AV60 - Rome September 22-24, 2014

A Brownian ratchet driven by Coulomb friction

Alberto Petri

Consiglio Nazionale delle Ricerche – CNR
Institute for Complex Systems
Rome - Italy

institute for complex systems

Andrea Gnoli
Fergal Dalton[1]
Giorgio Pontuale

Andrea Puglisi
Giacomo Gradenigo[2]
Alessandro Sarracino[3]

presently at:
[1]FireEye Ireland Inc., Cork, Ireland
[2]CEA Saclay, Gif-sur-Yvette, France
[3]Université Paris 6, Paris, France

FIRB-IDEAS Project: "*Granular Chaos*" and PRIN Project: "*Friction laws for granular media: ageing, memory and microscopic dynamics*"

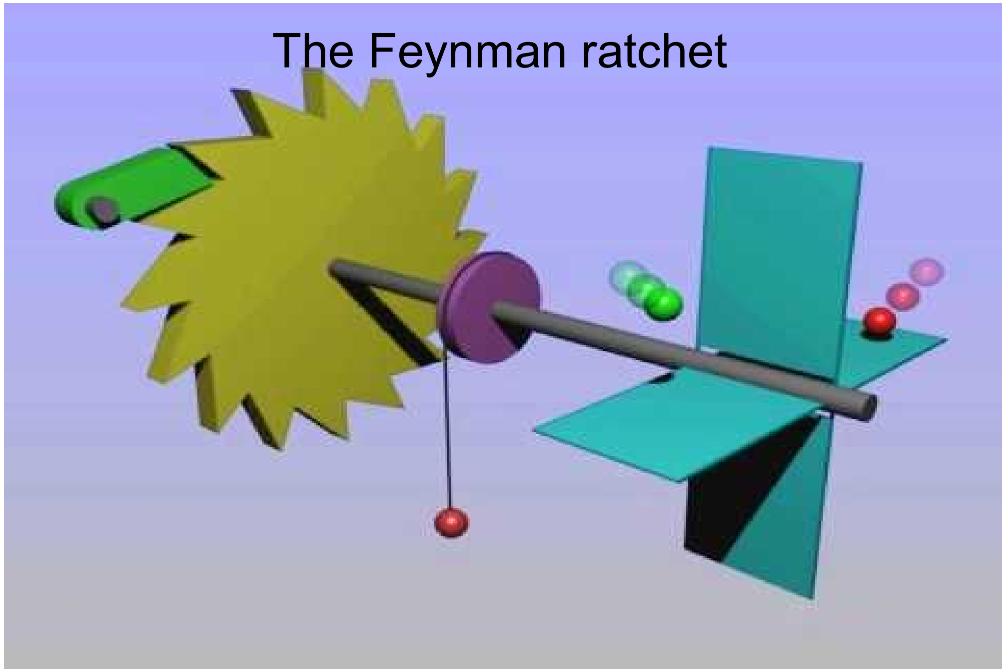
Collaborations with Angelo were grounded on some specific kind of granular matter

a very promising young boy asked him to be his thesis advisor

he is now one of the most active in the area

How to extract work from molecular chaos

An old problem: ***the ratchet*** (Smoluchowski 1912, Feynman 1963)



Rectification of thermal fluctuations requires
- breaking some spatial symmetry
- **operate under non-equilibrium** (Maxwell demon, 2nd principle of TD, etc.).

institute
for complex
systems

How to extract work from molecular chaos

An old problem: ***the ratchet*** (Smoluchowski 1912, Feynman 1963)

me.utdallas.edu

Rectification of thermal fluctuations requires
- breaking some spatial symmetry
- **operate under non-equilibrium** (Maxwell demon, 2nd principle of TD, ...)

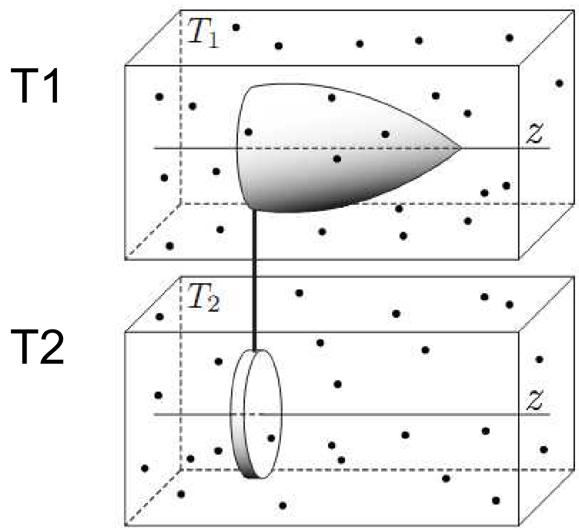
Often referred to as a ***Brownian motor*** drives and controls activities at small scale (biological systems, nanodevices, etc)

institute
for complex
systems

Breaking temporal (and spatial) symmetry

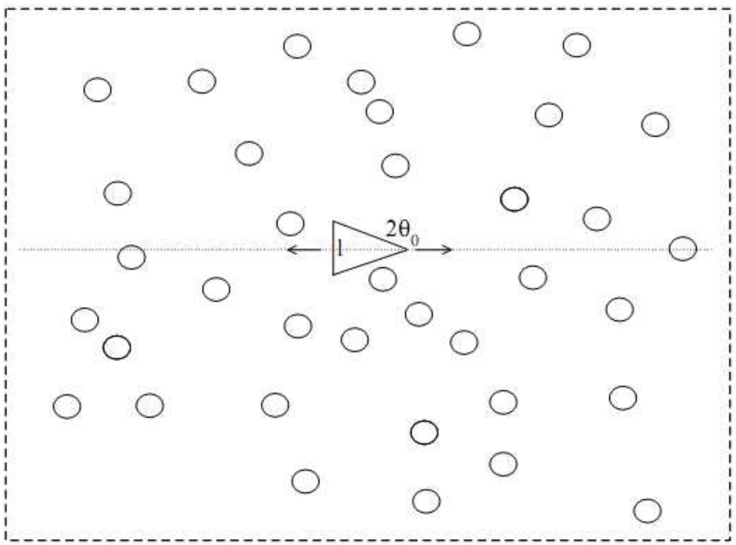
Computer simulations

molecular gas



temperature difference

granular gas



inelastic collisions

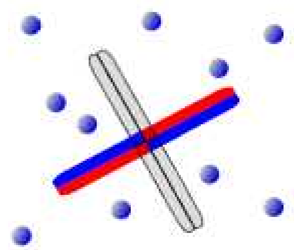
C. Van den Broeck, R. Kawai, P. Meurs,
Phys. Rev. Lett. **93**, 090601 (2004)
Phys. Rev. E 78, 011102 (2008)

