

# Robustness of complexity estimation in event-driven signals against accuracy of event detection method

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## ABSTRACT

Complexity has gained recent attention in machine learning for its ability to extract synthetic information from large datasets. Complex dynamical systems are characterized by temporal complexity associated with intermittent birth–death events of self-organizing behavior. These rapid transition events (RTEs) can be modeled as a stochastic point process on the time axis, with inter-event times (IETs) revealing rich dynamics. In particular, IETs with power-law distribution mark a departure from the Poisson statistics and indicate the presence of nontrivial complexity that is quantified by the power-law exponent  $\mu$  of the IET distribution. However, detection of RTEs in noisy signals remains a challenge, since false positives can obscure the statistical structure of the underlying process. In this paper, we address the problem of quantifying the effect of the event detection tool on the accuracy of complexity estimation. This is reached through a systematic evaluation of the Event-Driven Diffusion Scaling (EDDiS) algorithm, a tool exploiting event-driven diffusion to estimate temporal complexity. After introducing the event detection method RTE-Finder (RTEF), we assess the performance of the RTEF-EDDiS pipeline using event-driven synthetic signals. The reliability of the RTEF is found to strongly depend on parameters such as the percentile and the number of false positives can be much higher than the number of genuine complex events. Despite this, we found that the complexity estimation is quite robust with respect to the rate of false positives. In many of the studied cases, the second moment scaling  $H$  appears to even improve as the rate of false positives increases, reaching estimation errors of about 4 – 7%.

## 1. Introduction

The present-day focus on data science is linked, on the one hand, to the great availability of huge datasets and, on the other hand, to the increased computational power that makes it possible to use machine learning methods on these same datasets. In this framework, complexity is living a renewed interest due to the need for extracting synthetic information from the large amount of available data [1,2].

A complex system consists of many interacting elements, or nodes in a network, with nonlinear, cooperative dynamics that give rise to emergent, self-organized, structures, typically displaying some coherence among different elements and in time [3]. This coherence property can be quantified by proper *complexity* measures, or *metrics* in the information science jargon, computed from data that are obtained by observational probes embedded in the complex system. Complexity measures are often referred to the

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topological properties of the emerging coherent structures and are investigated in the framework of complex networks [4–7], with wide and interesting applications in the field of neuroscience [8–16]. However, when dealing with time evolving systems, e.g., time-varying networks, each network node is represented by a temporal signal. In this case, the investigation focus is also on the temporal dynamics of the network and, thus, on the temporal evolution of self-organized emerging states [17,18]. Actually, the cooperative dynamics of a wide set of complex systems trigger the emergence of coherent, or self-organized, structures that are not equilibrium states of the system, but, on the contrary, display peculiar metastability characterized by random lifetimes [19–23]. The complex metastability of these coherent states is a condition that is referred to as Temporal Complexity (TC) [17,24–26] or Intermittency-Driven Complexity (IDC) [18,27]. In this case, the self-organized structures are not equilibrium states but are characterized by a lifetime after which the instabilities of the nonlinear interactions determine a rapid decay of the self-organized state. After that, the system experiences a transition, typically a fast one, to a new dynamic condition, which may be another self-organized state or a disordered state. The latter case is clearly defined by the absence of well-defined internal coherence, whereas, on the contrary, a self-organized state is represented by a state with high internal coherence, e.g. long memory and/or long-range correlations between nodes. The overall displayed behavior is that of a sequence of intermittent events marking the transition among different dynamical regimes. It is essential to emphasize that transitions between self-organized states or between high and low coherence phases are often so fast as to be reduced to an instant in time. In this framework we can introduce the concept of Rapid Transition Event (RTE), and the temporal sequence of RTEs can be modeled as a stochastic point process in the time axis [28], a concept that was largely exploited in brain studies [19,20,29–32]. A point process is mathematically represented as a set of strictly increasing time instants  $\{t_n; n \in \mathcal{N}\}$ , being  $\mathcal{N}$  the set of integer numbers  $n \geq 0$ . The most important features of the intermittent birth–death process of self-organizing behavior are then the Probability Density Function (PDF)  $\psi(\tau)$  computed from the sequence of Inter-Event Times (IET-PDF)  $\tau_n = t_n - t_{n-1}$ ,  $n \geq 1$ , and the statistical dependence among RTEs and, consequently, among IETs. The *renewal* condition, which is defined by the mutual independence of IETs in the point process, is a further important feature that often characterizes RTEs [33]. The most interesting examples of IET-PDFs are given by an exponential, indicating a Poisson process, stretched exponential and power-law decays:

$$\psi(\tau) \sim e^{-r_p \tau}; \quad e^{-(r_p \tau)^\alpha}; \quad \frac{1}{\tau^\mu}$$

with  $0 < \alpha < 1$  and  $1 < \mu < 3$ . While the Poisson process is associated with the lack of self-organization, the last two behaviors, in particular the power-law, identify important non-Poisson processes marking the intermittent emergence of self-organized states [18].

In IDC systems, the testing of the renewal condition and the IET-PDF shape can be exploited to define the level of complexity [18,29]. Inverse power-law decay in the IET-PDF represents the most interesting case, as it links to self-similarity spanning over a wide range of time scales. In this case, the power exponent  $\mu$  is defined as the IDC index, thus being associated with the capacity of the cooperative dynamics to trigger self-organized metastable states [18]. Some prototypical models of signals with complex intermittency are given by the Pomeau–Manneville map, where the presence of a marginally stable point determines a sequence of fast transitions between a laminar and a chaotic region, [34–36], and by heteroclinic channels [21,22,37], that is, a sequence of fast passages through several saddle points that define the fast transitions between two different regions of the state space.

Event-based or intermittency-based approaches, i.e., based on the detection and statistical characterization of RTEs, were extensively applied in several fields. A greatly interesting case is given by brain dynamics, which are probably the most important prototype of a complex system [8–10,23]. In brain studies, the authors of Refs. [19,20] firstly introduced the idea of RTE<sup>1</sup> in the neuroscience field and applied it to the processing of ElectroEncephaloGrams (EEGs) and MagnetoEncephaloGrams (MEG). Other authors applied similar event-based approaches and similar RTE definitions to investigate brain complexity [29,30], functional Magnetic Resonance Imaging (fMRI) data [31,32], neural dynamics [24,38,39]. The concept of RTE was also applied in several other research contexts and to different kinds of data, e.g., atmospheric turbulence [40]. In general, many different RTE definitions were exploited including, more in general, the definition of crucial transition events not directly linked to rapid passages but, e.g., to some threshold passage in the signal, often after proper processing of the signal itself [40].

The event-based approach is a basic building block of the TC/IDC framework, such as the RTE definition and the associated event detection algorithm. In this respect, the effect of noise and the presence of false positives in the event detection are unavoidable. However, in complex systems theory, the main interest lies in the reliable estimation of the system's complexity. Regarding the IDC class of complex systems, the proper evaluation of complexity indices can be affected by the goodness of the event detection algorithm, whose accuracy can be affected by noise and by the unavoidable presence of false positives. Noisy events can originate from an erroneous detection (false positives), from electronic noise in the instrumentation or from an intrinsic noise of the system and are also denoted *secondary* events. The presence of noisy events, i.e., not complex events genuinely emerging from the underlying cooperative dynamics, were shown to affect the IET-PDF in such a way to hidden the genuine exponent  $\mu$ . In fact, secondary and complex events are mixed and indistinguishable and, consequently, the IETs of complex events are broken up in a non-trivial way by the presence of interposed secondary events, and similarly, the IETs of secondary events are split up by the presence of interposed complex events. The presence of secondary events may also determine the appearance of a different exponent, which turns out to be a statistically significant fit of the distribution. This condition is discussed by Allegrini et al. [30] with a specific application to the case of brain data (see also Refs. [40] for similar application to turbulence and [27] for a complete theoretical treatment). As a consequence, these findings foreclose the possibility of a reliable estimate of the IDC index  $\mu$  from the IET-PDF. However, a method to treat with noisy or secondary events is the Event Driven Diffusion Scaling (EDDiS) algorithm [18,27,29], a method

<sup>1</sup> These authors actually named Rapid Transition Processes (RTPs) the here introduced concept of RTEs.

that is based on the concept of anomalous diffusion and on related well-known results found in classical literature [41–51]. This algorithm is based on the well-known finding that secondary events drive a normal diffusion process while crucial complex events trigger anomalous diffusion [18,41,52–54], this last one being defined by a non-Gaussian distribution and/or a nonlinear growth of the variance<sup>2</sup>:

$$\langle X(t)^2 \rangle \sim t^{2H} .$$

$H$  is the second-moment scaling, which corresponds to the Hurst exponent for mono-scaling signals. EDDiS algorithm is based on well-known theoretical results about diffusion processes driven by crucial events, in particular referring to the Continuous Time Random Walk (CTRW) model [43,44,52,55–57]. In particular, these theoretical results follow from the assumption of the renewal condition introduced above [33].

Even if the problem of noisy events in complexity evaluation was identified and the potentiality of EDDiS was investigated [27, 29,30], a systematic evaluation of the accuracy in the complexity estimation is still lacking.

In this paper, we carry out a systematic study of estimation errors in complexity indices derived from the EDDiS algorithm. In particular, we investigate the role of the event detection algorithm and characterize the robustness of complexity estimation concerning the choice of parameters that need to be fixed *a priori* in the event detection algorithm. To carry out this study we here propose and develop an extension of an event detection algorithm first proposed by Kaplan et al. [20]. Our approach is essentially a generalization of that algorithm and of later variations proposed and applied by Allegrini et al. [29,58]. The proposed algorithm of event detection is then applied to a time series generated by a stochastic model that combines a sequence of complex events with a damped oscillator and an additive white Gaussian noise component.

The paper is organized as follows. In Section 2 we introduce the event detection algorithm, hereafter referred to as RTE Finder (RTEF), we briefly recall the EDDiS tool and we define the metrics used to evaluate the estimation errors. In Section 3 we introduce our stochastic model used to generate artificial signals driven by events with IET distributed according to an inverse power-law with power exponent  $\mu$  (complex events) or to an exponential (Poisson events). In Section 4 we illustrate the results of numerical simulations and, in particular, the application of the RTEF-EDDiS pipeline of statistical analyses to the simulated event-driven signals. In Section 5 we give a detailed discussion of the results. Finally, in Section 6 we discuss our findings.

## 2. Methods of data analysis

Our goal is to test the accuracy in the estimation of IDC index  $\mu$  by the combined applications of an event detection algorithm, herein denoted as RTE-Finder (RTEF), and of the EDDiS algorithm.

### 2.1. The RTEF algorithm

Our proposed algorithm is inspired by and generalizes the work of Kaplan et al. [20] and of Allegrini et al. [29,58]. The main differences lie in the evaluation of the signal steepness, of the signal envelope and on the possibility of extending the analysis to different signal frequency bands.

