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A Minimal Operational Criterion for No-Signaling Assessment and Near-Identity Binary Transmission

Lorenzo Albanese

Institute of Bioeconomy, National Research Council of Italy, Via Madonna del Piano 10, 50019 Florence, Italy; lorenzo.albanese@cnr.it

Abstract

Possible violations of the no-signaling constraint in Weinberg-type nonlinear extensions motivate the question of whether entanglement could, under suitable conditions, become a resource for operational signaling. A minimal binary model is introduced in which, in each run, a sender selects a binary input bit and a receiver locally records a binary output bit. Signaling is defined operationally as a dependence of the local output statistics on the remote input and is summarized by a single channel parameter that can be estimated from data. An estimator and a corrected confidence interval are then introduced to assess this dependence quantitatively, while transmission reliability is expressed through the minimum decision error. On this basis, a conservative criterion is formulated, using an upper bound on the error and a threshold fixed in advance, to characterize a near-identity channel regime. Minimal reporting requirements are also proposed to document the conditions under which artifacts and classical leakage may reasonably be excluded.

Keywords: no-signaling in quantum mechanics; operational signaling; binary communication channel; minimum decision error; conservative operational criterion

1. Introduction

In standard quantum mechanics, entangled correlations do not provide a controllable communication channel between spatially separated regions [1–3]. Although distant measurement outcomes may be correlated, the local statistics accessible to one observer do not depend on measurement choices or control actions performed at a remote location [4,5]. This is the no-signaling constraint. The distinction between nonlocal correlation and controllable communication is essential, because the former does not by itself imply the latter.

The issue becomes especially relevant when one considers possible extensions of quantum mechanics. In particular, Steven Weinberg has proposed a nonlinear generalization of quantum dynamics and discussed possible experimental tests of departures from the standard theory [6,7]. It was subsequently pointed out that nonlinear dynamics of this kind may enable superluminal communication through entangled states [8,9].

Although Weinberg-type nonlinear extensions are generally regarded as physically problematic because of their tension with locality and their possible implication of superluminal communication, those extensions remain helpful as a conceptual benchmark for formulating operational tests of no-signaling. In such nonlinear settings, a possible violation of the no-signaling constraint can arise because the local evolution can depend on features of the global state or of its preparation in a way that is not captured by the ordinary linear reduced-state description. A remote choice that changes the ensemble

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realization while leaving the reduced density matrix unchanged in standard quantum mechanics can then, in general, affect the local output statistics after nonlinear evolution. In a spacelike configuration, such an effect has the operational meaning of a faster-than-light communication channel.

Related discussions have examined the relation between faster-than-light communication and the distinguishability of ensembles associated with the same density matrix or compressed density matrix [10,11]. These issues are used here only as background motivation. The present study does not defend Weinberg-type nonlinear models as realistic alternatives to standard quantum mechanics. Rather, the study is methodological: if an apparent dependence of local output statistics on a remote controlled input is reported, one has to have a transparent procedure for quantifying that dependence, estimating its uncertainty, and distinguishing a mere nonzero effect from a reliable binary transmission regime.

More recent studies have revisited this question from both theoretical and experimental perspectives. A causal framework for nonlinear quantum mechanics has been developed in Ref. [12], while experimental bounds and direct tests of nonlinear state-dependent effects have been reported in Refs. [13,14]. More generally, the operational and structural meaning of no-signaling continues to be investigated in the current papers on nonsignaling channels and on its relation to microcausality [15–17].

The present study is more limited in scope. The present study does not propose a dynamical modification of quantum theory, nor does it provide a general treatment of the no-signaling constraint. Rather, it formulates, in a minimal binary setting, a quantitative criterion that relates an observable input–output dependence to an inferentially controlled assessment of transmission quality. In this respect, the focus here is not on the underlying physical origin of a possible deviation, but on the conditions under which an observed dependence may be described and reported, in a quantitatively explicit way, as compatible with a near-identity binary channel in the restricted sense adopted.