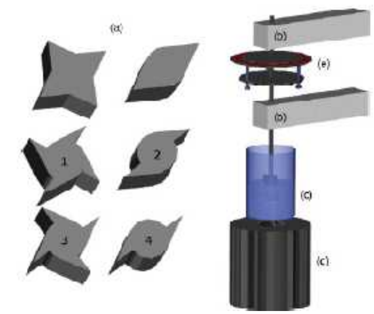
G. Costantini, U. M. B.Marconi, A. Puglisi.
Phys. Rev. E **75**, 4 (2006)

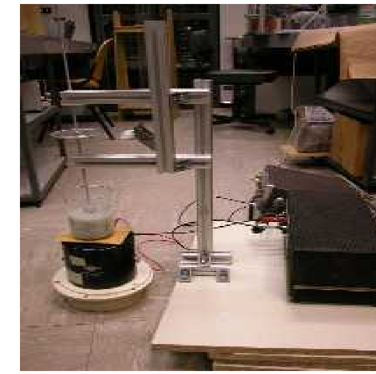
institute
for complex
systems

Breaking temporal (and spatial) symmetry

Experimental realizations







differential inelasticity in a granular gas

P. E. Eshuis, K. Van der Weele, D. Lohse, and D. Van der Meer, *Phys. Rev. Lett*. **104,** 248001 (2010)

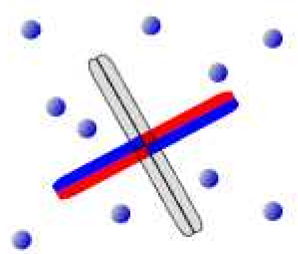
chiral rotator in a dense granular medium

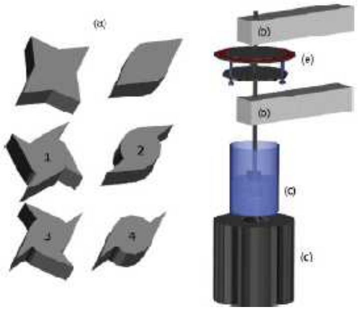
R. Balzan, F. Dalton, V. Loreto, AP, and G. Pontuale, *Phys. Rev. E* **83**, 031310 (2011)

institute
for complex
systems

Breaking temporal (and spatial) symmetry

Experimental realizations





differential inelasticity in a granular gas

P. E. Eshuis, K. Van der Weele, D. Lohse, and D. Van der Meer, *Phys. Rev. Lett*. **104,** 248001 (2010)

chiral rotator in a dense granular medium

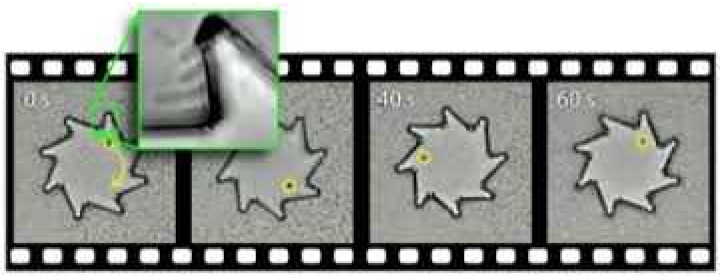
R. Balzan, F. Dalton, V. Loreto, AP, and G. Pontuale, *Phys. Rev. E* **83**, 031310 (2011)

Adopted sources of non equilibrium: two thermal baths, dissipative collisions

Breaking temporal (and spatial) symmetry

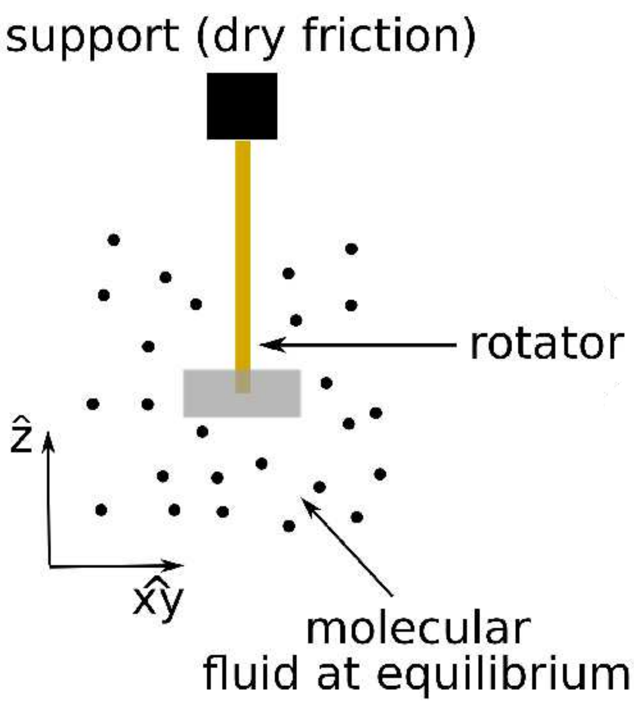
Diferent sources of non equilibrium can be used to obtain directed motion

Especially interesting is also to employ *active particles* like bacteria
(R. Di Leonardo at al., PNAS 107, 9541 (2010))



Another instance of directed motion from non-equilibrium act. particles: hunger driven

A granular ratchet



inelastic collisions

with asymmetric geometry

Unexpectedly problematic A first prototype displayed irregular behavior, motion inversion, etc.

Deeper theoretical investigation was needed

The angular motion of the rotator can be modelled by a stochastic equation

$$I\dot{\omega}(t) = -F_{friction}\sigma[\omega(t)] - \Gamma_{visc}\omega(t) + F_{coll}(t)$$

institute
for complex
systems

I = inertia, *σ* = sign function, *Γ* = viscous drag, *Fcoll* = collisional noise

Two relevant time scales

inter-collisional time

$$\tau_c \sim \frac{1}{n\Sigma v_0}$$

n = fluid particle number density

Σ = total rotator cross-section (perimeter x height)

average particle velocity

$$v_0 = \sqrt{\langle v^2 \rangle}$$

stopping time

$$\tau_\Delta = \frac{\langle|\omega|\rangle_{pc}}{\Delta} \sim \frac{\epsilon v_0}{R_I \Delta}$$

$R_I = \sqrt{I/M}$ inertial radius

$\epsilon = \sqrt{m/M}$ mass ratio

$\Delta = F_{\text{friction}}/I$ frictional damp

control parameter

$$\beta^{-1} = \frac{\epsilon n \Sigma v_0^2}{\sqrt{2\pi} R_I \Delta} \approx \frac{\tau_\Delta}{\tau_c}$$

