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A quadratic optimization approach  
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## ABSTRACT

Designing a supply chain to comply with environmental policy requires awareness of how work and/or production methods impact the environment and what needs to be done to reduce those environmental impacts and make the company more sustainable. This is a dynamic process that occurs at both the strategic and operational levels. However, being environmentally friendly does not necessarily mean improving the efficiency of the system at the same time. Therefore, when allocating a production budget in a supply chain that implements the green paradigm, it is necessary to figure out how to properly recover costs in order to improve both sustainability and routine operations, offsetting the negative environmental impact of logistics and production without compromising the efficiency of the processes to be executed. In this paper, we study the latter problem in detail, focusing on the CO<sub>2</sub> emissions generated by the transportation from suppliers to production sites, and by the production activities carried out in each plant. We do this using a novel mathematical model that has a quadratic objective function and all linear constraints except one, which is also quadratic, and models the constraint on the budget that can be used for green investments caused by the increasing

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internal complexity created by large production flows in the production nodes of the supply network. To solve this model, we propose a multistart algorithm based on successive linear approximations. Computational results show the effectiveness of our proposal.

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## 1. Introduction

Green management is a philosophy that is becoming prevalent in industry. It is a new way of thinking stimulated by the awareness of the need to respect the environment as it is exposed to the negative impacts, such as CO<sub>2</sub> emissions, caused by production and logistics operations worldwide. Indeed, industrial companies have a huge carbon footprint; their manufacturing and logistics activities account for more than half of all global carbon dioxide equivalent (CO<sub>2</sub>e) emissions from fuel combustion ([15]). Given current trends, emissions from manufacturing and logistics would need to fall by about 45% by 2030 to meet the 1.5°C target of the Paris Agreement to limit global temperature rise ([8], [9]). Leading companies have recognized the need for action and are implementing initiatives to decarbonize their operations. In addition, some companies have gone further and begun requiring their supply chain business partners to commit to decarbonization as well. The result is a convergence of environmental and economic imperatives for which all industrial companies must be prepared. Companies are planning to implement decarbonization measures and tend to allocate a portion of their manufacturing investment budget to decarbonization measures in the coming years. A study conducted in the literature states that companies will spend more than 10% of their available investment budget on decarbonization in the next few years.

Clearly, green management needs to be integrated into traditional supply chain management approaches, taking into account and complementing different paradigms to improve performance, such as manufacturing processes. This integration of management approaches leads to the goal of maximizing the efficiency of business processes while minimizing the impact on the environment, eliminating all types of waste, and ensuring better use of all types of resources.

As with all integrated approaches, managers working with them must pay particular attention to tradeoffs. Indeed, some measures may be beneficial for both process performance and environmental protection, while some others may not. In the latter cases, the overall impact of two different decisions may be the same in terms of economic costs, but one decision may result in higher costs for ensuring high efficiency/performance and lower environmental protection costs, while the second decision has the opposite effect. Finding a balance between the two costs is therefore an issue that should be explored to ensure a fair use of the total economic resources available to manage a supply chain. It

is therefore a matter of solving a strategic problem that involves understanding how to allocate a budget to each facility in the supply chain in order to achieve both objectives, i.e., maximizing the effectiveness of the activities to be carried out in each production center, and minimizing the environmental impact generated by the activities processed there.

Seeking to minimize the CO<sub>2</sub> emissions caused by the transportation from the suppliers to the production facilities and by the production activities carried out in each facility, in this paper we propose a novel mathematical model able to find an optimal budget allocation for each facility in a two-stage supply chain, where the first stage is represented by the suppliers and the second stage by the production centers. In modeling this system, economic resources are consumed by both green activities associated with transportation and production, and by activities needed to manage the complexity resulting from increasing commodity flows in each facility, which can adversely affect the efficiency of the system. The mathematical model includes capacity constraints on both sides of the supply chain, i.e., suppliers are allowed to serve plants within a certain service level and plants cannot serve market demand beyond a certain plant capacity. In addition to capacity constraints, each plant is given a budget to manage its processes. This budget is a linear function of the total amount of products reaching the plant from suppliers. However, we need to consider the effect of a growing incoming flow, which, on the one hand, increases the budget that can potentially be used to manage green operations by that plant, but, on the other hand, also increases the complexity associated with managing higher flows through the plant. The latter effect, i.e., the increasing costs caused by the direct and indirect cost components of an increasing flow, is modeled using a quadratic function that defines a constraint that reduces the available portion of a facility's budget that can be spent on green operations.

The resulting model is a quadratic programming problem with an additional quadratic constraint, known in the literature as quadratic constrained quadratic programming (QCQP) where the objective function has an indeterminate Hessian. To solve this model, we implemented an iterative scheme given by the hybridization of two known approaches. The core of the generic iteration is the linear approximation of the nonlinear constraint and the objective function; namely, we consider a feasible solution of the QCQP problem and approximate the quadratic constraint and the quadratic function around this feasible solution using a first order Taylor polynomial. We then solve the linear problem so obtained. This linear approximation phase is executed iteratively by a multistage general engine that ensures an effective diversification offering each time a different feasible solution to the problem. The algorithm stops when either a maximum number of iterations or a time limit is met. Computational results and a comparison with a commercial solver show the effectiveness of our proposal on synthetic instances. To give further insight on the performance of our proposal, we compared the latter to a state of art model on a real world instance.

The remainder of the paper is organized as follows. The literature review is reported in Section 2. Section 3 describes the formulation of the problem. Section 4 contains the

solution approach. Section 5 discusses the computational results and, finally, Section 6 draws conclusions.

## 2. Literature review

The literature on mathematical programming for green supply chain design and green logistics covers a wide range of methods, applications, and levels of decision-making, and it continues to grow. However, it is still difficult to find approaches that comprehensively consider financial, environmental, and operational aspects. [24] discuss combinatorial methods applied to green logistics, defined as all activities concerned with the sustainable production and distribution of goods, taking into account environmental and social factors. The paper examines combinatorial optimization methods used in reverse logistics, waste management, and vehicle routing problems with green aspects. In this paper, no attention is paid to facilities. In the review of [5], a different selection of articles is made to study the application of operations research methods to green logistics. The authors analyze OR methods for green logistics for the drivers of transportation, inventory, and asset supply chain. From the review emerges that, while several articles apply OR for green transportation and inventory, facilities are not yet sufficiently considered. However, often a facility is also a processing hub, and the impact of reducing green emissions for operational activities can be remarkable. In the review of [25], it can be noted that most of the works are related to emission control applications, paying particular attention to the transportation phase, using linear programming or linearization approaches. In some cases, emissions are considered nonlinearly, as in the work of [23], which use a concave function to model warehouse emissions. Concave relationships are also treated by [7], where emissions grow nonlinearly with transported weight. The resulting concave network design problem is solved using a Lagrangian relaxation based approach. In [12], the supply chain design problem is applied to a green product. In this case, emissions are considered as constraints along with the required service level. The model is solved using a decomposition approach where subproblems are solved independently. Multi-objective aspects are addressed in the work of [21], where financial and environmental objectives are considered. The resulting mathematical model is solved considering the weighted sum of different objective functions. In [4], multiobjective optimization for green corridors is treated using bilevel programming. In [17], a mixed-integer linear program is presented for designing a network with emissions accounting as a carbon tax in both production and transportation. Stochastic aspects are addressed in [20] to solve aggregate production scheduling of green supply chains. The authors also consider nonlinear convex relationships between the ordered quantity and the unit cost of the product, nonlinear shortage costs, and flexible lead times. These relationships are linearized using piecewise linear functions and the model is solved using a commercial solver.

