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ABSTRACT

An interactive method of image feature extraction based on B-Spline curves is proposed. B-Splines are piecewise polynomial curves that are guided by a sequence of points called the control points.

The proposed method differs from traditional one in that the features of an image are extracted automatically according to a limited sequence of control points inserted by user in an interactive way, and the extracted image features are described as a series of B-Spline curves. Consequentially, the analysis of image features becomes easy and effective because of the specific properties of the B-Spline curves, and at the same time a significant data compression is achieved. An image feature extraction software package based on the proposed method has been realized in C language.

1. Introduction

The extraction, representation, and analysis of image features are important research areas in computer vision and image processing. An image feature is a distinguishing primitive characteristic of an image field, such as the brightness of a region of pixels, the edge outlines of an object, and the gray scale texture regions; for example, changes or discontinuities of luminance values of pixels are fundamentally important primitive features of an image, as they often provide an indication of the physical extent of objects within the image.

Traditionally, the image features are extracted by the Edge Enhancement Detection Method according to the discontinuity of the image luminance values. First, edge point detectors which identify potential edge points according to a given threshold are used and an edge map identifying the location of the points is obtained. In order to use this information in higher level processing, for example image features analysis, it is necessary to identify those points which should be grouped together into edge segments, for the purpose of obtaining an image feature representation. Finally, the image feature analysis is achieved based on the representation of the image features. It is thus clear that the effectiveness of the early levels of processing which identify tokens to be used at higher levels of processing is crucial in determining how successful this higher level processing will be.

Different methods of image edge detection and image edge representation have been proposed [1-8]. But in practice, the characteristics of an image in the real world are very complicated, and the real features of an image[1] in the real world are degraded due to noise, low contrast and occlusions, and in fact sometimes we are only interested in some meaningful features rather than all. So there are a few weakness in the traditional method: (1) the threshold is difficult to be determined; (2) because of the existence of noise and background some pseudo features arise and some real features cannot be extracted; (3) the features of an image could not be extracted from disconnected edge points.

In this paper, different from traditional method, an efficient interactive method of image feature extraction based on B-Spline curves is proposed and achieved in terms of the characteristics of the real image feature. B-Splines are piecewise polynomial curves that are guided by a sequence of points called control points. B-Spline curve is attractive because it exhibits local control (Local control property) and can be used in the presence of incomplete boundary information (Curve fitting and fairing properties). Boundary representation and matching

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[1] In this paper, different from general concept of the image feature, the term *the real features of an image* means some meaningful features that we are interested in rather than all.

using B-Spline curves have also been known in the references [9-13].] the reference [14], the B-Spline curves have been used to extract ar represent local features of the boundary of an object, but edge detectic and edge segmentation must be done in order to obtain a sequence (control points, and how to get this sequence of control points from a s(of boundary points is not clear also. However, here an efficie1 interactive method and a new idea of image feature extraction a1 proposed. The interactive method is very important, because the huma visual system has the impressive ability to quickly extract meaningfı features from degraded images; the information which is given simpl and efficiently by user in an interactive way is very helpful in the imag feature extraction processing.

The proposed method may be summarized as follows:

(1) first, for each feature a sequence of control points {p'} i inserted by user in an interactive way according to the complexity of th image feature, and a control curve C is then created automatically b using the B-Spline curves. The control curve C is an approximation c the image feature curve F to be extracted;

(2) then another sequence of control points {p} is obtaine automatically according to the relative position and the relationshi between the control curve C and the control points {p'};

(3) finally, the image feature curve F is obtained by using agai the B-Spline curves according to the new sequence of control point {p}.

Repeating the steps described above, all the real features of a1 image can be extracted and represented by a set of B-Spline curves Consequentially, the analysis of the extracted image features become easy and efficient because of the specific properties of B-Spline curves and at the same time a significant data compression can be achieve(because the limited sequence of control points {p} are enough fo recovering the image features.

