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Noisy cooperative intermittent processes: From blinking quantum dots to human consciousness

Paolo Allegrini,^{1,2} Paolo Paradisi,^{1,3} Danilo Menicucci,^{1,3} Remo Bedini,^{1,2} Angelo Gemignani,^{1,4} Leone Fronzoni⁵

¹Istituto di Fisiologia Clinica (IFC-CNR), Via Moruzzi 1, 56124 Pisa, Italy

²Centro EXTREME, Scuola Superiore S. Anna, P.zza Martiri della Libertà 7, 56127 Pisa, Italy

³Istituto di Scienza e Tecnologie dell'Informazione "A. Faedo" (ISTI-CNR), Via Moruzzi 1, 56124 Pisa, Italy

⁴Dipartimento di Scienze Fisiologiche, Università di Pisa, Via San Zeno 31, 56127 Pisa, Italy

⁵Dipartimento di Fisica "E. Fermi"-Università di Pisa and INFM CRS-SOFT, Largo Pontecorvo 3, 56127 Pisa, Italy

E-mail: allegrini@ifc.cnr.it

Abstract. We study the superposition of a non-Poisson renewal process with the presence of a superimposed Poisson noise. The non-Poisson renewals mark the passage between metastable states in system with self-organization. We propose methods to measure the amount of information due to the two independent processes independently, and we see that a superficial study based on the survival probabilities yield stretched-exponential relaxations. Our method is in fact able to unravel the inverse-power law relaxation of the isolated non-Poisson processes, even when noise is present. We provide examples of this behavior in system of diverse nature, from blinking nano-crystals to weak turbulence. Finally we focus our discussion on events extracted from human electroencephalograms, and we discuss their connection with emerging properties of integrated neural dynamics, i.e. consciousness.

1. Introduction

Intermittence is an ubiquitous phenomenon in physics, and is an indication of the sporadic decrease in entropy production, due to formation of collective behaviors. In the special case when the classical Kolmogorov-Sinai Entropy (entropy-production rate) is null, a non-trivially evolving system is commonly referred to as as complex, and inverse-power-law (or fractal) statistics is found for both spatial and temporal correlations. The paradigms for these behaviors are found in turbulence. The classic theory of Kolmogorov [1] provides an analytical treatment, based on Gaussian statistics and self-similarity, for fractal behavior of the statistics of velocity fluctuations in fully developed turbulence. These fluctuations, through a branching process of energy transport, are due to bursts of activity at different scales. Deviations to the theory, due to extreme velocity excursions that happen more often than expected on the basis of Gaussian statistics are referred to as "intermittency" [2, 3]. Time is also intermittent, since bursts of activity are separated by temporal epochs of quiescence, termed "laminar regions" whose durations are in turn distributed as inverse-power laws [4, 5].

Large interest has been recently devoted to the fluorescent intermittence of quantum dots [6]. Quantum jumps have been first revealed by Dehmelt [7]. However, the Dehmelt's experiment

was on a single atom in a trap, and a Poisson intermittence [8] was observed. The blinking quantum dot phenomenon is in fact produced by a collection of atoms in a nanostructure, and the intermittent effect is non-Poissonian. Although a definitive theoretical explanation of the phenomenon is still missing, it seems obvious that collective fluctuations in the nano-crystal structure is ultimately responsible for the non-Poisson blinking [6]. The time distance between two consecutive transitions from the light-on (off) to the light-off (on) condition is given by an inverse power law distribution density with a power index $\mu < 2$. Recently, a random-walk approach (in the nanostructure configuration space) [9] theoretically yields $\mu = 1.5$, a typical index for return times, i.e. the Sparre-Andersen theorem [10]. A careful analysis of the experimental data reveals that μ can be different from 1.5 and that the process is renewal [11].

Moreover, the authors of [12] developed a theoretical model of synchronizing units generating intermittence similar to that produced by blinking quantum dots. Remarkably, the intermittent phenomenon correspond to the global properties produced by the cooperation among many units, each of which, in the absence of cooperation, may be described very well by using the Poisson statistics of quantum jumps. It is only when the cooperation parameter reaches the critical value, corresponding to a phase transition from the uncorrelated to the correlated condition, that the signal generated by the sum of all individual contributions exhibits the typical intermittent behavior of blinking quantum dots. A theoretical explanation, again based on returns of a random walker in a shallow potential, again yields $\mu = 1.5$ [13]. The proposed model can be looked at as a paradigm of decision making, namely the emergence of intelligent behavior out of non-intelligent but cooperating agents, and has been proposed as a model for a critical brain.