As for the solution approaches of non convex problems, one of the most general and widely used methods is proposed by [2], who develop a very effective approach

based on approximated duals used in general algorithmic frameworks. [13] determine specific optimality conditions in nonconvex quadratic problems and applied them to the weighted least squares minimization problems with ellipsoidal constraints. [10] use quadratic convex relaxation on 0-1 quadratic constrained quadratic programming dealing with equality constraints, in conjunction with methods for convexifying the objective value based on the minimum eigenvalue. [14] propose the use of 0-1 mixed integer programs to reformulate binary QCQP; they showed that this approach can lead to tighter linear relaxations. Several heuristic approaches based on relaxations and local methods are combined by [22] in a software tool. [11] exploit the use of linear relaxations in a branch-and-bound reduction algorithm and improved the dual relaxation gap for some specific problems. One of the most effective approaches is the use of semidefinite programs as relaxation-based approaches to QCQP, which can be further improved if certain conditions are met. Non-negative relaxation semidefinite programs and first-order relaxation linearization techniques are compared by [1] and [3]. [27] generate convex relaxations of the original QCQP using decomposition of nonconvex quadratic functions. [18] develop a branch-and-bound approach; they pointed out that improvements can be obtained by considering sensitive eigenvalues that have a greater impact on the relaxation gap.

### 3. Problem definition and mathematical formulation

In this section we formally define the framework of the problem, starting with the notations. We then present the mathematical model.

#### 3.1. Problem notation

Given is a two-layer supply chain network modeled by a bipartite graph  $G = (S, F, A)$ , where  $S$  is a partite set of nodes representing the set of suppliers,  $F$  is the other partite set modeling the set of facilities, and  $A$  is the set of arcs modeling the links between pairs of nodes belonging to the Cartesian product  $S \times F$ . We assume that there is a customer zone associated with an aggregate customer demand  $d$  that must be satisfied by the facilities. The latter have a finite capacity, say  $c_j$ , with  $j \in F$ , and we further assume that each supplier  $k \in S$  also has a finite capacity  $s_k$  to serve the facilities' requests.

In an effort to minimize CO<sub>2</sub> emissions caused both by transportation from suppliers to facilities and by the facilities themselves during the production phase, we aim at finding a strategy to allocate a budget  $b$  for the implementation of green strategies to achieve the green goals of each facility, taking into account that coordination and flow processing activities must also be carried out and, therefore, a percentage of this budget is also consumed by the latter activities, which proves to be an obstacle to the use of the entire budget for green activities. In other words, we assume that the budget  $b_j$  that must be allocated to plant  $j$  depends on the amount of flow that reaches that plant

from the suppliers, and that an increase in the incoming flow in a plant is simultaneously associated with increasing costs to coordinate the operations and to ensure the right level of effectiveness of the system. This results in the need to allocate a portion of the budget to handle the increasing complexity of the system as larger flows are processed in a facility. Therefore, the total budget  $b_j$  allocated to the plant  $j \in F$  can only theoretically be used to implement green strategies, since the actual amount that can be used is less than or equal to a limit represented by a quadratic function as defined below.

Let us first define the problem notation. The sets and parameters are:

- $S$ : the set of supplies;
- $F$ : the set of facilities;
- $k$ : index for suppliers;
- $j$ : index for facilities;
- $d$ : the demand the supply chain must satisfy;
- $b$ : the budget available for emission containmentment;
- $s_k$ : the supply capacity of supplier  $k \in S$ ;
- $c_j$ : the capacity of facility  $j \in F$ ;
- $\phi$  emission conversion factor equal to  $et \cdot ef$  where  $et$  is the amount of CO<sub>2</sub> emission generated by each unit of flow associated on arc  $(i, i') \in A$  and  $ef$  is the emission reduction factor per unit of budget invested in green technologies.

The decision variables are:

- $b_j$ : economical budget available at facility  $j \in F$ ;  $b_j \geq 0$ ;
- $x_{kj}$ : the flow of product from node  $k \in S$  to node  $j \in F$ ;  $x_{kj} \geq 0$ ;
- $z_j$ : the environment protection investment in facility  $j \in F$ ;  $z_j \geq 0$ .

The variables  $z_j, \forall j \in F$ , represent the investments made in each facility  $j \in F$  for environmental protection. More precisely, a higher value of  $z_j$  corresponds to a larger environmental protection investment and results in a lower CO<sub>2</sub> emission.

### 3.2. The mathematical model

Once defined sets, parameters, and variables, we define the objective function  $f(\mathbf{x}, \mathbf{z})$  of our model as follows:

$$\min f(\mathbf{x}, \mathbf{z}) = \sum_{k \in S} \sum_{j \in F} \phi x_{kj} (b_j - z_j) \quad (1)$$

The objective  $f(\mathbf{x}, \mathbf{z})$  measures the total CO<sub>2</sub> emission in the entire supply chain. The constraints of the problem are as follows:

**Table 1**  
Units of measurement of parameters and variables.

Parameter/Variable	Unit of measurement
$d$	units of flow
$s_k$	units of flow
$x_{kj}$	units of flow
$c_j$	units of flow
$\phi$	[gCO <sub>2</sub> /(€·unit of flow)]
$et$	[gCO <sub>2</sub> / unit of flow]
$ef$	€ <sup>-1</sup>
$f(\mathbf{x}, \mathbf{z})$	gCO <sub>2</sub>
$z$	€
$b$	€

$$\begin{aligned}
 b_j &= \frac{b \sum_{k \in S} x_{kj}}{d}, & \forall j \in F, \\
 \sum_{j \in F} x_{kj} &\leq s_k, & \forall k \in K, \\
 \sum_{k \in S} x_{kj} &\leq c_j, & \forall j \in F, \\
 z_j &\leq b_j \left[ 1 - \frac{\sum_{k \in S} x_{kj}}{c_j} \right], & \forall j \in F, \\
 \sum_{k \in K} \sum_{j \in F} x_{kj} &= d.
 \end{aligned} \tag{2}$$

The first constraint defines the maximum budget allocated to each facility  $j \in F$ . This is the total budget multiplied by the flow entering the facility and divided by the total demand  $d$ . The second constraint limits the flow from each supplier  $k \in S$  to its maximum capacity, while the third constraint imposes a maximum capacity on each facility  $j \in F$ . The fourth constraint limits, for each plant  $j \in F$ , the investment  $z_j$  dedicated to CO<sub>2</sub> reduction to a percentage, i.e.  $\left( 1 - \frac{\sum_{k \in S} x_{kj}}{c_j} \right)$  of the budget  $b_j$  allocated to plant  $j$  to model the coordination costs associated with large outflows in  $j$  that require additional investment in logistical operations in manufacturing. The fifth constraint ensures the satisfaction of total customer demand  $d$ .