2. The B-Spline Curves and Their Properties

The B-Splines are piecewise polynomial curves, that are guided by a sequence of points called "control points": in fact a B-Spline curve approximate the polygon obtained by joining its control points (see Fig.2-1).

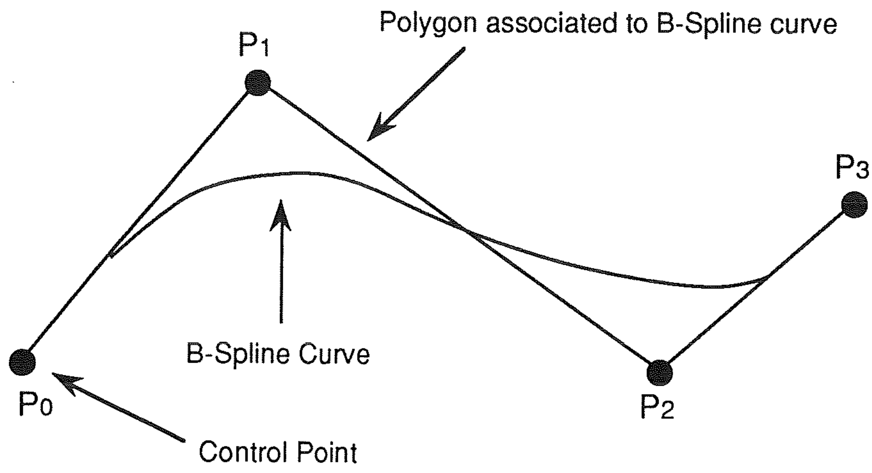

Fig.2-1: Control points, B-Spline curve and corresponding polygon

The influence of each control point on the curve is blended by a function called "blending function" or "basis function".

The parametric formula of a general B-Spline curve is given by

$$C(t) = \sum_{i=0}^{n} P_i \, N_{i,k}(t) \qquad (2\text{-}1)$$

where:

$C(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$ is the position vector of the curve;

$P_i = \begin{bmatrix} x_i \\ y_i \end{bmatrix}$ $0 \le i \le n$ is the position vector of the n+1 control points;

k $2 \le k \le n+1$ is the B-Spline curve order. The limit $k \ge 2$ denotes that at least two control points are necessary to define a B-Spline curve;

$N_{i,k}(t)$ is the normalized B-Spline basis function of order k(degree k-1).

The basis functions are defined by the Cox-deBoor recursiv formulas:

$$N_{i,1}(t) = \begin{cases} 1 \text{ if } x_i \leq t < x_{i+k} \\ 0 \text{ otherwise} \end{cases}$$

$$N_{i,k}(t) = \frac{t - x_i}{x_{i+k-1} - x_i} N_{i,k-1}(t) + \frac{x_{i+k} - t}{x_{i+k} - x_{i+1}} N_{i+1,k-1}(t) \qquad (2\text{-}2)$$

with $t \in [x_{min}, x_{max}]$ and $0 \leq i \leq n$

The points x_i are called knots, while $X^T = [x_{min},..,x_i,..,x_{max}]$ denote the knot vector. The only requirement for a knot vector is that it satisf the two rules

(1) $x_i \in \Re$;

(2) $x_i \leq x_{i+1}$.

The most important properties of the B-Spline curves are:

(1) the function $C(t)$ is a polynomial of degree k-1 on each spai $[x_i,x_{i+1})$;

(2) $C(t) \in C^{k-2}$ that is $C(t)$ and its derivatives up to k-2 order are al continuous over the entire curve;

(3) the generic basis function $N_{i,k}(t)$ is $\neq 0$ only over k successive spans which are $[x_i, x_{i+1})$, $[x_{i+1}, x_{i+2})$, ..., $[x_{i+k-1}, x_{i+k})$, so the variation of position of a control point does not modify all the curv but only k curve segments (Local Control Property);

(4) any geometric transformation can be applied to the curve by applying it to the associate control points;

(5) the curve lies within the convex hull of its defining polygon. Really the B-Spline curve of k-order lies within the union of all convex hulls formed by taking k successive defining polygon vertices (Convex Hull Property), as shown in Fig.2-2.

