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FEA FOR MASONRY STRUCTURES AND VIBRATION-BASED MODEL UPDATING USING NOSA-ITACA

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Abstract. *NOSA-ITACA is a finite-element code developed by the Mechanics of Materials and Structures Laboratory of ISTI-CNR for the structural analysis of masonry constructions of historical interest via the constitutive equation of masonry-like materials. The latest improvements in the software allow applying model updating techniques to match experimentally measured frequencies in order to fine-tune calculation of the free parameters in the model. The numerical method is briefly presented and applied to two historical buildings in Lucca, the Church of San Francesco and the Clock Tower.*

1 INTRODUCTION

Finite element model updating is a procedure aimed at calibrating a finite-element (FE) model of a structure in order to match numerical and experimental results. Introduced in the 1980s, it has earned a crucial role in the design, analysis and maintenance of aerospace, mechanical and civil engineering structures [1], [2], [3], [4]. In structural mechanics, model updating techniques are used in conjunction with vibration measurements to determine unknown system characteristics, such as material properties, constraints, etc. The resulting updated FE model can then be used to obtain reliable predictions on the dynamic behavior of the structure subjected to time-dependent loads. A further important application of model updating within the framework of structural health monitoring is damage identification [5], [6], which is based on the assumption that the presence of damage is associated with a decrease in the stiffness of some elements, with consequent changes in the structure's modal characteristics. FE model updating involves solving a constrained minimum problem, whose objective function is generally expressed as the discrepancy between experimental and numerical quantities, such as natural frequencies and mode shapes.

Application of FE model updating to ancient masonry buildings is relatively recent. In [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], a vibration-based model updating is conducted, and preliminary FE models are fine-tuned using the dynamic characteristics determined through system identification techniques. In the papers cited above the modal analysis of FE models are conducted via commercial codes, while the model updating procedure is implemented separately.

An efficient numerical method aimed at minimizing the discrepancy between the numerical and experimental natural frequencies has been proposed in [19]. The algorithm, based on the construction of local parametric reduced-order models embedded in a trust region scheme, has been implemented in a single numerical program, the NOSA-ITACA code [20]. In particular, the algorithms for solution of the constrained minimum problem, integrated in NOSA-ITACA, exploit the structure of the stiffness and mass matrices and the fact that only a few of the smallest eigenvalues have to be calculated. This new procedure reduces both the total computation time of the numerical process and user effort, thus providing the scientific and technical communities with efficient algorithms specific for FE model updating.

In this paper the method proposed in [19] is outlined and then tested on two actual structures of historical interest located in the Tuscan city of Lucca - the Church of San Francesco and the Clock Tower (*Torre delle Ore*).

2 MODAL ANALYSIS OF MASONRY BUILDINGS AND FREQUENCY MATCHING

Although the masonry materials constituting historical buildings have different strengths under tension and compression, modal analysis, which is based on the assumption that the materials are linear elastic, is widely used in applications and provides important qualitative information on the dynamic behavior of such structures. Modal analysis consists in solving the constrained generalized eigenvalue problem

$$K\,\mathrm{v} = \omega^2 M\,\mathrm{v}, \quad \text{subject to} \quad C\,\mathrm{v} = 0, \tag{1}$$

with $C \in \mathbb{R}^{h \times n}$ and $h << n$. K and $M \in \mathbb{R}^{n \times n}$ are respectively the stiffness and mass matrices of the structure discretized into finite elements. K is symmetric and positive-semidefinite, M is symmetric and positive-definite, and both are sparse and banded. $v \in \mathbb{R}^n$ is the vector of the degrees of freedom of the structure, the integer n is the system's total number of degrees

of freedom, which is generally very large, since it depends on the level of discretization of the problem.
The right part of equation (1) expresses the fixed constraints and the master-slave relations assigned to v. Imposing such constraints and boundary conditions is equivalent to projecting the matrices K and M on a subspace where they are symmetric positive-definite (the right kernel of the operator C). Henceforth, we assume that this projection has already been done, and refer the reader to [21] for further details. Note, in particular, that if K and M depend *linearly* on some parameters $\mathrm{x} = (x_1,...,x_l)$, the same holds true for their projections. More generally, the smooth dependency of K and M is preserved by the projection, since it is a linear operation.

The eigenvalues ω_i^2 of (1) are linked to the natural frequencies, or eigenfrequencies f_i of the structure via the relation $f_i = \omega_i / 2\pi$, and the eigenvectors $v^{(i)}$ are the corresponding mode shape vectors, or eigenmodes. Together with the natural frequencies, the mode shapes furnish qualitative information on the structure's deformations under dynamic loads.