If we now put the first constraint into the objective function and into the fourth condition, we get the mathematical program QP :

$$\begin{aligned}
 \min \quad & \sum_{k \in S} \sum_{j \in F} \phi x_{kj} \left( \frac{b}{d} \sum_{k' \in S} x_{k'j} - z_j \right) \\
 & \sum_{j \in F} x_{kj} \leq s_k, & \forall k \in K, \\
 & \sum_{k \in S} x_{kj} \leq c_j, & \forall j \in F, \\
 & z_j \leq \frac{b \sum_{k \in S} x_{kj}}{d} \left[ 1 - \frac{\sum_{k \in S} x_{kj}}{c_j} \right], & \forall j \in F, \\
 & \sum_{k \in S} \sum_{j \in F} x_{kj} = d.
 \end{aligned} \tag{3}$$

Note that the objective function is a quadratic function, as is the right-hand side of the third constraint, which bounds the investment  $z_j$  that can be used to implement green strategies in facility  $j \in F$ .

For the sake of completeness in Table 1 we report the units of measurement of parameters and variables.

### 3.3. Analysis of the objective function

In the proposed objective function, we have two contributions defining emissions. On the one hand there is the term  $b_j - z_j$  which decreases over increasing values of the investment in green technologies. In other words, we assume that there is contribution given by technologies available in each production plant in reducing the emission level.

Indeed, manufacturing machines produce different amount of CO<sub>2</sub> per unit of product worked based on their energy consumption which, in turn, depends on their technology level. Establishing the right level of technology to be used in a plant to limit the emissions is an issue, and should be carefully taken into account in association to the amount of products worked in the plant. The relations between green investment and carbon emissions have been analysed in several studies, in particular applied to the manufacturing industry. [16] presented the results of a study in China demonstrating the inverse relation between green investments and carbon emissions. The short and long-term green investments elasticity of carbon emissions showed that increasing of 1% the level of the green investments conducts to a 0.071% and a 0.085% reduction of the short and long-term carbon emission levels, respectively. This reveals that green investments have negative and statistically significant relationship with carbon emissions. [19] conducted a similar empirical study concluding that a 1% increase in technology innovations leads to reduce CO<sub>2</sub> emissions by 0.14635% and 0.28375% the same terms.

Further pursuing this aspect, we can find the study by [6] focusing on milling machines which are among the most critical equipment in manufacturing supply chains. The authors assess the carbon emissions of different milling machines in different production settings. Considering the emissions produced by these machines in their operative life, they range from a minimum of 93 grams of CO<sub>2</sub> per worked part to 160 grams of CO<sub>2</sub> per worked part. Using these values, for ease of exemplification of the objective function behavior, considering an average emission per part equal to 126.5 grams of CO<sub>2</sub> along with an emission reduction factor per unit of budget invested in green technologies equal to  $0.2 \cdot 10^{-4} \text{ €}^{-1}$ , we get a conversion factor  $\phi = et \cdot ef = 126.5 \cdot 0.00002 = 0.00253 \text{ gCO}_2 / \text{€} \cdot \text{part}$ .

Now, once assumed that, based on the level of the green investment  $z_j$  in a plant  $j$ , we can transform by means of  $\phi$  this investment in an emission level, we can also see that the number of worked products plays a relevant role in determining the overall level of emissions. Indeed, since  $\phi \cdot (b_j - z_j)$  is an emission level per unit of product, in order to wholly define the objective function  $f$  we have to take into account the overall number of worked products in plant  $j$  which equals the flow entering that facility, i.e.,  $\sum_{k \in S} x_{kj}$ .

If we consider a simple scenario with just one plant, a number of worked parts equal to 100,000 units, and an assigned budget  $b_1 = b = 1,000,000 \text{ €}$ ,  $f$  can be depicted as in the chart of Fig. 1, when the green investment  $z_1 = z$  ranges from 0 to 500,000 €. As it can be seen, an investment of 500,000 € can decrease the emissions from around 18 to 10 tons of CO<sub>2</sub>.



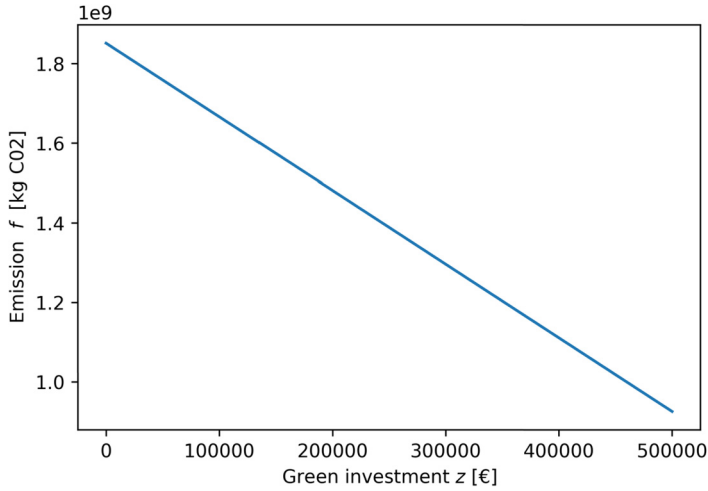


Fig. 1. The behavior of the objective function over varying green investment values for a fixed flow pattern.

For ease of completeness, we conclude the section defining the objective function explicitly on a gadget network with  $|S| = 3$ ,  $|F| = 2$ , and  $\phi = 1$ . We have

$$\begin{aligned}
 f(\mathbf{x}, \mathbf{z}) &= \sum_{k=1}^3 \sum_{j=1}^2 x_{kj} \left( \frac{b}{d} \sum_{k'=1}^3 x_{k'j} - z_j \right) \\
 &= x_{11} \left( \frac{b}{d} (x_{11} + x_{21} + x_{31}) - z_1 \right) && + x_{21} \left( \frac{b}{d} (x_{11} + x_{21} + x_{31}) - z_1 \right) \\
 &+ x_{31} \left( \frac{b}{d} (x_{11} + x_{21} + x_{31}) - z_1 \right) && + x_{12} \left( \frac{b}{d} (x_{12} + x_{22} + x_{32}) - z_2 \right) \\
 &+ x_{22} \left( \frac{b}{d} (x_{12} + x_{22} + x_{32}) - z_2 \right) && + x_{32} \left( \frac{b}{d} (x_{12} + x_{22} + x_{32}) - z_2 \right)
 \end{aligned}$$