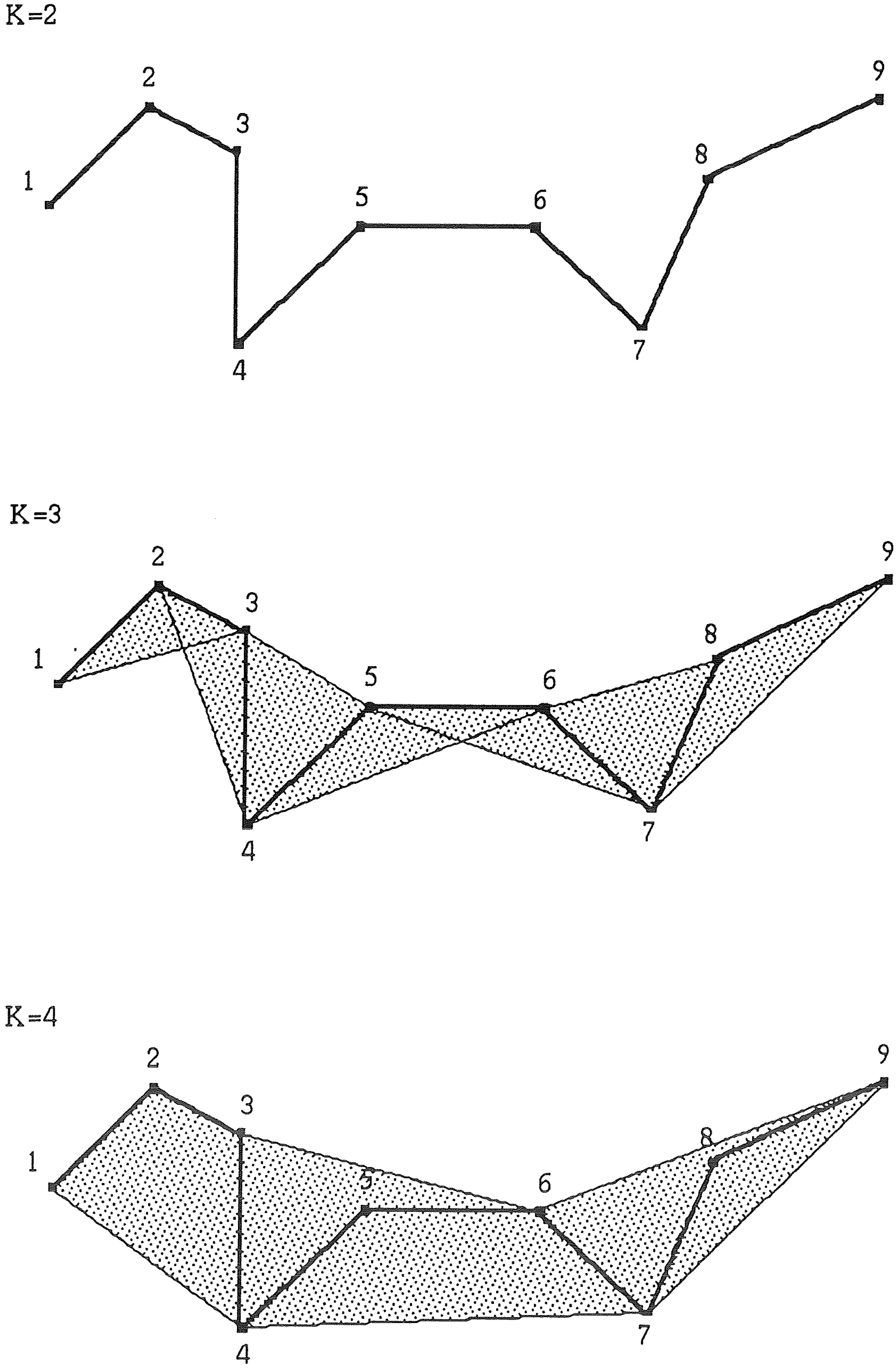

Fig.2-2: The Convex Hull Property of B-Spline curves.