The RTEF algorithm consists of two main steps: (1) calculation of the signal envelope and (2) extraction of the RTEs. In the first step, we estimate the signal envelope by using the modulus of the analytical signal associated with the Hilbert transform  $\mathcal{H}$ , i.e.:

$$S_a(t) = S(t) + i \mathcal{H}[S](t)$$

so that the envelope is given by  $E(t) = |S_a(t)|$ . Following the convention by Kaplan et al. [20], the signal envelope  $E(t)$  is hereafter denoted as “Test Signal”. In the second step, we start from the Test Signal and calculate the approximation of its derivative to see the rapid changes in the signal. Clearly, the Test Signal  $E(t)$  is given with the same sampling frequency of the original signal  $S(t)$ . Thus, given the signal sampling time  $T_s$ , the derivative of the Test Signal at time  $t_k = k T_s$  is estimated by the following formula:

$$\dot{E}(t_k) = \frac{1}{N_d} \sum_{j=1}^{N_d} \frac{E(t_k + jT_s) - E(t_k - jT_s)}{2 \cdot j \cdot T_s} \tag{1}$$

being  $N_d$  the number of sampling times before and after the time  $t_k$ , so that the time window considered to evaluate the derivative is given by a total  $2N_d$  times.

Eq. (1) is an average of  $N_d$  difference quotients, centered at  $t_k$ , taken at different time intervals  $2 \cdot j \cdot T_s$  around  $t_k$  itself. Kaplan et al. [20] use a slightly different approach to evaluate the rapid changes in the signal steepness that, however, apply formulas involving a number of sampled times up to five. In order to investigate the role of  $N_d$ , we here compare the value  $N_d = 5$ , which is the same as in Kaplan et al. [20], with the value  $N_d = 12$ . The models used to generate IET sequences, which will be introduced in the next Section, will be set with a unitary average IET and the modeled signals will be sampled with a 100 Hz rate. Thus, the choice  $N_d = 5$  in Eq. (1) gives, roughly speaking, a smoothing at about 10 Hz, which is much above the reference frequency scale given by the unitary average IET (1 Hz). Analogously,  $N_d = 12$  gives a smoothing of the derivative at about 4 Hz, which is still

<sup>2</sup> Here  $X(t)$  is the integral of an observed signal  $S(t)$ , i.e.,  $\dot{X}(t) = S(t)$ .

above the reference frequency scale of 1 Hz. We here report the results obtained with  $N_d = 5$ , while the analyses carried out with  $N_d = 12$  can be found in the Supplementary Material. We anticipate that, regarding accuracy of the complexity estimation, the two choices give very similar results, with a very slight worsening for  $N_d = 12$ .

Finally, we compute the absolute value of the derivative and we determine a threshold by selecting a specific percentile value from the distribution of these absolute values. The RTEs are then defined by the threshold crossings of the absolute value of the derivative.

### 2.2. EDDiS algorithm

The EDDiS method is a combination of two scaling analyses applied to three different event-driven diffusion processes, i.e., three random walks driven by a sequence of events by means of different walking rules (see Supplementary Material for details).

The general application of the EDDiS method involves the three walking rules. However, the Asymmetric Jump (AJ) walking rule, which is defined by the walker just making a unitary jump ahead at each event occurrence time, has been largely proven to be the most robust and reliable rule [18,27,29,30,58,59]. For this reason, we here limit to investigate the AJ rule and, in the following, we recall the two scaling analyses carried out on the resulting AJ diffusion process. Further details of the EDDiS algorithm can be found in Refs. [18,27,29] and, for reader's convenience, in the Supplementary Material.

The two scaling analyses are given by the Diffusion Entropy [41,53,54], which evaluate the similarity  $\delta$  of the diffusion PDF, and by the Detrended Fluctuation Analysis (DFA) [60], estimating the second moment scaling  $H$ , essentially corresponding to the so-called Hurst exponent [61]. In formulas, the scaling exponent  $H$  of the second moment is defined as:

$$F^2(t) = \sigma^2(t) = \left\langle \left( X(t) - \bar{X}(t) \right)^2 \right\rangle \sim t^{2H} \tag{2}$$

being  $X(t)$  the diffusion variable resulted after applying the walking rule, and  $\bar{X}(t)$  is a proper local trend of  $X(t)$ . The self-similarity index  $\delta$  is defined as:

$$P(x, t) = \frac{1}{t^\delta} F\left(\frac{x}{t^\delta}\right) \tag{3}$$

being  $P(x, t)$  the PDF of the diffusion variable  $X(t)$ .

In general, the diffusion variable is written as the integral of a temporal signal:

$$X(t) = \int_0^t \xi(t') dt' \approx \sum_{n=0}^N \xi_n$$

being  $t = t_N$ . In the AJ walking rule here applied, the impulsive, time-sampled, variable  $\xi_n$  consists of making a unitary jump ahead when an event occurs. The resulting event-driven diffusion process built according to the AJ rule corresponds to the counting process generated by the event sequence:

$$X(t) = \#\{n : t_n < t\}. \tag{4}$$

The functional relationships  $\delta = \delta(\mu)$  and  $H = H(\mu)$  in the case of AJ are the following [18,41]:

$$H_{AJ} = \begin{cases} \frac{\mu}{2} & \text{if } 1 < \mu < 2 \\ 2 - \frac{\mu}{2} & \text{if } 2 \leq \mu < 3 \\ \frac{1}{2} & \text{if } \mu \geq 3 \end{cases} \tag{5}$$

$$\delta_{AJ} = \begin{cases} \mu - 1 & \text{if } 1 < \mu < 2 \\ \frac{1}{\mu - 1} & \text{if } 2 \leq \mu < 3 \\ \frac{1}{2} & \text{if } \mu \geq 3 \end{cases} \tag{6}$$

For this study, we apply this method directly to the detected RTE sequences. This type of analysis is a powerful tool for scaling detection and, under the assumption of superdiffusion, i.e.,  $H > 0.5$  and of sufficiently long sequence of events, is effective in detecting the anomalous diffusion scaling, thus giving useful information on the underlying dynamics that indeed generate the events. In the following,  $H$ ,  $\delta$  and  $\mu$  will be generically referred to as *complexity indices*.

### 2.3. Error estimates

To have a metric of performances of the proposed algorithm, we calculate the relative error for different metrics:

$$RE = \left| \frac{EV - RV}{RV} \right| \tag{7}$$

where  $EV$  is the estimated value, and  $RV$  is the real value. When the  $RE$  is referred to a set of numerical simulations, we compute the average of the  $RE$  over these same simulations and denote it as Mean RE ( $MRE$ ).

### 3. The time series model

The model used to simulate the time series, or temporal signals, is inspired to the work of Guardabasso et al. [62]. In particular, Guardabasso and co-workers studied a noisy relaxation process triggered by a sequence of random events with Poisson statistics with the aim of modeling hormone signals. Other works use similar approaches based on dynamical systems, with or without a noise component and/or triggering events to model real-world biological signals, e.g., cortisol [63,64] and glucose signals [65]. The model by Guardabasso et al. [62] could also model EEG signals according to the approach firstly proposed by Fingelkurts et al. [19], which essentially involves the emergence of crucial transition events [29,30,59,66]. Then, we introduce here an extension of the Guardabasso model by introducing two elements:

- A Non-Poisson distribution of events according to a IET-PDF with inverse power-law decay.<sup>3</sup>
- A damped oscillator that is triggered by each event and whose dynamics are modeled through a second-order differential equation.

Similarly to Guardabasso et al. [62], a Gaussian white noise component is then added to the synthetic time series.

In summary, the time series model is divided into four main steps:

- Generation of an event sequence, described by the corresponding set of event occurrence times:  $\{t_n; 1 \leq n \leq M\}$ .
- Generation of a pulse amplitude  $A_n$  for each event, with  $A_n$  uniform random number in  $[1, 10]$ . At this point, we get a train of pulses with different amplitudes and the point process is described by the set  $\{(t_n, A_n); 1 \leq n \leq M\}$  and by the formula:

$$S_{pr}(t) = \sum_{i=0}^M A_i \delta(t - t_i) \tag{8}$$

- Convolution with pulse response: for each event, this step generates a damped oscillation, which then generalizes the relaxation process of Guardabasso et al. [62].
- Addition of Gaussian white noise to the generated time series.

In step (a) a sequence of IETs, also known as Waiting Times (WTs), is first drawn from a given IET-PDF. In particular, we choose two types of probability distributions [18]:

- Inverse power-law distribution for complex events [27,41,54]:

$$\psi_n(\tau) = \frac{\mu - 1}{T} \frac{1}{(T + \tau)^\mu} \tag{9}$$

being  $\mu$  the power-law exponent and  $T$  the time scale at which the power law emerges.<sup>4</sup>

- Exponential distribution for Poisson events

$$\psi_p(\tau) = r_p e^{-r_p \tau} \tag{10}$$

where  $r_p$  is the Poisson's event rate and  $\langle \tau_p \rangle = 1/r_p$ .

For both cases we set the mean IET equal to 1 ( $\langle \tau_p \rangle = \langle \tau_n \rangle = 1$ ) and we draw a sample of  $M = 20000$  IETs to get a sequence of random events. In the case of power-law distribution, we draw the IETs by using the following formula [67]:

$$\tau_i = T \left( \xi_i^{\frac{1}{1-\mu}} - 1 \right); i = 0, 1, \dots \tag{11}$$

being  $\xi_i$  a random number uniformly distributed in  $[0, 1]$ .<sup>5</sup>

The IETs from the exponential distribution are drawn using the usual expression:

$$\tau_i = -\frac{1}{r_p} \log \xi_i; i = 0, 1, \dots \tag{12}$$

where, as in Eq. (11),  $\xi_i$  is a random number uniformly distributed in  $[0, 1]$ . In Fig. 1 some generated IET-PDFs are reported together with the related theoretical curves, making it evident that the qualitative comparison is very good. Then, the sequence of event occurrence times is simply defined by:

$$t_n = t_{n-1} + \tau_n; t_0 = 0; n = 1, 2, \dots, M$$

<sup>3</sup> The analysis of complex events, associated with an inverse power-law IET-PDF, is here compared with the standard Poisson events investigated by Guardabasso et al. [62].

<sup>4</sup> When  $\mu > 2$ , the mean IET is given by  $\langle \tau_n \rangle = T/(\mu - 2)$ .

<sup>5</sup> This formula is also obtained by applying the cumulative function method.

