Characteristics, mechanisms and perceivability of combination tones in violins

Gabriele Caselli,[1] Giovanni Cecchi,[2] and Giulio Masetti[3]

[1] *Conservatorio Musicale Lorenzo Perosi, Campobasso, Italy*

[2] *University of Florence, Florence, Italy*[a]

[3] *Istituto di Scienza e Tecnologia dell'Informazione, Consiglio Nazionale delle Ricerche, Pisa, Italy*

Combination tones generated by the inner ear have been widely investigated in the past, starting from the famous Tartini's "third tone". Much less attention has been instead dedicated to the combination tones generated by musical instruments. In this paper we have investigated the combination tones generated by a set of violins of different quality and age when playing a selected set of dyads. Combination tones were found in all the violins and the strongest of them occurred at frequency below the lower note of the dyad. Its amplitude was strongly dependent on violin and dyad played, and was greatest in two old Italian violins and decreased down to a minimum in a factory made violin. All these findings are well explained by the boosting action of A_0, the main air resonance of the violin that correlates well with the strongest combination tone. A listening test, performed using selected dyads and violins, showed that the differences between dyads with and without combination tones were correctly recognized by a group of professional and amateur musicians, suggesting a possible musical significance of the main combination tone. Results investigating the possible source of violin non-linearity are also described.

[a]giovanni.cecchi@unifi.it

I. INTRODUCTION

In music and in musical acoustics, combination tones can be described as weak additional tones that can be perceived when two musical notes are played at the same time. These tones arise from a non-linearity of the inner ear and are perceived with different intensity by different individuals. Following Helmholtz (1877), they are known as subjective combination tones. In spite of their low intensity, combination tones have been considered important in musical theory and composition and have been assumed to contribute to the timbre of the intervals (Hindemith, 1942). Combination tones can also be generated by some musical instruments such as, for example, the harmonium or the violin. These combination tones are present in the air and are known as objective combination tones (Helmholtz, 1877). Unlike subjective combination tones, they have been rarely studied, possibly because they have been judged less perceivable than subjective combination tones and therefore of lesser importance in music. The examination of the combination tones generated by a set of different violins, when playing various intervals, was the aim of our study. In addition to discuss the properties and the mechanisms of the combination tones generated by the violins, this paper reports the results of a listening test performed on a group of professional and amateur musicians. The test was aimed at establishing the possible audibility of the combination tones produced by the violins. Also discussed is the possible influence of violin combination tones on the perception of the intervals as occurs for subjective combination tones. Part of the audio material used for the test is included as additional supplementary material[1] and is downloadable for those interested in checking the perceivability of the combination tones by themselves.

In 1714 Giuseppe Tartini, the famous Italian violinist and composer, discovered that when playing two simultaneous notes (a dyad) on the violin, he could hear a third note called by him "terzo suono" (Tartini, 1754). Tartini's discovery started a wealth of research about the origin, characteristics and musical applications of the third tone which went on throughout the Eighteen century and are still alive today. These studies showed the existence of several third tones and the term Combination Tones (CTs) was introduced to better describe the complexity of the phenomenon (Vieth, 1805). A great step forward regarding the CTs nature was made by Helmholtz (1877, Appendix XII), with the formal demonstration that any non-linearity in the sound production or transmission can generate the combination tones. Helmholtz showed that two sinusoidal sound waves at frequencies f_1 and f_2 acting upon a nonlinear system, give rise to a series of new waves (combination tones) at frequencies of : $2f_1$, $2f_2$, $3f_1$, $3f_2$ (called harmonic distortion products), $f_2 - f_1$, $f_2 + f_1$ (quadratic distortion) and $2f_2 + f_1$, $2f_2 - f_1$, $2f_1 - f_2$, $2f_1 + f_2$ (cubic distortion). Since the source of non-linearity was postulated to be in the middle ear, these combination tones were called by Helmholtz Subjective Combination Tones (SCTs). The validity of the Helmholtz hypothesis has been confirmed by more recent research but the source of auditory non-linearity has been attributed to the amplification mechanism of the cochlea (Robles and Ruggero, 2001).

Helmholtz (1877) also showed that combination tones could be generated by some musical instruments in which two sound sources are coupled through a non-linear element as occurs, for example, in sirens and to a lesser extent in the harmonium. In contrast to SCTs, these combination tones, that Helmholtz called Objective Combination Tones (OCTs), were present in the air and could be amplified by sensitive resonators (Helmholtz, 1877). The characteristics of the OCTs are the same as of SCTs, the only difference is that the non-linear structure responsible of the OCTs is outside the ear and thus OCTs can be detected in the air and measured. Often the CT that is most easily heard has a frequency equal to $f_2 - f_1$ (when f_2 and f_1 are less than one octave apart) mainly because in this case the $f_2 - f_1$ interval ($f_2 > f_1$) has a lower frequency than f_1. In contrast to subjective combination tones which were extensively studied (Goldstein, 1967; Plomp, 1965; Smoorenburg, 1972; Vieth, 1805), but see also Lohri (2016) for an extensive bibliography, studies on objective combination tones after Helmholtz are scarce. This may be also due to the Helmholtz observation that, apart from the sirens, objective combination tones in other musical instruments, as for instance in the harmonium or violin, were perceived as less intense than subjective combination tones and therefore of less musical importance. To our knowledge, apart from Waetzmann (1906), quoted by Lohri *et al.* (2011), only Caselli *et al.* (2020); Lohri (2016); Lohri *et al.* (2011), investigated the phenomena recently. The paper by Caselli et al. was mostly dedicated to Tartini's third tones whereas Lohri et al. investigated the objective combination tones, referred to as extraaural combination tones. They studied the characteristics and the possible musical relevance of OCTs produced by the violin, viola and cello. Combination tones produced by the violin are from now on called Violin Combination Tones (VCTs). As expected from the theory, when frequencies in integer ratio relation are played, VCTs occur with a regular pattern with frequencies multiplies of the Greatest Common Divisor (GCD) of the two primary frequencies. A listening test was performed with a group of musicians trying to establish if the CTs generated by the three bowed instruments during playing of selected musical intervals were audible. To this end, the musicians were asked to distinguish between a dyad as originally recorded and the same dyad where CTs were filtered out. Even with the dyad played with the violin (a minor sixth $C_5 - A_5$) which showed the strongest CTs, the audience failed to recognize the difference for 27% of the times (Lohri, 2016). Thus, even trained musicians under optimal listening conditions were unable to detect the presence of combination tones without a significant number of errors. These results, however, cannot be generalized because of two experimental limitations present in Lohri et al's. observations: 1. the use of a single instrument, either a violin, viola or cello and, 2. the use of a single dyad (a minor sixth, for the violin) for the listening test. Due to well-known variability among instruments, it is likely that different violins, as well as different dyads, will produce combination tones of different intensity. In the study published recently by Caselli *et al.* (2020) in which two violin players were involved, it was observed that VCTs intensities depended on both the violin and the dyad played. Although some differences might have been due to the different players, there was no doubt that the two different violins produced combination tones with strongly different intensity. The same occurred when different dyads were played. It should be pointed out that Lohri *et al.* (2011) were well aware of these possible effects, as indicated by their suggestion that VCTs intensity may be different in different violins and perhaps correlated with the quality of the violin.

These observations, as well as a further investigation of the nature and characteristics of the combination tones generated by the violin and their possible audibility, motivated the experiments reported here. We investigated the combination tones generated by a set of 5 violins of different quality and age when playing several dyads.

II. ACOUSTICAL ANALYSIS

A. Methods

A professional violinist standing on the stage of a musical auditorium (approximate dimensions: 12m length, 7m wide and 4m height), played a series of selected intervals with five different violins of different age and quality. To reduce sound reflections, the auditorium walls were partially covered with acoustic panels and thick blankets and the stage floor was covered with carpets. Recordings were made with a cardiod microphone (KM 184, Neumann, Germany) placed in front of the player at about 0.6m distance and at the same vertical height of the violin. The microphone was directed to the violin to emphasize the capture of the direct sound from the instrument. The signal from the microphone was fed to a low noise flat response preamplifier (Line Audio 8MP, Sweden) and from there to a sound card (Traveler Mk3, Motu, USA) connected to a PC for recording. The audio signals were digitized at 192kHz with 24 bits resolution and recorded with the Logic Pro Software (Apple, USA) without any filtering. Processing of the audio signals and Fast Fourier Transform algorithm (FFT), were performed excluding completely the initial and final transient, on a recorded period of 0.8-1s selected for the best note intonation and stability. FFTs of 153600-192000 samples length were made with the Hann window of 0.8-1s duration. The signal processing and graphing were made with the SIGVIEW (USA) and Origin (USA) software. All the plotted FFT spectra were filtered with the moving average method (5 points) to remove high frequency noise. FFT magnitude was expressed in dB with 0 dB corresponding to the maximum value. In graphs including various dyads and violins, 0 dB was the mean amplitude of the two fundamentals at f_1 and f_2. The intervals played (reported in scientific pitch) were the following: P_5: perfect fifth $C_5 - G_5$; $P_{4,1}$: perfect fourth $C_5 - F_5$; $P_{4,2}$: perfect fourth $D_5 - G_5$; M_3: major third $C_5 - E_5$; m_3: minor third $C_5^{\#} - E_5$; M_6: major sixth $E_5 - C_6^{\#}$; m_6: minor sixth $A_4 - F_5$. In addition, the following intervals with open strings were recorded: a perfect fifth, $A_4 - E_5$, a perfect fourth, $A_4 - D_5$, obtained by lowering by a tone the E strings, and a minor sixth, $A_4 - F_5$, obtained by stretching the E_5 string to F_5. All the dyads were played $forte$ without vibrato with Pythagorean tuning with A_4=440Hz. The selected dyads span a representative range of double stops with various degree of consonance. To favour the comparison, most of the dyads coincide with those played by Tartini (1754) when studying the third tones. They are all played exclusively on the A and E strings. Dyads played on G and D strings with double stops were not investigated mainly because of the limited time availability of the old Italian violins. However, it is likely that VCTs are also produced on G and D or D and A strings. Each dyad was played with an up and down bow stroke lasting in total about 5s. The violin tested were the following: violin A: maker Carlo Annibale Tononi, (Bologna, 1700); violin B: Italian anonymous (1778); violin C: Henry Lokey Hill (London, 1820); violin D: Giustino Dal Canto (Pisa, 1971); violin E: factory made cheap violin (2013).