Now we focalize our attention on cubic periodic[2] B-Spline curve that are very important for two reasons:

(α) they provide continuity of slope and curvature (se property (2));

(β) their equations can be easily expressed in the followin matrix form:

$$C_j(t) = T^T N G_j \qquad 1 \le j < n-2 \qquad (2\text{-}3)$$

$$C_j(t) = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \frac{1}{6}\begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix} \begin{bmatrix} P_{j-1} \\ P_j \\ P_{j+1} \\ P_{j+2} \end{bmatrix}$$

$C_j(t) = C(t)_{t \in [x_j, x_{j+1}]}$ is the j-th curve segment;

Note that the matrices T and N are constant for all curve segments Only the vector G_j changes from segment to segment.

Using the matrix form is immediate to evaluate the first anc second derivative.

$$C_j'(t) = \begin{bmatrix} t^2 & t & 1 \end{bmatrix} \frac{1}{2}\begin{bmatrix} -1 & 3 & -3 & 1 \\ 2 & -4 & 2 & 0 \\ -1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} P_{j-1} \\ P_j \\ P_{j+1} \\ P_{j+2} \end{bmatrix} \qquad (2\text{-}4)$$

$$C_j''(t) = \begin{bmatrix} t & 1 \end{bmatrix} \begin{bmatrix} -1 & 3 & -3 & 1 \\ 1 & -2 & 1 & 0 \end{bmatrix} \begin{bmatrix} P_{j-1} \\ P_j \\ P_{j+1} \\ P_{j+2} \end{bmatrix} \qquad (2\text{-}5)$$

Now the slope of the curve C(t) at a given point $t=t_1$ is obtained from the first derivative $C'(t) = \begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix}$, in this way : $\frac{dy}{dx} = \frac{y'(t)}{x'(t)}$.

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[2]A B-Spline curve is called periodic if its knots are equally spaced as [0, 1 2, 3].

3. The Developed Procedure

The developed procedure for image feature extraction and analysis is shown in Fig.3-1. Three main phases are executed in interactive and automatic way:

Data acquisition. An input matrix is obtained by acquiring a three-dimensional scene or a two-dimensional image. The input matrix contains samples of a luminance functions L(x,y), where x and y coordinates are expressed by the row and column indices of the matrix; in our application L is quantized on 256 levels, while x and y are quantized on 512 values.

Feature extraction. The interactive method described in chapter 4 is applied by the user to the input matrix in order to extract features of interest. A series of feature curves F(x,y) is obtained, each of them is a related to a selected feature in the input image.

Data analysis. Two types of data analysis can be performed.

Type I: Metric and morphological measurements are executed on the feature curve vectors: for example, the length of a curve or the distance between couples of curves can be determined. Higher order results can also be obtained: for example, parameters describing closed curves (e.g. perimeter, area, barycentre and so on) are easily computed; the equations of other curves (e.g. lines which cross perpendicularly feature curves) can be determined.

Type II: Luminance measurements on the input image are executed along paths related to features curves. For example, luminance values of pixels lying on a line segment orthogonal to a feature curve can be extracted from the input matrix, giving the possibility to study the shape of the feature, to find its width, and so on.

A prototype version of the described procedure has been implemented on a workstation based on a personal computer (see Fig. 2.3).

The image to be processed can be acquired by means of a video digitizer board (Matrix PIP 1024) or by using a MFA high resolution scanner [15], obtaining input matrices defined by 512x512 elements with 256 gray levels. A RGB monitor displays digital images defined by input or processed pictorial data; a VGA graphic monitor, a keyboard and a mouse are used for the man-machine interaction.