These developments motivate the need for criteria that are explicitly operational and directly applicable to experimental data. In the current study, a minimal binary model is considered. In each experimental run, a sender S selects a binary input $W \in \{0,1\}$, and a receiver R locally records a binary output $B \in \{0,1\}$. In what follows, $P(\cdot)$ denotes a probability, and $P(X = x \mid Y = y)$ denotes the conditional probability that the random variable X takes the value x given that Y takes the value y.

Operational signaling is present if the local distribution of the output depends on the remote choice of the input; in particular, this occurs when

$$P(B = 1 \mid W = 0) \neq P(B = 1 \mid W = 1). \tag{1}$$

The channel strength is summarized by a single parameter σ_{SR}:

$$\sigma_{SR} = P(B = 1 \mid W = 1) - P(B = 1 \mid W = 0), \tag{2}$$

which is zero in the absence of any observable dependence of the output on the input and nonzero when such a dependence is present.

The present study is concerned not only with detecting operational signaling, but also with characterizing a regime in which the output provides a reliable estimate of the input. A mere nonzero dependence is not sufficient for this stronger claim, since the induced channel may still be insufficiently strong to support meaningful transmission. To capture this point, the transmission error probability ε is defined as the minimum error obtained by choosing the more favorable decoding between the two elementary binary maps:

$$\varepsilon := 1 - \max\{P(B = W),\ \ P(B \neq W)\}. \tag{3}$$

By construction, $0 \leq \varepsilon \leq 1/2$. The near-identity channel condition is formalized as

$$B \approx W \Leftrightarrow \varepsilon < \varepsilon^*, \tag{4}$$

where ε^* is an experimental threshold fixed a priori. This formulation has two advantages. First, it keeps the discussion strictly operational, because all relevant quantities are defined in terms of observable input–output statistics. Second, the formulation separates two logically distinct claims: the detection of signaling and the stronger claim that the observed dependence is sufficient to support reliable binary transmission. In what follows, the minimal model assumptions are specified, an estimator for σ_{SR} is constructed, and a conservative criterion is introduced to assess whether $B \approx W$ can be supported in the restricted operational sense adopted here.

The resulting contribution is therefore intended as a minimal operational framework, rather than as a general claim about the underlying physical mechanism.

2. Definitions and Model Assumptions

Any possible violation of the no-signaling constraint is treated here exclusively in operational terms, that is, in terms of observable input–output statistics. In the present model, the input W identifies the choice between two controlled procedures applied by the sender to the same upstream prepared state. The receiver records a local binary output B. The study does not rely on a specific microscopic mechanism for the dependence of B on W. It only asks whether the conditional output distribution changes when the input is changed under experimental control. In this sense, the problem is formulated directly at the level of an effective binary channel associated with the pair (W, B) [15].

Under this viewpoint, operational signaling is present when the conditional distributions $P(B \mid W = 0)$ and $P(B \mid W = 1)$ differ. The presence of signaling is therefore defined by condition (1). This formulation is intentionally minimal. It does not assume a particular dynamical model, nor does it require any commitment about the origin of the observed dependence. The formulation only identifies the empirical signature that must be present if the output is to carry information about the remote input.

The model adopted here is deliberately restricted to the simplest binary setting in which the distinction between observable dependence and reliable binary transmission can be formulated explicitly. Its purpose is not to cover, without further extension, more general scenarios involving multivalued inputs or outputs, continuous variables, memory effects, or more elaborate decoding strategies. Rather, the point is to isolate a minimal operational framework in which the presence of an input-dependent effect and the stronger claim of near-identity transmission can be stated separately and assessed on a common quantitative basis.

The channel strength is quantified by the parameter σ_{SR} introduced in Equation (2). By definition, $\sigma_{SR} \in [-1, 1]$. Its sign indicates which value of W increases the probability of observing $B = 1$, whereas the magnitude measures how strongly the local output distribution changes when the remote input is switched. In the binary setting considered here, the dependence of B on W is fully specified by the two conditional probabilities

$$P(B = 1 \mid W = 0) := p_0 \quad \text{and} \quad P(B = 1 \mid W = 1) := p_1. \tag{5}$$

Accordingly, $\sigma_{SR} = p_1 - p_0$, consistently with Equation (2). This representation makes explicit that the model is completely determined by the two conditional output probabilities, without the need for additional parameters.