The hypothesis of a critical brain, on the other hand, is becoming increasingly popular. Since the seminal ideas of Turing [14] more experimental data have been reported supporting this view, a notable example being the discovery of intermittent avalanches by Beggs and Plenz [15], who invoke self-organized criticality (SOC) [16] to explain their findings [17]. In this case the model consists in a branching process of information transport through layers of neurons, which, in terms of a parameter describing the probability of successfully transmitting a signal at each step, can be subcritical (information quenching, yielding null information transfer), critical (optimal information exchange) and supercritical (information explosion yielding saturation). This particular model predicts, at the critical point, neural avalanches whose size and durations are inverse-power-law distributed, with indexes respectively 1.5 and 2.0, in agreement with the experimental data of Beggs and Plenz.

Criticality in human brain has been recently reviewed by Ref. [18], focusing again on inverse-power laws and the presence of avalanches (as in SOC), and also by the scale-free-network representation of cross correlations between pairs of units, superimposable to what can be found in a dynamical Monte-Carlo of a critical Ising model. Another illuminating review is the work of Werner [19], showing that inverse-power laws and intermittency are ubiquitous in the brain, from single cells to complex behavior. At one of the highest levels, i.e. at the level of consciousness, it has been recently hypothesized that the dynamical pattern of “thoughts” is a serial conscious output which integrates non-conscious units which are in fact working in a highly parallel way, and that the serial, operational time is in fact characterized by the system visiting meta-stable states, with information only increasing at the transitions between a metastable state and the next [20]. This hypothesis may be linked to the heteroclinic orbits of Ref. [21], where it was proven that reproducible metastable states correspond to neural response to stimuli (e.g. odors) and can be associated to segmentation in electroencephalogram (EEG), namely the fact that EEG signals frequently looks like juxtapositions of epochs with fixed frequency and amplitude, with abrupt changes, called rapid transition processes (RTP), from epoch to epoch [22].

We point out that an aspect which is frequently overlooked is the “renewal” character of intermittency, namely that the time durations of the laminar regions, or of the operational epochs, are mutually independent. This means that long-range autocorrelations in the signals

are dominated by the waiting time distribution $\psi(\tau)$, i.e. by the distribution density of the temporal durations of the metastable states. From an entropic point of view, this means that entropy increases during transitions, while is kept frozen when cooperative states are activated. It has been recently shown that the fluctuations of the order parameter in a critical system undergoes similar dynamics [23], so that there is a strong connection between critical systems and temporal intermittency, while space intermittency was seen at criticality a few decades ago [24], to model massless-to-mass transition (intermittency in hadron formation [25]).

Having discussed the ubiquity of intermittency in cooperative structures in this Section, we focus, respectively in Section 2 and 3, on two particular aspects, namely on formally deriving a new form of linear response theory for intermittent systems, as it allows a new kind of control, and, second, at the level of signal analysis, on how to assess intermittency in the presence of superimposed noise.

2. Linear Response for intermittent processes

The fluctuation-dissipation of first kind [26] plays a quite important role in physics, as the response to external perturbation, which can be experimentally detected, brings information on the systems dynamics, thereby helping the foundation of proper models for the processes under study. The most general form of linear response theory is given by

$$\Pi(t) = \epsilon \int_0^t \chi(t, t') \xi_P(t') dt', \quad (1)$$

where $\Pi(t)$ is the susceptibility, $\xi_P(t)$ is a function describing the perturbation time dependence and ϵ is the perturbation strength. The Fluctuation-Dissipation Theorem (FDT) is a theoretical proposal for the two-times function $\chi(t, t')$, the response function. In the literature of non-Poisson systems (e.g. glasses) [27], the form ordinarily adopted for the response $\chi(t, t')$ is

$$\chi(t, t') = \frac{d}{dt'} \langle A(t) B(t') \rangle, \quad (2)$$

where A is the variable of which we study the response to perturbation and B is the variable through which the system is coupled to the external perturbation. This simple formula is very attractive. First of all, it does not imply that the correlation function depends on $t - t'$, thereby going beyond the stationary restriction of the conventional linear response theory [26]. Then, it fits the physical intuition insofar as the derivative with respect to $t' < t$ is claimed [27] to be equivalent to enforcing the causality principle. The ultimate theoretical foundation of this theory is given [27] by the Onsager perspective [28], based on variable probabilities.