β >> 1: friction dominated dynamics: **Rare Collisions Limit (*RLC*)**

β << 1: collision dominated dynamics: **Frequent Collisions Limit (*FLC*)**

Assuming Molecular Chaos (diluted gas) one can derive the Boltzmann eq. for the angular velocity pdf

$$\partial_t p(\omega, t) = \partial_\omega[(\Delta\sigma(\omega) + \gamma_a \omega)p(\omega, t)] + J[p, \phi]$$

two limit solutions assuming Maxwellian particle velocities

institute for complex systems

Frequent Collision Limit (FCL)
Continuum noise

$\beta \ll 1 \quad \tau_\Delta \gg \tau_c$

reduced probe drift $\Omega = \frac{R_I}{\epsilon v_0}\omega$ $\quad v_0 = \sqrt{\langle v^2 \rangle}$ average particle velocity

$$\langle \Omega \rangle = \epsilon \sqrt{\frac{\pi}{2}} \frac{1-\alpha}{4} \mathcal{A}_{\mathrm{FCL}}$$

α = restitution coefficient (= 1 if elastic)

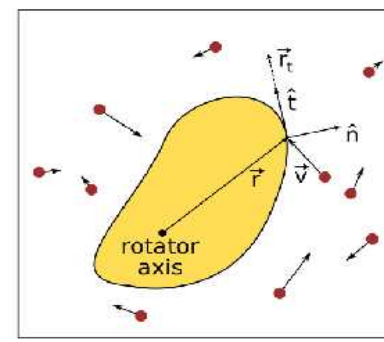
$\epsilon = \sqrt{m/M}$

$$\mathcal{A}_{\mathrm{FCL}} = -\frac{\langle g^3 \rangle_{\mathrm{surf}}}{\langle g^2 \rangle_{\mathrm{surf}}}$$

probe form factor = 0 for symmetric probe

$$g = \frac{\mathbf{r} \cdot \hat{t}}{R_I}$$

$$R_I = \sqrt{I/M}$$



- *zero drift for symmetric probe (A_{FCL}=0)*
- *zero drift for elastic collisions (α=1)*
- $\omega \approx v_0$ *for an asymmetric shape*

institute
for complex
systems

Rare Collision Limit (RCL)

Each collision produces an independent increment

$\beta \gg 1 \quad \tau_\Delta \ll \tau_c$

reduced probe drift $\Omega = \frac{R_I}{\epsilon v_0}\omega$ $\quad v_0 = \sqrt{\langle v^2 \rangle}$ average particle velocity

α = restitution coefficient

$$\langle \Omega \rangle = \sqrt{\pi}(1+\alpha)^2 \beta^{-1} \epsilon^2 \mathcal{A}_{\mathrm{RCL}}$$

$$\beta^{-1} = \frac{\epsilon n \Sigma v_0^2}{\sqrt{2\pi} R_I \Delta} \approx \frac{\tau_\Delta}{\tau_c} \qquad \epsilon = \sqrt{m/M}$$

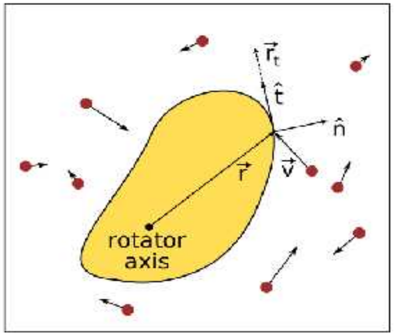
sign

$$\mathcal{A}_{\mathrm{RCL}} = \left\langle \frac{\sigma(g) g^2}{(1+\epsilon^2 g^2)^2} \right\rangle_{\mathrm{surf}}$$

probe form factor = 0 for symmetric probes

$$g = \frac{\mathbf{r} \cdot \hat{t}}{R_I}$$

$$R_I = \sqrt{I/M}$$

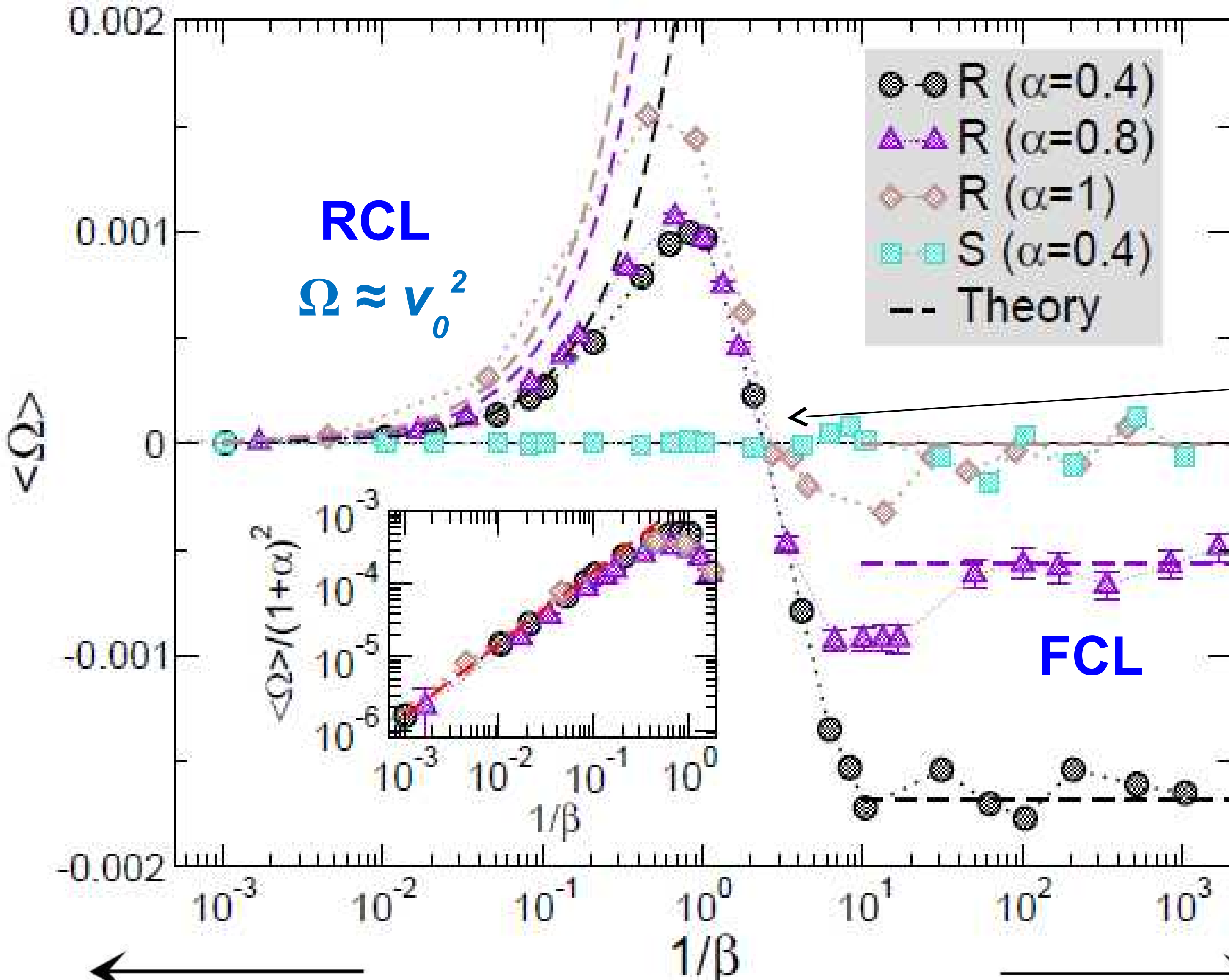


- *zero drift for symmetric probe (A_{RCL}=0)*
- *$\omega \approx v_0^3$ for asymmetric shape*
- *rotation* ***can invert direction*** *wrt FCL (A_{FCL}* can be negative)
- **since now $\omega \approx v_0^3$ while $\omega \approx v_0$ for $\beta \ll 1$, one expects a *maximum at intermediate β*** (kind of ***stochastic resonance***)