$$\begin{aligned}
 f(\mathbf{x}, \mathbf{z}) &= \frac{b}{d} (x_{11}^2 + x_{11}x_{21} + x_{11}x_{31} + x_{21}x_{11} + x_{21}^2 + x_{21}x_{31} + x_{31}x_{11} + x_{31}x_{21} + x_{31}^2 \\
 &+ x_{12}^2 + x_{12}x_{22} + x_{12}x_{32} + x_{22}x_{12} + x_{22}^2 + x_{22}x_{32} + x_{32}x_{12} + x_{32}x_{22} + x_{32}^2) \\
 &- x_{11}z_1 - x_{21}z_1 - x_{31}z_1 - x_{12}z_2 - x_{22}z_2 - x_{32}z_2 \\
 &= \frac{b}{d} (x_{11}^2 + x_{12}^2 + x_{13}^2 + x_{21}^2 + x_{22}^2 + x_{32}^2 + 2x_{11}x_{21} + 2x_{11}x_{31} + 2x_{21}x_{31} \\
 &+ 2x_{12}x_{22} + 2x_{12}x_{32} + 2x_{32}x_{22}) - x_{11}z_1 - x_{21}z_1 - x_{31}z_1 - x_{12}z_2 - x_{22}z_2 \\
 &- x_{32}z_2
 \end{aligned}$$

Let us now merge  $\mathbf{x}$  and  $\mathbf{z}$  in a unique vector  $\mathbf{y}$  as follows:

$$\mathbf{y} = \begin{pmatrix} x_{11} \\ x_{12} \\ x_{21} \\ x_{22} \\ x_{31} \\ x_{32} \\ z_1 \\ z_2 \end{pmatrix}$$

We can write the objective function  $f(\mathbf{y}) = \frac{1}{2}\mathbf{y}^T\mathbf{H}\mathbf{y}$ , where  $\mathbf{H}$  is the Hessian of  $f(\mathbf{y})$  and, in detail, it can be written as:

$$\mathbf{H} = \left( \begin{array}{cc|cc|cc|cc} 2\frac{b}{d} & 0 & 2\frac{b}{d} & 0 & 2\frac{b}{d} & 0 & -1 & 0 \\ 0 & 2\frac{b}{d} & 0 & 2\frac{b}{d} & 0 & 2\frac{b}{d} & 0 & -1 \\ \hline 2\frac{b}{d} & 0 & 2\frac{b}{d} & 0 & 2\frac{b}{d} & 0 & -1 & 0 \\ 0 & 2\frac{b}{d} & 0 & 2\frac{b}{d} & 0 & 2\frac{b}{d} & 0 & -1 \\ \hline 2\frac{b}{d} & 0 & 2\frac{b}{d} & 0 & 2\frac{b}{d} & 0 & -1 & 0 \\ 0 & 2\frac{b}{d} & 0 & 2\frac{b}{d} & 0 & 2\frac{b}{d} & 0 & -1 \\ \hline -1 & 0 & -1 & 0 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & -1 & 0 & 0 \end{array} \right)$$

Vertical and horizontal bars are used to analyse patterns of the matrix, which can be divided into blocks. In particular,

- blocks defined by pairs of variables which do not belong to  $\mathbf{z}$ , are diagonal matrices with dimension  $|F| \times |F|$  ( $|F| = 2$  in the example) and diagonal elements all equal to  $2\frac{b}{d}$ ;
- blocks defined by pairs of variables from both  $\mathbf{x}$  and  $\mathbf{z}$  are diagonal matrices with dimension  $|F| \times |F|$  and diagonal elements all equal to  $-1$ ;
- the block defined by only components in  $\mathbf{z}$  is a  $|F| \times |F|$  zero matrix.

#### 4. The solution approach

In this section, we describe the solution approach, which, as introduced in the first section of the paper, puts together two known approaches in order to solve successive linear approximations of the QP model.

Let  $\mathbf{y}^{(0)}$  be a starting initial feasible solution of the model. Since  $\nabla f(\mathbf{y}) = \mathbf{H}\mathbf{y}$ , we can write the first order approximation of the function  $f(\mathbf{y})$  as follows:

$$f(\mathbf{y}) \approx \bar{f}(\mathbf{y}) = f(\mathbf{y}^{(0)}) + \phi(\mathbf{H}\mathbf{y}^{(0)})^T(\mathbf{y} - \mathbf{y}^{(0)}).$$

Let us now consider the quadratic constraints

$$g_j(\mathbf{y}) = z_j - \frac{b}{d} \left[ \sum_{k \in S} x_{kj} - \frac{(\sum_{k \in S} x_{kj})^2}{c_j} \right] \leq 0,$$

with  $j \in F$ , the gradient vector  $\nabla g_j(\mathbf{y})$ , and the first order approximation of constraints  $g_j(\mathbf{y})$ , i.e.,

$$g_j(\mathbf{y}) \approx g_j(\mathbf{y}^{(0)}) + \nabla g_j(\mathbf{y}^{(0)})^T(\mathbf{y} - \mathbf{y}^{(0)}),$$

where

$$\nabla g_j(\mathbf{y}) = \begin{pmatrix} \frac{\partial g_j(\mathbf{y})}{\partial x_{11}} \\ \vdots \\ \frac{\partial g_j(\mathbf{y})}{\partial x_{1j}} \\ \vdots \\ \frac{\partial g_j(\mathbf{y})}{\partial x_{1|F|}} \\ \vdots \\ \frac{\partial g_j(\mathbf{y})}{\partial x_{|S|1}} \\ \vdots \\ \frac{\partial g_j(\mathbf{y})}{\partial x_{|S|j}} \\ \vdots \\ \frac{\partial g_j(\mathbf{y})}{\partial x_{|S||F|}} \\ \frac{\partial g_j(\mathbf{y})}{\partial z_1} \\ \vdots \\ \frac{\partial g_j(\mathbf{y})}{\partial z_j} \\ \vdots \\ \frac{\partial g_j(\mathbf{y})}{\partial z_{|F|}} \end{pmatrix} = \begin{pmatrix} 0 \\ \vdots \\ \frac{b}{d} \left( \frac{2 \sum_{k \in S} x_{kj}}{C_1} - 1 \right) \\ \vdots \\ 0 \\ \vdots \\ 0 \\ \vdots \\ \frac{b}{d} \left( \frac{2 \sum_{k \in S} x_{kj}}{c_j} - 1 \right) \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix},$$