In this Section we propose a new approach focusing on the events responsible for the variables time evolution, rather than on the variables probabilities. For simplicity's sake we make the assumption $A = B = \xi_S$ and we assume the variable ξ_S dichotomous. This restriction does not prevent us from applying our approach to real physical systems. In fact, our approach is inspired by the phenomenon of molecular intermittency, and the dichotomous blinking of quantum dots [6] has been proved to be non-ergodic and renewal [11]. We prove that, at variance with (2),

$$\chi(t, t') = -\frac{d}{dt} \Psi(t, t'), \quad (3)$$

where $\Psi(t, t') \equiv \langle \xi_S(t) \xi_S(t') \rangle$, and the system is prepared at $t = 0$.

A renewal system is characterized by a waiting-time distribution $\psi(\tau)$, meaning a succession of waiting times $\{\tau_i\}$; next we define a dichotomous variable $\xi(t)$

$$\xi(t) = \begin{cases} \text{Rand}[+1; -1] & \text{if } t = t_n \\ \xi(t - dt) & \text{otherwise,} \end{cases} \quad (4)$$

with $t_n = \sum_{i=0}^n \tau_i$, meaning that during the quiescent regions of lengths $\{\tau_i\}$ no change of sign can happen. Only at the end of these regions the variable ξ may change or keep the sign, with probability 1/2, according to a fair coin-tossing procedure, indicated above with the function $\text{Rand}[+1;-1]$. We now dynamically generate $\{\tau_i\}$ with an inverse-power law distribution as follows. We consider the following equation of motion for a real positive variable $y \in (0, 1)$

$$\dot{y} = ay^z, \quad (5)$$

and consider the time τ it takes for y to reach the border $y = +1$ starting from an initial condition y_0 . We integrate (5) by quadrature and obtain

$$\int_{y_0}^{+1} \frac{dy}{y^z} = a\tau, \quad (6)$$

yielding a relation $y_0(\tau)$. If now we impose that every time the border is reached the new initial condition is taken randomly, namely with a constant density $p(y_0) = 1$, we find

$$\psi(\tau) = \left| \frac{dy_0}{d\tau} \right| = \frac{(\mu - 1)T^{\mu-1}}{(T + \tau)^{\mu-1}}, \quad (7)$$

where $T = 1/[a(z - 1)]$ and $\mu = z/(z - 1)$. The renewal condition is ensured by the randomly selected re-injection in y_0 . This model, a continuous-time version of the map [4], depends on two parameters, T and μ . T is the lapse of time necessary for the expression in the r.h.s of (7) to become identical to an inverse-power law. μ , on the other hand, is a fractional index that marks the presence and measures the degree of complexity.

Here let us focus our attention to linear response to small external perturbations. It is arguable that the parameter to be perturbed in (7) is T , rather than μ , since the time scale of a perturbation is normally finite. This is true, for instance, both in the case of looking at the relaxation after a kick, and in the case of a periodic perturbation. In both cases the perturbation should not be able to change the scaling μ of the system, since this is the result of infinitely long scales. We therefore imagine that a linear perturbation to a complex system can be simply modeled as a perturbation on T , or equivalently, on a . The dynamical equation (5) becomes

$$\dot{y} = a[1 \pm \epsilon \xi_P(t)]y^z, \quad (8)$$

where ϵ is a small parameter ($0 < \epsilon \ll 1$), and $\xi_P(t)$ is the external forcing. Notice that this forcing does not change the weak repeller topology in the dynamical system. In other words, the trajectory is still laminar near the origin and its long-range properties are still dominated by a Pomeau-Manneville type of intermittency (this holds true also in multidimensional systems).