B. Results and discussion

Violin combination tones (indicated by the asterisk) of the perfect fifth interval played with the violin B can be seen on the FFT spectrum of Figure 1. The strongest one, called

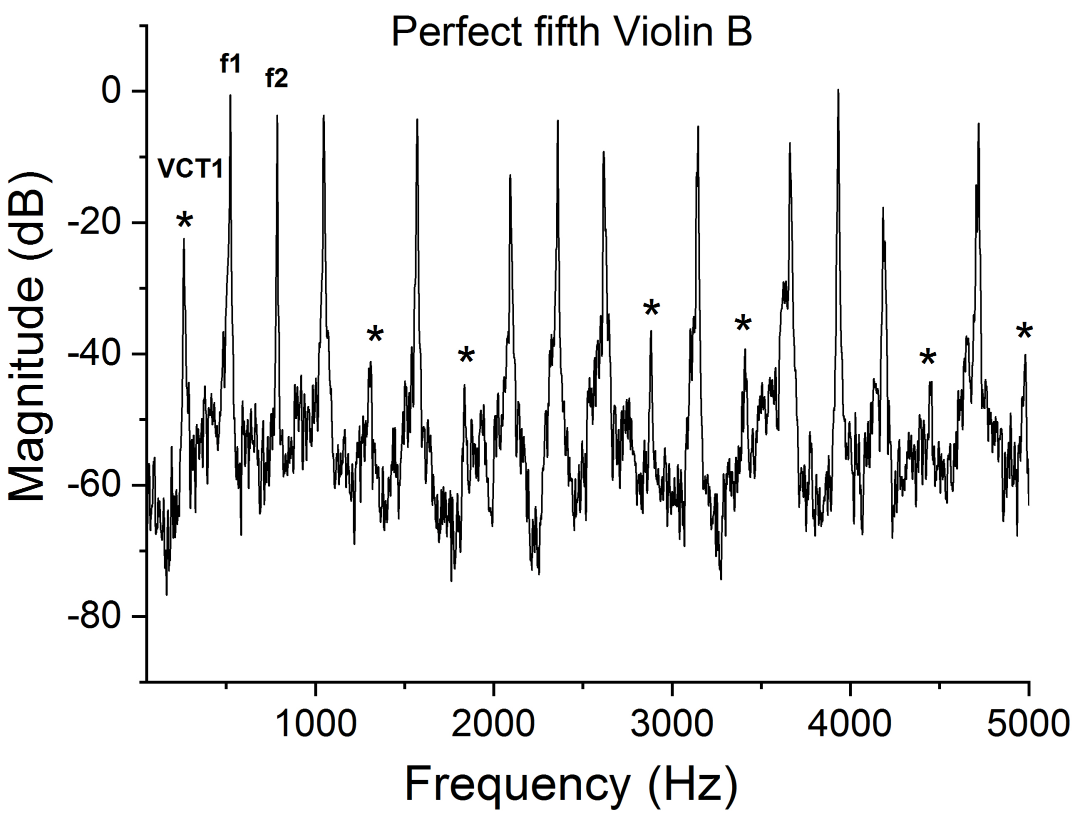

FIG. 1. FFT spectrum of B P_5, the perfect fifth dyad (523-786 Hz) played with violin B. Asterisks indicate the combination tones produced by the violin. The strongest one at 263 Hz (VCT_1) corresponds to $f_2 - f_1$. Abscissa limited to 5 kHz for clarity.

VCT_1, is found at frequency of 263 Hz corresponding to $f_2 - f_1$, while the weaker second peak

(VCT$_2$) occurs at $f_1 + f_2$ (1309 Hz). Many other VCTs are present at higher frequencies, they are all multiples of 263 HZ so that many of them are superimposed on the fundamentals and partials, possibly affecting their magnitude (for an extensive study on this effect, see Lohri (2016)). The presence of sizeable VCTs, even at frequencies well above the two fundamentals, is due to the interaction between the fundamentals, between the partials and between fundamentals and partials (Hallstrôm, 1832). VCT$_1$ corresponds to the GCD of f_1 and f_2, but this occurs only with a perfect intonation. If the dyad is played with an intonation error, even a small and not perceivable one, VCT$_1$ does not correspond anymore exactly to the GCD. A simple analysis of error propagation is provided in Caselli *et al.* (2020).

During listening to the recordings of the perfect fifth, played especially with violin A and B, we noticed that the strongest combination tone (VCT$_1$), which was well perceived during record playback, showed a fluctuating intensity whereas the same did not occur for the major third and minor sixth. Examination of the time course of the FFT of the perfect fifth played with violin A during about 1.5 s of recording (Figure 2), shows that VCT$_1$ amplitude changes in an apparent random way whereas f_1 and f_2 amplitudes were practically constant. If the dyad of a perfect fifth is played with a precise intonation ratio at 3/2, the combination tones at $f_2 - f_1$ and $2f_1 - f_2$ coincide, therefore the VCT$_1$ amplitude is contributed by both of them. In case of a small intonation error, even a very small one, the coincidence does not occur anymore and beats between the two VCTs are generated with a consequent modulation of VCT$_1$ amplitude. During bowing with double stops, small fluctuations of the note intonation will occur in a kind of random way due, for example, to small finger movements or pressure and so will the amplitude of VCT_1 due to beating.

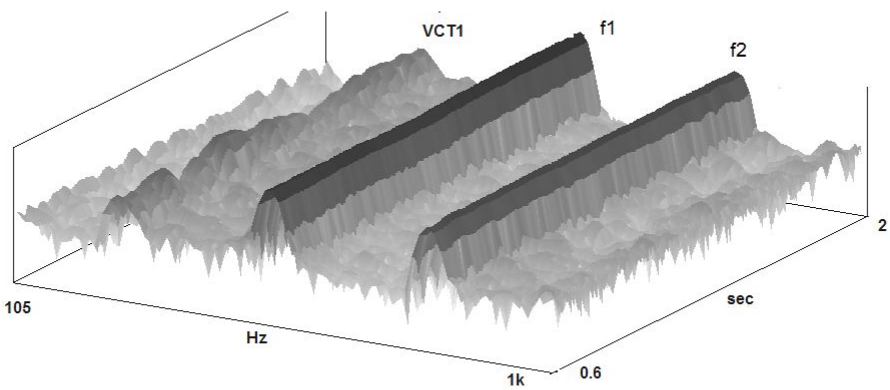

FIG. 2. FFT time course of A P_5, the perfect fifth dyad $C_5 - G_5$ (586-723 Hz) played with violin A, for a period of about 1.5 s. Vertical scale in dB. Note the changes of VCT_1 peak in spite of the almost constant amplitude of f_1 and f_2 fundamental peaks.

Although it was more difficult to see, because VCT_1 was smaller, a similar effect occurred also with the perfect fourth where VCT_1 occurred at $2f_1 - f_2$ and at $2f_2 - 2f_1$, the difference between the second harmonics. This effect did not occur during playing of the major third and minor sixth because in both cases VCT_1 is due to only one VCT at $2f_2 - 2f_1$ and $f_2 - f_1$, respectively. A further evidence of the beating (not shown here) arises from the observation that very small or no intensity changes of VCT_1 occur when a perfect fifth is played on open strings, a condition in which intonation it is much more stable than with adjustable stops (Gough, 1980).