which means that

$$\begin{aligned} g_j(\mathbf{y}) &\approx z_j^{(0)} - \frac{b}{d} \left[ \sum_{k \in S} x_{kj}^{(0)} - \frac{\left( \sum_{k \in S} x_{kj}^{(0)} \right)^2}{C} \right] + \sum_{k \in S} \left[ \frac{b}{d} \left( \frac{2 \sum_{k' \in S} x_{k'j}^{(0)}}{c_j} - 1 \right) (x_{kj} - x_{kj}^{(0)}) \right] \\ &+ z_j - z_j^{(0)} = z_j - \frac{b}{d} \left[ \sum_{k \in S} x_{kj}^{(0)} - \frac{\left( \sum_{k \in S} x_{kj}^{(0)} \right)^2}{c_j} \right] \\ &+ \sum_{k \in S} \left[ \frac{b}{d} \left( \frac{2 \sum_{k' \in S} x_{k'j}^{(0)}}{c_j} - 1 \right) (x_{kj} - x_{kj}^{(0)}) \right] \\ &= z_j + \frac{b}{d} \sum_{k \in S} \left[ \left( \frac{2 \sum_{k' \in S} x_{k'j}^{(0)}}{c_j} - 1 \right) x_{kj} \right] \\ &- \frac{b}{d} \left\{ \sum_{k \in S} x_{kj}^{(0)} - \frac{\left( \sum_{k \in S} x_{kj}^{(0)} \right)^2}{c_j} + \sum_{k \in S} \left[ \left( \frac{2 \sum_{k' \in S} x_{k'j}^{(0)}}{c_j} - 1 \right) x_{kj}^{(0)} \right] \right\}. \end{aligned} \tag{4}$$

Now, defining

$$\begin{aligned} \delta_j^{(0)} &= \frac{b}{d} \left\{ \sum_{k \in S} x_{kj}^{(0)} - \frac{(\sum_{k \in S} x_{kj}^{(0)})^2}{c_j} + \sum_{k \in S} \left[ \left( \frac{2 \sum_{k' \in S} x_{k'j}^{(0)}}{c_j} - 1 \right) x_{kj}^{(0)} \right] \right\}, \\ \gamma^{(0)} &= f(\mathbf{y}^{(0)}) - \phi(\mathbf{Hy}^{(0)})^T \mathbf{y}^{(0)}, \end{aligned}$$

and substituting (4) in the quadratically constrained quadratic formulation QP, we have the following linear program LAP( $\mathbf{t}$ ), with  $\mathbf{t} = 0$ :

$$\begin{aligned} \min \quad & \phi(\mathbf{Hy}^{(0)})^T \mathbf{y} + \gamma^{(0)} \\ & \sum_{j \in F} x_{kj} \leq s_k, & \forall k \in K, \\ & \sum_{k \in S} x_{kj} \leq c_j, & \forall j \in F, \\ & z_j + \frac{b}{d} \sum_{k \in S} \left[ \left( \frac{2 \sum_{k' \in S} x_{k'j}^{(0)}}{c_j} - 1 \right) x_{kj} \right] \leq \delta_j^{(0)}, & \forall j \in F, \\ & \sum_{k \in S} \sum_{j \in F} x_{kj} = d. \end{aligned} \tag{5}$$

Now, consider LAP( $\mathbf{t}$ ) solved in an iterative fashion as defined by Algorithm Multistart Successive Linear Approximations (MSLA), whose pseudocode is reported in Algorithm 1. The idea is to develop a multi-start procedure which works into phases. Each phase starts from a feasible solution  $\mathbf{y}^{(\mathbf{t})}$  of QP computed by Algorithm Starting Solution and solves LAP( $\mathbf{t}$ ). Denoting with  $\bar{\mathbf{y}}^*$  the optimal solution of the latter problem, if this solution is feasible for QP and improves the best solution found so far, then  $\bar{\mathbf{y}}^*$  is stored as the best solution and the best solution value is updated and stored as well. The algorithm therefore keeps on iterating assigning  $\bar{\mathbf{y}}^*$  to  $\mathbf{y}^{(\mathbf{t}+1)}$ . The stopping criterion of a phase is attained when the norm of the difference between the incumbent best (feasible) QP solution and the previous best (feasible) QP solution are less than or equal to a given threshold  $\epsilon$ . When a phase ends, the algorithm changes the starting feasible solution and enters a new phase carrying out the same steps as those performed in the previous phases. The overall stopping criterion is reached when either a prefixed number of restarts, i.e., *max\_restarts*, or a maximum time limit is met.

The computation of the starting feasible solution is done by means of Algorithm Starting Solutions SS, which is a randomized procedure whose pseudocode is reported in Algorithm 2. In its initialization phase, the algorithm orders at random nodes in  $S$  and next nodes in  $F$ . Then, the starting flows  $x_{kj}^0$ , with  $k \in S$  and  $j \in F$ , are allocated sequentially following the two orderings and taking into account capacity and demand constraints. In particular, for each  $j$  and  $k$ , a value  $t$  is generated uniformly at random between 0 and  $\frac{c_j}{2|S|}$  and assigned to  $x_{kj}^0$  as far as it is lower than the residual capacity of  $j$  and the residual demand to be served; otherwise, the assignment is made taking the minimum among the latter two quantities.

**Algorithm 1** Successive Linear Approximations.

---

```

 $f^{best} = +\infty;$ 
 $\tau \leftarrow 0;$ 
 $phase \leftarrow 1$ 
repeat
   $halt \leftarrow 0;$ 
  Find a starting feasible solution  $\mathbf{y}^{(\tau)}$  by means of algorithm SS;
  if  $f(\mathbf{y}^{(\tau)}) < f^{best}$  then
     $\mathbf{y}^* = \mathbf{y}^{(\tau)};$ 
     $f^{best} \leftarrow f(\mathbf{y}^*);$ 
  end if
  repeat
    Solve LAP( $\tau$ );
    Let  $\bar{\mathbf{y}}^*$  be the optimal solution of LAP( $\tau$ );
    if  $f(\bar{\mathbf{y}}^*) < f^{best}$  and  $\bar{\mathbf{y}}^*$  is feasible for QP then
       $f^{best} \leftarrow f(\bar{\mathbf{y}}^*);$ 
       $\mathbf{y}^\pi = \bar{\mathbf{y}}^*$ 
       $\mathbf{y}^* = \bar{\mathbf{y}}^*$ 
       $\mathbf{y}^\ell = \bar{\mathbf{y}}^*$ 
      if  $\|\mathbf{y}^\ell - \mathbf{y}^\pi\| \leq \epsilon$  then
         $halt \leftarrow 1;$ 
      end if
    end if
     $\mathbf{y}^{(\tau+1)} = \bar{\mathbf{y}}^*$ 
     $\tau \leftarrow \tau + 1;$ 
  until  $halt = 1;$ 
   $phase \leftarrow phase + 1;$ 
until  $phase = max\_restarts$  or a time limit is exceed;
return