Integrating (8) yields the analogous of (6)

$$\int_{y_0^\pm}^{+1} \frac{dy}{y^z} = a \int_{t'}^{t'+\tau_\pm} [1 \pm \epsilon \xi_P(t'')] dt'', \quad (9)$$

where y_0^\pm denotes the reinjection at t' corresponding to a choice of the state “plus” or “minus” for the variable ξ . If re-injection is kept flat, $y_0^\pm = y_0$ and the l.h.s's of (9) and of (6) coincide. Equating the r.h.s's results in an iterative relation for the perturbed waiting times τ_\pm

$$\tau_\pm = \tau \mp \epsilon [F(t' + \tau_\pm) - F(t')], \quad (10)$$

where the function F is the integral function of ξ_P . $t = t' + \tau$ denotes the end of the quiescent region, or, according to the model (8), the instant when $y = +1$.

Let us rephrase the model and the coin-tossing procedure using the density approach. Let us define a vector $\mathbf{p} = (p_1, p_2)$. We impose equiprobability after the coin tossing. If we toss the coin at any discrete time we have

$$\mathbf{p}_{m+1} = [I - A]\mathbf{p}_m, \text{ where } A = \frac{1}{2} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \quad (11)$$

is the transition matrix and I is the identity matrix. The sporadic tossing of the coin is easily modeled by imagining a scalar characteristic function $r(m)$ which assumes the value 1 when events occur (at t_n) and 0 elsewhere, multiplying A in (11). This means

$$\mathbf{p}_{m+1} = [I - r(m)A]\mathbf{p}_m, \text{ with } r(m) = \sum_n \delta_{m,t_n}, \quad (12)$$

where t_n are the transition times. In the continuous limit the Kroeneker delta's of (12) become Dirac delta's. Their intensity must so large that after the first coin tossing $p_1 = p_2$, thus

$$\mathbf{p}(t + dt) - \mathbf{p}(t) = \lim_{M \rightarrow 0} \delta(t - t_n) \frac{dt}{M} A \mathbf{p}(t) \quad (13)$$

and hence

$$\dot{\mathbf{p}}(t) = -r(t)A\mathbf{p}(t), r(t) = \lim_{M \rightarrow 0} \left[\sum_n \frac{\delta(t - t_n)}{M} \right]. \quad (14)$$

This means that $r(t)$ is a sum of $1/M$ -normalized density functions whose amplitudes tend to infinity in the continuous limit. This is correct since the formal solution of (13) tells us that $p_1(t)$ and $p_2(t)$ become $1/2$ after the first coin tossing and never move afterwards. Let us define $\sigma \equiv p_1 - p_2$ the equation for σ , $\dot{\sigma} = -r(t)\sigma$, is readily solved

$$\sigma(t) = \exp \left[- \int_0^t r(t') dt' \right] \sigma(0) \Rightarrow \langle \sigma(t) \rangle = \left\langle \exp \left[- \int_0^t r(t') dt' \right] \right\rangle \langle \sigma(0) \rangle, \quad (15)$$

where we used the fact that the system prepared at $t = t_p = 0$, so we can factorize the average over the initial condition. The first term in the r.h.s of (15) is a product of step function since

$$\exp \left[- \int_0^t r(t') dt' \right] = \lim_{M \rightarrow 0} \prod_n \exp \left[- \int_0^t \frac{\delta(t' - t_n)}{M} dt' \right], \quad (16)$$

and each exponential is unity for $t \leq t_n$, dropping to the vanishing value $\exp(-1/M)$ for $t > t_n$. Only the first coin tossing, at time t_1 is important, so we can formally write (15) in the form

$$\left\langle \exp \left[- \int_0^t r(t') dt' \right] \right\rangle = \text{Pr} [t \leq t_1]. \quad (17)$$

The r.h.s. in (17) is in fact a conditional probability, the condition being a coin tossing occurring at $t_p = 0$, and can be identified with the survival probability

$$\left\langle \exp \left[\int_0^t r(t') dt' \right] \right\rangle = \Psi(t) \equiv 1 - \int_0^t \psi(t') dt', \quad (18)$$

and therefore (15) is a zero-aged Onsager relation $\langle \sigma(t) \rangle = \Psi(t) \langle \sigma(0) \rangle$.