Figure 3 shows the spectrum of the major third played on the violin A. As in Figure 1, a strong VCT is present at a frequency below f_1, followed by a series of weaker VCTs at higher frequencies. In all the experiments, the strongest combination tone, VCT_1, occurred

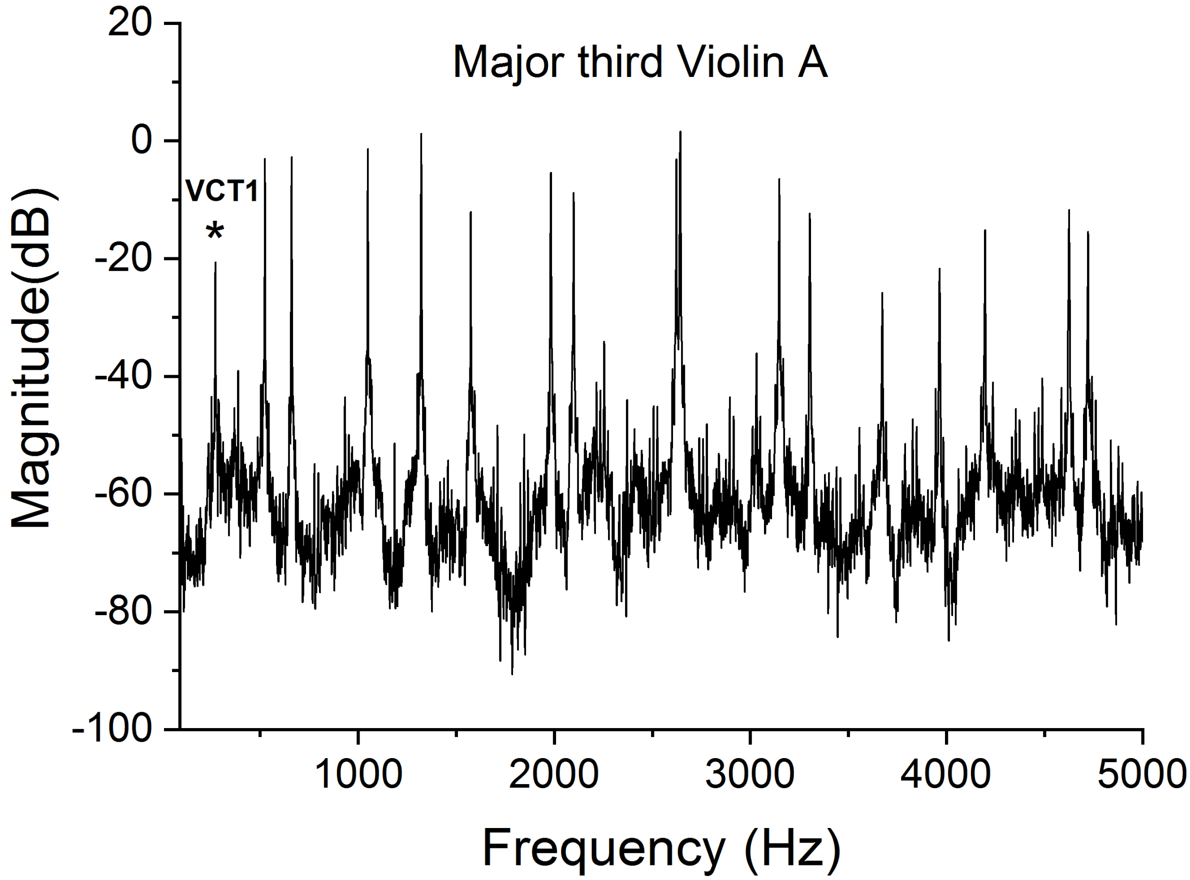

FIG. 3. Spectrum of A M_3, the major third $C_5 - E_5$ (524-661 Hz) played with violin A. Note the strong VCT_1 at frequency of 274 Hz (at $2f_2 - 2f_1$), below f_1.

at frequency lower than f_1, in the same range were Tartini's tones are heard. The same occurs with the minor sixth dyad shown in Figure 4: a strong VCT_1 occurs at 254 Hz, again corresponding to the difference between f_2 and f_1 (694-440Hz).

Many other combination tones at progressively increasing frequency are seen on all the records. Since one of the main objectives of this paper was to investigate the perception

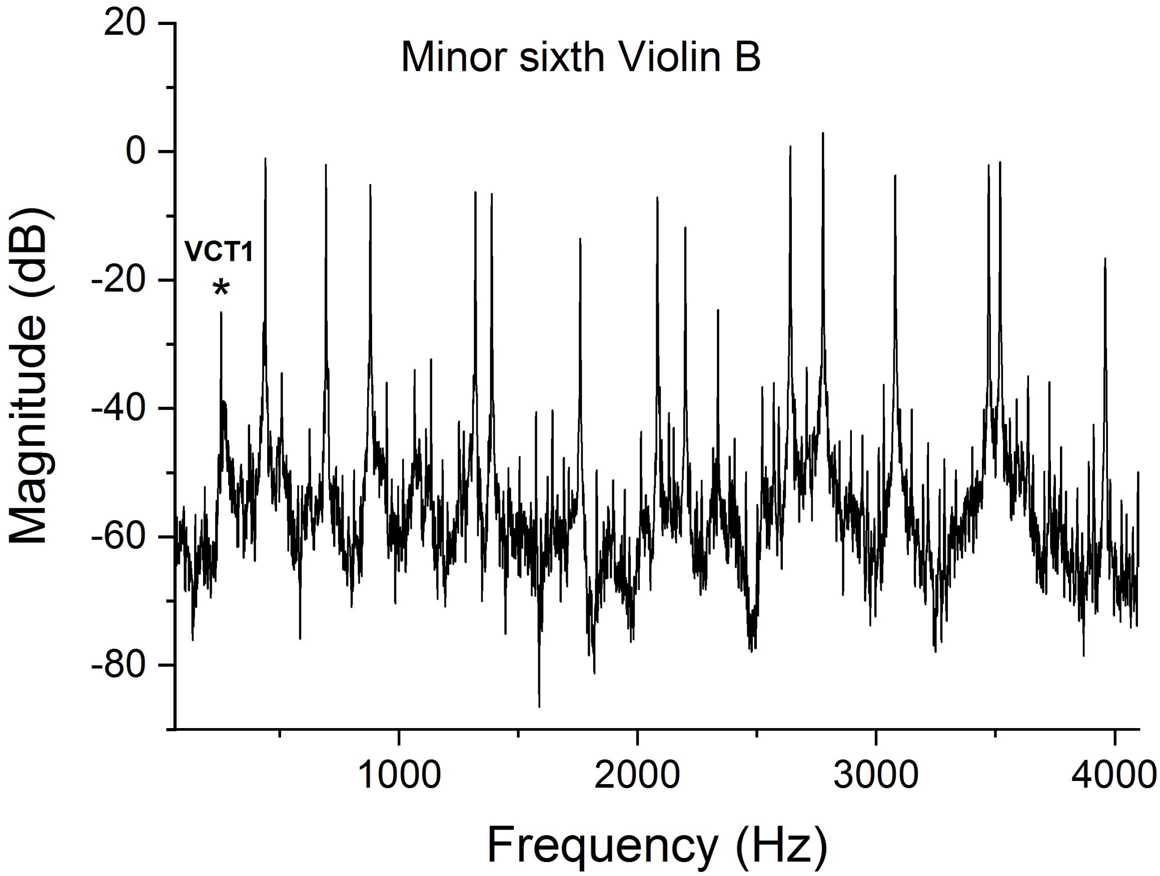

FIG. 4. Spectrum of the minor sixth interval played with the violin B.

of the VCTs during playing of the dyads, most of the following analysis is dedicated to VCT_1 that is the strongest of all VCTs. In addition, VCT_1 is easier to perceive than VCTs at higher frequencies. This is because low frequency tones as VCT_1s, are less masked by interfering sounds than high frequency tones (Wegel and Lane, 1924). A summary of the fundamentals and VCT_1 frequencies ($f VCT_1$) of all the dyads is shown in Table I. Naturally, some variability of f_1 and f_2 corresponding to a small intonation error, occurred when the

	M_3	m_6	P_5	$P_{4,1}$	m_3	M_6	$P_{4,2}$
f_1 (Hz)	524	440	523	524	551	662	585
f_2 (Hz)	661	695	786	698	662	1108	780
$f\text{VCT}_1$ (Hz)	273	255	263	348	440	446	390
VCT_1 source(s)	$2(f_2 - f_1)$	$f_2 - f_1$	$f_2 - f_1$, $2f_1 - f_2$	$2f_1 - f_2$, $2(f_2 - f_1)$	$2f_1 - f_2$	$f_2 - f_1$	$2f_1 - f_2$, $2(f_2 - f_1)$

TABLE I. Intervals, fundamental frequencies and combination tone frequencies ($f\text{VCT}_1$) for all the dyads played with the Tononi violin (A) .

same notes where repeated with all the five violins. However, with the exception of 3 dyads played with the cheap violin E (that according to the player was difficult to play), this variability was rather small. In no dyad it was greater than 6 cents, at or just below the JND (Pierce, 1983). Figure 5 shows a summary of VCT_1 magnitudes for all the violins and all the dyads played. Violins on the abscissa have been ordered according to their age with the oldest ones (violin A) to the left. Two general aspects emerge from the data: 1. VCT_1 decreases from violin A to E in all the dyads and, 2. three dyads, namely, perfect fifth, major third and minor sixth (strong dyads, filled symbols in Figure 5), have stronger VCT_1s than all the others. The strongest of all VCT_1s was about -20 dB and occurred with the major third played on Violin A. All the strongest VCTs of the dyads played with violins A, B and C are well above -40 dB. The graph suggests that the VCT_1 amplitude is determined by both the violin and the dyad played. Violins A, B and C are the ones producing the greatest