```

---

**Algorithm 2** Starting Solution.

---

```

set  $feas \leftarrow \text{False};$ 
Set  $d^0 \leftarrow d; x^0 \leftarrow 0, z^0 \leftarrow 0;$ 
Set  $s_k^0 \leftarrow s_k \forall k \in S; c_j^0 \leftarrow c_j \forall j \in F;$ 
Let  $\mathcal{S}$  be an ordering of nodes in  $S;$ 
Let  $\mathcal{F}$  be an ordering of nodes in  $F;$ 
repeat
  for  $k \in \mathcal{S}$  and  $j \in \mathcal{F}$  do
    Set  $t \leftarrow \text{uniform}\left(0, \frac{c_j}{2|S|}\right);$ 
     $\bar{x} \leftarrow \min\{d^0, S_k^0, t\};$ 
     $x_{kj}^0 \leftarrow x_{kj}^0 + \bar{x};$ 
     $d^0 \leftarrow d^0 - \bar{x};$ 
     $s_k^0 \leftarrow s_k^0 - \bar{x};$ 
     $c_j^0 \leftarrow c_j^0 - \bar{x};$ 
    if  $d^0 = 0$  then
       $feas \leftarrow \text{True};$ 
      break for;
    end if
  end for
until  $feas = \text{True};$ 
for  $j \in F$  do
   $z_j^0 \leftarrow \frac{b}{d} \left[ \sum_{k \in K} x_{kj}^0 - \frac{(\sum_{k \in K} x_{kj}^0)^2}{c_j} \right];$ 
end for

```

---

## 5. Computational analysis

The proposed algorithm has been tested on a set of randomly generated instances. In addition, a comparison with a literature based model and a real world instance has been performed to validate the model. In this section, we present results of these two test campaigns.

### 5.1. Tests on random instances

The baseline settings for random instances are as follows:

- $s_k \in \text{uniform}[100, 150], \forall k \in S$ ;
- $c_j \in \text{uniform}[100, 150], \forall j \in F$ ;
- $d = \frac{1}{2} \sum_{k \in S} s_k$ ;
- $\phi = 1$ .

The algorithm has been implemented in Python 3.7. The LAP model has been solved by means of GUROBI<sup>TM</sup> version 9.1.1. The computational tests have been executed on a laptop equipped with Intel Core<sup>TM</sup> i7-5600U CPU @ 2.60 GHz with 4 cores, 8 GB RAM, Operating System Ubuntu 20.04.

Table 2 summarizes the performance of the algorithm over different instance sizes with a maximum running time of 300 seconds and  $\frac{b}{d} = 2$ . The table reports, for each instance, the number  $|S|$  of suppliers, the number  $|F|$  of facilities, the value **bsv** of the best solution found, the iteration **it\_b** at which such a solution is found, and the time **t\_b** (in seconds) at which this has been recorded. The behavior of the algorithm is depicted, in particular, in Fig. 2 for instance with  $|S| = 5$  and  $|F| = 5$ , in Fig. 3 for instance with  $|S| = 20$  and  $|F| = 20$ , in Fig. 4 for instance with  $|S| = 50$  and  $|F| = 50$ , and in Fig. 5 for instance with  $|S| = 75$  and  $|F| = 100$ . In these figures we can see the progress of the best solution value over time.

In Table 3, we show the performance of our algorithm on the same instances with  $\frac{b}{d} = 10$ .

### 5.2. Comparison with a commercial solver

In order to evaluate the effectiveness of the model and the proposed approach, we solved the same instances with GUROBI<sup>TM</sup> solver with the **NonConvex** flag enabled. In Table 4, we compared the performance of our algorithm to that of the solver with the same time limit of 300 seconds on medium to large instances. The best solution value of our algorithm is named **MSLA best**, while the solver best solution value is denoted with **S0 best**. The column **gap** reports the ratio of the difference between the solution values achieved by the MSLA algorithm and the solver over the MSLA best solution value. Improvements of our algorithm w.r.t. the solver are reported in bold. In Table 5, we

**Table 2**  
Results of the MSLA algorithm on instances with  $\frac{b}{a} = 2$ .

$ S $	$ F $	bsv	it_b	t_b
5	5	17407	153	0.50
5	10	7187	35751	194.79
10	10	38226	3876	83.73
10	15	21865	7599	239.18
15	15	73106	5304	214.89
15	20	32678	2040	109.31
20	20	103752	2907	252.10
30	30	149259	102	16.37
30	40	69491	969	195.00
40	40	199615	306	79.39
40	50	91221	306	93.24
50	50	254319	663	263.24
75	75	392807	204	170.89
75	100	179325	204	218.00
100	100	522385	51	79.06

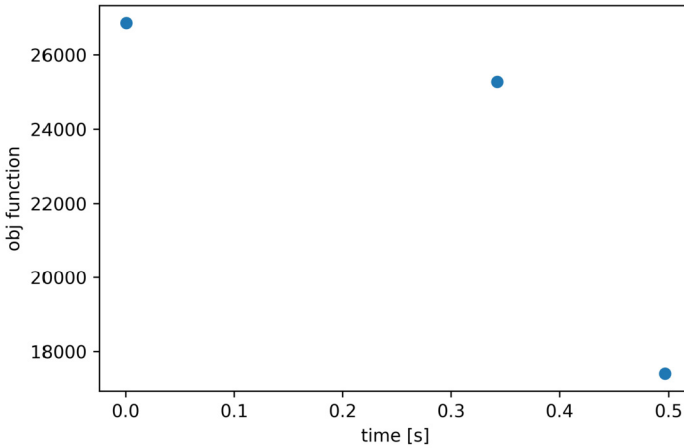


Fig. 2. Progress of the best solution value over time (instance with  $|S| = 5$  and  $|F| = 5$ ).

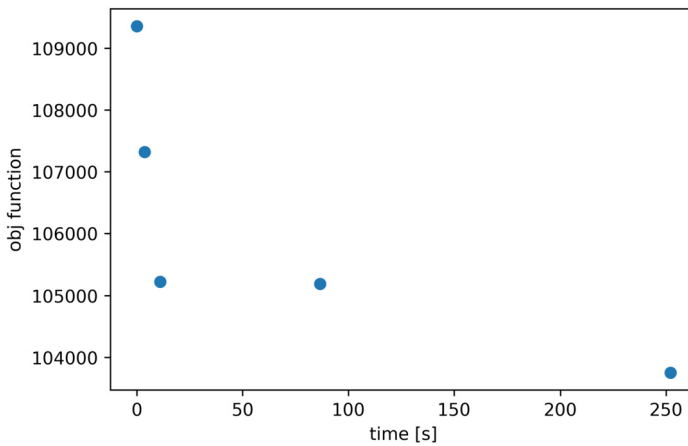


Fig. 3. Progress of the best solution value over time (instance with  $|S| = 20$  and  $|F| = 20$ ).