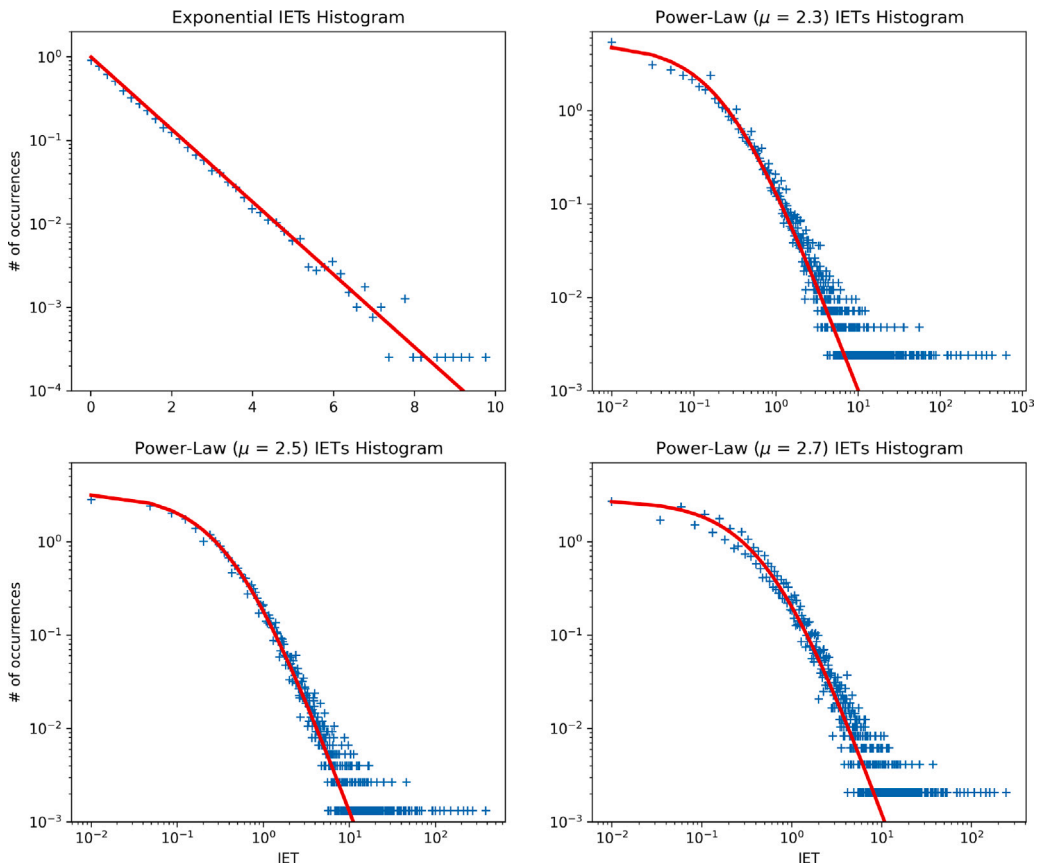


Fig. 1. Comparison of observed IET distributions with theoretical curves. IET in seconds.

The total simulation time is given by  $t_{max} = t_M$ . In order to reproduce a sampled signal, we then established a sampling grid spanning from 0 to  $t_{max}$  with a sampling time  $T_s = 0.01$ .<sup>6</sup> Starting from Eq. (8), the discrete time version of the pulse train is given as a sequence of zeros at each time step except for the steps that are nearer to an event occurrence time. In formulas, given  $t_i = i \cdot T_s$  with  $t_i$  the time step that is nearest to the event occurrence time  $t_n$ , it results:  $S_{p_T}(t_i) = A_n$ . Subsequently, according to above point (c), we generated a time series based on the pulse train using the following algorithm:

1. Take an angular frequency band:  $[\omega_{0_{min}}, \omega_{0_{max}}]$ , or equivalently, a frequency band:  $[v_{min}, v_{max}]$ , being that  $\omega = 2\pi v$ .
2. Choose  $N$  angular frequencies  $\omega_{0_k}$  equispaced in the band. In our case the grid size  $\Delta\omega_0 = 1$ .
3. For each  $\omega_{0_k}$  the pulse response function is given by a damped oscillator:

$$I_k(t) = \frac{e^{-\frac{\gamma}{2}t}}{\omega_{1_k}} \sin(\omega_{1_k} t) \tag{13}$$

where  $\omega_{1_k} = \frac{1}{2} \sqrt{4\omega_{0_k}^2 - \gamma^2}$ , and  $\gamma$  is the relaxation factor of the damped oscillator, which is here set to 1.1, which is a value compatible with under-damped oscillations. Notice that this is the solution of a second-order differential equation representing the dynamics of a particle in a harmonic potential with friction, i.e.:

$$\frac{dI_k}{dt} = v_k ; \quad \frac{dv_k}{dt} = -\gamma v_k - \omega_{0_k}^2 x_k + \delta(t) \tag{14}$$

being  $I_k(0) = 0$ ,  $v_k(0) = 1$ .<sup>7</sup> The Dirac function  $\delta(t)$  in Eq. (14) is a pulse at time  $t = 0$  and the solution to this equation, given in Eq. (13), is the fundamental solution (Green function) of the system of differential equations in Eq. (14) and, thus, the response to this initial pulse.

<sup>6</sup> When considering time units in seconds, this corresponds to a sampling rate of 100 Hz.

<sup>7</sup> The response function (13) is also referred to as a second-order linear filter.

- Then, for each event occurrence time  $t_n$ , the total pulse response function is given by the superposition of all contributions in the frequency band:

$$I(t) = \sum_{k=1}^N I_k(t) = \sum_{k=1}^N \frac{e^{-\frac{\gamma}{2}t}}{\omega_{1k}} \sin(\omega_{1k} t) \tag{15}$$

- The total generated time series is then given by the time convolution between  $I(t)$  and  $S_{PT}(t)$ :

$$S(t) = (I * S_{PT})(t)$$

The sampled signal  $S(T_i) = S(i)^8$  is calculated by Z-transforming the function  $I(t)$  and by multiplying it with the Z-transform of the pulse train  $S_{PT}(t)$ . At the end of this step, given the sampling time  $T_s$  and the time steps  $t_i = i \cdot T_s$ , we get a sampled signal  $S(i)$  by anti-transforming.

- A Gaussian noise  $W(t)$  is added to the time series, that is, a sequence of Gaussian random variables  $W(i)$ . The Gaussian noise has zero mean and variance computed as a fraction of the variance computed from the time series obtained in the previous step. In formulas:

$$S(i) = \mathcal{Z}^{-1} \left[ \mathcal{Z} \left[ S_{PT} \right] \mathcal{Z} [I] \right] (i) + W(i); \quad 0 \leq i \cdot T_s \leq t_{max} = t_M \tag{16}$$

The algorithm’s functionality is illustrated in Fig. 2, the noise variance is set to 10% of the variance computed from the time series without noise.

We evaluate the performance of the RTEF by generating time series in three distinct frequency bands  $[v_{min}, v_{max}]$ : [0.5, 4] Hz, [4, 8] Hz, and [8, 12] Hz. Fig. 3 shows an example of the same time series generated in these different frequency bands starting from the same event sequence, without noise.

#### 4. Numerical simulations and results

To test the RTEF algorithm we simulated different synthetic time series exploiting the model introduced in the previous Section 3. We limit our analysis to the range  $\mu \in (2, 3]$  for two reasons: (i) the statistical distribution of the IETs, Eq. (9), yields a finite mean IET, while  $\langle \tau \rangle = \infty$  for  $\mu \leq 2$ ; (ii) most applications of the EDDiS showed results compatible with a complexity index  $\mu \sim 2.1 \div 2.5$ .<sup>9</sup> The existence of a finite mean IET allows to get a stable mean in the random samples generated using Eq. (11), a condition that was found to be associated with ergodicity. Conversely, the infinite mean IET in the range  $\mu \in (1, 2]$  determines non-ergodic behavior [36,68,69], thus leading to wide variability in the mean of the sampled IETs, since it does not converge. Thus, we generated a total of 150 signals for each IET-PDF, which are: power-law with  $\mu = 2.3, \mu = 2.5, \mu = 2.7$ ; exponential with  $r_p = 1$ . We recall that all IET sequences are constrained to a unitary mean IET. This is immediate in the exponential case, being  $\langle \tau_p \rangle = 1/r_p = 1$ . Conversely, in the power-law case, the parameter  $T$  in Eqs. (9) and (11) must obey the relationship:  $T/(\mu - 2) = \langle \tau_n \rangle = 1$ .

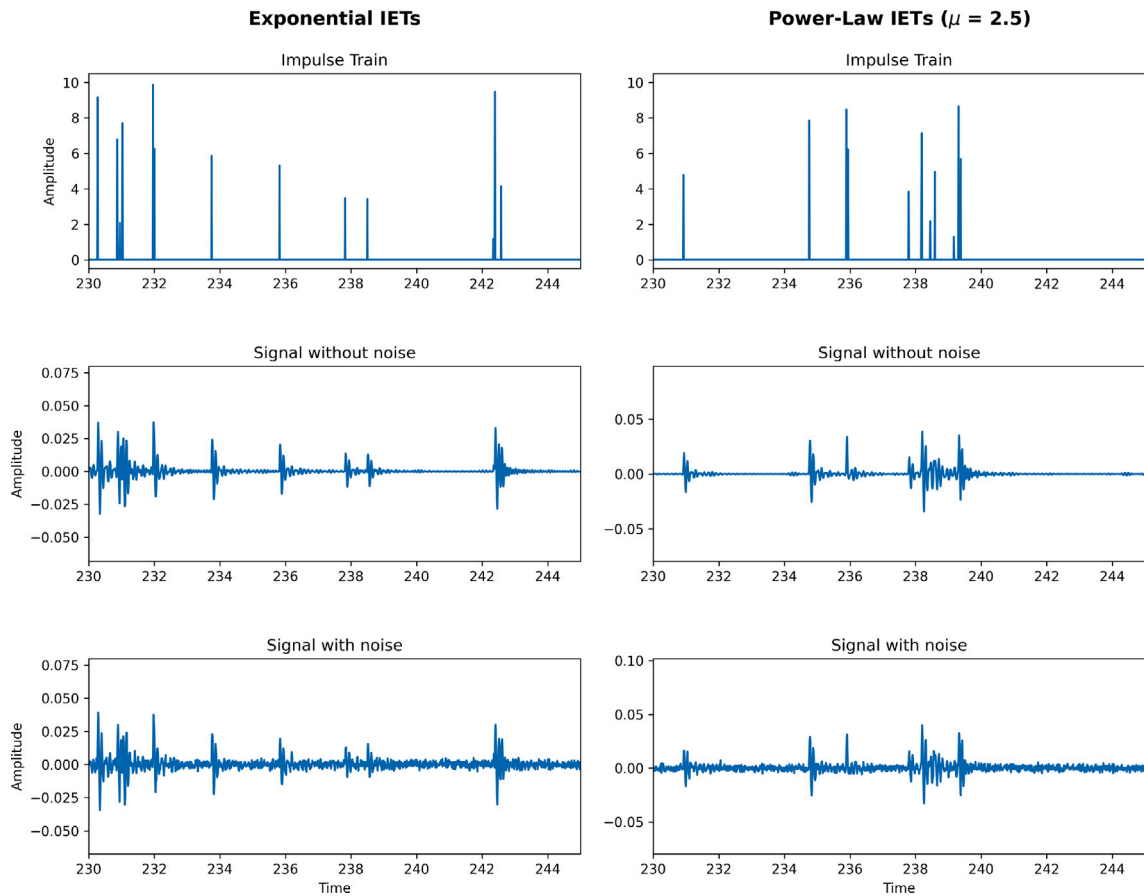
For each IET-PDF, 50 different sample sets of IETs were drawn:  $\{\tau_n; n = 1, M\}$  ( $M = 20000$ ). The associated sequence of event time occurrence is then given by:  $t_0 = 0, t_n = t_{n-1} + \tau_n, n \geq 1$ . For each sample set of IET, we apply the points (4–5) of the algorithm for three different frequency bands of the signal: [0.5, 4] Hz, [4, 8] Hz, and [8, 12] Hz. We finally add the Gaussian noise, as described in the point (6) of the previous section, to get the final temporal signal.