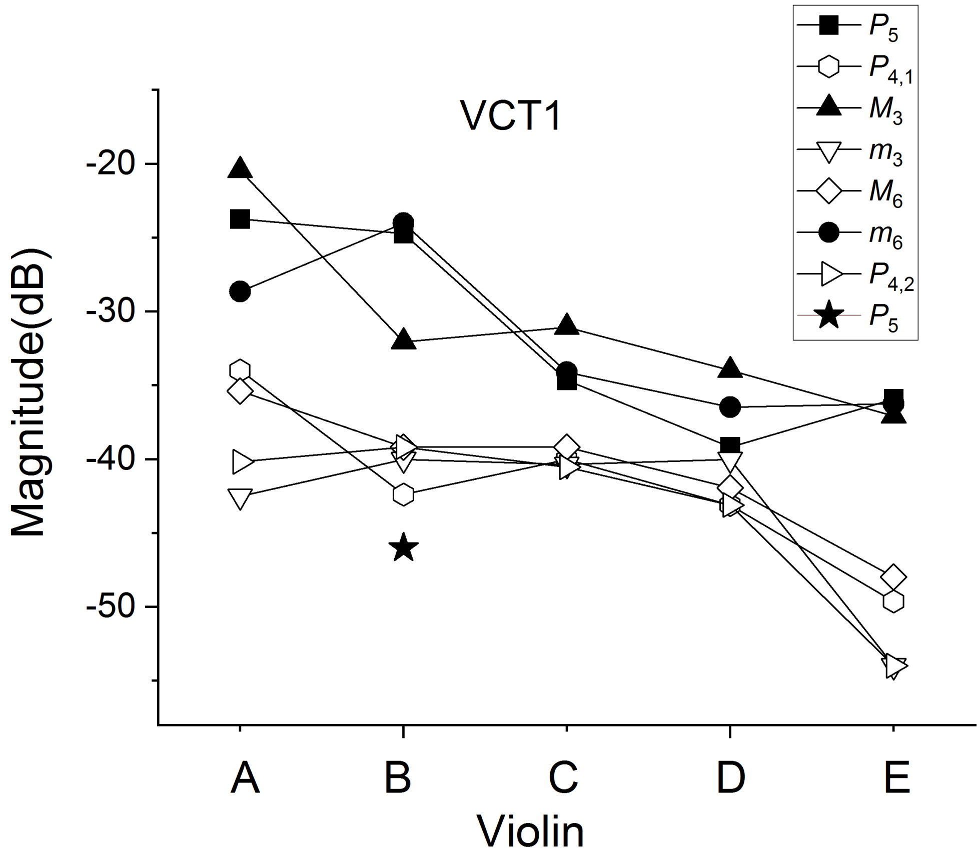

FIG. 5. Summary of all VCT_1 recorded in all the violins for all the dyads played. Filled symbols show the dyad with the strongest VCT_1. Violins are ordered according to their age from the oldest (A) to the youngest (E). Star symbol refers to the P_5 dyad played on open string $A - E$ (440-660Hz) with violin B. Note the great VCT_1 difference with P_5 played with double stops at $C_5 - G_5$ (523-786 Hz) on the same violin (filled square symbol).

211 VCT_1s when playing the dyads of a perfect fifth, a major third and a minor sixth while the same dyads and all the other dyads played with all violins, have smaller VCTs.

Looking at Table I, it can be seen that the strongest VCT_1s of the major third, minor sixth and perfect fifth, played with violin A, occur at frequencies of 273, 255 and 263 Hz, respectively. These values are rather close to A_0, the modified Helmholtz air resonance which occurs in all violins at about 280 Hz (Hutchins, 1962, 1983; Schelleng, 1963). This resonance is very important because it amplifies the sound emission in the low frequency range of the violin spectrum and has been empirically identified as an important quality discriminator between violins (Bissinger, 2008; Dünnwald, 1991; Hutchins, 1962; Rodgers, 2005). VCT_1s of the strongest dyads occur at a frequency near the air resonance whereas those of the perfect fourth 1, perfect fourth 2, minor third, and major sixth, are further away. This suggests that the VCT_1s of the strongest dyads are the strongest simply because their frequency is close to A_0 and, contrary to VCT_1s of other dyads, they are boosted by the A_0 resonance. To investigate this point, we plotted VCT_1 amplitude for each violin and dyad as a function of VCT_1 frequency, in Figure 6. The results show that in all violins, VCT_1 amplitude increases with frequency from about 200 Hz reaching a peak in a frequency range of between 253 and 278 Hz and falling again at higher frequencies, similarly to resonance curves. Assuming that for a given violin, the non-linearity responsible for VCTs does not change much in the frequency range of about 220-350 Hz, which seems likely, the curves in Figure 6 would then represent the A_0 resonance curve of the various violins. This confirms the suggestion above that VCT_1 of the strong dyads are the largest simply because their frequency is close to the peak region of their A_0 resonance and therefore they are the most amplified. At frequencies higher than about 350 Hz, with perhaps the exception of violin A, VCT_1 in all violins does not exceed 1% (-40 dB) of the maximum. Another interesting finding showed in Figure 6 is that the resonance curves of the A and B old Italian violins (filled points) have a much greater peak than all the other violins.

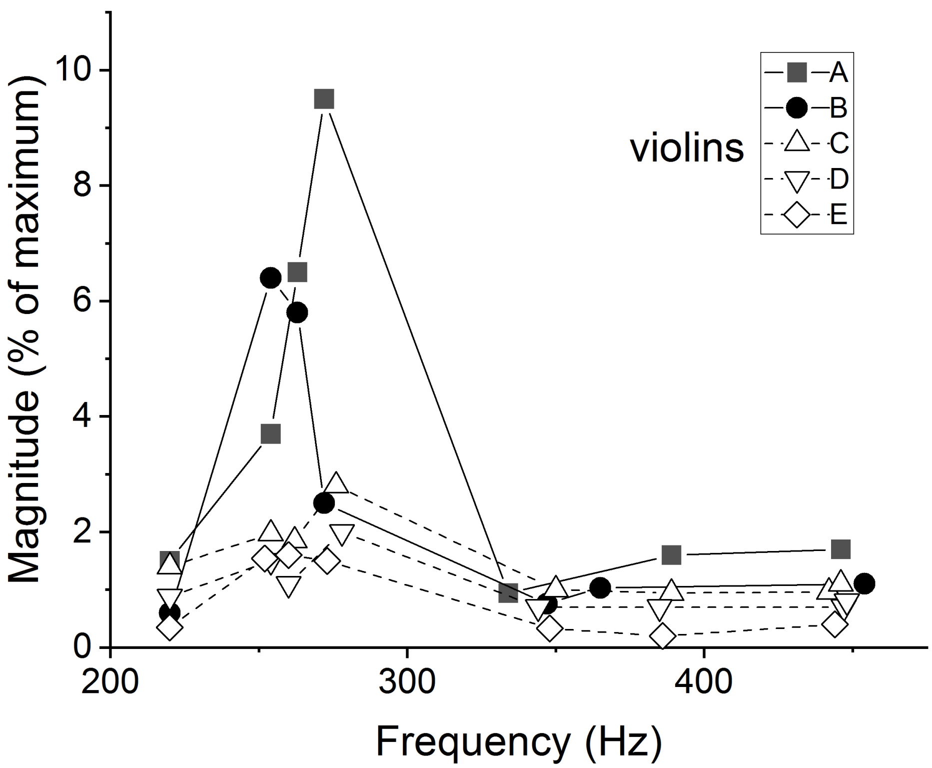

FIG. 6. Magnitude and frequency of all the VCT_1 for all the violins and all the dyads played. Although VCT_1 peaks are quite different in different violins, all of them have occur between 253 and 278 Hz. To emphasize the difference between the various peaks in this figure, as well as in fig 7 and 10, magnitude is expressed as % rather than in dB.

Results of Figure 6 suggest that combination tones generated somewhere in the violin are boosted more or less according to their closeness to the A_0 peak. Thanks to this amplification VCT_1 values increases well beyond -40 dB. Other experimental observations are well explained by this mechanism: the first one is that the same dyad played with a different height of the two fundamentals has a different VCT_1 amplitude. For example, the perfect fifth played on open string at $A_4 - E_5$ (440-660 Hz) in violin B gives a VCT_1 of -46 dB whereas the perfect fifth played at $C_5 - G_5$ (523-770 Hz) gives a much greater VCT_1 of -25 dB (stars and square symbols in Figure 5). This result is expected because $f_2 - f_1$ in the open string dyad is 220 Hz, rather far away from 280 Hz (A_0), whereas in the dyad 523-770 Hz, VCT_1 frequency rises to 263 Hz, much closer to A_0. The same mechanism also explains the observation that VCT_1 occurs at a frequency of $f_2 - f_1$ for the P_5 and the m_6 intervals, whereas for the M_3 interval VCT_1 frequency corresponds to $2f_2 - 2f_1$. In fact, for the M_3 interval, $2f_2 - 2f_1$ is 274 Hz, close to A_0 (about 280 Hz) and amplified by the air resonance, whereas $f_2 - f_1$ is 137 Hz, far away from A_0 and not amplified. All these observations strongly suggest that VCT_1 amplitude is crucially determined by the closeness of VCT_1 frequency to A_0. The following experiments made on violin E (the only one available for this test) further illustrate the role of A_0 resonance on VCT_1 amplitude. The violin was mounted horizontally and clamped between two stiff wooden supports with an interposed cork ribbon placed at the violin ribs close to the end button and at neck. G and D strings were damped with a felt ribbon. We tested the effect of changing the frequency of the two fundamentals f_2 and f_1, on VCT_1 amplitude. Five values of $f_2 - f_1$ intervals, from 255 up to 293 Hz were obtained by tuning appropriately both A and E open strings, and the resulting VCT_1s were measured. Some of these intervals do not belong to Pythagorean tuning, but this is not relevant here since it has been shown that combination tones occur equally well with any kind of interval. (Plomp, 1965; Smoorenburg, 1972; Tartini, 1754).