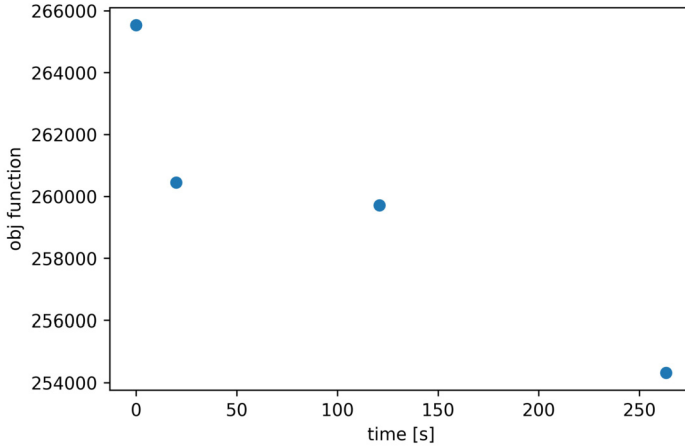


Fig. 4. Progress of the best solution value over time (instance with  $|S| = 50$  and  $|F| = 50$ ).

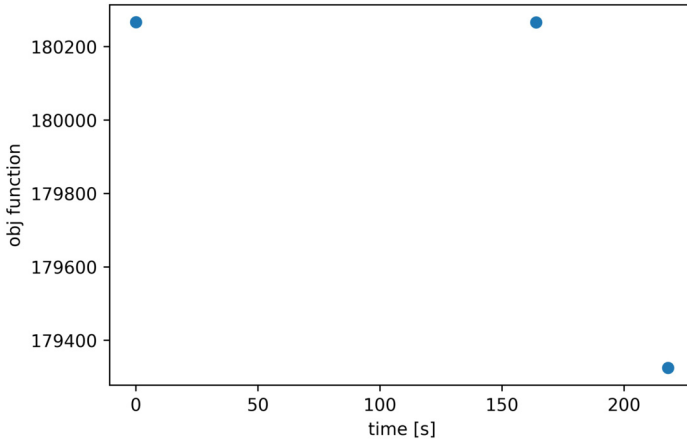


Fig. 5. Progress of the best solution value over time (instance with  $|S| = 75$  and  $|F| = 100$ ).

report the comparison between the MSLA algorithm and the commercial solver on the same instances with  $\frac{b}{a} = 10$ . As it can be noted, in both the two scenarios, MSLA is able to considerably improve the best solution value found by the solver.

### 5.3. Comparison with a state of the art model on a real world instance

In order to prove the effectiveness of our model, we compared the latter with the state of the art model and the real world problem detailed in [26]. The former is a network design model for green supply chain management with two objectives to be minimized, i.e., the cost and the emission, respectively. Similarly to what happens in our model, the higher green investments the lower the CO<sub>2</sub> emissions per unit of product handled in the facilities. In particular, the competing model is defined on a graph  $G = (V, A)$  where



**Table 3**

Performance of the MSLA algorithm on instances with  $\frac{b}{a} = 10$ .

$ S $	$ F $	bsv	it_b	t_b
5	5	92080	37842	128.02
5	10	37317	2907	16.01
10	10	174849	11373	261.46
10	15	108270	3315	107.13
15	15	397828	3366	157.47
15	20	182594	3060	191.74
20	20	515340	2856	234.77
30	30	775278	1326	216.14
30	40	350650	102	20.44
40	40	1019081	816	203.61
40	50	509924	306	90.67
50	50	1304508	714	297.09
75	75	1965578	51	44.13
75	100	912849	51	60.35
100	100	2676473	102	178.73

**Table 4**

Comparison between the performance of the MSLA algorithm and the commercial solver; max computation time is set to 300 seconds;  $\frac{b}{a} = 2$ .

$ S $	$ F $	MSLA best	S0 best	gap
30	30	149259	145927	2.28
30	40	69491	219108	<b>-68.28</b>
40	40	199615	200873	<b>-0.63</b>
40	50	91221	320786	<b>-71.56</b>
50	50	254319	403052	<b>-36.90</b>
75	75	392807	678049	<b>-42.07</b>
75	100	179325	725397	<b>-75.28</b>
100	100	522385	894257	<b>-41.58</b>

**Table 5**

Comparison between the performance of the MSLA algorithm and the commercial solver; max computation time is set to 300 seconds;  $\frac{b}{a} = 10$ .

$ S $	$ F $	MSLA best	S0 best	gap
30	30	775278	764151	1.46
30	40	350650	826665	<b>-57.58</b>
40	40	1019081	1738974	<b>-41.40</b>
40	50	509924	1239300	<b>-58.85</b>
50	50	1304508	2194397	<b>-40.55</b>
75	75	1965578	3657521	<b>-46.26</b>
75	100	912849	3521618	<b>-74.08</b>
100	100	2676473	5241653	<b>-48.94</b>

$V = S \cup F \cup C$ , where  $S$ ,  $F$ , and  $C$  are the sets of suppliers, facilities, and customers, respectively.  $A$  is the set of arcs connecting suppliers with facilities, and facilities with customers. The variables of the model are the amount of product  $x_{ii'}$  transported along arc  $(i, i') \in A$ , the amount of product  $x_{jl}$  processed in each facility  $j \in F$ , with an environmental protection level  $l \in L$ , and the binary variable  $z_{jl}$  which equals 1 whether the environmental protection level  $l$  is selected in facility  $j$ , and holds 0 otherwise. The

objective functions  $f_1$  and  $f_2$  defined in the model in [26] are defined in (6), and (7), respectively.

$$f_1 = \sum_{(i,i') \in A} c_{ii'} x_{ii'} + \sum_{j \in F, l \in L} g_{jl} z_{jl} + \sum_{j \in F, k \in S} \ell_j x_{kj} \tag{6}$$

$$f_2 = \sum_{j \in F, l \in L} w_{jl} x_{jl} + \sum_{(i,j) \in A} e_{ii'} x_{ii'} \tag{7}$$

Function  $f_1$  has three terms: the traveling cost, the cost associated with the green investment in the facilities, and the handling cost of the products in the facilities.  $f_2$  has two terms: the emissions in the production facilities, and the emissions associated with traversing a certain arc. The first term, as detailed in Table 6, is non linear w.r.t. green investment level  $l$ . The model constraints are defined in (8).

$$\begin{aligned} \sum_{k \in S} x_{kj} - \sum_{i \in C} x_{ji} &= 0 & \forall j \in F, \\ \sum_{l \in L} x_{jl} &= \sum_{k \in S} x_{kj} & \forall j \in F, \\ \sum_{j \in F} x_{ji} &= d_i & \forall i \in C, \\ \sum_{j \in F} x_{kj} &\leq s_k & \forall k \in S, \\ r_j x_{jl} &\leq u_j z_{jl} & \forall j \in F, l \in L, \\ \sum_{l \in L} z_{jl} &\leq 1 & \forall j \in F, \\ x_{ii'}, x_{jl} &\geq 0 & \forall (i, i') \in A, l \in L, j \in F, \\ z_{jl} &\in \{0, 1\} & \forall l \in L, j \in F. \end{aligned} \tag{8}$$

