seismogenic fault based on the ellipticity of the highest degree isoseismals; however, his and other researchers' work relied only on hand-drawn, hence inherently arbitrary, isoseismals. Our solution is closer to the attempt made by De Rubeis *et al*. (1992), that is, we obtain information just on strike and dimensions of the meizoseismal area by running an algorithm of the free R software. The method has been first tested on some volcanic earthquakes of Etna area for which the fault is known (Azzaro *et al.* (2013)) and the positive results were also supported by the scoring criteria used to compare the different estimated damage scenarios. Then we have also applied the algorithm to the macroseismic field of some strong Italian earthquakes.

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