Imagine now that observation starts at $t' > 0$. In this case (17) must be modified to account for a further condition in the probability, namely that no observation was made in the interval $[0, t']$. This probability is the t' -aged survival probability $\psi(t, t')$, which obeys the relation

$$\Psi(t, t') \equiv \int_t^\infty \psi(t'', t') dt'' \Rightarrow \psi(t, t') = - \frac{d\Psi(t, t')}{dt}, \quad (19)$$

the density $\psi(t, t')$ being the t' -aged waiting time distribution, defined as the probability density of observing a coin tossing in the interval $(t, t + dt)$, with $t > t'$, given a coin tossing at $t = 0$. Eq. (18) is thus generalized as follows

$$\left\langle \exp \left[\int_{t'}^t r(t'') dt'' \right] \right\rangle = \Psi(t, t'). \quad (20)$$

Applying a perturbation means changing the transition matrix. In general we can write

$$\dot{\mathbf{p}}(t) = -R(t)\mathbf{p}(t), \quad \text{with } R(t) = \frac{1}{2} \begin{pmatrix} r_+(t) & -r_-(t) \\ -r_+(t) & r_-(t) \end{pmatrix}. \quad (21)$$

It is straightforward to evaluate the equation for σ . Eq (21) becomes

$$\dot{p}_1 = \frac{-r_+ p_1 + r_- p_2}{2}, \quad \dot{p}_2 = \frac{r_+ p_1 - r_- p_2}{2}, \quad (22)$$

and taking the difference of the two equations yields $\dot{\sigma} = -S(t)\sigma - D(t)$, where

$$S(t) = \frac{r_+ + r_-}{2}, \quad D(t) = \frac{r_+ - r_-}{2}, \quad (23)$$

whose formal solution is

$$\sigma(t) = - \int_0^t dt' \exp \left[- \int_{t'}^t dt'' S(t'') \right] D(t') + \exp \left[- \int_0^t dt' S(t') \right] \sigma(0). \quad (24)$$

Let us go back to (10). The bias is given by the fact that the duration of the quiescent regions become shorter (longer) according to the system being in a status with the same (opposite) sign of the perturbation, after which a fair coin is tossed. It is clear that the overall bias cannot be due to the cumulative effect of previous drawings happened in the past, but one should focus his attention on the unperturbed coin-tossing times t , which will be turned into the perturbed coin-tossing time t' . In other words, the probability of a bias at time t will depend on the probability of having an unperturbed event at that same time t . We can use Eq. (10) to write the perturbed time t' in terms of the unperturbed t . To simplify our treatment let us focus on the first event time, $t = t_1$. According to this notation

$$r(t) = \lim_{M \rightarrow 0} \frac{1}{M} \delta(t - \tau), \quad \text{and } r_{\pm}(t') = \lim_{M \rightarrow 0} \frac{1}{M} \delta(t' - \tau_{\pm}), \quad (25)$$

and Eq. (10) yields $t = t' \pm \epsilon \int_0^{t'} dt'' \xi_P(t'')$. Using the Jacobian methods for the Dirac's deltas,

$$r_{\pm}(t') = r[t(t')] \left| \frac{dt}{dt'} \right| \Rightarrow r_{\pm}(t') = r(t) [1 \pm \epsilon \xi_P(t')], \quad (26)$$

therefore, plugging (26) into the formal solution (24), we have

$$\sigma(t) = - \int_0^t dt' \exp \left[- \int_{t'}^t dt'' S(t'') \right] \frac{r_+(t') - r_-(t')}{2} + \exp \left[- \int_0^t dt' S(t') \right] \sigma(0). \quad (27)$$

We can now assume that $S \equiv (r_+ + r_-)/2 = r$, this approximation being exact for vanishing ϵ , since the two symmetric deltas converge to the unperturbed one. Making use of (26) we write

$$\sigma(t) = -\epsilon \int_0^t dt' \exp \left[- \int_{t'}^t dt'' r(t'') \right] r(t) \xi_P(t') + \exp \left[- \int_0^t dt' r(t') \right] \sigma(0). \quad (28)$$

Making averages and taking $\langle \sigma(0) \rangle = 0$ Eq. (28) becomes identical to (1) with a response function given by (3), which is the result we are seeking: making use of (20),

$$\chi(t, t') = \left\langle \exp \left[- \int_{t'}^t dt'' r(t'') \right] r(t) \right\rangle = \left\langle \frac{d \exp \left[- \int_{t'}^t dt'' r(t'') \right]}{dt} \right\rangle = - \frac{d\Psi(t, t')}{dt}. \quad (29)$$