A relabeling of the output, $B \longmapsto 1 - B$, changes the sign of σ_{SR} but does not change the channel reliability. This observation is relevant because the identification of which output value is called "1" is conventional. What matters for transmission quality is not the sign of σ_{SR}, but the extent to which the output supports a reliable inference about the input. For this reason, the relevant quality measure must depend on $|\sigma_{SR}|$, not on its sign alone.

To connect σ_{SR} with the near-identity channel condition introduced in Equation (4), balanced inputs are assumed, namely $P(W = 0) = P(W = 1) = 1/2$.

Two elementary binary decoding conventions are then considered: $\hat{W} = B$ and $\hat{W} = 1 - B$. In the first case, the output is directly interpreted as an estimate of the input. The corresponding average decoding error is

$$P(\hat{W} \neq W) = \frac{1 - \sigma_{SR}}{2}. \tag{6}$$

In the second case, the opposite convention is adopted by interpreting the complemented output as an estimate of the input; the corresponding average decoding error $P(\hat{W} \neq W)$ is

$$P(\hat{W} \neq W) = \frac{1 + \sigma_{SR}}{2}. \tag{7}$$

The decoding rules (6) and (7) exhaust the elementary binary possibilities. The first is favorable when $\sigma_{SR} > 0$, whereas the second is favorable when $\sigma_{SR} < 0$. This is precisely why the sign of σ_{SR} is not itself the relevant figure of merit for transmission quality. Once the better decoding convention is selected, the appropriate quantity is the minimum achievable error.

By selecting the more favorable decoding, the minimum transmission error is therefore

$$\varepsilon = \frac{1 - |\sigma_{SR}|}{2}. \tag{8}$$

The relation (8) provides the key link between operational signaling strength and transmission quality. In particular, $|\sigma_{SR}| = 0$ corresponds to $\varepsilon = 1/2$, that is, to the absence of helpful information about the input beyond random guessing. At the opposite extreme, $|\sigma_{SR}| = 1$ corresponds to $\varepsilon = 0$, that is, to perfect binary transmission under the optimal decoding rule.

Under the present assumptions, the condition $B \approx W$ is therefore reduced to the requirement $\varepsilon < \varepsilon^*$. The present formulation should be understood as a minimal operational benchmark for the elementary binary case considered here, rather than as an attempt to provide a general characterization of signaling in more complex settings. The purpose is not to address multivalued or continuous-input scenarios, but to isolate the simplest setting in which the distinction between detectable signaling and reliable binary transmission may be formulated quantitatively and assessed directly from experimental data.

3. Estimation of σ_{SR} and a Quantitative Criterion for $B \approx W$

Starting from the observed counts under the two input conditions, an estimate of the channel parameter σ_{SR} is obtained and a confidence interval is constructed to quantify statistical uncertainty. The resulting interval is then used to define a conservative upper bound on the transmission error and to state the near-identity criterion $B \approx W$. This construction separates evidence of input–output dependence from the firmer claim that such a dependence is sufficient to support reliable binary transmission.

3.1. Finite-Sample Inference for σ_{SR}

Consider N runs described by the sequence $\{(W_k, B_k)\}_{k=1}^{N}$. The input W is assumed to be controlled by the sender and chosen independently across runs, ideally with equal probabilities, while the output B is recorded locally by the receiver. The following construction moves from the operational definitions introduced above to a finite-sample procedure that can be applied directly to experimental data. The data are first separated according to the value of the input. Let

$$n_1 = \#\{k : W_k = 1\} \text{ and } n_0 = \#\{k : W_k = 0\}, \tag{9}$$

where $\#\{\bullet\}$ denotes set cardinality. These quantities represent the numbers of runs performed under the two input conditions. The corresponding counts of outputs equal to 1 are

$$k_1 = \#\{k : W_k = 1,\ B_k = 1\} \text{ and } k_0 = \#\{k : W_k = 0,\ B_k = 1\}. \tag{10}$$