An efficient implementation of the numerical solution of the constrained eigenvalue problem (1) has been embedded into NOSA-ITACA [21].

Measuring the vibrations of masonry buildings is common practice for assessing their dynamic behaviour and determining their natural frequencies and mode shapes. Model updating techniques are used in conjunction with vibrations measurements to determine the structure's characteristics, such as the material's properties (Young's modulus, Poisson's ratio, mass density), constraints, and so forth, which are generally unknown.

The model updating problem can be reformulated as an optimization problem by assuming that the (projected) stiffness and mass matrices K and M are functions of the parameter vectors x. We use the notation

$$K = K(\mathrm{x}), \qquad M = M(\mathrm{x}), \qquad \mathrm{x} \in \mathrm{R}^l \tag{2}$$

to denote this dependency. The set of valid choices for the parameters is denoted by Ω. Within this framework, we assume that the set Ω is an l -dimensional box, that is

$$\Omega = [a_1, b_1] \times [a_2, b_2] \times ... \times [a_l, b_l], \tag{3}$$

for certain values $a_i < b_i$, $i = 1...l$.

Our ultimate aim is to determine the optimal value of x that minimizes a certain cost functional $\phi(\mathrm{x})$ within the box Ω. Generically, this is an instance of the optimization problem

$$\min_{\mathbf{x} \in \Omega} \phi(\mathrm{x}) . \tag{4}$$

The choice of the objective function $\phi(\mathrm{x})$ is related to the frequencies that we wish to match. If we need to match s frequencies of the model, we choose a suitable weight vector $\mathrm{w} = [w_1,...,w_s]$, with $w_i \ge 0$ and define the functional $\phi(\mathrm{x})$ as follows:

$$\phi(\mathrm{x}) = \left\| \frac{\sqrt{\Lambda_s(K,M)}}{2\pi} - \mathrm{f} \right\|_{\mathrm{w},2}^2, \qquad \|\mathrm{y}\|_{\mathrm{w},2} = \sqrt{\mathrm{y}^T \,\mathrm{diag}(\mathrm{w})\, \mathrm{y}} \tag{5}$$

where f is the vector of the measured frequencies, and $\Lambda_s(K,M)$ the one containing the smallest s eigenvalues of (1), ordered according to their magnitude. The vector w encodes the weight that should be given to each frequency in the optimization scheme. If the goal is to minimize the distance between the vectors of the measured and the computed frequencies in the usual Euclidean norm, then $w = 1_s$, the vector of all ones, should be chosen. If, instead, relative accuracy on the frequencies is desired, $w_i = f_i^{-1}$ is a natural choice. If some frequencies need to be ignored, we can set the corresponding component of w to zero. To avoid scaling issues, the weight vector is always normalized in order to have $\|w\|_2 = 1$.

When the FE model is very large, it is convenient to use model reduction techniques to reduce its size to a more manageable order.
If, as in equation (2), the model depends on the parameters x, it is not a trivial matter to obtain a reduced parametric model that accurately reflects the behavior of the original one for all possible parameter values. An efficient Lanczos-based projection strategy tailored to the needs of the FE analysis of masonry structures is presented in [19]. By modifying the projection scheme used to compute the first eigenvalues (and corresponding eigenvectors) of problem (1), we obtain local parametric reduced-order models that, embedded in a trust region scheme, are the basis for an efficient algorithm that minimizes objective function (5).

3 CASE STUDIES

In this section we verify the performance of our approach in two example applications. In the first, the Church of San Francesco in Lucca, we perform model updating with a large number of parameters and fit the first ten natural frequencies of the church computed via the NOSA-ITACA code (by using fixed parameter values). In the second, the Clock Tower, also in Lucca, we use the proposed algorithm to fit the tower’s first four natural frequencies, which have been measured during an experimental campaign conducted in November 2016 [22]. The two case studies have been run on a computer with an Intel Core i7-920 CPU running at 2.67 GHz and 18 GB of RAM clocked at 1066 MHz and the convergence tolerance is set to 10^{-4} (the trust-region procedure is stopped when the norm of the projected gradient falls below 10^{-4}).