- ***directed motion shown also for α=1*** **(elastic collisions)**

Another way to non-equilibrium: friction

Theory vs molecular dynamics simulation



R = right
S = symmetric

inversion predicted for suitable shape

$\Omega \approx$ const

← Friction dominates (RCL)

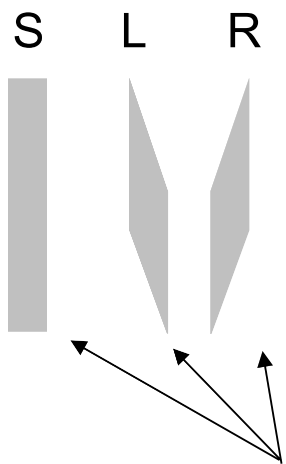
Collisions dominate (FCL) →

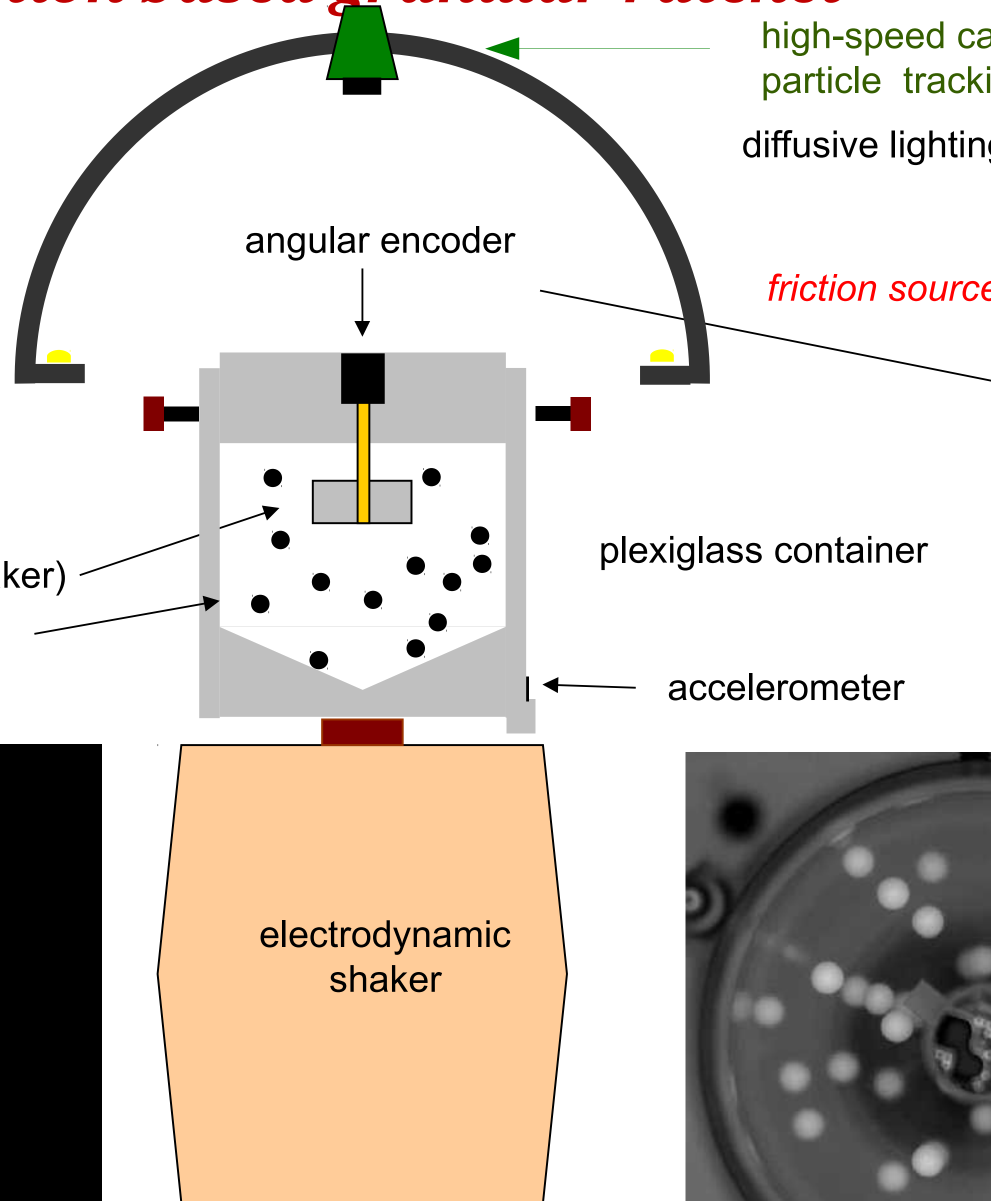
$$\beta^{-1} = \frac{\epsilon n \Sigma v_0^2}{\sqrt{2}\pi R_I \Delta} \approx \frac{\tau_\Delta}{\tau_c}$$