We first evaluated the complexity indices  $H$  and  $\delta$  directly on the IET samples, giving an estimation of relative errors due only to the random drawing, that is, to the random number generator, and to the best fit algorithm. This is obtained by applying the EDDiS, limited to the AJ walking rule, to the IET samples. These estimated relative errors, which will be denoted in the following as *reference errors*, are presented in Table 1. This is a useful estimation of the statistical error associated with the random generator and the best fit procedure that can be used as a set of reference values for the combined RTEF-EDDiS pipeline.

As illustrated before, we used two different time windows to estimate the derivative of the envelope of the absolute value of the signal. In Fig. 4 we display an example of how the RTEF algorithm works. In particular, these panels highlight that there are important differences in choosing  $N_d = 5$  or  $N_d = 12$  symmetrical points to estimate the derivative. In fact, for all the percentile values that we used, the estimation of the derivative with  $N_d = 5$  gives more precise results than the one with  $N_d = 12$ . Moreover, by passing from the 85th to the 98th percentiles, the number of false positives decreased, but some real events were missed. In Fig. 4 we report only the results with the 85th and the 98th percentiles, whereas other results with 90th and 95th percentiles are reported in the Supplementary Material. The IET distribution of the real and of the detected RTEs are similar but with some differences. In Fig. 5 we can see that there are some differences in the IET-PDFs estimated by using 5 or 12 symmetrical points to estimate the derivative. The estimated distributions by using 5 symmetrical points are more similar to the real one with respect to the one estimated with 12 symmetrical points. By passing from the 85th percentile to the 98th percentile we noticed that the exponential distribution and the initial part of the power-law distributions got worse, while the tails of the power-law distributions became more similar to the real one. Even if not trivial, this result is expected by observing that a higher percentile value select time events with greater signal steepness. It is worth noting that the estimations of the IET distributions are quite robust concerning the change in

<sup>8</sup> Since there are no ambiguities, it is possible to use the concise notation  $S(i)$  for the sample signal.

<sup>9</sup> Clearly, these values emerge for systems that are then recognized to be complex, otherwise exponential decays in the IET distributions are usually found for non-complex behavior, corresponding to normal diffusion scaling  $H = \delta = 0.5$ .



**Fig. 2.** Example of time series generated by our model. Left and right panel refer to exponential and power-law (with  $\mu = 2.5$ ) IET distributions, respectively. Top: comparison of the impulse trains. Middle: time series generated without the noise in frequency band  $[8, 12]Hz$ . Bottom: final time series with noise (with 10% variance of the deterministic component).

both  $N_d$  and percentile. However, the presence of noisy, secondary, events determine a blurring of the original IET-PDF that can give a statistically significant power-law fit that does not correspond to the genuine complexity index. This condition is similar to that found in previous works [30,40].

To estimate the diffusion scaling parameter derived from the detected events, we employed the DFA and DE analyses. In Figs. 6 and 7, we report the average results of the DFA and DE analysis with error bars and with all the percentile values chosen. We report here the results obtained for signals generated in the frequency band  $[8, 12] Hz$ , while the results obtained for signals generated in frequency bands  $[0.5, 4] Hz$  and  $[4, 8] Hz$  are reported in the Supplementary Materials. Tables 2, 3, 4, and 5 present the Mean Relative Errors (MREs) with Standard Deviations (STDs) of estimated scaling parameters  $H$  and  $\delta$  obtained for each percentile and each frequency band, limited to  $N_d = 5$ . We chose to report only the 5-point results due to their better performance compared to the 12-point results, which are included in the Supplementary Material.

## 5. Discussion of results

We here give a detailed discussion of the numerical results regarding the application of EDDiS algorithm to the IET samples and of the RTEF-EDDiS pipeline to the artificial signals driven by the same IET samples.

### 5.1. EDDiS analysis of IET samples and reference errors

Reference errors are due to: finite size of the statistical sample; limitations of the random number generator; and intrinsic errors of the best-fit procedure. These errors were estimated by applying the EDDiS algorithm to the IET samples. The ideal values of  $H$ ,  $\delta$  and  $\mu$  are given by Eqs. (5) and (6) and are compared to the corresponding estimated values by applying the RE formula, Eq. (7), and taking the average over the 50 samples to get the MRE.

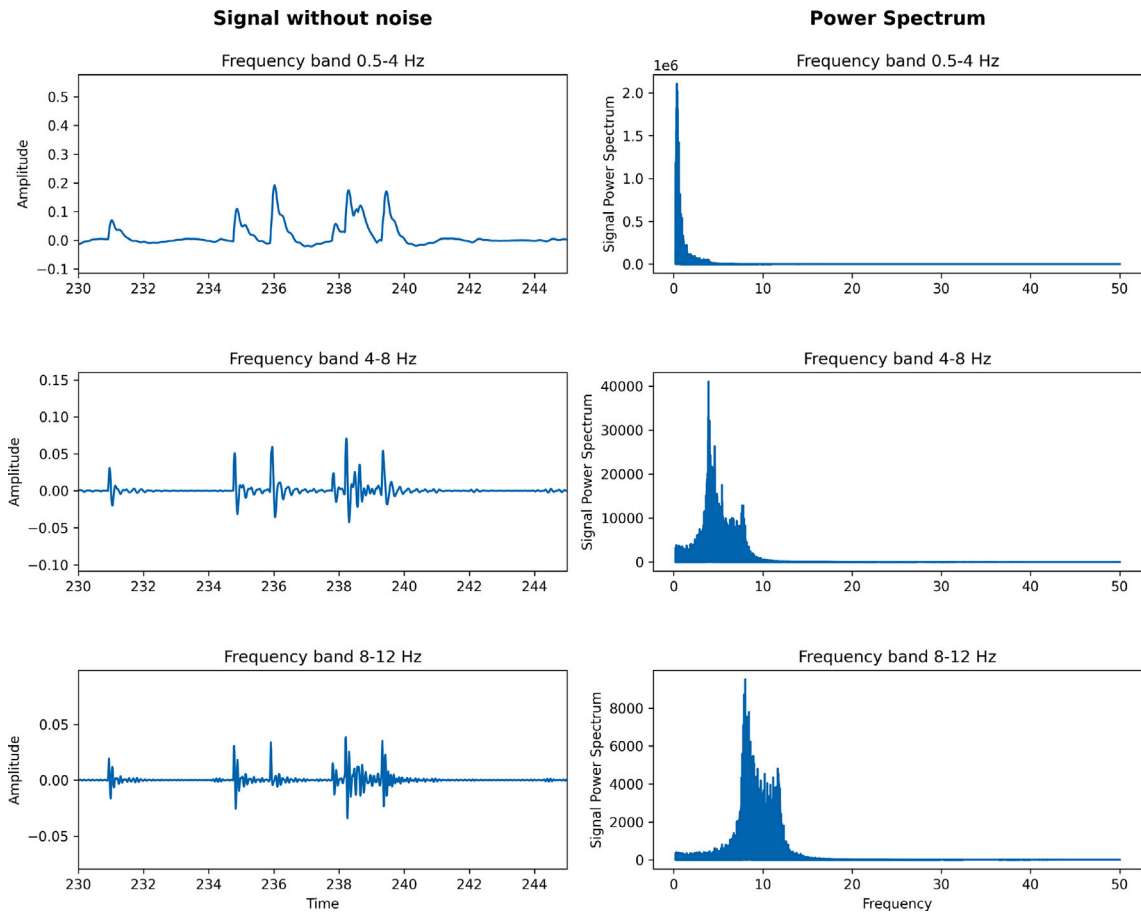


Fig. 3. Example of a time series generated in different frequency bands. Left panels: generated time series; same event sequence as in the top right panel of Fig. 2 (power-law,  $\mu = 2.5$ ). Right panels: power spectra corresponding to the time series reported in the left panels.

Table 1

Reference errors for complexity analysis carried out directly on the IET samples (average and standard deviation are computed on the 50 IET samples). (a) Mean Relative Errors (MREs) with Standard Deviation (STD) of DFA and DE; (b) MREs and STDs on  $\mu$  for Power-Law and  $r_p$  for exponential distributions computed on the 50 IET samples.