Figure 7 shows that VCT_1 increases as $f2 - f1$ rises from 255 Hz reaching a peak around 275 Hz to decrease again as $f_2 - f_1$ continues to increase, following the envelop of the

A_0 resonance curve. At 275 Hz, VCT_1 reached the greatest value of -32 dB, greater than any other values for the violin E, but still 11 dB below the greatest VCT_1 of violin A (see Figure 5). Furthermore, we closed the f-holes of the violin with a sticky paper tape,

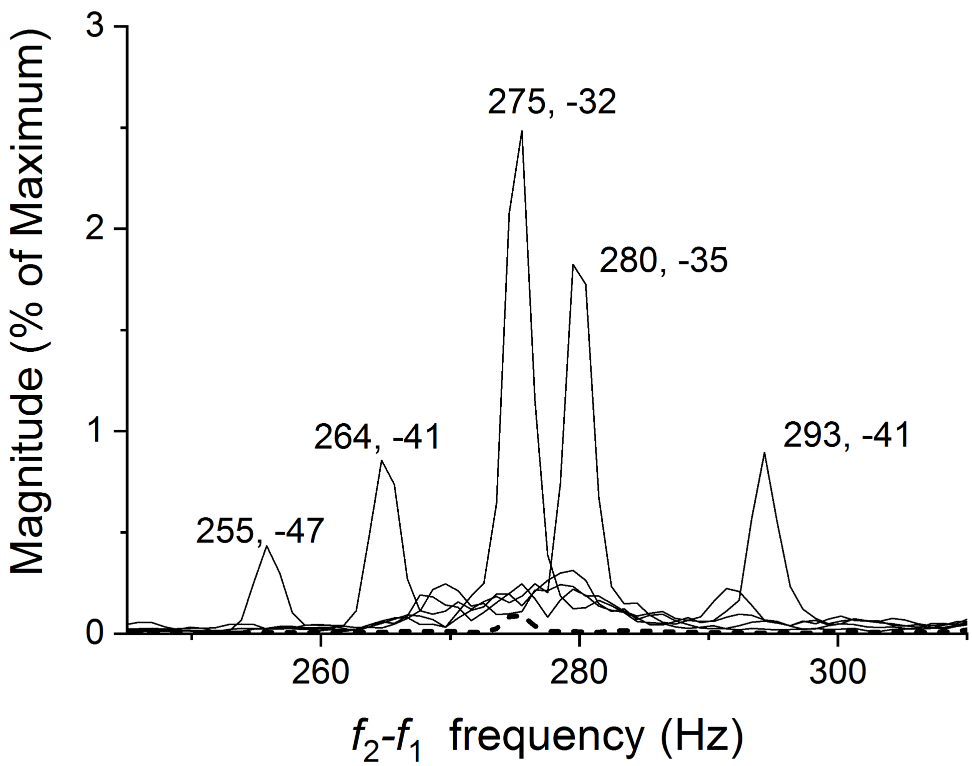

FIG. 7. Expanded FFT spectra in the frequency range 245-310 Hz showing the effect of changing $f_2 - f_1$ on VCT_1 peak on the violin E. Figures close to the peaks indicate frequency (Hz) and magnitude (dB). The thick dashed trace at the bottom is the response at 275 Hz with the f-hole closed.

greatly reducing the A_0 resonance and played again the dyad 430-705 Hz (the one giving the maximum VCT_1 at 275 Hz). The result was quite striking: VCT_1 dropped by about 30 dB (thick dashed line in Figure 7) confirming the fundamental role of A_0 resonance on VCT_1 amplitude. The effect of f-hole closure is also shown in Figure 8 where the spectra with f-hole closed and open are compared. Again, VCT_1 peak at 275 Hz almost disappears when the f-holes are closed. As expected, other VCTs at higher frequencies (above f_2) which are not amplified by A_0 air resonance, were not affected by the closure of the f-holes.

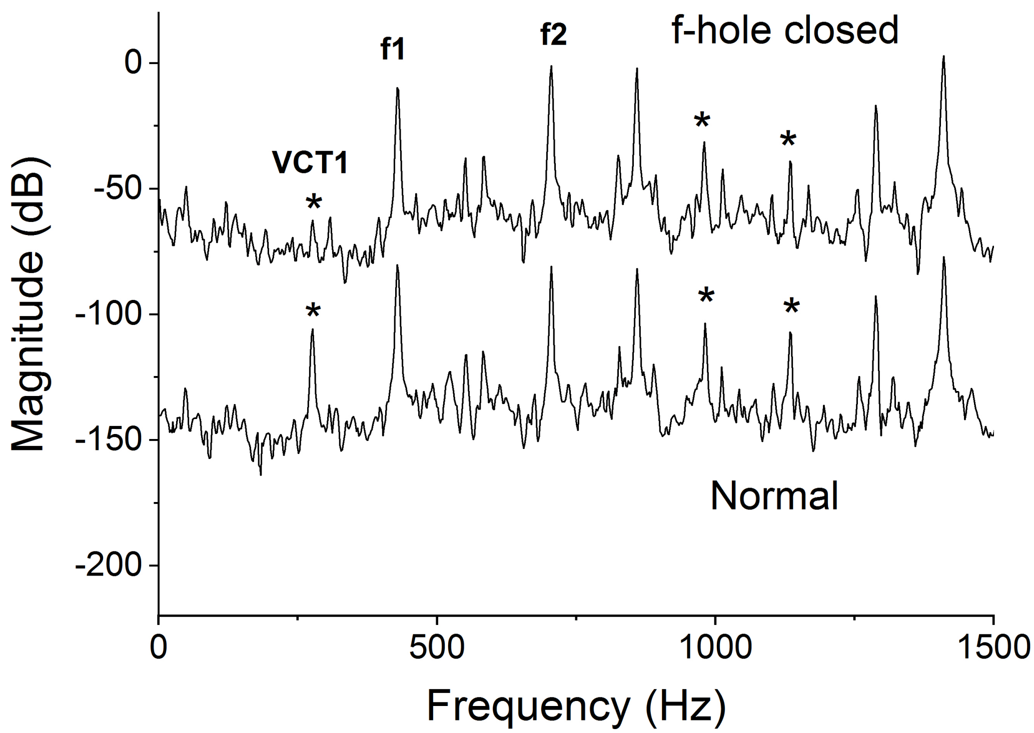

FIG. 8. Effect of the f-hole closure on the spectrum of the dyad with $f_2 - f_1 = 275$ Hz, on violin E. The f-hole closure almost abolishes VCT_1, but does not affect VCTs at higher frequencies. Lower trace shifted downward for clarity.

For a given violin, the effect of VCT_1 closeness to A_0 seen above, explains well the changes of VCT_1 observed with the different dyads, but differences between violins when playing the same dyad with the same $f_2 - f_1$, require a different explanation. At least two mechanisms working together can be envisaged: 1. different degrees of non-linearity among various violins and, 2. different heights of the A_0 resonance peak in different violins which leads to different VCT_1 amplification. VCT_1s occurring at frequency above $\sim$ 350 Hz (see Figure 5), being away from $\sim$280 Hz, are only little amplified by A_0 resonance with perhaps a small contribution from the main body resonance (Schelleng, 1963), therefore they are mostly dependent on violin non-linearity. In contrast, VCT_1s occurring at around $\sim$280 Hz are amplified according to the A_0 resonance which likely becomes the main determinant of their amplitude. Thus, the greatest VCT_1 of the violins A and B, seems mainly explained by their largest A_0 resonance curves shown in Figure 6 compared to the others.

Dünnwald (1991), in examining the frequency response of a great number of violins, showed that A_0 amplitude had a great variability. On average, in classical Italian old violins, the ratio A_0 amplitude over the max amplitude in the 650-1120 Hz frequency region, was greater than those of modern violins and even more with respect to factory violins. He suggested that this ratio could be used as a good empirical indicator, among others, of violin quality. Similarly, Rodgers (2005) reported that that none of the new violins tested on occasion of the 2004 VCA competition had an A_0 resonance peak of comparable amplitude to that of the Stradivari tested. Further, according to Bissinger (2008), A_0 amplitude is the sole robust parameter that distinguishes good from poor violins. In general, it is accepted that a relatively strong response in the lowest frequency range is a common characteristic of violins preferred by musicians (McIntyre and Woodhouse, 1978; Meinel, 1957). Our observations show that the Tononi and the Italian anonymus violins, reputedly the best and second best respectively, among our violins (some properties of a Tononi violin were evaluated in Curtin (2006)) have the greatest VCT_1s and the greatest A_0. This seems in agreement with the idea above that good violins are characterized by a strong A_0. The correlation between VCT_1 amplitude and A_0 peak and possibly with violin quality, is an interesting finding important also for luthiers.