3.1 The Church of San Francesco

The Church of San Francesco (Figure 1) is a typical single-nave Franciscan masonry building, about 70 m long, 16 m wide and 19 m high. The nave is closed on the west by the façade overlooking the square of San Francesco and on the east by the apse. The northern wall leans against the portico of the monastery cloister; the southern wall, completely free, runs along the street named via della Quarquonia. The earliest walls of the church date back to the 13th century, but the building underwent several changes and enlargements over the centuries. At about 15 m from the façade, there is a transverse joint, the result of an extension added during the 13th - 14th century. The perimeter walls are about 1 m thick and are mainly made of masonry bricks and lime mortar, except for the top of the longitudinal walls, where a band of poor quality masonry is present, probably the result of heightening the nave. The apsidal chapels date back to the 15th century, and the original rectangular windows in the longitudinal walls were closed and new ones opened in 1848. The façade was completed during the 1930’s [23].

Some reinforcement operations, aimed mainly at improving the quality of the masonry and the connections between the walls, were concluded in July 2013. To increase the building’s resistance to horizontal actions, considering how slender the nave walls are, a metal framework was constructed at roof level in order to brace the structure, and the roof layer was stiffened by means of a crossed double-layer wooden deck. Some numerical simulations aimed at assessing the seismic vulnerability of the church before and after the reinforcement operations were conducted via the NOSA-ITACA code and can be found in [24].

Figure 1: The Church of San Francesco.

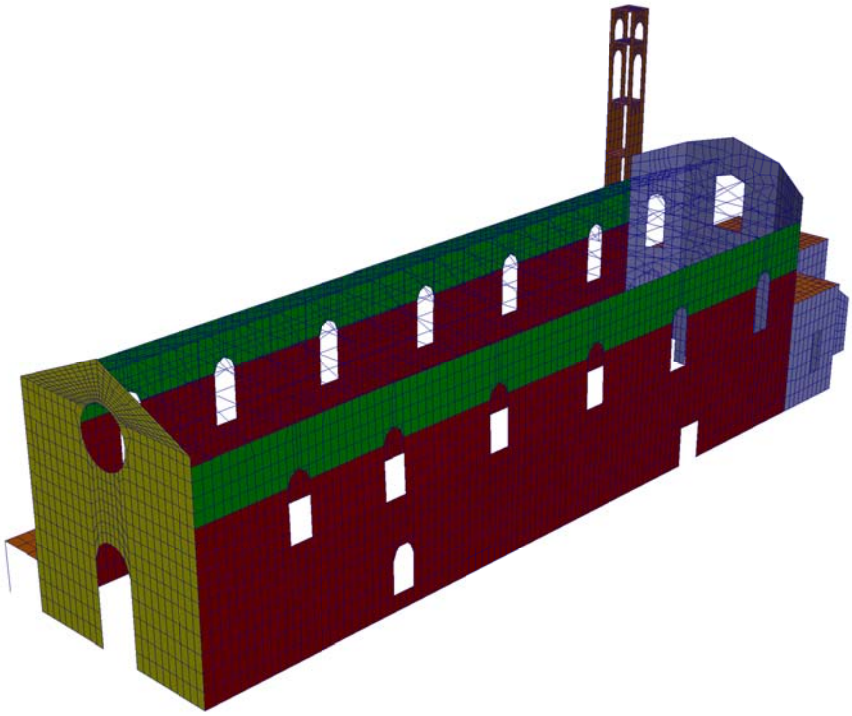

Figure 2: FE mesh of the church.

In the present paper the FE model of the church (Figure 2) is used to test the model updating algorithm. In the model, thick shell elements and beam elements are used [20], for a total number of 7295 elements and 40416 degrees of freedom. We update six parameters: Young's modulus and the mass density of the masonry constituting the lower part of the longitudinal walls (E_d, ρ_d); the upper band of the longitudinal walls (E_u, ρ_u) and the apsidal part of the church (E_a, ρ_a). This choice of parameters partly reflects the structure's stages of construction. The parameters are allowed to vary within the intervals:

$$2000\,\text{MPa} \le E_d \le 6000\,\text{MPa}\,, \qquad 1700\frac{\text{kg}}{\text{m}^3} \le \rho_d \le 2200\frac{\text{kg}}{\text{m}^3}$$

$$1000\,\text{MPa} \le E_u \le 4000\,\text{MPa}\,, \qquad 1600\frac{\text{kg}}{\text{m}^3} \le \rho_d \le 1900\frac{\text{kg}}{\text{m}^3}$$

$$2000\,\text{MPa} \le E_a \le 6000\,\text{MPa}\,, \qquad 1700\frac{\text{kg}}{\text{m}^3} \le \rho_a \le 2200\frac{\text{kg}}{\text{m}^3}\,.$$