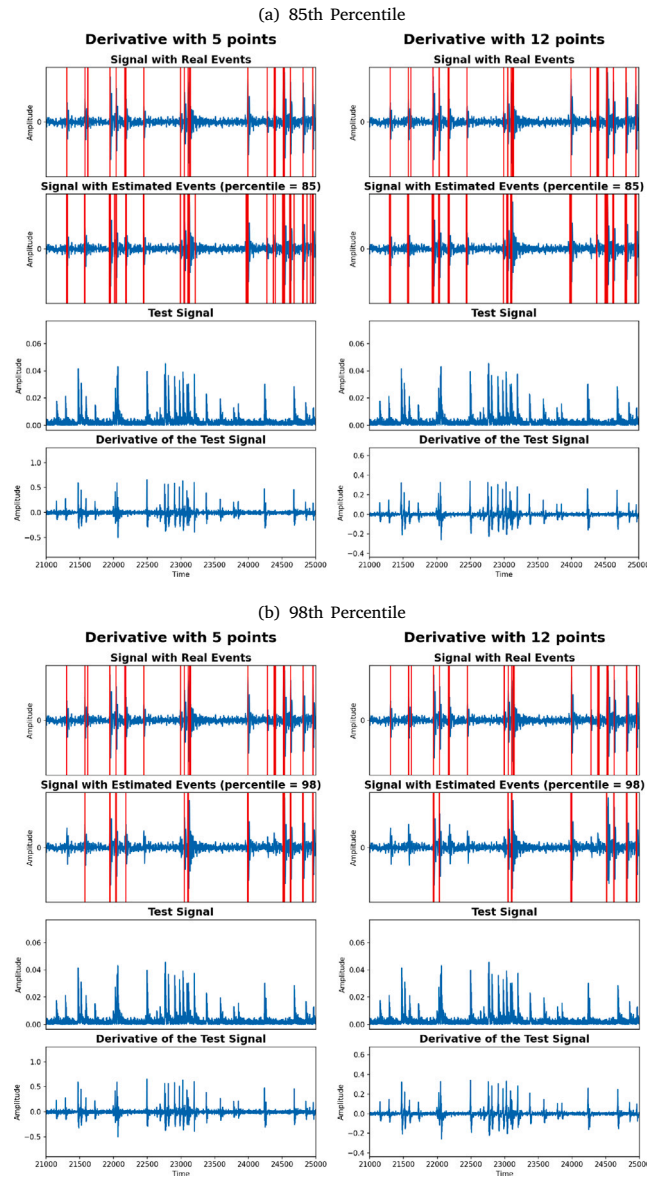
Case	MRE $H$ ( $\pm$ STD)	MRE $\delta$ ( $\pm$ STD)
Exponential	0.0052 ( $\pm$ 0.0042)	0.024 ( $\pm$ 0.011)
Power-Law ( $\mu = 2.3$ )	0.088 ( $\pm$ 0.040)	0.040 ( $\pm$ 0.035)
Power-Law ( $\mu = 2.5$ )	0.040 ( $\pm$ 0.030)	0.062 ( $\pm$ 0.021)
Power-Law ( $\mu = 2.7$ )	0.070 ( $\pm$ 0.040)	0.15 ( $\pm$ 0.030)

(a) DFA and DE Mean Relative Errors

Case	MRE $\mu$ or $r_p$ ( $\pm$ STD)	Min and Max Relative Error
Exponential	0.019 ( $\pm$ 0.012)	[0.001, 0.044]
Power-Law ( $\mu = 2.3$ )	0.068 ( $\pm$ 0.055)	[0.0031, 0.25]
Power-Law ( $\mu = 2.5$ )	0.061 ( $\pm$ 0.046)	[0.0002, 0.22]
Power-Law ( $\mu = 2.7$ )	0.051 ( $\pm$ 0.041)	[0.0015, 0.16]

(b) Fit Mean Relative Errors

Table 1a shows the mean relative errors in the estimation of the scaling factors  $H$  and  $\delta$  obtained by applying DFA and DE algorithms on the real events drawn from all considered IET-PDFs. Notably, the relative errors for both scaling factors across all IET distributions remain below 10%, except for the  $\delta$  scaling factor estimated for the inverse power-law distribution with  $\mu = 2.7$ , which reaches 15%. In particular, the estimated  $H$  values consistently exhibited a relative error below 8.8%, with the lowest error

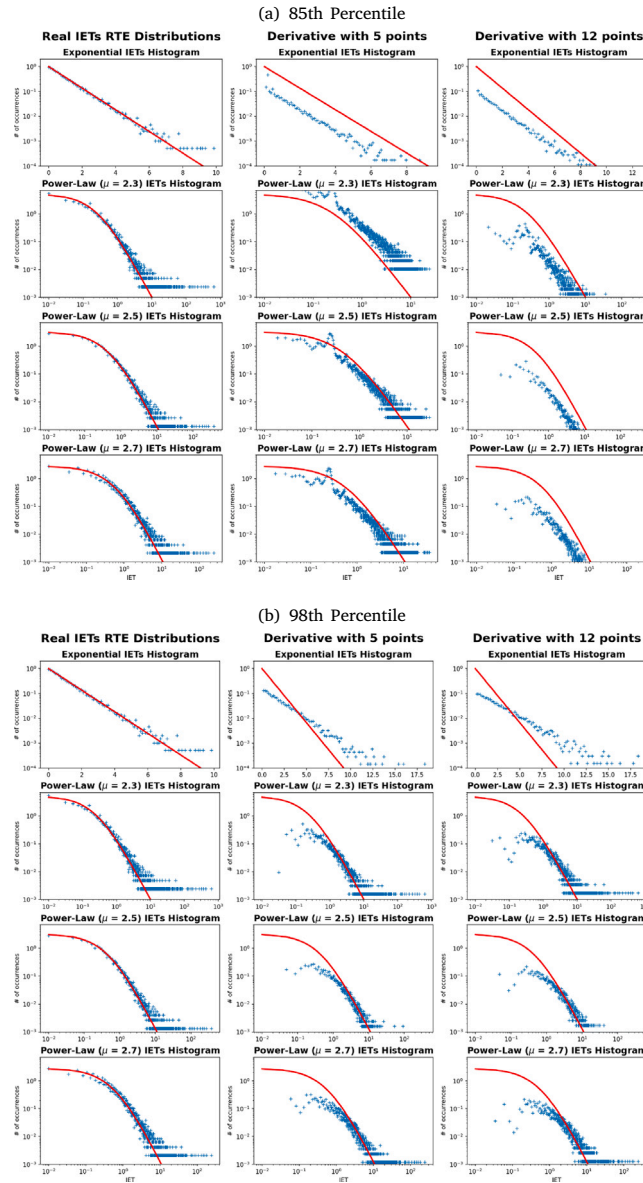


**Fig. 4.** Example of application of the RTEF algorithm. The results are obtained using 5 or 12 symmetrical points to estimate the derivative for a signal generated in frequency band [8, 12] Hz. On the first two top panels, vertical red lines indicate the real (above) or estimated (below) events. (a) RTEF results for the 85th percentile of the derivative distribution. (b) RTEF results for the 98th percentile of the derivative distribution.

occurring for the exponential distribution and the highest for the power-law distribution with  $\mu = 2.3$ . Conversely, the  $\delta$  estimated values have higher MRE than  $H$  values, reaching a maximum of 15% for the power-law distribution with  $\mu = 2.7$ . There is only one exception: the power-law distribution with  $\mu = 2.3$ , where the relative error of  $\delta$  was lower than that of  $H$  (4.0% compared to 8.8%).

Table 1b highlights that the mean relative errors in the direct  $\mu$  (or  $r_p$ ) evaluation, i.e., from the fit of IET-PDFs, for all the cases do not pass the 7% on average. As expected, the best performance is given by the evaluation of  $r_p$  in the exponential case, while the accuracy in  $\mu$  evaluation increases (MRE decreases) as  $\mu$  increases, thus giving the worst performance for the slower power-law decay, i.e.,  $\mu = 2.3$ . Moreover, an analysis of the confidence intervals revealed a maximum relative error of 25% in the worst-case scenario and of 0.44% in the best-case scenario. In contrast, the minimum relative errors were assessed at 0.31% in the worst-case scenario and 0.02% in the best-case scenario.

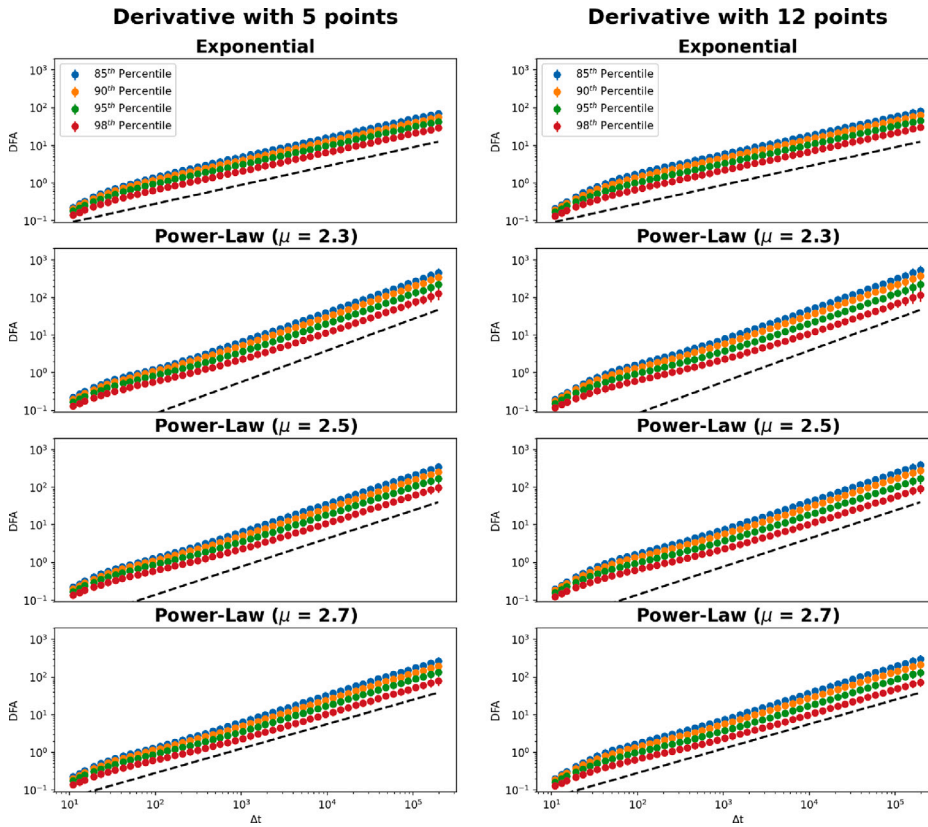
In general, the  $\delta$  and  $H$  evaluations have the highest accuracy for the exponential case. For the power-law case,  $\delta$  is better estimated for smaller  $\mu$ , while the accuracy of  $H$  estimation is not monotonic with  $\mu$ , giving the best result for  $\mu = 2.5$  (4%).



**Fig. 5.** Comparison of real IET distributions to the estimated ones for a signal generated in frequency band [8, 12] Hz and using 5 or 12 symmetrical points to estimate the derivative. The red lines represent the theoretical curves. (a) Comparison of real IETs to the estimated one from the 85th percentile of the derivative distribution. (b) Comparison of real IETs to the estimated one from the 98th percentile of the derivative distribution.

### 5.2. RTEF-EDDiS analysis of event-driven artificial signals

The artificial signals generated according to the model of Section 3 were processed with the RTEF algorithm and the detected events were exploited to estimate the scaling exponents  $H$  and  $\delta$  through the EDDiS algorithm. We found that the RTEF algorithm's performance is primarily affected by the number  $N_d$  of symmetrical time points used for derivative estimation of the absolute value of the signal's envelope. In particular, we highlight that the case  $N_d = 5$  generally outperforms the case  $N_d = 12$ . For this reason, we here focus the discussion on the case  $N_d = 5$ , while interested readers can find the results obtained using  $N_d = 12$  in the Supplementary Material. Actually, it is worth noting that the difference between  $N_d = 5$  and  $N_d = 12$  is not really significant. This finding seems to confirm that the average IET is the most important time scale, while the presence of the other time scales associated with the frequency bands determine a higher-order effect.