C. Mechanism underlying non-linearity distortion

Finding the physical mechanism responsible for violin combination tones' formation is not an easy task, especially considering the small amount of distortion involved. Unfortunately, we haven't been able to localize precisely the violin structure(s) responsible for the non-linearity leading to the combination tones. However, the new data we show here allowed us to verify some of the possible reasons for nonlinearity among those postulated previously (Lohri *et al.*, 2011). Non-linearity can be due to many factors including, for example: non-linear coupling of the two strings through the bow, nonlinear behavior of bridge or of the arching of the top plate or non-linear properties of the wood and others. The possible role of the string coupling through the bow was investigated on the violin E by comparing VCT_1 seen when bowing normally two open strings (A and E) and when bowing the same two strings independently with two bows so as to eliminate the coupling. The two open strings were appropriately tuned so that $f_2 - f_1$ was closer to violin A_0 to increase VCT_1 amplitude,

VCT_1 freq.	VCT_1 ampl. 1 bow	VCT_1 ampl. 2 bow	p
275Hz	6.8 ± 0.5	6.6 ± 0.3	0.75
278Hz	5.3 ± 0.6	4.5 ± 0.3	0.26

TABLE II. .Comparison of VCT_1 amplitude (mean and SEM) when the same dyad is played with 1 or 2 bows in violin E. Data shows no significant differences between the two conditions. VCT_1 amplitude is in % of max spectrum amplitude. t-student test for paired values was used to determine the p-values.

making it easily measurable. Two sets of seven measurements each were obtained with one and two bows, for VCT_1 frequency of 275 Hz and 278 Hz. The results are shown in Table II.

It can be seen that at both frequencies, VCT_1s are not different when the dyads are played with one or two bows. Therefore, we can conclude that for violin E, the combination tones do not originate from bow-strings interaction. More violins need to be tested to generalize this conclusion.

Given the coupling between the strings, bridge and body, combination tones generated somewhere in the violin will propagate to the other components. The analysis of changes occurring during this propagation, can be useful to locate the source of VCT_1. In the following experiments, we investigated the presence of the VCT_1 at strings level. To do so a high strength magnet was mounted on the violin, similarly to Gough (1980) with the difference that the magnetic field included two adjacent strings (A and E) instead of one, for a length of 14mm starting 1mm away from the bridge. The two strings were connected electrically in series and the voltage output was fed to a high impedance analog integrator circuit to obtain string displacement when playing the dyads.

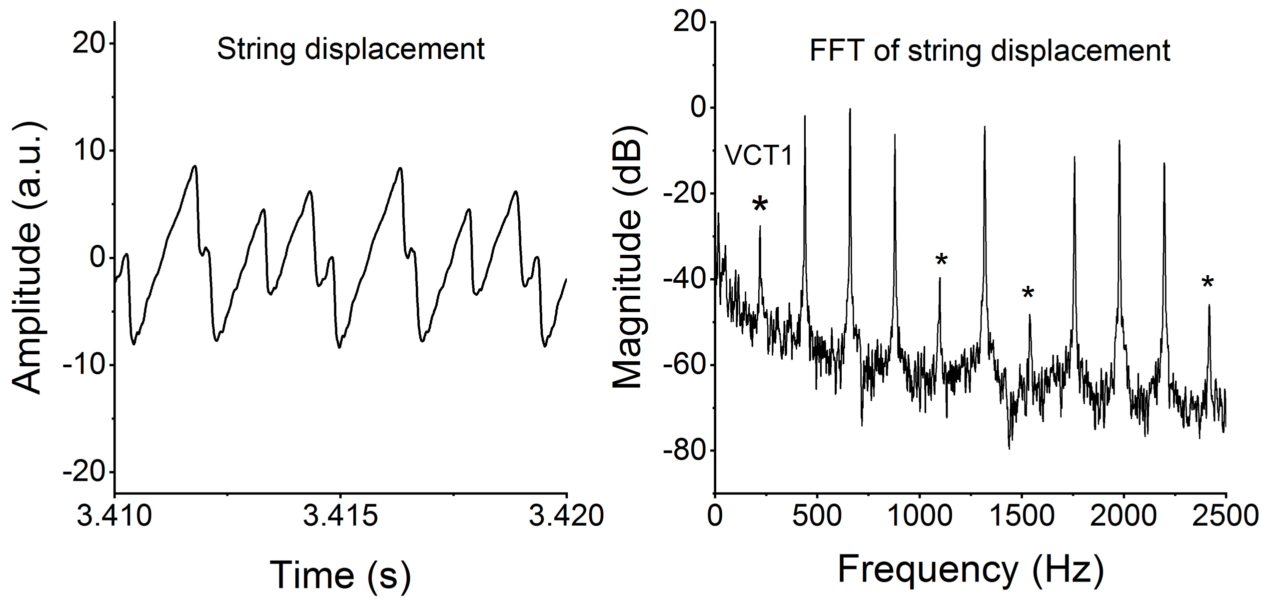

FIG. 9. (left) Electric signal of strings displacement during playing of a perfect fifth dyad with f_1 =440 Hz (A4) and f_2 =660 Hz (E5) with violin E; (right) Spectrum of the displacement signal showing relatively strong VCTs at 220, 1099, 1538 and 2418 Hz .

A typical displacement record and its spectrum are shown in Figure 9. It can be seen that VCTs are well present on the string displacement spectrum. These VCTs can be either generated directly by the strings or generated elsewhere and propagated to the strings through the bridge.

It is known that some non-linearity arises directly from the strings, especially during strong vibrations. This kind of non-linearity, however, generates only harmonic distortion, in particular third harmonics (Gough, 2007), and cannot explain VCTs non- coincident with harmonics as VCT_1. The vibrations generated by two strings need to be transmitted through a non-linear structure to give rise to the VCTs. Therefore, VCTs seen at strings level shown in Figure 9, are generated somewhere else in the violin and then propagated back to the strings through the bridge.

Figure 10 shows the comparison of string displacement and acoustic VCT_1s as a function of $f_2 - f_1$ either under normal conditions or with the f-holes closed. Different $f_2 - f_1$ values were obtained as described previously. As seen before in Figure 7, acoustic VCT_1 greatly increases as $f_2 - f_1$ rises approaching A_0 and decreases again as frequency continues to rise. On the contrary, very small or no resonance effect is seen on the displacement VCT_1 records. Closure of the f-hole almost abolishes the acoustic VCTs whereas it does almost nothing to the VCTs on string displacement that remain unaltered. This result excludes the possibility that VCTs are generated directly by the A_0 resonance mechanism.

The two strings are coupled through the bridge, however it seems unlikely that the bridge itself could be responsible for non-linearity. This is because at frequencies as f_1, f_2 and fVCT1, all below about 3 kHz (the first bridge resonance), the bridge behaves as a rigid component (Reinicke and Cremer, 1970). Thus the more reasonable possibility seems to be that non-linearity is due to the contact between the bridge and the top plate of the violin or the arched top plate itself.