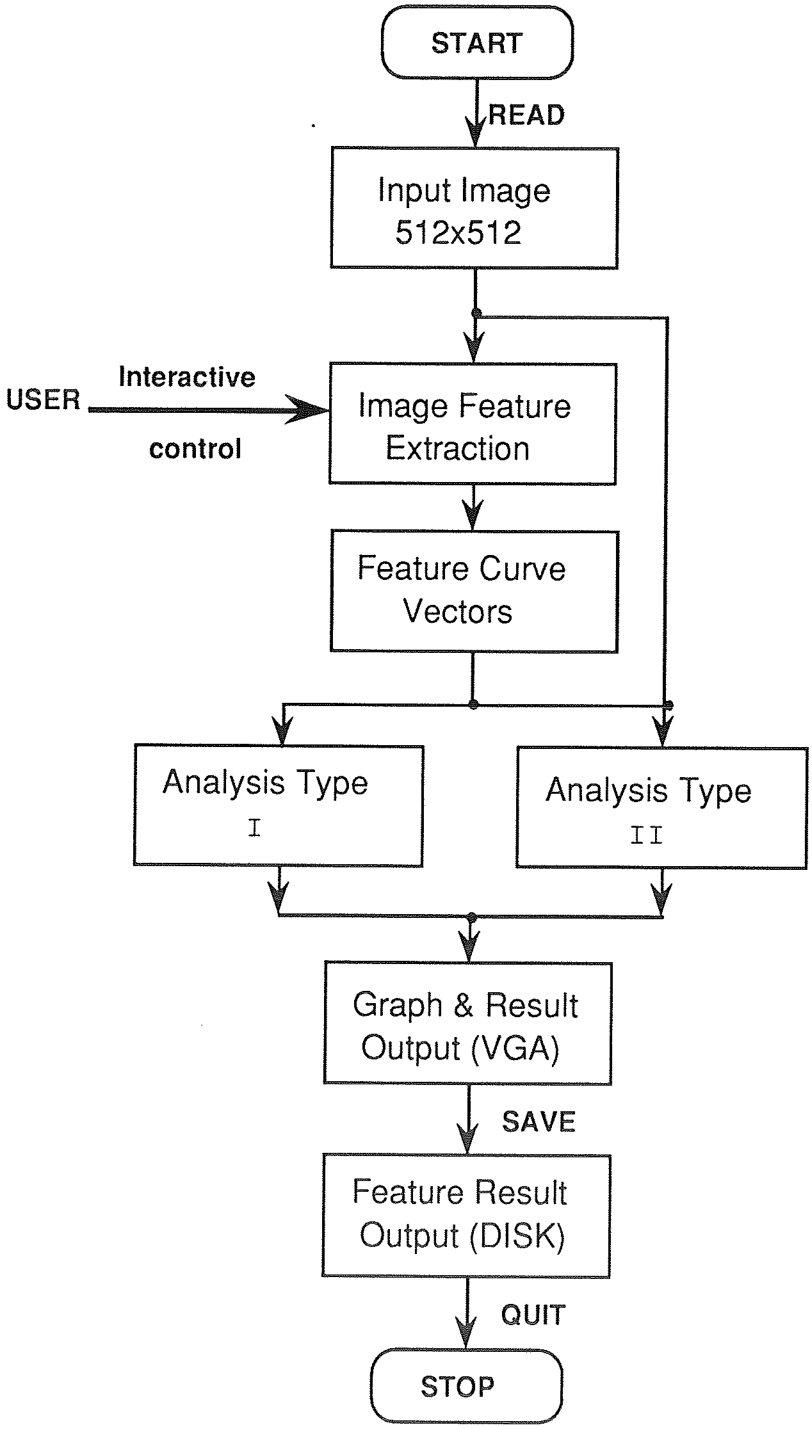

Fig.3-1: Flow diagram of the developed procedure