**Fig. 6.** DFA mean curves with error bars for signals generated in frequency band [8, 12] Hz. Mean and error bars are computed over the 50 IET samples. This figure compares the DFA curves obtained using  $N_d = 5$  and  $N_d = 12$  symmetrical points to estimate the derivative. The dashed black lines represent the theoretical slope, that is:  $H = 0.5$  for the Exponential Distribution,  $H = 0.85$  for Power-Law distribution with  $\mu = 2.3$ ,  $H = 0.75$  for  $\mu = 2.5$ , and  $H = 0.65$  for  $\mu = 2.7$ .

**Table 2**

MREs and STDs of DFA and DE computed from detected events in signals driven by exponential IET distributions (MREs and STDs computed from the 50 IET samples). All parameter configurations of the RTEF algorithm are reported for comparison. Results were obtained by using  $N_d = 5$ .

Frequency band	Percentile	MRE H ( $\pm$ STD)	MRE $\delta$ ( $\pm$ STD)
[0.5, 4]	85	0.033 ( $\pm$ 0.016)	0.056 ( $\pm$ 0.050)
	90	0.038 ( $\pm$ 0.017)	0.061 ( $\pm$ 0.051)
	95	0.037 ( $\pm$ 0.016)	0.081 ( $\pm$ 0.065)
	98	0.033 ( $\pm$ 0.018)	0.12 ( $\pm$ 0.084)
[4, 8]	85	0.031 ( $\pm$ 0.015)	0.073 ( $\pm$ 0.052)
	90	0.014 ( $\pm$ 0.010)	0.082 ( $\pm$ 0.054)
	95	0.010 ( $\pm$ 0.010)	0.085 ( $\pm$ 0.064)
	98	0.012 ( $\pm$ 0.010)	0.11 ( $\pm$ 0.083)
[8, 12]	85	0.028 ( $\pm$ 0.015)	0.069 ( $\pm$ 0.052)
	90	0.013 ( $\pm$ 0.010)	0.072 ( $\pm$ 0.048)
	95	0.010 ( $\pm$ 0.010)	0.080 ( $\pm$ 0.054)
	98	0.012 ( $\pm$ 0.010)	0.091 ( $\pm$ 0.069)

Another crucial factor affecting the RTEF-EDDiS pipeline’s performance is the percentile value used to select the RTEs. As expected, the lower the percentile value, the more false positives are found. From Fig. 8 we can see that the number of detected RTEs spanned from two times to almost ten times the number of real RTEs going from 95th to 85th percentile values, thus revealing a very large number of false positives. Only the 98th percentile gives RTE counts comparable with the real number of RTEs (20000), but still with an overestimation error of about 20 – 30%.

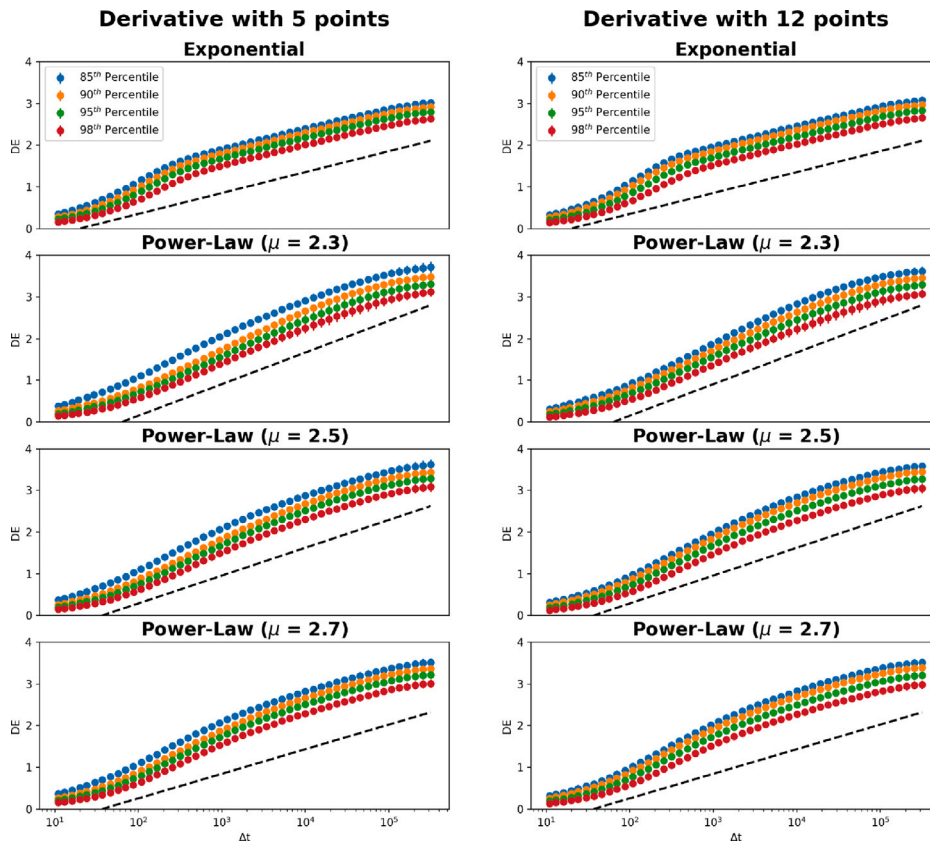


Fig. 7. DE mean curves with error bars for signals generated in frequency band [8, 12] Hz. Mean and error bars are computed over the 50 IET samples. This figure compares the DE curves obtained using  $N_d = 5$  and  $N_d = 12$  symmetrical points to estimate the derivative. The dashed black lines represent the theoretical slope that is:  $\delta = 0.5$  for the Exponential Distribution,  $\delta \simeq 0.77$  for Power-Law distribution with  $\mu = 2.3$ ,  $\delta \simeq 0.67$  for  $\mu = 2.5$ , and  $\delta \sim 0.59$  for  $\mu = 2.7$ .

Table 3

MREs and STDs of DFA and DE computed from detected events in signals driven by Power-Law IET distribution with  $\mu = 2.3$  and  $T = 0.3$  (MREs and STDs computed from the 50 IET samples). All parameter configurations of the RTEF algorithm are reported for comparison. Results were obtained by using  $N_d = 5$ .

Frequency band	Percentile	MRE $H$ ( $\pm$ STD)	MRE $\delta$ ( $\pm$ STD)
[0.5, 4]	85	0.073 ( $\pm$ 0.044)	0.060 ( $\pm$ 0.011)
	90	0.079 ( $\pm$ 0.049)	0.10 ( $\pm$ 0.013)
	95	0.10 ( $\pm$ 0.064)	0.12 ( $\pm$ 0.014)
	98	0.10 ( $\pm$ 0.052)	0.044 ( $\pm$ 0.017)
[4, 8]	85	0.065 ( $\pm$ 0.044)	0.17 ( $\pm$ 0.040)
	90	0.077 ( $\pm$ 0.050)	0.13 ( $\pm$ 0.033)
	95	0.081 ( $\pm$ 0.046)	0.090 ( $\pm$ 0.030)
	98	0.10 ( $\pm$ 0.056)	0.040 ( $\pm$ 0.024)
[8, 12]	85	0.064 ( $\pm$ 0.040)	0.17 ( $\pm$ 0.036)
	90	0.073 ( $\pm$ 0.043)	0.12 ( $\pm$ 0.034)
	95	0.077 ( $\pm$ 0.043)	0.090 ( $\pm$ 0.031)
	98	0.092 ( $\pm$ 0.050)	0.046 ( $\pm$ 0.025)

### 5.2.1. Exponential case

For the exponential case, summarized in Table 2, the accuracy in  $H$  improves as the percentile increases, or remains almost constant for the band [0.5, 4]. This last band is also that giving the worst MREs, which however remain below 4%. At variance with  $H$ , in all bands, the accuracy in the  $\delta$  evaluation decreases as the percentile increases.

**Table 4**

MREs and STDs of DFA and DE computed from detected events in signals driven by Power-Law IET distribution with  $\mu = 2.5$  and  $T = 0.5$  (MREs and STDs computed from the 50 IET samples). All parameter configurations of the RTEF algorithm are reported for comparison. Results were obtained by using  $N_d = 5$ .

Frequency band	Percentile	MRE H ( $\pm$ STD)	MRE $\delta$ ( $\pm$ STD)
[0.5, 4]	85	0.073 ( $\pm$ 0.035)	0.17 ( $\pm$ 0.055)
	90	0.080 ( $\pm$ 0.031)	0.19 ( $\pm$ 0.076)
	95	0.095 ( $\pm$ 0.036)	0.23 ( $\pm$ 0.074)
	98	0.12 ( $\pm$ 0.054)	0.19 ( $\pm$ 0.014)
[4, 8]	85	0.041 ( $\pm$ 0.014)	0.24 ( $\pm$ 0.092)
	90	0.061 ( $\pm$ 0.030)	0.28 ( $\pm$ 0.065)
	95	0.063 ( $\pm$ 0.042)	0.24 ( $\pm$ 0.044)
	98	0.071 ( $\pm$ 0.045)	0.17 ( $\pm$ 0.018)
[8, 12]	85	0.040 ( $\pm$ 0.020)	0.24 ( $\pm$ 0.092)
	90	0.052 ( $\pm$ 0.033)	0.28 ( $\pm$ 0.060)
	95	0.051 ( $\pm$ 0.033)	0.24 ( $\pm$ 0.043)
	98	0.062 ( $\pm$ 0.042)	0.18 ( $\pm$ 0.020)

**Table 5**

MREs and STDs of DFA and DE computed from detected events in signals driven by Power-Law IET distribution with  $\mu = 2.7$  and  $T = 0.7$  (MREs and STDs computed from the 50 IET samples). All parameter configurations of the RTEF algorithm are reported for comparison. Results were obtained by using  $N_d = 5$ .