3. Superposition of Poisson noise to non-Poisson signals

To study the effect of noise on signals we study a model adopted in [29] for human language and in bioinformatics [30], with the name Copying Mistake Map (CMM). The CMM model $\{\xi(t)\}$ consists of the superposition of two sequences, a non-Poisson renewal Sequence n , or $\{\xi_n(t)\}$, and a Poisson Sequence p , or $\{\xi_p(t)\}$, with $\psi_n(\tau) \propto \tau^{-\mu}$ for $\tau \rightarrow \infty$, and $\psi_p(\tau) = \gamma \exp(-\gamma\tau)$. The sequences of event times are

$$t_i^{(n)} = \sum_{k=1}^i \tau_k^{(n)} \quad \text{and} \quad t_i^{(p)} = \sum_{k=1}^i \tau_k^{(p)}. \quad (30)$$

The signal is unity in these points, and zero otherwise, namely

$$\xi_n(t) = \delta_{t, t_i^{(n)}}, \quad \xi_p(t) = \delta_{t, t_i^{(p)}}, \quad (31)$$

where $\delta_{i,j}$ is a Kroeneker delta. The CMM model is

$$\xi(t) = \xi_p \vee \xi_n \approx \xi_p + \xi_n \quad (32)$$

where \vee is the boolean OR, and we used the fact that $\text{Prob}[\xi_p = \xi_n = 1]$ is vanishingly small.

The survival probability $\Psi(\tau)$ for the combined sequence (32) reads

$$\Psi(\tau) = P(n)e^{-\gamma\tau}\Psi_n(\tau) + P(p)e^{-\gamma\tau}\Psi_n^\infty(\tau), \quad (33)$$

where $\Psi_n^\infty(t)$ is the stationary correlation function of the non-Poisson process. In (33) $P(p)$ denotes the probability for an event to belong to the Poisson series, while $P(n)$ to the non-Poisson one. In the stationary regime $\mu > 2$ The probabilities $P(p)$ and $P(n)$ are inversely proportional to the respective average times:

$$P(n) \propto \frac{1}{\langle \tau_n \rangle} \quad ; \quad P(p) \propto \frac{1}{\langle \tau_p \rangle}, \quad (34)$$

where $\langle \tau_{p,n} \rangle = \int_0^\infty \tau \psi_{p,n}(\tau) d\tau$, namely

$$\langle \tau_p \rangle = \frac{1}{\gamma} \quad ; \quad \langle \tau_n \rangle = \frac{T}{\mu - 2}. \quad (35)$$

After normalization, and using (33) to evaluate the global mean time $\langle \tau \rangle$

$$P(n) = \frac{\langle \tau_p \rangle}{\langle \tau_p \rangle + \langle \tau_n \rangle} = (1 - \gamma)\langle \tau \rangle; \quad P(p) = \frac{\langle \tau_n \rangle}{\langle \tau_p \rangle + \langle \tau_n \rangle} = \gamma\langle \tau \rangle. \quad (36)$$

$\Psi_n^\infty(t) \equiv \int_t^\infty \psi_n^\infty(\tau) d\tau$ denotes the infinitely aged survival probability, where $\psi_n^{t_a}(t)$ is the infinitely aged waiting-time distribution, with a change notation with respect of Section 2 to allow infinite age. It is calculated using $R(t)$, the probability of having an event at t , as [31]

$$\begin{aligned} \psi_n^{t_a}(\tau) &\equiv \psi(\tau + t_a, t_a) = \int_{-t_a}^0 R(t' + t_a) \psi_n(\tau - t') dt', \text{ where} \\ R &\equiv \sum_{i=0}^{\infty} \psi_n^i(t), \quad \psi_n^i(t) = \int_0^\infty dt' \psi_n^{i-1}(t - t') \psi_n(t'), \end{aligned} \quad (37)$$

and $\psi_n^0(t) = \delta(t)$ is a Dirac δ . For $t_a = \infty$ $R = 1/\langle \tau_n \rangle$ [31], so

$$\psi_n^\infty(\tau) = \frac{1}{\langle \tau_n \rangle} \int_{-\infty}^0 \psi_n(\tau - t') dt' = \frac{\Psi_n(\tau)}{\langle \tau_n \rangle}. \quad (38)$$

Notice that before the exponential cutoff in (33) we have the joint action of two inverse-power laws, with indexes $\mu - 1$ and $\mu - 2$. If γ were vanishingly small, the asymptotic behavior would be dictated by the index $\mu - 1$. We recall that the treatment herein holds when $\mu > 2$, but it can be generalized for $\mu < 2$, where $\Psi_n^\infty(t)$ tends to a non-vanishing constant, while $\langle \tau \rangle \rightarrow \gamma^{-1}$ as $t \rightarrow \infty$. Numerically, the presence of two inverse-power decays and the exponential cut-off can make the decay of the survival probability indistinguishable from a stretched exponential.