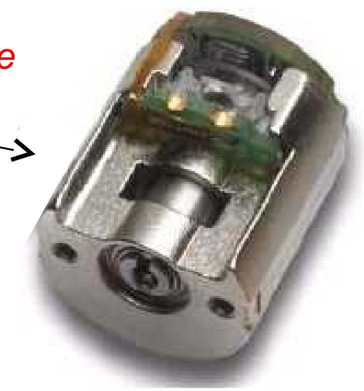
institute
for complex
systems

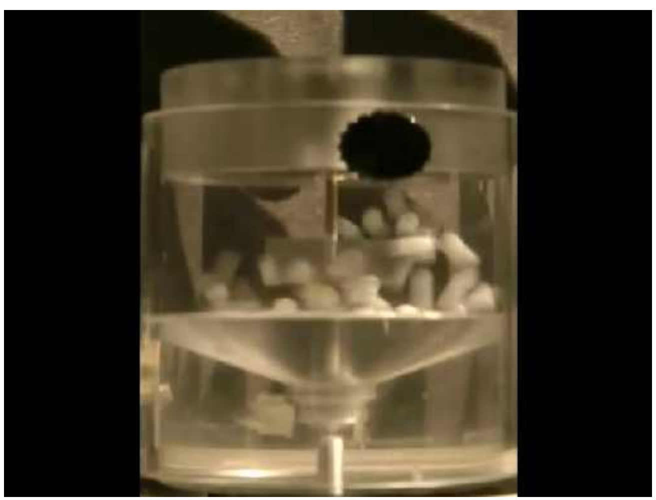
Experimental realization of the rotator

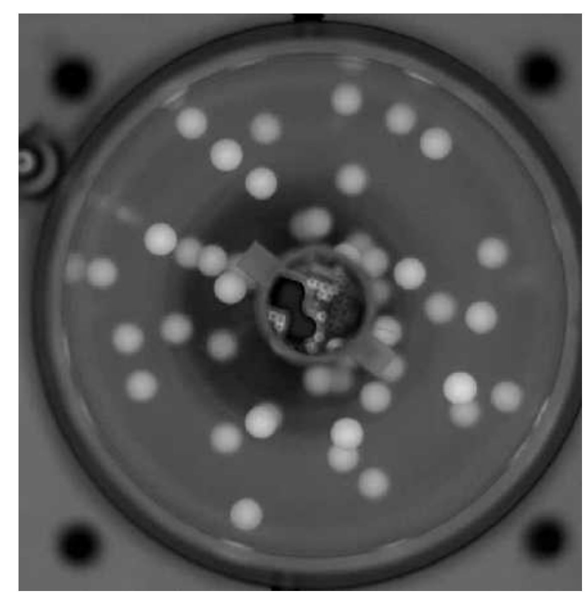
Experimental Realization
friction based granular ratchet