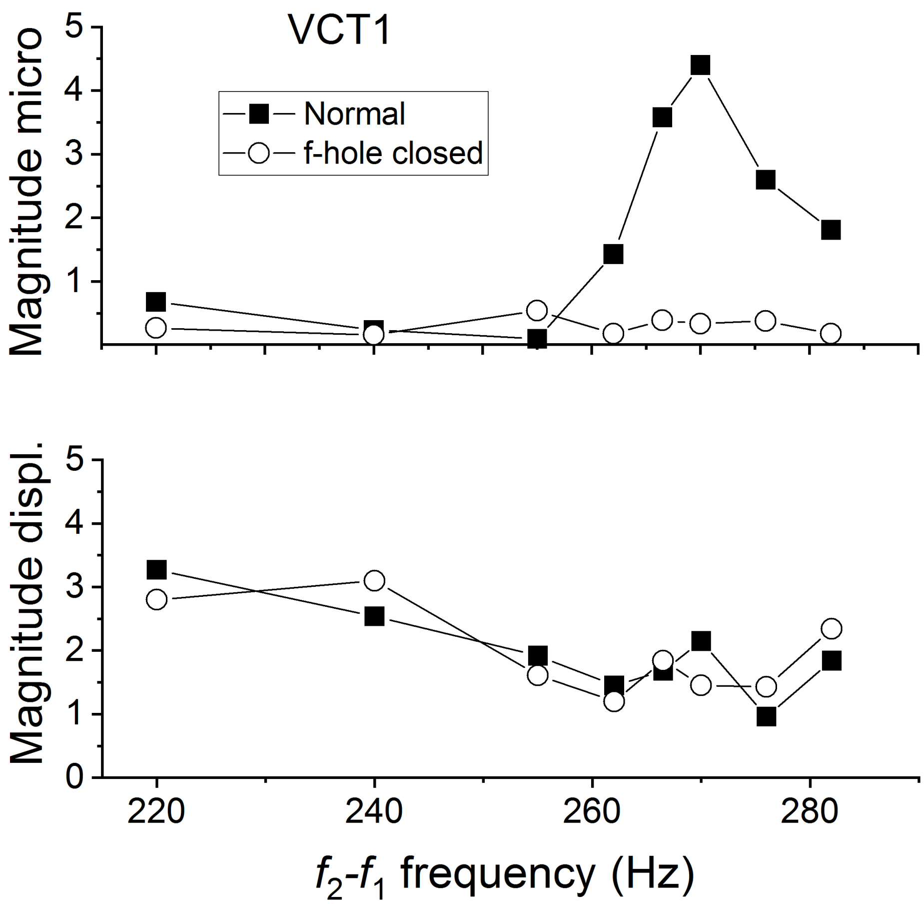

FIG. 10. Effects of the interval $f_2 - f_1$ on VCT_1 of the string displacement spectrum (bottom graph) and of the microphone spectrum (upper graph) under normal conditions (filled symbols) and with the f-hole closed (open symbols). Magnitude is expressed as % of maximum spectrum amplitude. No resonance and no effect of f-hole closure are seen on the displacement VCT_1, whereas VCT_1 resonance peak on the microphone signal is abolished by closing f-holes. Data from violin E.

III. LISTENING EXPERIMENTS

A. The audibility of Violin Combination Tones

The results of the acoustic experiments seen up to now, show that VCT1s can be relatively strong, being in some dyads and in some violins, only 20 dB weaker with respect to the average amplitude of the two fundamentals at f_1 and f_2. This suggests the possibility that VCT1 could be audible. This could be an important finding, potentially relevant also in a musical context. To verify this possibility we performed a listening test in which the audibility of the strongest VCTs was tested in a group of professional and amateur musicians.

B. Subjects and test method

Eleven subjects, nine males and two females, participated in the listening test. Median age was 34 and age range was 18-75. Six of them were amateur musicians and five were professional musicians. None of them reported any hearing impairment. The test was carried out exclusively on 6 selected dyads showing the strongest VCTs, although other dyads might produce perceivable VCTs.

The dyads selected were the following: 1. Violin A: M_3, m_6 and P_5; 2. violin B: m_6 and M_3; 3. violin C: M_3. For each dyad we prepared an audio file of a complete up and down bow stroke period of about 5 s duration. The procedure for making the files was the following. A spectrogram (FFT size 65536, window size 19201 and hop size 2400) was made for each dyad with the SPEAR software (https//ww.klingbeil.com), then the file was re-synthesized (by inverse FFT) twice: 1. with the original file unaltered (ORIG) and 2. after filtering out all the VCTs up to 10kHz (FILT). In this way, any possible small distortion due to the SPEAR software is present on both files and it will not influence the difference. An example of the spectrum of the perfect fifth, after the re-synthesis with and without VCTs, is shown in Figure 11. VCTs coincident with fundamentals and partials could not be eliminated by this procedure but this was considered acceptable given that the perception of violin combination tones is mostly related to VCT_1. The file preparation procedure above and the listening test illustrated below, also allowed the compensation for the effects of Tartini's tones generated by the distortion of the cochlea (Robles and Ruggero, 2001). These tones are in some cases perceivable and have the same frequency of VCTs, potentially affecting their perception. During playing of an ORIG file, in principle we will hear both VCTs and Tartini's tone superimposed, whereas with FILT files only Tartini's tone will be heard. Thus, if a difference is heard, it is necessarily due to VCTs.

For each of the 6 selected dyads for the listening test, we prepared a file with 12 pairs of sound files that comprised three repetitions of the four pairwise combinations ORIG+ORIG, FILT+FILT, ORIG+FILT and FILT+ORIG, randomly distributed in the file. The distribution of same and different pairs was equally balanced which gives a 50% chance level. The dyads of each pair were separated by about 1 s of silence whereas different pairs were separated by about 5 s. Although the silences between pairs varied somewhat they were always well within 0.5-1.5s, the region were the echoic memory operates with the maximum effectiveness (Crowder, 1982). Sound emission time during the playing of a dyad, was the whole up and down bow stroke including attack and decay. Like the silences between the pairs, sound time too was somewhat variable. These conditions are perhaps not optimal for

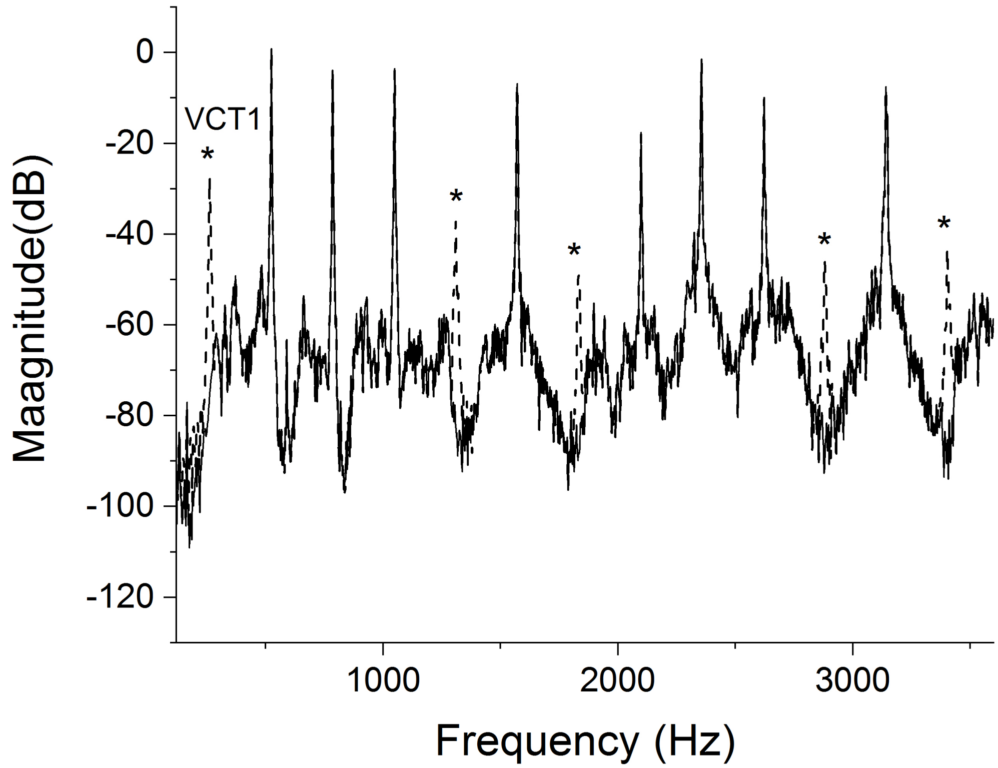

FIG. 11. Superposition of re-synthesized P_5 spectra from violin A before (dashed lines) and after filtering out the combination tones up to 10 kHz (continuous line). Abscissa limited to 3700 Hz for clarity. Except for the VCT peaks, the dashed line is completely superimposed to the continuous line.

detecting differences between sounds, as shorter and equal silence and sound times, would probably be a better choice (Fritz *et al.*, 2007). Our choice was motivated by the desire to have listening test conditions similar to those of the original recordings, retaining the musical aspects of the sound. Files were played by a computer with the SIGVIEW software and an external audio card (Behringer, mod. UMC292HD). Signals from the audio card were fed to a high-fidelity flat response amplifier (Yamaha RX395) driving a full range loudspeaker (Markaudio, mod. CHR70A gen.3) mounted on a closed box of 4 liters volume. Listening tests were performed with approximately the same sound volume and in the same auditorium of the original recordings. Sound stimuli were presented to each individual listener sitting in front of the loudspeaker at about 2 m distance. Participants were asked to tell if the two records of each pair were equal or different. If requested by the listener, the same file was played more than once up to 3 times. Table III reports the results of the analysis. The proportion of correct answers in identifying different/same for each dyad and for each subject was very high: from 0.909 to 0.992 for dyads, and for 0.903 to 1 for subjects. The chi-squared tests for the goodness of fit against the chance level of 50% , span from 126.07 to 86.74 with p-values always < 0.0001. For each dyad, we also measured the correct rejection, the false alarm, the hit and the miss responses. Table IV shows the results. The chi square tests for the independence between two categorical variables (stimulus vs. response) span from 89 to 128 with p-values smaller than 0.0001 for all the dyads. The statistics can be considered quite good, however, from a musical point of view a good statistic is not enough. Optimal listening conditions as those of the test, are in fact very unlikely to be encountered during a musical performance. However, the very good results of the listening test for M_3 and m_6 dyads, suggest that VCT_1s of these intervals, if played with a good violin, are likely to be perceived also in musical contexts .