This is apparent in Fig. 1, showing $\Psi(t)$ for typical cascading events (RPT in more than one channel) stemming from multi-channel EEG. Here we show Subject 5 of the experiment discussed in Ref. [32]. We show two sets of data points, one stemming from EEG data (open squares), and another with a Monte Carlo simulation of the model (open circles) with ψ_n stemming from (7), with $T = 6.0s$ and $\mu = 2.05$, $\gamma = 0.006Hz$, and the same length of the data analyzed (5 minutes, sampling ratio 2ms). The curves stemming from the two point sets are indistinguishable. Notably, the dotted line, indicating a best stretched-exponential fit for $\Psi(t)$ seem to accurately describe both data sets, as earlier stated. It is in fact indistinguishable from data for $t > 30ms$.

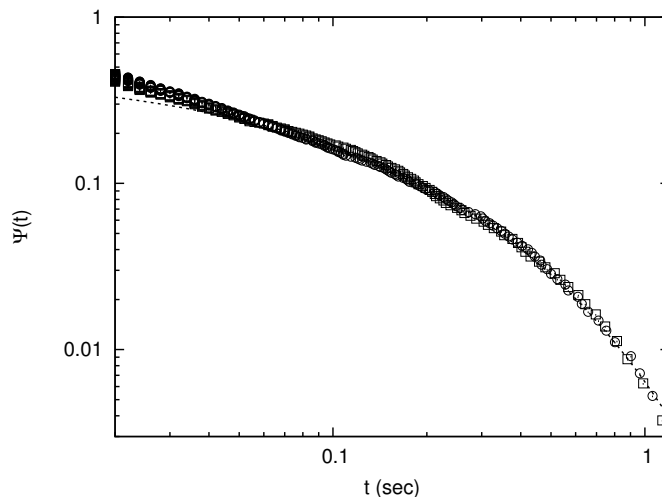


Figure 1. Survival probability. Open squares stem from EEG RTPs data. Open circles stem from a numerical simulation of the CMM with $r = 0.006$ Hz, $T = 6.0s$ and same statistics of the data. Dotted line is a stretched exponential $A \exp[(t/B)^\alpha]$, with $A = 0.5$.

Let us use a more sensible correlational analysis on the CMM model. Following [32] we build a diffusing trajectory from $\xi(t)$ using two different rules. We make use of DFA to unravel long-range correlations. The correlation function of the signal $\xi(t)$ is proportional to the second derivative of $\sigma^2(t) \equiv \langle x^2(t) \rangle$, where $\dot{x} = \xi$ or, equivalently $x(t) = \int_{t_0}^{t_0+t} dt' \xi(t')$, and average $\langle \dots \rangle$ is over all possible values of t_0 (moving windows). Therefore, if $\langle \xi(t_0) \xi(t_0 + t) \rangle$ is not integrable, namely it decays as $t^{-\beta}$ with $0 < \beta < 1$, then asymptotically $\sigma^2(t) \sim t^{2H}$, with $H = 1 - \beta/2$.

In Rule #1 $\xi(t)$ is simply (32). As earlier stated, $\xi(t) = \xi_p(t) + \xi_n(t)$ and since the two sequences p and n are mutually independent, it is possible to separate their second moment contributions x_p and x_n and we have that

$$\sigma^2(t) = \langle x_p^2(t) \rangle_p + \langle x_n^2(t) \rangle_n, \quad (39)$$

where the suffix p (n) on the angular brackets means the average on the p (n) sequence alone. The Poisson process yields standard diffusion $\langle x_p^2 \rangle_p \propto t$, while for renewal non-Poisson processes

$\langle x_n^2(t) \rangle_n \propto t^{4-\mu}$. The non-Poisson process dominates in the asymptotic limit. However, the presence of the Poisson Process yields a transient $H = 0.5$ in the short-time limit. This behavior is illustrated in Fig. 2, bottom curves, where we compare the analyses same EEG data of Fig. 1 with the theoretical prediction, with $\mu = 2.05$.