Fig.3-2: The workstation

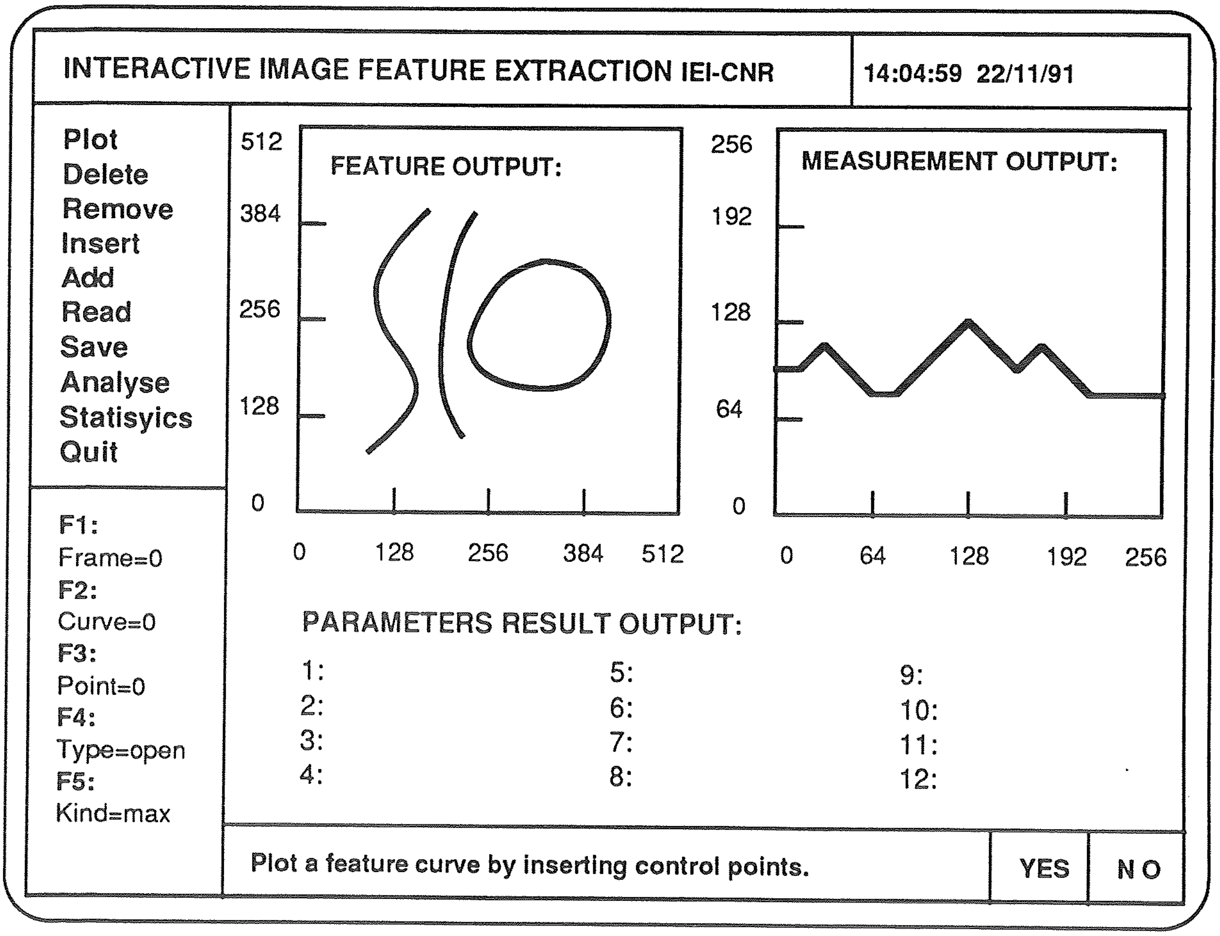

Fig.3-3: The man-machine interface.