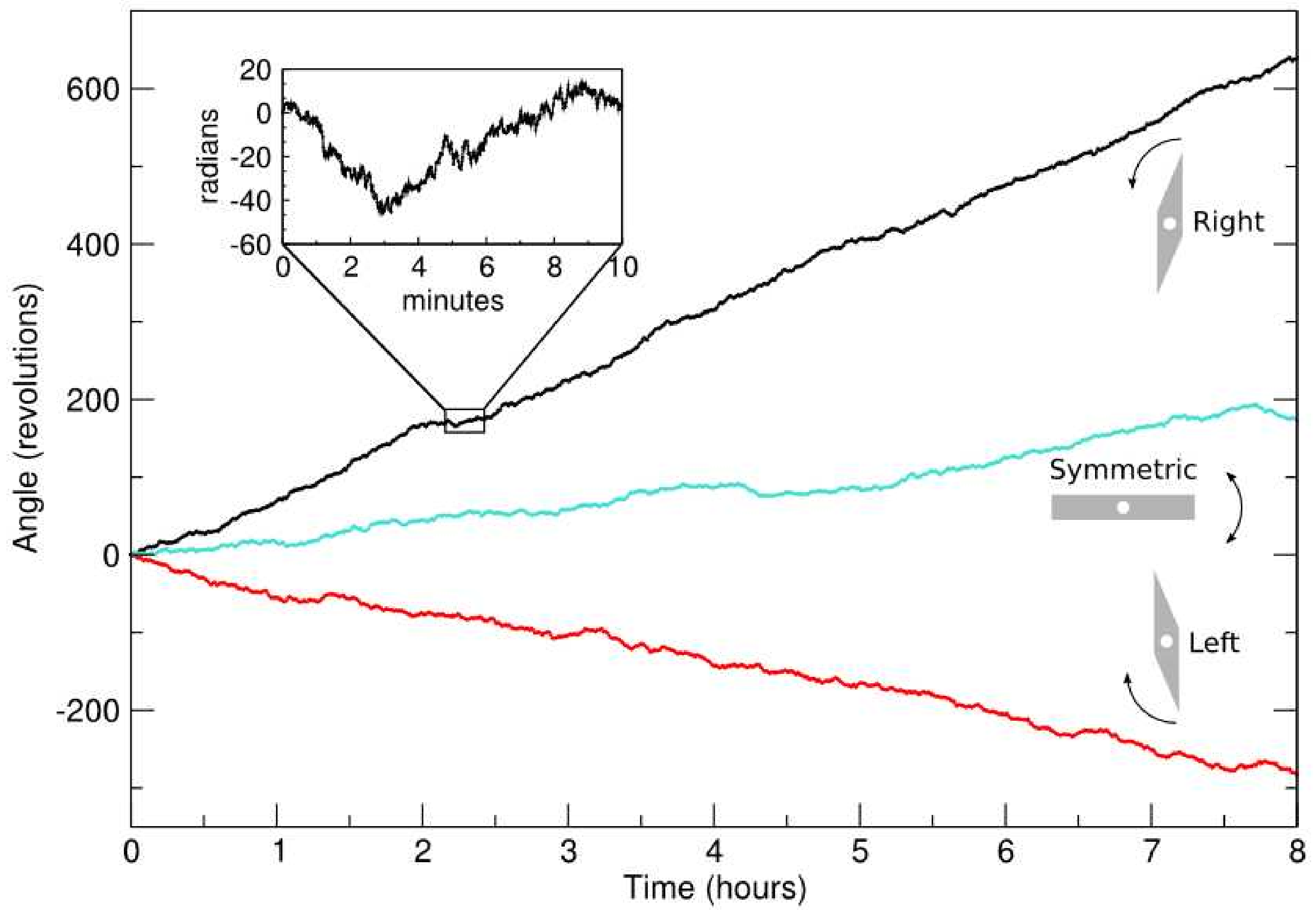






institute
for complex
systems

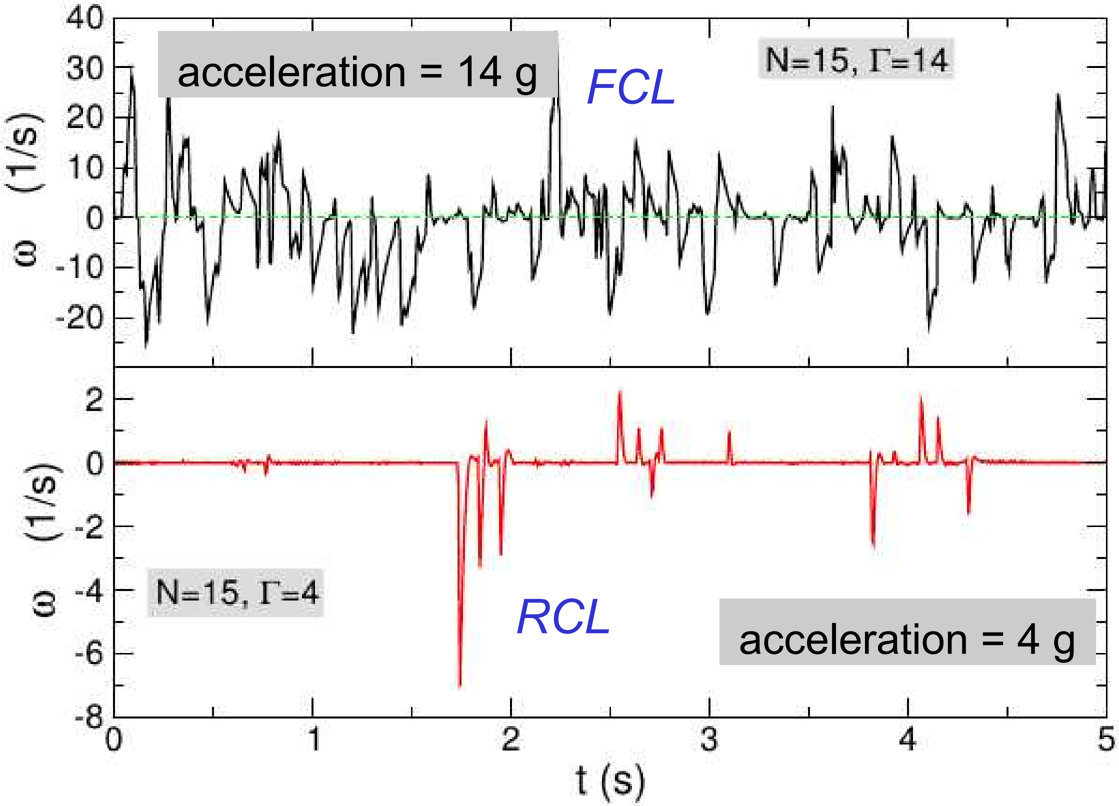
Experimental observation of a net effect



slow drift due to mechanical manufacturing imperfections

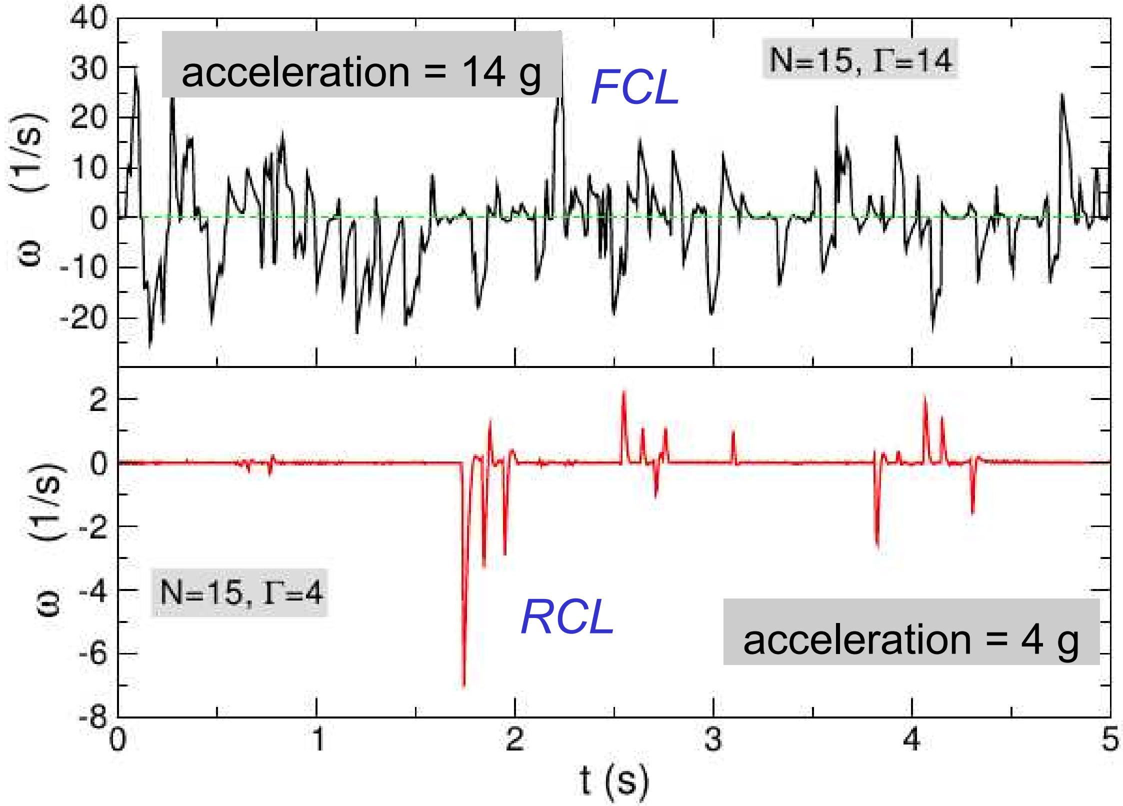
institute for complex systems

Experimental measured drift



institute
for complex
systems

Experimental measured drift



Predicted probe drift

Frequent Collision Limit

$$\langle \Omega \rangle = -\epsilon \sqrt{\frac{\pi}{2}} \frac{1-\alpha}{4} \left\langle \frac{g^3}{g^2} \right\rangle_S$$