Rule #2 consists of making a walker assume a constant velocity ± 1 within laminar regions, chosen with a coin-tossing procedure for every region. In this case (39) does not hold: While for this rule we again have $\langle x_p^2 \rangle_p \propto t$ and $\langle x_n^2(t) \rangle_n \propto t^{4-\mu}$, the asymptotics are dominated by the Poisson process, while the non-Poisson scaling is confined in the short-time limit. Fig. 2 illustrates this behavior both for the model and the experimental data.

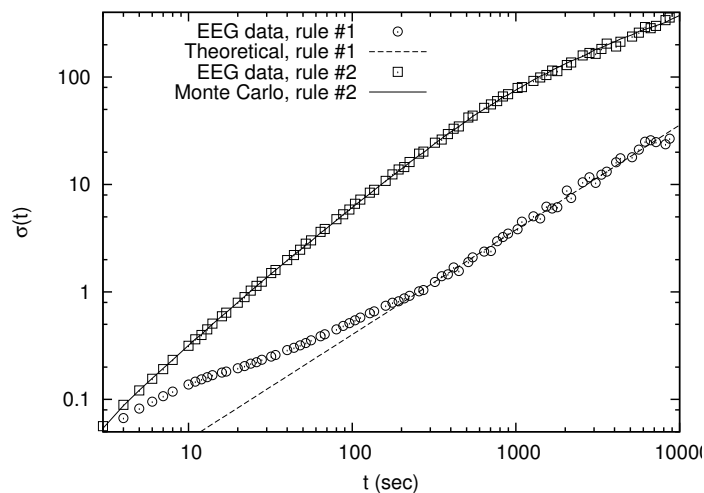


Figure 2. DFA Analysis: Rule #1 for EEG data (open squares) compared with theoretical prediction $\sigma(t) \propto t^{2-\mu/2}$ (dashed line); Rule #2 for EEG data (open circles) compared with same rule on the same Monte Carlo simulation of Fig. 1 (solid line).

In conclusion, studying concurrent (multi-channel) abrupt transitions in EEGs we conclude that a single complex renewal process (plus Poisson noise) underlies non sensory-constrained mental activity. Since (results will be reported elsewhere) single-channel transitions, although with similar $\psi(\tau)$, do not obey this simple dynamics, we claim that a serial (i.e. computationally non parallel) binding activity among neural groups is at the basis of the holistic output called consciousness. It is important to acknowledge that classifying the thought-related high level activity of the brain as a non-Poisson renewal process is of a great importance, since, as discussed in detail in Section 2, these processes obey a new form of Linear Response theory, of which we provide a detailed derivation in Section 2. The proof exposed here is mathematically simpler than what has been published elsewhere and it does not suffer from limitations due to discrete time as in [33] or to perturbations which are analytical in time as in [34]. It can be generalized to other forms of $\psi(\tau)$, as in the treatment of Ref. [35]. We think that the new Linear Response is important both for its implications and for its generality, from blinking quantum dots to human consciousness. The authors of Ref. [36] theoretically predict a response to abrupt and persistent stimuli, which is extremely slow in all the range $1 \leq \mu \leq 3$. These predictions have been experimentally confirmed in physical systems (nematic liquid crystals) [37] studying slow relaxation upon an abrupt changes of μ ; at the level of cognitive neuroscience, it is worth noting that this slow response was conjectured to dominate the phenomenon of memory losing, as discussed in [38]. As far as periodical perturbations are concerned, we refer the reader to the experimental observations of Refs [39] and [40], respectively again on liquid crystals and

neuroscience. Finally, non-Poisson renewal systems effectively respond to perturbations only if the stimulus is of the same class of complexity, with a maximal information transport for $\mu \approx 2$, as recently theoretically assessed by Aquino *et al.* [41]. This phenomenon is termed “complexity matching”. While an experimental assessment of these latter predictions is still missing, we note that this maximal information exchange can constitute a basis for explaining the surprising coincidence of $\mu \approx 2.1$ both in EEGs and in human language [29]. In general, the complexity matching effect can provide the basis for studying the interactions between complex systems or networks, for instance between the central nervous system and the peripheral autonomic system.

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