The natural estimates of the conditional probabilities are the sample proportions $\hat{p}_1$ and $\hat{p}_0$:

$$\hat{p}_1 = \frac{k_1}{n_1} \text{ and } \hat{p}_0 = \frac{k_0}{n_0}, \tag{11}$$

which estimate $P(B = 1 \mid W = 1)$ and $P(B = 1 \mid W = 0)$, respectively. The direct estimator of the channel parameter is therefore

$$\hat{\sigma}_{SR} = \hat{p}_1 - \hat{p}_0. \tag{12}$$

This estimator is the empirical counterpart of Equation (2). It provides a direct measure of the observed dependence of the local output distribution on the remote input. A positive value indicates that the event $B = 1$ is more likely when $W = 1$, whereas a negative value indicates the opposite. However, for the purposes of experimental assessment, a point estimate alone is not sufficient. A quantitative statement about signaling strength or channel quality requires an uncertainty estimate and a confidence-based criterion.

For inference on σ_{SR}, a confidence interval more robust than the standard Gaussian approximation is adopted through the Agresti–Caffo correction [18]. This choice is especially practical for finite samples, where the naive Gaussian interval for the difference in proportions may be unstable or poorly calibrated. The corrected counts and sample sizes are defined as

$$\widetilde{k}_1 = k_1 + 1,\quad \widetilde{n}_1 = n_1 + 2,\quad \widetilde{k}_0 = k_0 + 1 \text{ and } \widetilde{n}_0 = n_0 + 2, \tag{13}$$

respectively, and the corresponding corrected proportions $\widetilde{p}_1$ and $\widetilde{p}_0$ are

$$\widetilde{p}_1 = \frac{\widetilde{k}_1}{\widetilde{n}_1} \quad\text{and}\quad \widetilde{p}_0 = \frac{\widetilde{k}_0}{\widetilde{n}_0}. \tag{14}$$

The corrected estimate $\widetilde{\sigma}_{SR}$ and its standard error $\mathrm{SE}\left(\widetilde{\sigma}_{SR}\right)$ are then defined by

$$\widetilde{\sigma}_{SR} = \widetilde{p}_1 - \widetilde{p}_0 \text{ and } \mathrm{SE}\left(\widetilde{\sigma}_{SR}\right) = \sqrt{\frac{\widetilde{p}_1\left(1-\widetilde{p}_1\right)}{\widetilde{n}_1} + \frac{\widetilde{p}_0\left(1-\widetilde{p}_0\right)}{\widetilde{n}_0}}. \tag{15}$$

Fixing a level $\alpha \in (0,1)$, a confidence interval $[L,\ U]$ at the level $1-\alpha$ is given by

$$[L,\ U] = \widetilde{\sigma}_{SR} \pm z_{1-\frac{\alpha}{2}} \mathrm{SE}\left(\widetilde{\sigma}_{SR}\right), \tag{16}$$

where $z_{1-\alpha/2}$ is the corresponding standard normal quantile.

The interval (16) serves two related purposes. First, it allows one to assess whether the data are compatible with the absence of signaling, namely with $\sigma_{SR} = 0$. Second, the interval provides the basis for a more demanding statement, namely whether the observed dependence is strong enough to support the near-identity condition $B \approx W$. The latter requires a conservative bound on the transmission error ε, rather than only a test against zero.

To assess $B \approx W$ conservatively, an upper bound ε_{UB} on ε is constructed. From the interval $[L,\ U]$, a lower bound $|\sigma_{SR}|_{LB}$ on $|\sigma_{SR}|$ is obtained by defining

$$|\sigma_{SR}|_{LB} = \begin{cases} 0 & \text{if } L \leq 0 \leq U; \\ \min\{|L|,\ |U|\} & \text{if } LU > 0. \end{cases} \tag{17}$$

The definition (17) has the following interpretation. If the confidence interval crosses zero, then the data do not support a strictly positive lower bound on $|\sigma_{SR}|$, and the conservative choice is therefore 0. If the interval lies entirely on one side of zero, then the smallest magnitude compatible with the interval is the smaller of $|L|$ and $|U|$. In this way, $|\sigma_{SR}|_{LB}$ captures the minimum signaling strength supported by the data at the chosen confidence level.