Frequency band	Percentile	MRE H ( $\pm$ STD)	MRE $\delta$ ( $\pm$ STD)
[0.5, 4]	85	0.020 ( $\pm$ 0.010)	0.14 ( $\pm$ 0.098)
	90	0.023 ( $\pm$ 0.014)	0.13 ( $\pm$ 0.090)
	95	0.018 ( $\pm$ 0.013)	0.18 ( $\pm$ 0.10)
	98	0.051 ( $\pm$ 0.018)	0.25 ( $\pm$ 0.088)
[4, 8]	85	0.071 ( $\pm$ 0.021)	0.16 ( $\pm$ 0.090)
	90	0.029 ( $\pm$ 0.015)	0.23 ( $\pm$ 0.11)
	95	0.028 ( $\pm$ 0.011)	0.23 ( $\pm$ 0.11)
	98	0.055 ( $\pm$ 0.030)	0.24 ( $\pm$ 0.087)
[8, 12]	85	0.066 ( $\pm$ 0.015)	0.17 ( $\pm$ 0.095)
	90	0.024 ( $\pm$ 0.012)	0.23 ( $\pm$ 0.011)
	95	0.030 ( $\pm$ 0.010)	0.23 ( $\pm$ 0.011)
	98	0.049 ( $\pm$ 0.027)	0.23 ( $\pm$ 0.095)

5.2.2. Power-law  $\mu = 2.3$

For the power-law distribution with  $\mu = 2.3$ , we find the counter-intuitive result that the relative errors of  $H$  are generally smaller than the reference errors of Table 1. Further, accuracy improves as percentile decreases, a condition corresponding to a much larger number of false positives in the RTE detection. Additionally, the variation in  $H$  relative errors between frequency bands is minimal. Conversely, the relative errors of  $\delta$  are typically higher than reference errors and generally decrease as the percentile increases. In the band [0.5,4] a non-monotonic behavior is seen, but the best accuracy is still found at the 98th percentile with a 4.4% relative error. Errors in  $\delta$  are slightly larger than those in  $H$ .

5.2.3. Power-law  $\mu = 2.5$

For the power-law distribution with  $\mu = 2.5$ , we find that the relative errors of  $H$  and  $\delta$  are generally higher than the reference errors. Similarly to previous cases, the differences in the relative errors between frequency bands are minimal, even if we found slightly better results for  $H$  in the [4, 8] Hz and [8, 12] Hz bands.

Regarding  $\delta$ , the accuracy is non-monotonic with the percentile. The best results are given by the 98th percentile in the [4, 8] and [8, 12] bands, and by the 85th percentile in the [0.5, 4] band. Interestingly, the errors in  $\delta$  are much higher than those in  $H$ .

5.2.4. Power-law  $\mu = 2.7$

Surprisingly, for the power-law distribution with  $\mu = 2.7$ , the relative errors of  $H$  are consistently lower than reference errors given in Table 1a. Conversely, the  $\delta$  relative errors are generally greater or equal to the corresponding reference errors. Regarding frequency band differences, minimal variations were observed in the relative errors of both  $H$  and  $\delta$  between [4, 8] and [8, 12] bands, while the [0.5, 4] Hz band exhibits slightly better results. Finally, the 90th and 95th percentiles yielded the best results for  $H$  estimation, while the  $\delta$  estimation improves as the percentile decreases, reaching the best performance at the 85th percentile. Similarly to the case  $\mu = 2.5$ , the errors in  $\delta$  are much higher than those in  $H$ .

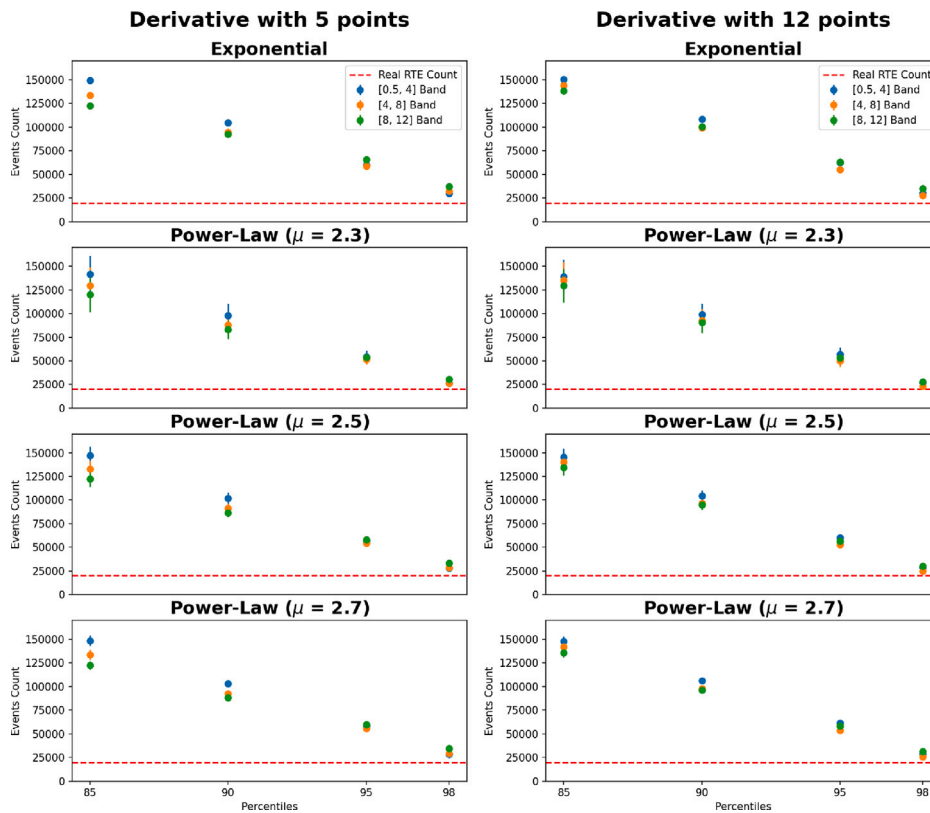


Fig. 8. Mean estimated RTE counts with error bars for each IET distribution and frequency band vs percentile values. Left panels and right panels refer to  $N_d = 5$  and  $N_d = 12$ , respectively. The dashed red lines represent the number of real RTEs that is equal to 20000.

### 6. Concluding remarks

In this work, we evaluated the effect of an event detection algorithm on the estimation accuracy of event-based complexity indices  $\mu$ ,  $H$  and  $\delta$ . In the present study, we were not interested in the efficiency of single-event detection, but in the possibility of getting a good estimation of the genuine temporal complexity. In particular, our main interest lies in the evaluation of signals driven by complex events, that is, events with inverse power-law IET distributions.

In Table 6 we summarize the best and worst relative errors for  $H$  and  $\delta$  compared to the corresponding reference errors, while in Figs. 9 and 10 we give a synthetic picture of the mean estimated values of  $H$  and  $\delta$  with error bars, respectively, compared to their corresponding real values, for all IET distributions, frequency bands and percentiles. From these figures, it is easy to appreciate the better performance of  $H$  estimations with respect to the  $\delta$  estimations, which display larger fluctuations and greater differences with the theoretical values. A general remark is the tendency to underestimate the real  $H$  and  $\delta$  values, even if with some exceptions. This is probably related to the initial short-time transient, which usually displays a slower increase. The automatic best-fitting procedure is able to distinguish the two different regimes, but the short-time slower increase could slightly affect the fit in the long-time regime, where the real scaling exponent emerges. Interestingly, looking at Table 6 it is clear that the estimation errors (MREs) of  $H$  in the power-law cases are comparable to the corresponding reference errors. In the best cases, some MREs are surprisingly smaller than the reference errors. Conversely, in the exponential case, the MRE is larger or much larger than the reference error.

When looking at the RTE counting, the best accuracy in RTE detection seems to come from the higher percentiles, in particular the 98th one (see Fig. 8). Then, we would not expect to find good performances in the evaluation of complexity indices when the number of estimated RTEs is very high concerning the number of real RTEs in the signal. Surprisingly, Tables 3–5 show that, for power-law IET distributions, better results for the diffusion scaling exponent  $H$  are usually obtained for smaller percentiles (85, 90), even if the application of the algorithm results in a very high number of false positives. Conversely, the 98th percentile, which better approximates the RTE counts, gives the worst performances in the  $H$  evaluation.<sup>10</sup> Actually, power-law cases see an improvement in the estimate of  $H$  as the percentile decreases and, therefore, as the number of false positives increases (see, e.g., Tables 3–5),

<sup>10</sup> A slight worsening in accuracy is only seen for  $\mu = 2.7$ , bands [4, 8] and [8, 12], when passing from 90th to 85th percentiles.

**Table 6**

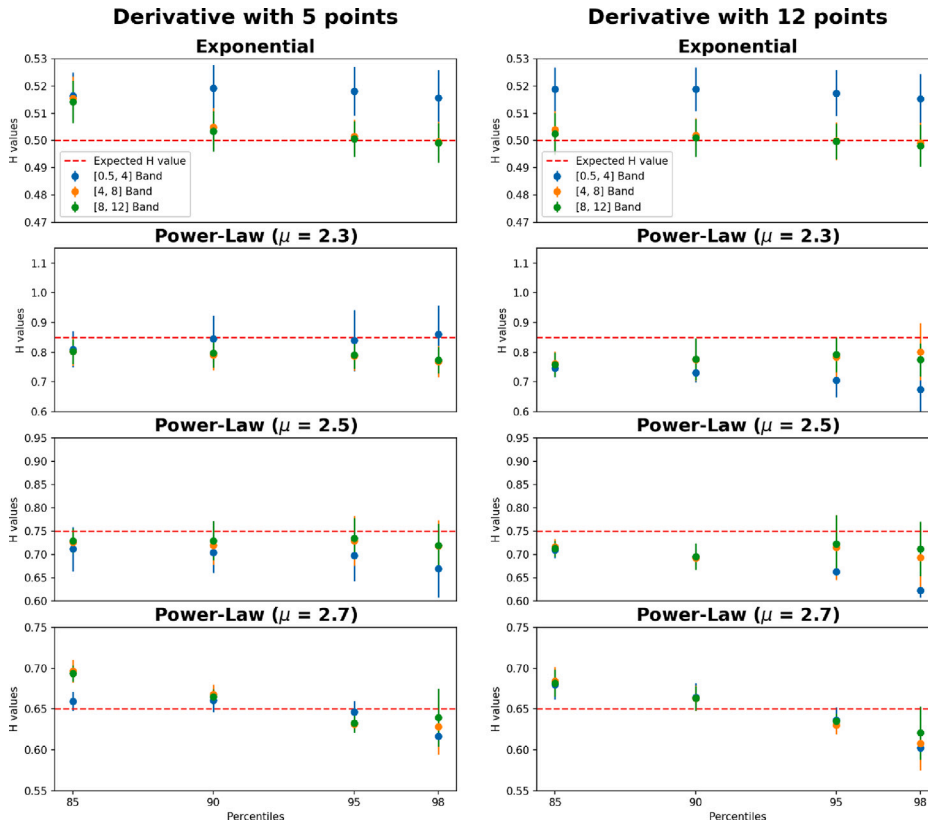
Comparison of MRE for  $H$  and  $\delta$  estimated from RTEs detected with the RTEF algorithm versus corresponding reference errors, i.e., relative errors of  $H$  and  $\delta$  estimated from the real RTEs. (a) Reference Errors in  $H$  versus the best and worst estimated  $H$  from detected RTEs; (b) Reference Errors in  $\delta$  versus the best and worst estimated  $\delta$  from detected RTEs.