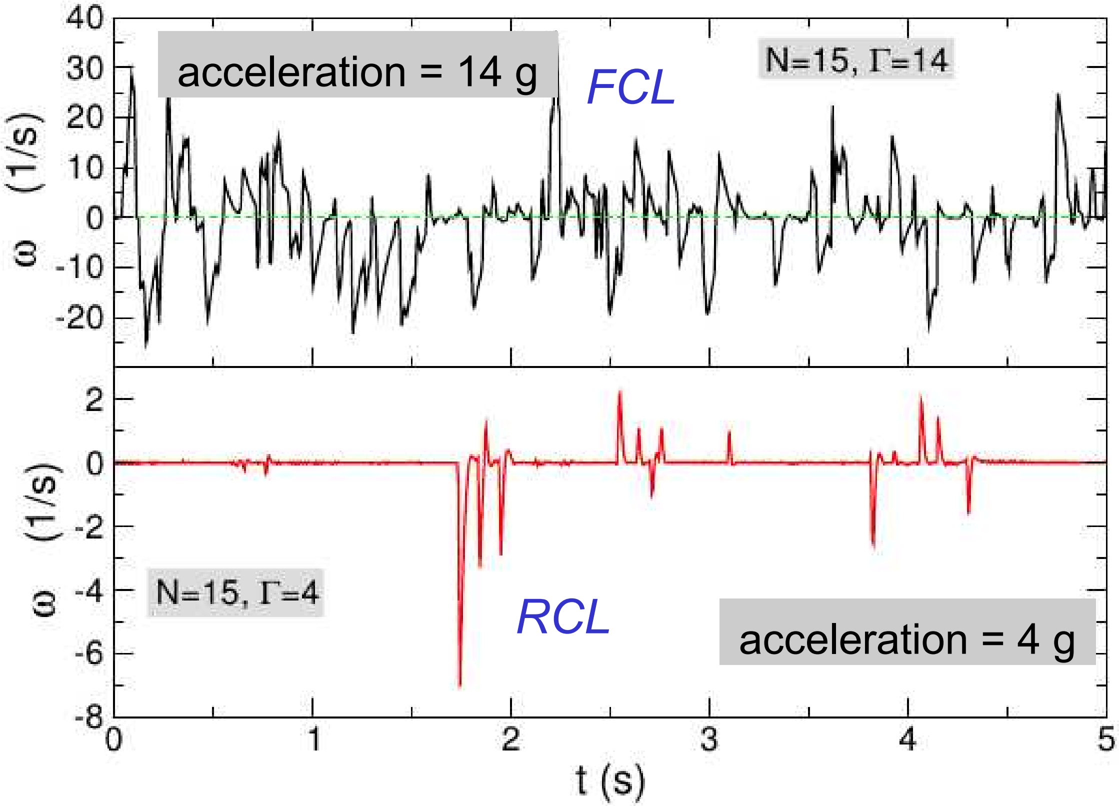
Rare Collision Limit

$$\langle \Omega \rangle = \frac{\epsilon^2 \sqrt{2\pi}(1+\alpha)^2}{\beta} \left\langle \frac{\sigma(g) g^2}{(1+\epsilon^2 g^2)^2} \right\rangle_S$$

for Gaussian pdf of grains velocity

institute
for complex
systems

Experimental measured drift



Predicted probe drift

Frequent Collision Limit

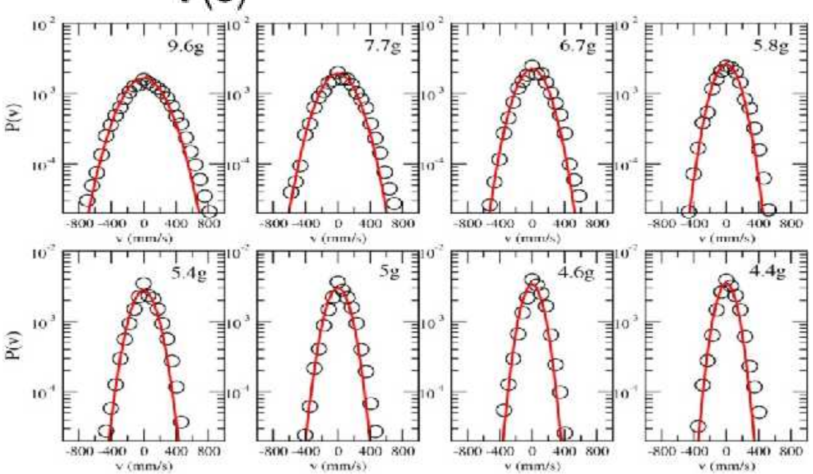
$$\langle \Omega \rangle = -\epsilon \sqrt{\frac{\pi}{2}} \frac{1-\alpha}{4} \left\langle \frac{g^3}{g^2} \right\rangle_S$$

Rare Collision Limit

$$\langle \Omega \rangle = \frac{\epsilon^2 \sqrt{2\pi}(1+\alpha)^2}{\beta} \left\langle \frac{\sigma(g) g^2}{(1+\epsilon^2 g^2)^2} \right\rangle_S$$

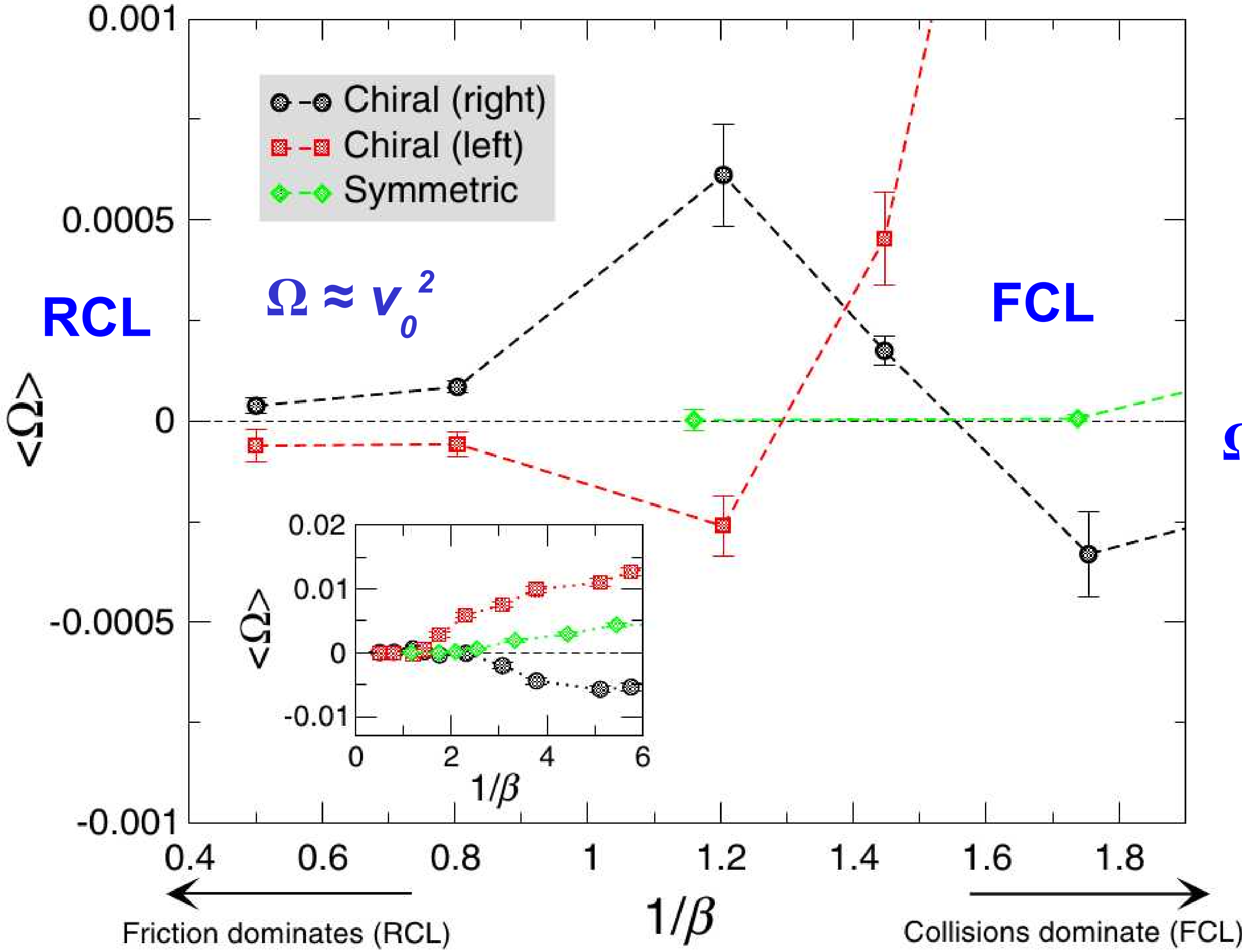
for Gaussian pdf of grains velocity

Measured grains velocity pdf is Gaussian



institute for complex systems

Experimentally measured drift



$\Omega \approx const$

ω is varied by changing shaking acceleration from 5 to 20 g

stochastic resonances ***velocity inversion***

predicted by the theory

institute for complex systems

Resume

Friction can supply the out-of-equilibrium condition necessary to produce noise rectification