The masonry’s Poisson’s ratio is fixed at 0.2, while the mechanical characteristics chosen for the iron, wood and remaining masonry parts of the church (including the façade) are

$$E_i = 210000\,\text{MPa}\,,\ \rho_i = 7850\frac{\text{kg}}{\text{m}^3}\,,\ \nu_i = 0.3,\ E_w = 8000\,\text{MPa}\,,\ \rho_w = 800\frac{\text{kg}}{\text{m}^3}\,,\ \nu_w = 0.3,$$

$$E_m = 3500\,\text{MPa}\,,\ \rho_m = 2000\frac{\text{kg}}{\text{m}^3}\,.$$

The frequency values [Hz] to match (henceforth, the reference frequencies) are fixed as

$$\text{f} = [0.9824,\ 1.063,\ 1.722,\ 2.0621,\ 2.2936,\ 2.330,\ 2.528,\ 2.758,\ 3.172,\ 3.198]$$

and correspond to the first ten frequencies calculated by NOSA-ITACA using the following parameter values (reference parameters):

$$E_d^{\,r} = 3000\,\text{MPa}\,,\ \rho_d^{\,r} = 2000\frac{\text{kg}}{\text{m}^3}\,,\ E_u^{\,r} = 2000\,\text{MPa}\,,\ \rho_u^{\,r} = 1800\frac{\text{kg}}{\text{m}^3}\,,$$

$$E_a^{\,r} = 2500\,\text{MPa}\,,\ \rho_a^{\,r} = 1900\frac{\text{kg}}{\text{m}^3}\,.$$

The objective function (5) is evaluated using $w_i = f_i^{-1},\ i = 1,\ldots,10$. The total computation time is **41.7 s**. For comparison’s sake, a single modal analysis (with fixed parameters) using NOSA-ITACA takes 28.2s. Table 1 reports the parameter values found by the algorithm and their relative errors with respect to the reference values. The match between the calculated parameter values and the reference values is quite good: the relative errors are on the order of 0.5%. With regard to the model frequencies, they are also matched by the algorithm with very good accuracy, the corresponding relative errors being on the order of 0.01%.

Reference parameters [MPa, $\frac{\text{kg}}{\text{m}^3}$]	Parameters [MPa, $\frac{\text{kg}}{\text{m}^3}$]	Relative errors
$E_d^{\,r} = 3000$	$E_d = 3019.1$	0.6356%
$\rho_d^{\,r} = 2000$	$\rho_d = 2007.5$	0.3773%
$E_u^{\,r} = 2000$	$E_u = 1993.6$	0.3215%
$\rho_u^{\,r} = 1800$	$\rho_u = 1804.9$	0.2743%
$E_a^{\,r} = 2500$	$E_a = 2499.5$	0.0200%
$\rho_a^{\,r} = 1900$	$\rho_a = 1884.5$	0.8160%

Table 1: Results of the optimization algorithm: parameters and their relative errors with respect to the reference values.

Figure 3 shows on the left side the convergence of the objective function to the minimum for each new reduced model; the dashed line is the tolerance. On the right, the convergence of the model's frequencies to the fixed values during the process is shown.

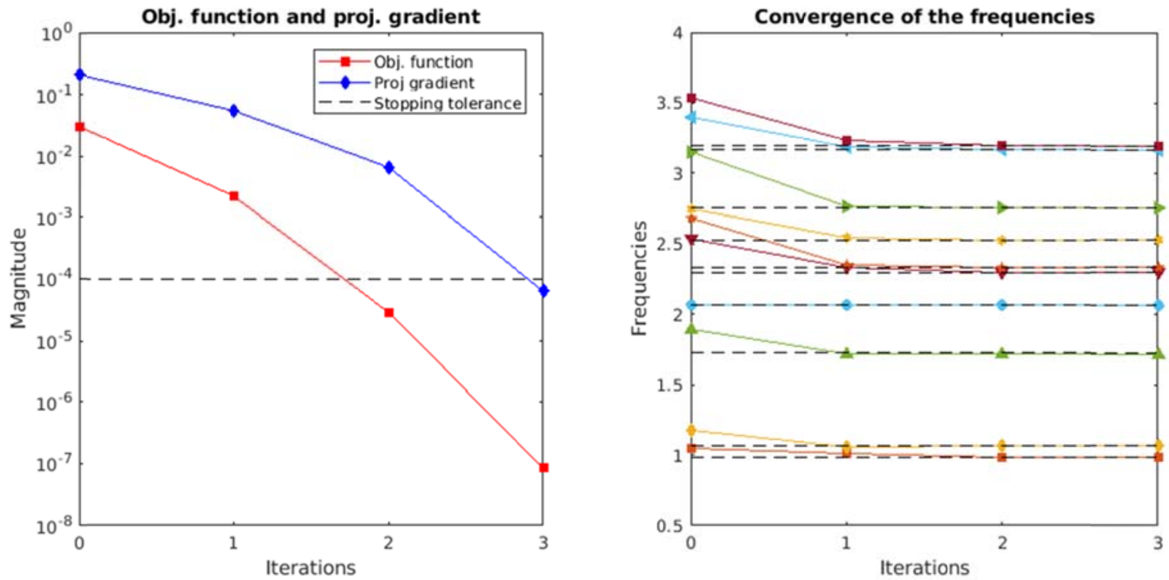

Figure 3: Convergence history of the optimization algorithm.