Case	MRE $H$ ( $\pm$ STD) Reference Errors	MRE $H$ ( $\pm$ STD) Best Case	MRE $H$ ( $\pm$ STD) Worst Case
Exponential	0.0052 ( $\pm$ 0.0042)	0.010 ( $\pm$ 0.010)	0.038 ( $\pm$ 0.017)
Power-Law ( $\mu = 2.3$ )	0.088 ( $\pm$ 0.040)	0.064 ( $\pm$ 0.040)	0.10 ( $\pm$ 0.064)
Power-Law ( $\mu = 2.5$ )	0.040 ( $\pm$ 0.030)	0.040 ( $\pm$ 0.020)	0.12 ( $\pm$ 0.054)
Power-Law ( $\mu = 2.7$ )	0.070 ( $\pm$ 0.040)	0.018 ( $\pm$ 0.013)	0.071 ( $\pm$ 0.021)

(a)  $H$  relative errors

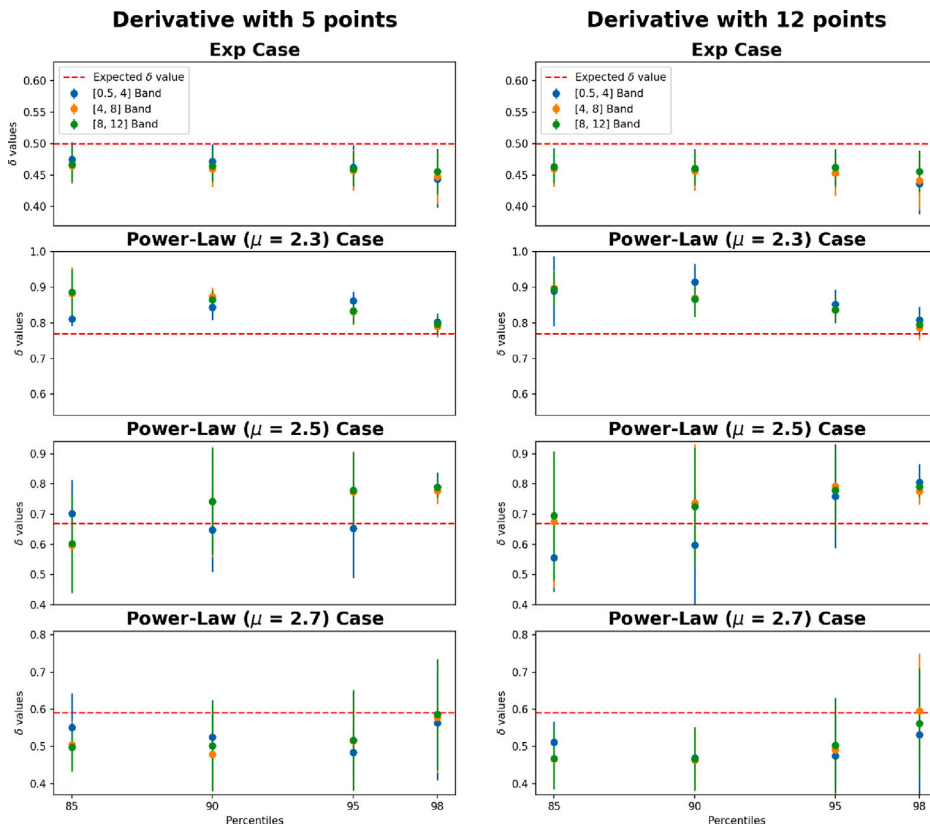
Case	MRE $\delta$ ( $\pm$ STD) Reference Errors	MRE $\delta$ ( $\pm$ STD) Best Case	MRE $\delta$ ( $\pm$ STD) Worst Case
Exponential	0.024 ( $\pm$ 0.011)	0.056 ( $\pm$ 0.050)	0.12 ( $\pm$ 0.084)
Power-Law ( $\mu = 2.3$ )	0.040 ( $\pm$ 0.035)	0.040 ( $\pm$ 0.024)	0.17 ( $\pm$ 0.040)
Power-Law ( $\mu = 2.5$ )	0.062 ( $\pm$ 0.021)	0.17 ( $\pm$ 0.018)	0.28 ( $\pm$ 0.065)
Power-Law ( $\mu = 2.7$ )	0.15 ( $\pm$ 0.030)	0.13 ( $\pm$ 0.090)	0.25 ( $\pm$ 0.088)

(b)  $\delta$  Relative Errors



**Fig. 9.** Mean estimated  $H$  values with error bars for each IET distribution and frequency band vs percentile values. Both  $N_d = 5$  and  $N_d = 12$  cases are reported. The dashed red lines represent the theoretical  $H$  values: 0.5 for the Exponential Distribution, 0.85 for Power-Law distribution with  $\mu = 2.3$ , 0.75 for  $\mu = 2.5$ , and 0.65 for  $\mu = 2.7$ .

while an opposite trend is seen in the exponential case (see Table 2). In the last case, the  $H$  estimation gets worse as the percentile decreases in [4, 8] and [8, 12] bands and remains about the same in the [0.5, 4] band.



**Fig. 10.** Mean estimated  $\delta$  values with error bars for each IET distribution and frequency band vs percentile values. Both  $N_d = 5$  and  $N_d = 12$  cases are reported. The dashed red lines represent the theoretical  $\delta$  values: 0.5 for the Exponential Distribution,  $\sim 0.77$  for Power-Law distribution with  $\mu = 2.3$ ,  $\sim 0.67$  for  $\mu = 2.5$ , and  $\sim 0.59$  for  $\mu = 2.7$ .

The counter-intuitive result, in the case of power-law distributed IETs, of getting better estimates for lower percentile values, i.e., for increased number of false positives, is related to the property of the AJ diffusion process of separating the contribution of false positives, and of other noisy events, from the genuine complex events [18,27,30,40]. This is also in agreement with the observation that the anomalous diffusion scaling determined by complex events essentially comes from the long IET range, where the power-law tail of IET emerges.

Conversely, the trend in the  $\delta$  evaluation is more intricate, with estimates either getting worse as the percentile decreases or remaining about the same in the power-law case. The only exception we found is given by  $\mu = 2.7$ , where the  $\delta$  estimation improves as the percentile decreases. This last result is similar to the exponential case:  $\delta$  evaluation improves as percentile decreases, that is, as the number of false positives increases. Generally speaking, the exponential case shows the best performances in the evaluation of all parameters, i.e., not only  $H$  and  $\delta$ , but also the direct estimation of  $r_p$ . This is an expected result, as the exponential distribution has not a strong tail, so that the rare events have a negligible probability, being most of the IET included in 2–3 times the average IET  $\langle \tau_p \rangle = 1/r_p$ .

On the contrary, the opposite trends of both  $H$  and  $\delta$  estimation accuracy in the exponential case with respect to the inverse power-law distributed IETs are somewhat surprising and are surely related to the very low probability of large IETs of the exponential distribution. Moreover, we observe that being theoretically  $H = \delta = 0.5$  in this case, the relative errors are mostly affected by limitations of the fitting procedure. Further, our results showed that the case  $N_d = 5$  performs slightly better than the case  $N_d = 12$ . This could be related to the more extended cut in frequencies in the case  $N_d = 12$  than for  $N_d = 5$ , thus getting a worst estimation of the derivative. However, the errors for  $N_d = 12$  are only marginally larger than those for  $N_d = 5$ . In many cases, they are essentially the same and, in very few cases, they are even smaller. This proves that the average IET is the crucial reference time scale for the event detection algorithm.

In general, it is important to underline that, in all cases, the mean relative errors in the  $\delta$  estimation are much higher than in the  $H$  estimation. In fact, being based on the computation of distribution, the DE method needs a greater size of the statistical sample with respect to the DFA method, which essentially evaluates the statistical moments.<sup>11</sup> However, as proven by the authors of

<sup>11</sup> In this work the second moment, while, in the more general multifractal DFA algorithm [70], the moments of generic order  $q$ , including non-integer orders.

Ref. [71], the scaling  $\delta$  is more affected by the regions around distribution maxima (the central region for single-mode distributions) and the scaling  $H$  is more affected by the distribution tails. As a consequence,  $\delta$  scaling gives different information with respect to scaling  $H$  and are both needed in temporal complexity analyses, but the different level of accuracy at the same statistical sample size has to be taken into account.

In a nutshell, the main finding of the present work is that the EDDiS algorithm, which is based on the concept of event-driven diffusion scaling, is able to decrease the masking effect of false positives and, consequently, the error in the estimation of temporal complexity [17,18]. This is in agreement with the Time Mixed Model (TMM), where a mixing of complex events and Poisson noisy events is investigated [27,30]. However, these studies were limited to the case of a low rate of noisy events with respect to that of complex events. Here, the noisy events are given by the numerous false positives in the event-detection algorithm and, in some case, the rate of noisy events is much larger than that of the genuine complex events. This shortcoming of the event detection algorithm was here found not to compromise the possibility for the EDDiS algorithm of obtaining a good estimate of complexity, particularly of the scale of the second  $H$  moment, in the power-law case. In fact, regarding the application of DFA, the main, but counter-intuitive, result is a net improvement in the performances of complexity estimation, in the power-law case, as the number of false positives in the RTEF algorithm increases. Conversely, the trend is the opposite for the exponential case, as it would be expected for all cases.

This work has been focused on the ergodic range  $\mu \in (2, 3]$ , which allows to get a well-defined time averaging. This is also the range of temporal complexity index that is most found in applications. However, the non-ergodic range  $\mu \in (1, 2]$  is also an interesting one [36,69] that introduces some subtle aspects regarding the infinite mean IET and, thus, the comparison of different statistical samples. In fact, the random generator has a non-negligible probability of giving at least one IET as long as the entire simulation time, whatever is the length of the simulation time itself. A possible approach could be to consider power-law IET distribution with an exponential cut-off in a very long IET range, thus getting a finite average. This aspect is of some interest and surely deserves future investigations.

### CRedit authorship contribution statement

**Marco Cafiso:** Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Methodology, Investigation, Data curation, Conceptualization. **Paolo Paradisi:** Writing – review & editing, Writing – original draft, Supervision, Methodology, Funding acquisition, Conceptualization, Project administration, Resources, Validation. Marco Cafiso and Paolo Paradisi are both main authors of this paper.

### Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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### Appendix A. Supplementary data

Supplementary material related to this article can be found online at <https://doi.org/10.1016/j.chaos.2026.118264>.

### Data availability

No data was used for the research described in the article.

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