Using Equation (8), the corresponding coherent upper bound ε_{UB} is

$$\varepsilon_{UB} = \frac{1 - |\sigma_{SR}|_{LB}}{2}\,. \tag{18}$$

The condition $B \approx W$ is taken to hold whenever

$$\varepsilon_{UB} < \varepsilon^*. \tag{19}$$

The criterion (19) is intentionally stronger than the mere observation of a nonzero point estimate. A dataset may suggest a nonzero value of $\widetilde{\sigma}_{SR}$ and yet fail to support a sufficiently small upper bound on ε. The proposed procedure therefore separates two distinct claims: evidence for a dependence between input and output, and evidence that such a dependence is large enough to support the interpretation of the induced channel as near-identity.

Within the adopted inferential construction, ε_{UB} is intended to provide a conservative upper bound on ε associated with the chosen confidence level $1 - \alpha$. The final claim is therefore reduced to a finite-sample and directly testable statement: estimate $\widetilde{\sigma}_{SR}$, construct the interval $[L,\ U]$, derive $|\sigma_{SR}|_{LB}$, compute ε_{UB}, and compare it with the threshold ε^* fixed in advance.

3.2. Minimal Operational Criterion for No-Signaling Assessment

The procedure described in Section 3.1 defines a minimal operational criterion in the binary setting considered here. At the first level, no-signaling is assessed by asking whether the confidence interval for σ_{SR} is compatible with zero. At the second level, the stronger near-identity claim $B \approx W$ is evaluated by using the same interval to obtain a conservative upper bound on the minimum transmission error.

Accordingly, a nonzero point estimate of σ_{SR} is not sufficient, by itself, to support the near-identity claim. The condition $B \approx W$ is supported only when the confidence-based upper bound ε_{UB} lies below the a priori threshold ε^*. This separates the detection of an input–output dependence from the stronger claim of reliable binary transmission.

4. Minimal Experimental Requirements and Reporting Criteria

The experimental assessment concerns two distinct aspects. The first is the reliable estimation of the channel parameter σ_{SR}, based on the corrected point estimate $\widetilde{\sigma}_{SR}$ and on the confidence interval $[L,\ U]$. The second is the exclusion of artifacts that may generate a spurious dependence between B and W. This distinction is essential because a nonzero estimate of σ_{SR} is not, by itself, sufficient to support the interpretation of the observed dependence as genuine operational signaling.

Within the operational model adopted here, transmission quality is summarized by the minimum error ε through Equation (8), and the target condition $B \approx W$ is translated into the conservative criterion given by Equation (19). Accordingly, the experimental claim is not exhausted by the observation of a nonzero point estimate, but requires both a quantitative bound on the transmission error and an explicit control of possible experimental biases.

At a minimum, reporting must include the following elements:

- the corrected point estimate $\widetilde{\sigma}_{SR}$ defined in Equation (15);
- the confidence interval $[L,\ U]$ defined in Equation (16) for the chosen confidence level $1-\alpha$;
- the lower bound $|\sigma_{SR}|_{LB}$ defined in Equation (17);
- the upper bound ε_{UB} defined in Equation (18);
- the values of α and ε^*, fixed a priori.

Under these conditions, the statement $B \approx W$ may be supported when $\varepsilon_{UB} < \varepsilon^*$, as expressed in Equation (19).

To rule out the possibility that a nonzero estimate of $\widetilde{\sigma}_{SR}$ arises from artifacts rather than from a genuine dependence of the output on the input, a minimal set of operational constraints must also be satisfied and documented. This requirement is consistent with the fact that operational channel claims must be supported not only by statistical evidence, but also by an explicit control of alternative local explanations [13–15].

First, the generation of the input W must be independent across runs. In addition, the dataset needs to be approximately balanced, with comparable values of n_0 and n_1. A strong imbalance does not invalidate the model in itself, but it reduces the transparency of the binary communication task and can weaken the robustness of the finite-sample analysis.

Second, the receiver's acquisition chain has to be invariant with respect to W.

Triggering, gating, acquisition windows, and event-validation criteria need to be defined independently of the input value. Acceptance and rejection rates conditioned on W must also be reported, and the absence of a systematic dependence has to be explicitly verified. Otherwise, an apparent input–output dependence can be generated at the level of data selection rather than at the level of the underlying physical process.