Dyad	Proportion	Confidence interval
A M_3	0.992	$[0.942, 1]$
A m_6	0.985	$[0.929, 1]$
B m_6	0.985	$[0.929, 1]$
B P_5	0.985	$[0.929, 1]$
A P_5	0.955	$[0.887, 1]$
C M_3	0.909	$[0.829, 1]$

Subject	Proportion	Confidence interval
1	0.958	$[0.855, 1]$
2	1.000	$[0.918, 1]$
3	1.000	$[0.918, 1]$
4	1.000	$[0.918, 1]$
5	0.903	$[0.783, 1]$
6	0.944	$[0.836, 1]$
7	0.931	$[0.818, 1]$
8	0.972	$[0.874, 1]$
9	1.000	$[0.918, 1]$
10	1.000	$[0.918, 1]$
11	0.944	$[0.836, 1]$

TABLE III. Proportion of correct responses to the listening test and confidence intervals ($\alpha = 0.01$). The proportion is over 132 for dyads and 72 for listeners.

C. Comparison between violin combination tones and Tartini third tones (subjective combination tone)

Given the perceivability of the strongest VCT_1s of the dyads shown above, it is natural to ask if violin combination tones could contribute to the perceived musical quality of the

DYAD	*STIMULI*	*RESPONSES* Same	Different
AM_3	Same	66	0
(χ^2=128)	different	1	65
Am_6	same	66	0
(χ^2=124)	different	2	64
Bm_6	same	64	2
(χ^2=124)	different	0	66
BP_5	same	66	0
(χ^2=124)	different	2	64
AP_5	same	64	2
(χ^2=106)	different	5	61
CM_3	same	63	3
(χ^2=89)	different	9	57

TABLE IV. Number of correct rejections (stimulus same- response same), false alarms (same-different), hits (different-different) and misses (different-same) responses. Twelve sound pairs (stimuli) were presented to each of the 11 listeners, so that the total number of pairs presented was $12 \cdot 11 = 132$. χ^2 values are also shown. All the calculated p-values where less then 0.0001.

intervals as proposed for the subjective combination tones (Hindemith, 1942). It is therefore interesting to compare the characteristics of Tartini's third tones with those of VCT_1s, as shown in table V.

Dyad	Intervals	TT	TT source	VCT_1	VCT_1 source
P_5	$C_5 - G_5$	C_4	$f_2 - f_1$ $2f_1 - f_2$	C_4	$f_2 - f_1$ $2f_1 - f_2$
M_3	$C_5 - E_5$	C_3	$f_2 - f_1$	C_4^*	$2(f_2 - f_1)$
m_6	$A_4 - F_5$	F_3	$2f_1 - f_2$	C_4^*	$f_2 - f_1$

TABLE V. Comparison of notes and source(s) of the Tartini third tones (TT) and the VCT_1s. Asterisks show VCT_1 notes that are not perfectly in tune with the generating dyads (see Table VI)

.

The third tone of the P_5 dyad reported by Tartini, was one octave below the lower note of the dyad, exactly as our VCT_1 for the same dyad. The third tone of M_3 was two octaves below the lower note of the dyad whereas VCT_1, was one octave below. In this case the third tone frequency differs from that of VCT_1. A similar situation is found for the m_6 dyad: the third tone was one tenth below the lower note of the dyad, whereas VCT_1 was one minor sixth below. These differences occur because VCT_1 amplitude, in contrast to third tones, is mainly determined by the A_0 resonance of the violin. This means that among all the possible VCTs, only those with a frequency around A_0 will be amplified reaching a significant amplitude, whereas the others will remain negligibly small. For example, Tartini tone for the M_3 dyad shown in table V, corresponds to $(f_2 - f_1)$ whereas VCT_1 corresponds to $2(f2-f1)$. This occurs simply because $2(f_2-f_1)(273\ \text{Hz})$ is much closer to A_0 than f_2-f_1 (136.5 Hz). Due to the amplification of A_0 resonance, unlike third tones, only VCT_1s with frequency around 280Hz will reach a significant amplitudes. Results from Tartini's book (Tartini, 1754) obtained with the just intonation, show that the third tones were perfectly in tune with the notes of the generating dyads in all the intervals. Actually, based on this property of the third tones, Tartini introduced a method for violin tuning still in use today. In contrast, our results obtained with the Pythagorean intonation, show that VCT_1 is in tune only for P_5, whereas for M_3 and m_6 dyads, VCT_1 shows a significant microtonal deviation from the Pythagorean tuning (Table VI). These deviations are related to the note ratio characterizing the interval and are intrinsic to the Pythagorean tuning. This can be seen for example, from the data of the M_3 dyad in Table VI: f_1 and f_2 of the Pythagorean tuning for C_5 and E_5 are 521.5 and 660 Hz respectively, thus $2(f_2 - f_1)$ corresponding to VCT_1, is 277 Hz. This value strongly deviates from 260.7 Hz, the frequency of C_4 of the Pythagorean intonation, being 104 cents sharp (the interval between C_4 and $C_4^{\#}$ in the Pythagorean tuning is 114 cents). Experimentally we found a smaller deviation of 85.2 cents because of the note played slightly deviated from the exact Pythagorean tuning (see Table VI). Similar considerations apply also to the m_6 dyad, where the deviation from the calculated Pythagorean value is 37 cents flat and the experimental deviation is 47 cents flat. As show in Table VI there is no significant deviation for the P_5 dyad in which the average value of VCT_1 was 261.4 Hz, very close (4.4 cents sharp) to the expected value of 260.7 Hz. This occurs because of the note ratio for this interval (3/2) is the same as that of the just intonation. Had our dyads been played with the just intonation, all VCT_1s would have been perfectly in tune exactly as the third tones. A summary of these data is shown in Table VI. Given the above results we can assume that with Pythagorean tuning, only the VCT_1 of the perfect fifth contributes the harmonic content of the interval as occurs for the third tones. The possible effect of the mistuned VCT_1 on the timbre of the major third and minor sixth intervals, remains to be established. It is possible that the good audibility of VCT_1 of the m_6 and M_3 dyads is at least partially enhanced by their mistuning. Because of the dominant and selective effect of A_0 amplification, strong VCT_1s occur almost exclusively in a relatively narrow range of frequencies around A_0 resonance at about 280 Hz. This explains why the P_5, M_3 and m_6 dyads shown in Table V, have a VCT_1 frequency corresponding (ignoring the microtonal deviation) in all cases to C_4.

IV. CONCLUSIONS

Combination tones generated by a set of five violins of different age and quality when playing a number of selected dyads were investigated. The results show that combination tones are present in all the violins during playing of all the dyads; the greatest one (VCT_1), was found at frequency below the lower note of the dyad. VCT_1 amplitude was determined by both non-linearity and by A_0, the main air resonance of the violin. The greatest VCT_1s, found in the old Italian violins, were crucially determined by the boosting action of A_0. This explains well the findings that different dyads and different violins have different VCT_1 amplitudes and that the strongest VCT_1s are correlated with A_0 peak amplitude. In the literature, a powerful A_0 resonance has been linked to violin sound quality, hence violin

Dyad	Parameter	Note	Frequency played (Hz)	Frequency of Pythagorean intonation (Hz)	Difference (cents)
	f_1	C_5	524 ± 0.4	521.5	8.3
P_5	f_2	G_5	785.7 ± 0.4	782.2	7.7
	f_{VCT_1}	$\mathbf{C_4}$	$\mathbf{261.4 \pm 0.5}$	**260.7**	**4.4**
	f_1	C_5	522.9 ± 0.7	521.5	-4.6
M_3	f_2	E_5	659.4 ± 0.3	660.0	-1.6
	f_{VCT_1}	$\mathbf{C_4}$	$\mathbf{273.9 \pm 0.7}$	**260.7**	**85.2**
	f_1	A_4	440.3 ± 0.2	440.0	1.2
m_6	f_2	F_5	694.3 ± 0.2	695.2	2.2
	f_{VCT_1}	$\mathbf{C_4}$	$\mathbf{253.7 \pm 0.3}$	**260.7**	$\mathbf{-47.0}$

TABLE VI. Comparison of note frequencies of the P_5, M_3 and m_6 dyads (experimental vs theoretical) and of VCT_1 (experimental vs VCT_1 source described in Table V) in Pythagorean intonation. Mean ± SEM for all the 5 violins.

quality and VCT_1 amplitude seem to be correlated. Indeed, the two old Italian violins, judged the best of our group, have the strongest VCT_1s. Combination tones seem not to originate by the bow string(s) interaction, as shown by comparing VCT_1 produced with one and two independent bows. Listening tests showed that VCTs of the two old Italian violins A and B, when playing P_5, M_3 and m_6 dyads, were well perceived by a group of musicians.

This occurred also for M_3 of violin C, although with slightly worse statistics. These results suggest a possible musical significance of violin combination tones which would add to the weaker Tartini's third tones. Combination tone frequencies of the M_3 and m_6 dyads, do not correspond exactly to musical notes of the Pythagorean temperament, with a significant microtonal deviation, perhaps enhancing the richness of violin sound.

Future work is foreseen in several directions: the extension of the analysis to a much greater number and variety of violins to confirm the hypothesis that strongest combination tones are correlated with the A_0 peak amplitude; the identification of the exact origin of VCTs which was not identified yet. This is a challenging task because of the very nature of coupling that makes impossible to analyze violin parts separately. The analysis of the same violin with a normal and a flat top plate could give some clues.

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[1]see supplemental material at [ ]for listening to an audio file with 4 pairs of dyads with and without violin combination tones

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