3.2 The Clock Tower

The Clock Tower (Figure 4) is one of the best known and most often visited monuments in the old city of Lucca, thanks to the peculiar shape of its bell chamber, which is clearly visible and recognizable throughout the entire historic centre. Built in the 13th century by local families, since the last decade of the 15th century the Clock Tower has served as a civic building, taking its name from the big clock visible on its southern facade. The Clock Tower is 48.4 m high; it has a rectangular cross section of about 5.1x7.1 m and walls of thickness varying from about 1.77 m at the base to 0.85 m at the top. The bell chamber, made up of four brick masonry pillars connected by elliptical arches, is covered by a pavilion roof constructed of wooden trusses and rafters. With regard to the materials constituting the masonry tower, visual inspection reveals that the masonry from the base up to a height of 15 m is made up of regular stone blocks and thin mortar joints, as are the corners of the walls above this level, as well. The upper walls' central portions are instead made up of regular stone blocks and bricks. On 25 November, 2016 the ambient vibrations of the Clock Tower were monitored for a few hours via four seismometric stations made available by the Seismological Observatory of Arezzo (INGV). This monitoring campaign allowed identifying some of the tower's natural vibration frequencies and mode shapes [22]. The measured frequencies (Hz) are

$$f = [1.05,\ 1.3, 4.19, 4.50].$$

Figure 4: The Clock Tower

Figure 5: FE mesh of the Clock Tower

The FE model of the Clock Tower (Figure 5) is used here to test the proposed algorithm. The model employs 11383 eight-node brick elements for the masonry [20], while the tie rods and the wooden roof are modelled via beam elements, for a total number of 45511 degrees of freedom. We update two parameters, Young's modulus of the masonry constituting the tower, E_{low}, and that of the bell chamber, E_{up}. These parameters are allowed to vary within the intervals:

$$2500\,\text{MPa} \le E_{low} \le 5500\,\text{MPa}\,, \quad 1000\,\text{MPa} \le E_{up} \le 5500\,\text{MPa}\,.$$

The masonry's Poisson's ratio has been fixed at 0.2; the assumed mass densities of the tower and the bell chamber are ρ_{low} = 2100 kg/m^3 and ρ_{up} = 1700 kg/m^3, respectively, while the mechanical characteristics chosen for the iron and wood components are

$$E_i = 210000\,\text{MPa}\,,\ \rho_i = 7850\frac{\text{kg}}{\text{m}^3}\,,\ \nu_i = 0.3,\ E_w = 8000\,\text{MPa}\,,\ \rho_w = 800\frac{\text{kg}}{\text{m}^3}\,,\ \nu_w = 0.35\,.$$

The total computation time was **76.32 s**. A single modal analysis (with fixed parameters) using NOSA-ITACA takes 29.59 s. In this case the algorithm's performance is shown for two different expressions of objective function (5), which has been evaluated for $w_i = f_i^{-1}$ (default choice) and $w_i = 1$, $i = 1,\dots,4$. The parameters found are:

$$E_{low} = 3076\,\text{MPa}\,,\ E_{up} = 1950\,\text{MPa}$$

for $w_i = f_i^{-1}$ and

$$E_{low} = 3182\,\text{MPa}\,,\ E_{up} = 1873\,\text{MPa}$$

for $w_i = 1$. Table 2 shows the tower's frequencies found in the two cases, and their relative errors with respect to the experimental values. In the case of $w_i = f_i^{-1}$, the accuracy of frequency matching tends to increase for the lowest modes; analogously, using Operational Modal Analysis, the lowest frequencies are generally determined with greater accuracy [25]. For $w_i = 1$ the matching of the lower frequencies is less accurate.

Exp. freq.	Freq. ($w_i = f_i^{-1}$)	Rel. errors ($w_i = f_i^{-1}$)	Freq. ($w_i = 1$)	Rel. errors ($w_i = 1$)
1.05	1.0449	0.49%	1.0621	1.15%
1.3	1.315	1.15%	1.3366	2.82%
4.19	4.2154	0.61%	4.2041	0.33%
4.5	4.4409	1.31%	4.4729	0.60%

Table 2: Results of the optimization algorithm: frequencies and their relative errors with respect to the experimental values, in the case of $w_i = f_i^{-1}$ and $w_i = 1$. Frequencies are expressed in Hz.