Third, classical pathways that may carry information about W from the sender to the receiver need to be excluded, as far as reasonably possible. These include direct wiring, synchronization signals or shared clocks, electromagnetic side channels, and software-level dependencies. Any such mechanism can produce $\sigma_{SR} \neq 0$ without supporting the channel interpretation adopted in the present model. In this respect, the operational significance of the claim depends crucially on excluding, and documenting the exclusion of, ordinary local leakage channels as far as reasonably possible [13,14].

Finally, an internal permutation control must be performed. If the input labels W are randomly permuted and the same analysis pipeline is repeated, the resulting estimate must be compatible with $\sigma_{SR} = 0$ within the corresponding uncertainty. This provides quite a simple internal consistency check against analysis-induced structure and helps distinguish a genuine input-dependent effect from a spurious pattern produced by the processing pipeline.

With these reporting elements and control conditions, the final claim may be stated in a quantitatively explicit and, actually, verifiable form: $B \approx W$ in the restricted sense that $\varepsilon_{UB} < \varepsilon^*$. The proposed criterion may therefore be understood not only as a statistical test of dependence, but also as an operational basis for assessing when an observed binary input–

output relation may be regarded as compatible with near-identity within the assumptions of the model.

Once these minimum control and reporting conditions are satisfied, it remains significant to clarify the scope of the proposed criterion, its methodological limitations, and the proper interpretation of the conclusions that may be drawn from the criterion.

5. Scope, Limitations, and Proper Use of the Criterion

The criterion introduced in Sections 3 and 4 provides a quantitative and conservative procedure for assessing, within a minimal binary setting, whether the observed data may be regarded as compatible with the condition $B \approx W$. Precisely because this procedure is formulated in operational and inferential terms, it is appropriate to state explicitly its domain of applicability, its methodological limitations, and the proper interpretation of the conclusions that it is intended to support.

5.1. Scope of the Criterion

The proposed criterion is strictly operational in character and is formulated for a minimal setting in which a sender selects a binary input and a receiver locally records a binary output. Within this framework, any dependence of the local output statistics on the remote input choice is summarized by the parameter σ_{SR}, while the transmission quality of the channel is expressed, in the case of balanced inputs, through the minimum decision error ε. The contribution of the present study lies in defining a quantitative procedure, formulated entirely in terms of observable quantities, that relates estimation of the channel parameter to a conservative criterion intended to support, in operational terms, the condition $B \approx W$.

In this sense, the criterion is not proposed as a general theory of signaling, nor as an exhaustive characterization of every possible violation of the no-signaling constraint. Rather, the criterion is intended in a narrower sense as a minimal methodological tool for quantifying, testing, and reporting, in a verifiable manner, a possible transmissive dependence in an elementary binary case. Its function is therefore to provide a transparent and reproducible operational rule, without presupposing any specific dynamical account of the underlying physical mechanism, should such a mechanism be at issue.

More precisely, the criterion is constructed for cases in which the aim is not merely to detect a difference between conditional distributions, but also to assess whether such a difference is compatible with a regime in which the output may serve as a sufficiently reliable estimate of the input, relative to an error threshold fixed in advance. Within this scope, the proposed formalism makes it possible to relate, in a transparent way, the notion of observable dependence, its inferential estimation, and a quantitative near-identity channel condition formulated in operational terms.

5.2. Relation to Bell-Type Correlations

It is practical to distinguish the present criterion from the assessment of Bell-type correlations. In a standard Bell scenario, the joint probabilities may violate a Bell inequality, showing that the observed correlations cannot be explained by a local hidden-variable model [1]. However, this does not imply operational signaling. The no-signaling condition requires the local marginal distribution observed by one party to be independent of the measurement choice or controlled input selected by the distant party [2–5].

In the notation of the present model, Bell-type correlations may therefore coexist with $P(B = 1 \mid W = 0) = P(B = 1 \mid W = 1)$, and hence with $\sigma_{SR} = 0$. The proposed criterion is sensitive only to changes in the receiver's local output statistics associated with changes in the sender's controlled input. The criterion is not intended as a test of Bell nonlocality, nor does it interpret nonlocal correlations alone as a communication channel.