A sketch of the man-machine interface is shown in Fig.3-3. O the left upper side of the screen, a function menu can be activated b using the mouse.

The PLOT function is used to extract a feature by inserting sequence of control points in an interactive way, generally, satisfactory feature curve can be obtained; otherwise REMOVI INSERT and ADD functions can be used to correct the feature curve b editing the control points and DELETE function is used to discard feature curve.

READ and SAVE functions are used to input and output th digital images from or to the disk.

ANALYSE and STATISTIC functions are used to process dat pertaining to the extracted feature curve.

On the left lower side of the screen a function menu can b activated by using function keys F1-F5:

F1 is used to access a frame buffer (0 / 1 / 2 / 3); F2 is used t select a feature curve; F3 is used to select a control point of a featur curve; F4 is used to define the type of the feature curve to be extracte (OPEN / CLOSE) and F5 is used to define the kind of the luminanc value of features to be extracted (MAXIMUM / MINIMUM).

Two areas of the screen are used to display graphics results: th feature output, which shows the extracted feature curves F(x,y) and th measurements output, which shows a parameter calculated along th curvilinear coordinates of the selected curve.

4. The Interactive Method of Image Feature Extraction

According to the previously described properties, an interactive curve interpolation procedure by means of B-Spline curves has been implemented and applyed to a test image

The test image (see Fig.4-3a) consists of concentric circles with different luminance L, defined by:

$$\begin{aligned} &(x - 255)^2 + (y - 255)^2 = r^2 \\ &L(r) = \text{int}[128 + 64 \cdot \sin(2\pi r/32)] \end{aligned} \qquad (4\text{-}1)$$

where $0 \leq x, y \leq 511$ is the coordinate of the pixel on the image; $r=1,2,\ldots,255$ is the radius of circles.

So the test image looks like a set of shaded rings: in each ring the luminance decreases from a maximum value in the central zone to a minimum value at the ring borders, if we assume that a ring is an image feature then the extracted feature curves will be a set of concentric circles.