Figure 6 shows the objective function $\phi(\text{x})$ evaluated for $w_i = f_i^{-1}$ (the function's values are in log-scale). Figure 7 shows instead a section of $\phi(\text{x})$ (blue line) passing through the function's minimum point; the plot for $w_i = 1$ is also shown in red. Finally Figure 8 shows, on the left, the convergence of the objective function to the minimum for each new reduced model (dashed line is the tolerance), and on the right, the convergence of the model's frequencies to the experimental values.

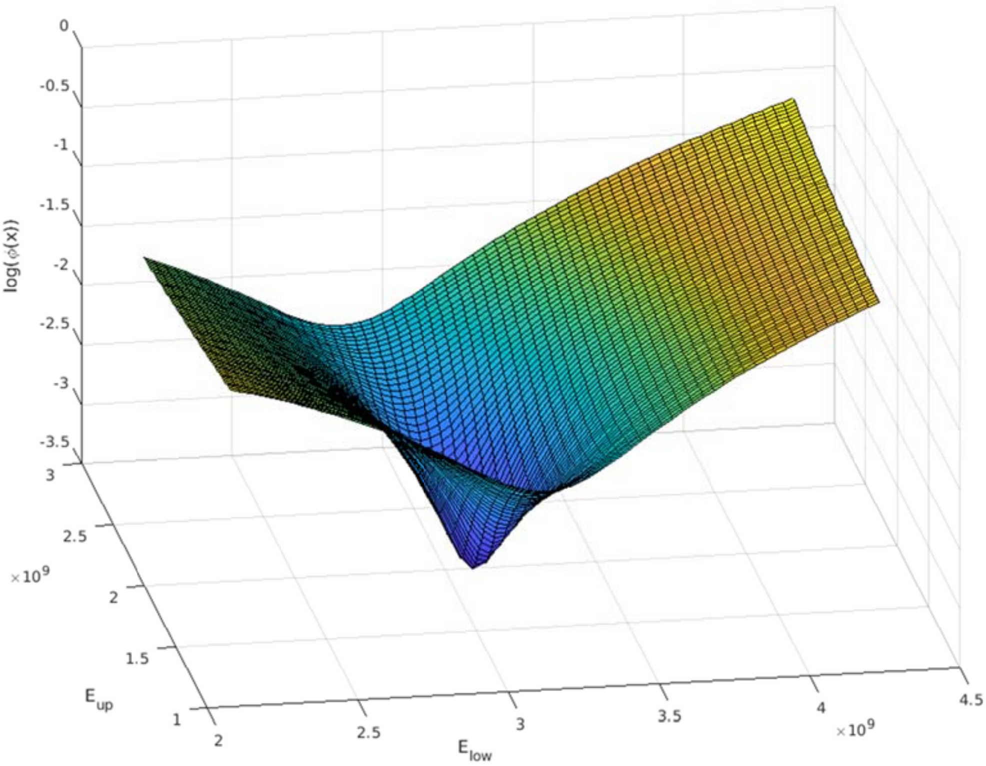

Figure 6: Objective function built for the Clock Tower for $w_i = f_i^{-1}$. Young's moduli are expressed in Pa.

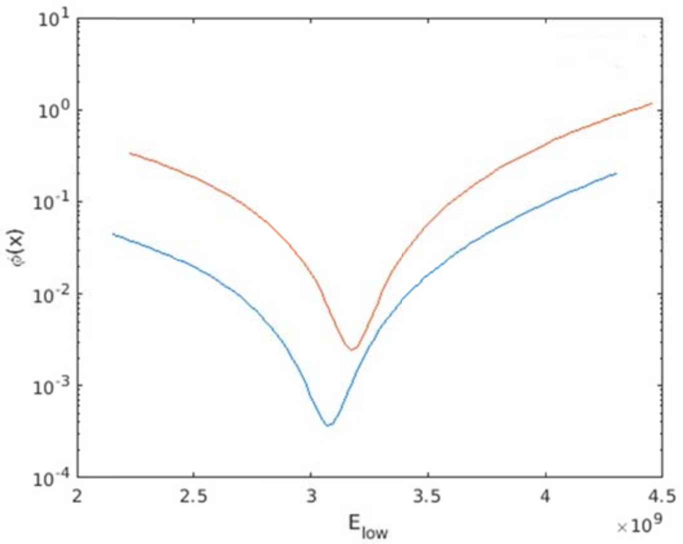

Figure 7: Sections of the objective functions for $w_i = f_i^{-1}$ (blue line) and for $w_i = 1$ (red line), passing through the corresponding minimum points. The tower's Young's modulus is expressed in Pa.