5.3. Limitations of the Criterion

The proposed criterion is subject to several limitations that have to be kept in mind in interpreting its scope and possible use. In its present formulation, the criterion is restricted to an elementary binary setting and is not meant to cover, without further extension, scenarios involving multivalued inputs or outputs, more structured forms of dependence, memory effects, or more general decoding tasks. Its use is therefore confined to a minimal operational case in which the relation between observable dependence and transmission quality can be stated in a relatively transparent form.

In addition, the relatively simple relation between $|\sigma_{SR}|$ and the minimum decision error ε is derived here under the assumption of balanced inputs.

This assumption is appropriate for the specific decision problem considered in the present study, but the resulting formulation should not be taken to extend automatically to cases with non-uniform input priors. In such cases, the corresponding error criterion generally requires a separate treatment. For this reason, no claim is made here beyond the balanced-input setting explicitly adopted.

The criterion is also inferential in character. Accordingly, the proposed decision rule is expressed in conservative terms through an upper bound on the decision error rather than through a point estimate alone. Even under this formulation, however, the strength of any conclusion remains dependent on sample size, data quality, the adequacy of the underlying statistical assumptions, and the correct implementation of the inferential procedure.

Finally, the criterion is not, by itself, sufficient to distinguish between a genuine physical effect and an artefactual source of apparent dependence. Its use therefore remains conditional on the exclusion, as far as reasonably possible, of ordinary sources of spurious correlation, including classical leakage, acquisition-chain asymmetries, and other procedural dependencies of the kind discussed in the preceding section. Any interpretation of the result must therefore remain limited by the adopted controls, assumptions, and reporting conditions.

5.4. Proper Interpretation of the Criterion

The criterion proposed here is intended to support an operational assessment within the specific framework in which it is defined. Accordingly, the outcome should be interpreted with considerable care and only within the limits of the adopted assumptions, controls, and inferential formulation.

If the conservative condition based on the upper bound on the decision error is satisfied, this may be taken to indicate that, under the present operational definition, the observed data are compatible with a regime in which the output provides a sufficiently reliable estimate of the input relative to the threshold fixed in advance. Such a result may therefore support the use of the expression $B \approx W$ in the restricted quantitative sense adopted here.

At the same time, such a finding should not be interpreted more broadly than the present framework warrants. In particular, the finding does not by itself establish a general conclusion regarding the physical origin of the observed dependence, nor does it amount, in isolation, to a general statement about signaling beyond the operational setting explicitly considered here.

Conversely, failure to satisfy the proposed condition must not be read as excluding every possible form of dependence between input and output. The failure indicates only that, under the present criterion and at the chosen confidence level, the available data do not support the near-identity channel condition in the conservative sense adopted here.

6. Conclusions

A quantitative and conservative operational criterion has been proposed for assessing, in a minimal binary setting, whether an observed input–output relation may be regarded as compatible with the condition $B \approx W$.

The framework is formulated entirely in terms of observable quantities. Operational signaling is summarized by the channel parameter σ_{SR}, estimated from data through the corrected estimator $\widetilde{\sigma}_{SR}$, while transmission quality is expressed, in the balanced-input case considered here, through the minimum decision error ε, defined independently of the output-labeling convention. On this basis, the condition $B \approx W$ is quantified by fixing a threshold ε^* in advance and by adopting a conservative decision rule based on the upper bound ε_{UB}.

Accordingly, the proposed procedure reduces the experimental claim to a directly testable form: estimate $\widetilde{\sigma}_{SR}$, construct a confidence interval for σ_{SR}, derive the corresponding upper bound ε_{UB} on the decision error, and assess whether $\varepsilon_{UB} < \varepsilon^*$. If this condition is satisfied, the available data may be taken to support, within the present framework, the interpretation of the induced binary channel as near-identity in the restricted quantitative sense adopted here.

The scope of this conclusion remains limited by the assumptions stated above. In particular, the present formulation is restricted to an elementary binary setting, relies on the balanced-input case for the comparably simple relation between $|\sigma_{SR}|$ and ε, and does not by itself distinguish between a genuine physical effect and an artefactual source of apparent dependence. Within these limits, the proposed framework can provide an operationally transparent basis for the quantitative assessment and reporting of possible transmissive dependence in elementary binary scenarios.

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