The related ratchet effect has been computed, simulated and experimentally observed

Directed drift is maximum at a stochastic resonance of the friction and collisions competing time-scales,
also separating possible velocity inversion

Expected to be observed even with non-dissipative collisions,
thus potentially suitable for devices at the micro and nanoscale

A. Gnoli, AP, F. Dalton, G. Pontuale, G. Gradenigo, A. Sarracino and A. Puglisi,
Phys. Rev. Lett. 110, 120601 (2013)

A. Gnoli, A. Sarracino, A. Puglisi, and AP,
Phys. Rev. E 87, 052209 (2013)

institute
for complex
systems

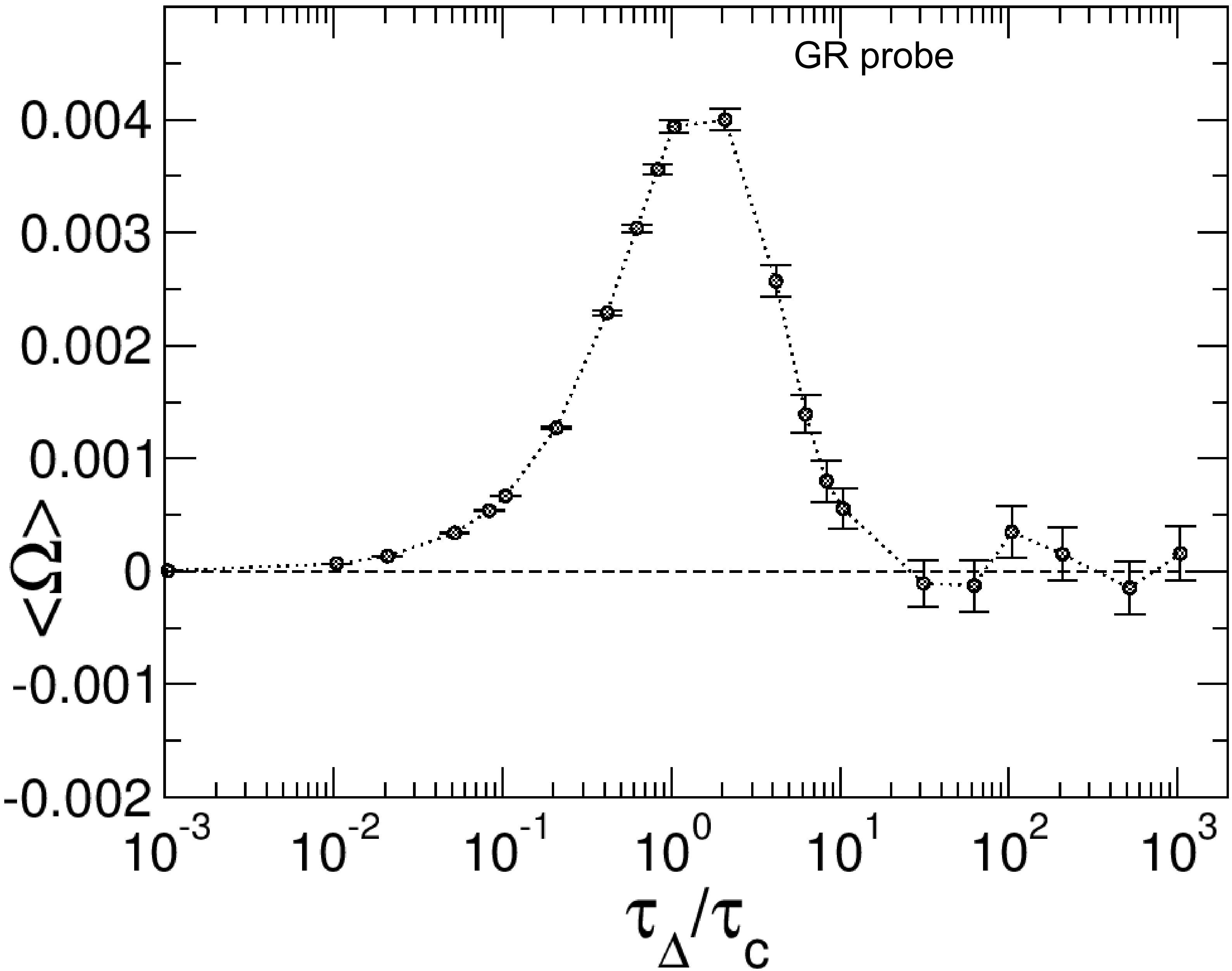
There are some, king Gelon, who think that the number of the sand is infinite in multitude; and I mean by the sand not only that which exists about Syracuse and the rest of Sicily, but also that which is found in any region wether inhabited or uninhabited (Archimedes, The Sand-Reckoner)

Many wishes Angelo of countless happy days

Some useful references

- R. Feynman, R. Leighton, and M. Sands, “The Feynman Lectures on Physics” I (Addison-Wesley, Reading, MA, 1963) p. Chap. 46
- C. Van den Broeck, R. Kawai, P. Meurs, "Microscopic Analysis of a Thermal Brownian Motor", *Phys. Rev. Lett*. **93**, 090601 (2004)
- G. Costantini, U.M.M. Bettolo, A. Puglisi, “Granular Brownian ratchet model”, *Phys. Rev. E* **75**, 4 (2006)
- P. G. de Gennes, “Brownian motion with dry friction”, *J. Stat. Phys*. **119**, 953 (2005)
- J. Talbot, R.D. Wildman, and P. Viot, “Kinetics of a Frictional Granular Motor” *Phys. Rev. Lett.* **107**, 138001 (2011)
- J. Talbot, A. Burdeau, and P. Viot, “Kinetic analysis of a chiral granular motor” *J. Stat. Mech.* P03009 (2011)
- B. Cleuren, R. Eichhorn, “Dynamical properties of granular rotors”, *J. Stat. Mech*. P10011 (2008)

Stochastic resonance expected from elastic collisions



Molecular dynamics simulations