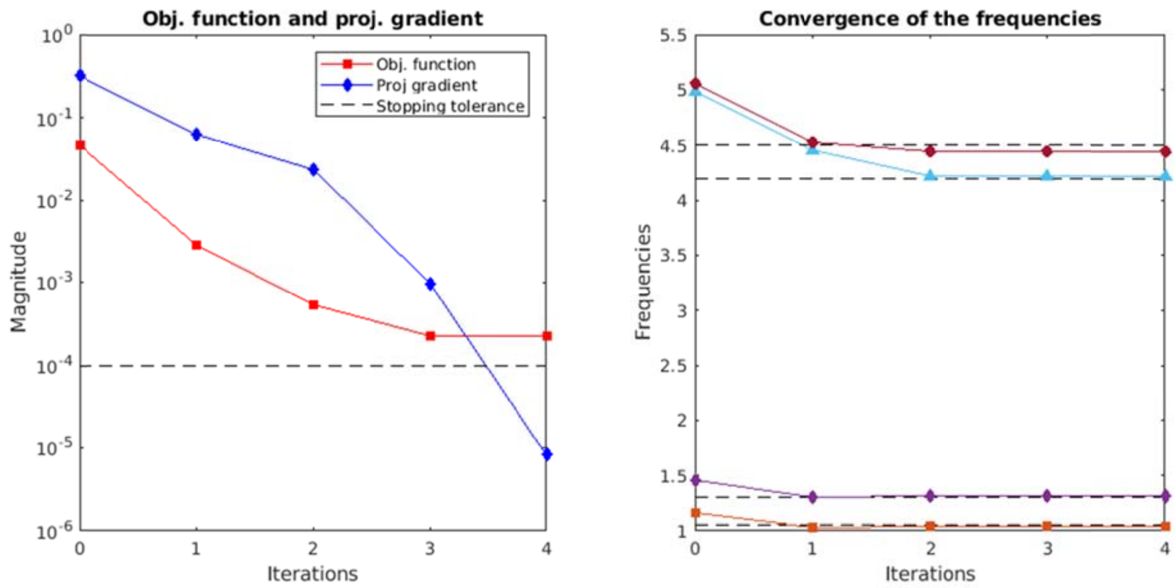

Figure 8: Convergence history of the optimization algorithm.

4 CONCLUSIONS

A new algorithm for solving FE model updating problems has been presented. The strategy followed is to calibrate the parameters of a FE model (mechanical properties or boundary conditions) in order to minimize the discrepancies between a set of experimentally determined natural frequencies and the corresponding frequencies evaluated numerically. The algorithm, which relies on construction of local parametric reduced order models embedded in a trust region scheme, has been coupled with the NOSA-ITACA code for calculation of the eigenfrequencies. The resulting procedure, which is completely automatic, turns out to be very efficient and reduces both the total computation time of the numerical process and the user effort. The paper illustrates its applications to two monumental buildings in Lucca, the Church of San Francesco and the Clock Tower. Although the results are very encouraging, several problems remain open, and deserve further study, which has been planned for future research. Among these, special attention will be devoted to optimization of the eigenmodes, in order to obtain even more accurate simulations of the dynamic behavior of masonry structures.

REFERENCES

[1] J. C. Chen e J. A. Garba, Analytical model improvement using modal test results. *AIAA journal*, **18**, 684-690, 1980.

[2] B. M. Douglas, W. H. Reid, Dynamic tests and system identification of bridges. *Journal of the Structural Division*, **108**, 1982.

[3] M. Friswell, J. E. Mottershead, *Finite element model updating in structural dynamics*, vol. 38, Springer Science & Business Media, 2013.

[4] T. Marwala, *Finite element model updating using computational intelligence techniques: applications to structural dynamics*, Springer Science & Business Media, 2010.

[5] E. Simoen, G. De Roeck, G. Lombaert, Dealing with uncertainty in model updating for damage assessment: A review. *Mechanical Systems and Signal Processing,* **56**, 123-149, 2015.

[6] A. Teughels, G. De Roeck, Damage detection and parameter identification by finite element model updating. *Revue européenne de génie civil*, **9**, 109-158, 2005.

[7] T. Aoki, D. Sabia, D. Rivella, T. Komiyama, Structural characterization of a stone arch bridge by experimental tests and numerical model updating. *International Journal of Architectural Heritage,* **1**, 227-250, 2007.