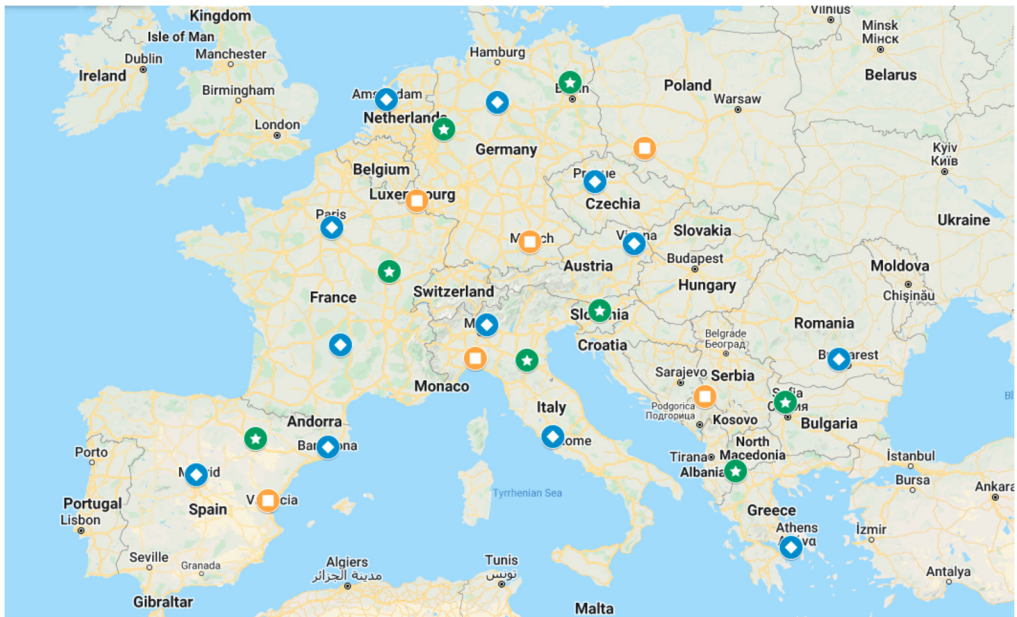
The first constraint of (8) is the balance of product flows at the facilities. The second constraint defines the relation between  $x_{ii'}$  and  $x_{jl}$  variables. The third constraint satisfies customer demand. The fourth constraint limits capacity for suppliers, while the fifth constraint limits the handling capacity for each facility. The sixth constraint imposes that at most one environmental protection level can be set for each facility. The other constraints define the domains of the variables. With respect to the [26] model, we explicitly inserted the link between  $x_{kj}$  and  $x_{jl}$ , and we represented the problem with a single commodity.

In order to compare with the model in [26], we adapted our model to consider customers, traveling costs, traveling emissions, and handling costs. This extension is straightforward.

We took the input data, i.e., network topology, range of emissions, and costs, verbatim from [26] and built the instance for comparison. The parameter settings are detailed in Table 6. With respect to [26], since there is no information on arc distances, we redefined a set of geographic points as shown in Fig. 6 for which it was possible to calculate arc lengths. By solving the [26] model with the two objectives it was possible to evaluate the maximum and minimum costs and the emissions, respectively. This information has been useful to define the budget  $b_j$  and the  $\phi$  value necessary to implement our model, as detailed in the last two rows of Table 6.

**Table 6**  
Parameter settings for the [26] case study.

Parameter	Setting
Number of suppliers	$ S  = 6$
Number of facilities	$ F  = 8$
Number of customers	$ C  = 12$
Investment levels	$l \in L = \{0, \dots, 4\}$
Demand [units]	$d_j = 3000$
Supply capacity [units]	$s_k = 3000 \cdot \frac{ C }{ S } \cdot 3$
Arc geographic distance [km]	$\text{dist}_{ii'}$
Transportation cost [€/unit]	$c_{ii'} = 0.1 \cdot \text{dist}_{ii'}$
CO <sub>2</sub> per transported product [€/unit]	$e_{ii'} = 0.1 \cdot \text{dist}_{ii'}$
CO <sub>2</sub> per worked product [gCO <sub>2</sub> /unit]	$w_{jl} = 60 / (2^{(l-1)})$
Cost factor [€]	$f \cdot r_{cost}, f = 65, r_{cost} = 1000$
Environment investment per product [€/unit]	$g_{jl} = f \cdot r_{cost} \cdot l$
Handling cost per product [€/unit]	$\ell_j = 75$
Processing required capacity per product	$r_j = 8$
Processing capacity per facility [unit]	$u_j = 45,000$
CO <sub>2</sub> per investment per product [gCO <sub>2</sub> /(€·unit)]	$\phi = 1.167 \cdot 10^{-5}$
Max green investment [€]	$b = 1,800,000$



**Fig. 6.** The network associated with the case study. The diamond symbols represent customers, the star symbols represent facilities, and the square symbols represent the suppliers.

Table 7 reports the results of our model compared to those found by the [26] model solved with only the  $f_2$  objective function. The first and the second columns report the minimum emissions found by our model (a) and that in [26] (b), respectively. The third and the fourth columns report the cost for the green investment found by our model and by the [26] model, respectively. The fifth and the sixth columns report a cross comparison between the two models. In particular, the fifth column reports the solution

**Table 7**

Results of the proposed model (a), compared with the results of the [26] model (b). G.I. is the green investment. (a)vs(b) measures the solution of the proposed model with the [26] emission objective function. (b)vs(a) measures the solution of the [26] model with our objective function. CO<sub>2</sub> in grams, green investment in €.

min CO <sub>2</sub> <sup>(a)</sup>	min CO <sub>2</sub> <sup>(b)</sup>	G.I. <sup>(a)</sup>	G.I. <sup>(b)</sup>	CO <sub>2</sub> <sup>(a)vs(b)</sup>	CO <sub>2</sub> <sup>(b)vs(a)</sup>
2,044,220	2,070,000	1,800,000	1,820,000	2,192,154	2,090,244

of the proposed model measured by the [26] emission objective function, while the sixth column reports the solution of the [26] model measured with our objective function. The results show how our model produces promising results both in terms of emissions and investments compared to the competing state of the art model on the same instance.

## 6. Conclusions

Green management in supply chains requires the decarbonisation of operations and a proper allocation of a budget for green investments in order to attain both effectiveness and environmental sustainability. In this paper, we formulated this problem in terms of quadratic optimization with a quadratic constraint. The objective function is non convex, being the Hessian indefinite. The quadratic constraint gives a limit to the budget allocable for emission mitigation which depends on the product flow entering the downstream supply chain layer. A multistart algorithm based on successive linear approximations has been proposed to solve the problem. The approach has been tested on instances with different sizes, proving its capability of providing promising solutions in limited running times. These results have been also compared to those obtained with a commercial solver on the same instances. Furthermore, we implemented a competing approach from the state of the art and compared its performance to that of our model on an instance depicting a real world application. Future work may focus on improving the algorithm in order to get rid of the quadratic constraint and the negativity of some of the eigenvalues of the Hessian matrix of the objective function.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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