[8] A. S. Araujo, P. B. Lourenço, D. V. Oliveira, J. C. Leite, Seismic assessment of St. James church by means of pushover analysis: before and after the New Zealand earthquake. *The Open Civil Engineering Journal,* **6**, 160-172, 2012.

[9] A. Cabboi, C. Gentile, A. Saisi, From continuous vibration monitoring to FEM-based damage assessment: Application on a stone-masonry tower. *Construction and Building Materials*, **156**, 252-265, 2017.

[10] R. Ceravolo, G. Pistone, L. Z. Fragonara, S. Massetto, G. Abbiati, Vibration-based monitoring and diagnosis of cultural heritage: a methodological discussion in three examples. *International Journal of Architectural Heritage*, **10**, 375-395, 2016.

[11] V. Compan, P. Pachon, M. Camara, P. B. Lourenço e A. Saez, Structural safety assessment of geometrically complex masonry vaults by non-linear analysis. The Chapel of the Würzburg Residence (Germany), *Engineering Structures,* **140**, 1-13, 2017.

[12] Y. S. Erdogan, Discrete and Continuous Finite Element Models and Their Calibration Via Vibration and Material Tests for the Seismic Assessment of Masonry Structures, *International Journal of Architectural Heritage*, 2017.

[13] L. Z. Fragonara, G. Boscato, R. Ceravolo, S. Russo, S. Ientile, M. L. Pecorelli, A. Quattrone, Dynamic investigation on the Mirandola bell tower in post-earthquake scenarios, *Bulletin of Earthquake Engineering*, **15**, 313-337, 2017.

[14] C. Gentile, A. Saisi, Ambient vibration testing of historic masonry towers for structural identification and damage assessment. *Construction and Building Materials,* **21**, 1311-1321, 2007.

[15] T. Kocaturk, Y. S. Erdogan, C. Demir, A. Gokce, S. Ulukaya, N. Yuzer, Investigation of existing damage mechanism and retrofitting of Skeuophylakion under seismic loads. *Engineering Structures,* **137**, 125-144, 2017.

[16] V. Pérez-Gracia, D. Di Capua, O. Caselles, F. Rial, H. Lorenzo, R. Gonzalez-Drigo, J. Armesto, Characterization of a romanesque bridge in Galicia (Spain). *International Journal of Architectural Heritage,* **5**, 251-263, 2011.

[17] L. F. Ramos, L. Marques, P. B. Lourenço, G. De Roeck, A. Campos-Costa, J. Roque, Monitoring historical masonry structures with operational modal analysis: two case studies. *Mechanical systems and signal processing*, **24**, 1291-1305, 2010.

[18] W. Torres, J. L. Almazan, C. Sandoval, R. Boroschek, Operational modal analysis and FE model updating of the Metropolitan Cathedral of Santiago, Chile. *Engineering Structures,* **143**, 169-188, 2017.

[19] M. Girardi, C. Padovani, M. Porcelli, D. Pellegrini, L. Robol, Finite element model updating for structural applications, *arXiv:1801.09122*, 2018.

[20] V. Binante, M. Girardi, C. Padovani, G. Pasquinelli, D. Pellegrini, M. Porcelli, L. Robol, *NOSA-ITACA 1.1 documentation,* 2017.

[21] M. Porcelli, V. Binante, M. Girardi, C. Padovani, G. Pasquinelli, A solution procedure for constrained eigenvalue problems and its application within the structural finite-element code NOSA-ITACA. *Calcolo,* **52**, 167-186, 2015.

[22] D. Pellegrini, M. Girardi, C. Padovani e R. Azzara, A new numerical procedure for assessing the dynamic behaviour of ancient masonry towers, in *COMPDYN2017 - 6th ECCOMAS Thematic Conference on Computational Methods in Structural Dynamics and Earthquake Engineering*, 2017.

[23] A. Guidi, *Il Convento di San Francesco alla Fratta*, Lucca: PubliEd, 2008.

[24] M. Girardi, C. Padovani, G. Pasquinelli, Numerical modelling of the static and seismic behaviour of historical buildings: the church of San Francesco in Lucca, in *CC2013 - Fourteenth International Conference on Civil, Structural and Environmental Engineering Computing. Proceedings, article n. 80. B.H.V. Topping, P. Ivanyi (eds.),* (Cagliari, Italy, 3-6 September 2013), 2013.

[25] C. Ventura, R. Brinker, *Introduction to Operational Modal Analysis*, John Wiley and Sons, Ltd, 2015.