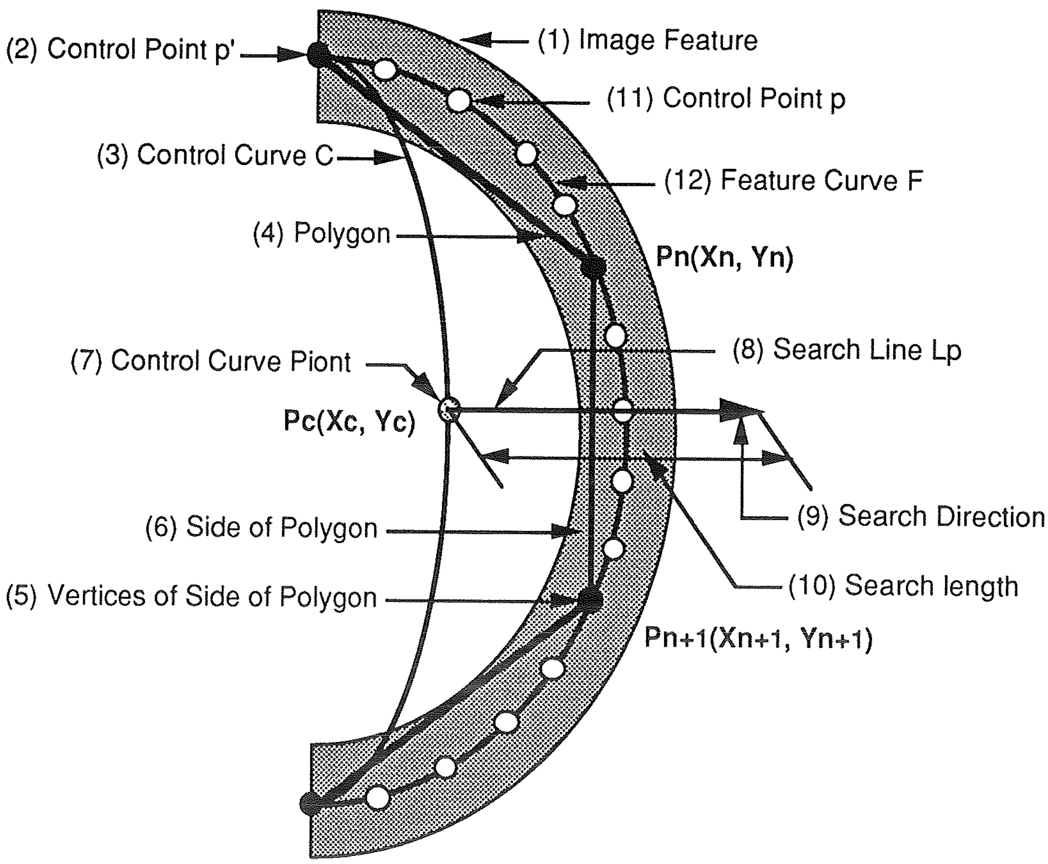

Fig.4-1: The image feature extraction based on B-Spline curves.

Each circle has to be determined by examining each featur within a region of interest and by looking for maximum luminanc values.

The feature curve can be obtained by using the proposed metho in the following steps:

(1) A sequence of control points {p'} defined by a limited numbe is inserted by the user in an interactive way.

The user points the cursor on the input image near the feature t be extracted and clicks the mouse button, then moves the cursor acros the feature and a line segment is drawn. The line is automaticall scanned and the point with maximum luminance value is assumed a control point p'.

A control curve C which is determined by the sequence of contrc points {p'} is then calculated by using B-Spline curves; C is a firs approximation of a curve F which is used to describe the image featurε The more the amount of control points is, the better C approximates t the feature curve F; and if the amount of control points is large enougł the C coincides with the F. But in terms of the practicability of th method, the amount of control points must be limited, according to th complexity of the image feature and care has to be taken that the contrc curve does not cross and overlap with another features of the image.

In the example shown 8 control points have been inserted to extrac a feature (see Fig.4-3b).

(2) Another sequence of control points {p} defined by a greate number should be obtained in order to create the feature curve F. Du to the convex hull property of the B-Spline curves, the control curve (lies within the convex hull of the polygon which is defined by th control points {p'}, or it is a straight line and coincides with th polygon itself if all the control points are colinear. According to th relative position and relationship between the control curve C and th polygon, the sequence of control points {p} can be searched by startin from each point on the control curve C and by moving along predefined path and a specified direction, as shown in Fig.3-3c.

If $p_c(x_c, y_c)$ is a point on the control curve C', and $p_n(x_n, y_n)$ an $p_{n+1}(x_{n+1}, y_{n+1})$ are the vertices of the corresponding polygon side L_p then the equation of the straight line corresponding to L_p is given by:

$$y-ax-b = 0 \qquad (4\text{-}2)$$

where:

$a=(y_{n+1}-y_n)/(x_{n+1}-x_n)$

$b=(y_n \cdot x_{n+1}-y_{n+1} \cdot x_n)/(x_{n+1}-x_n)$

Let be: $T = y_c-a \cdot x_c-b$ (4-3)

T can be used to determine the relative position of the point p_c with respect to the corresponding side L_p. If T>0, p_c is over the side L_p; if T=0, p_c lies on the side L_p and if T<0, p_c is under the side L_p. In terms of the slope of side, all the sides of the polygon are divided into four categories, and there are two search directions for each category (see Fig.4-2). The relationship between the point p_c and the corresponding side L_p, and the search directions is summarized in Table I. For example, let us consider a side of category I: if T<0, the search direction is N (i.e. Upper); if T=0, the search directions are N and S (i.e. Upper and Down); if T>0, the search direction is S (i.e. Down).

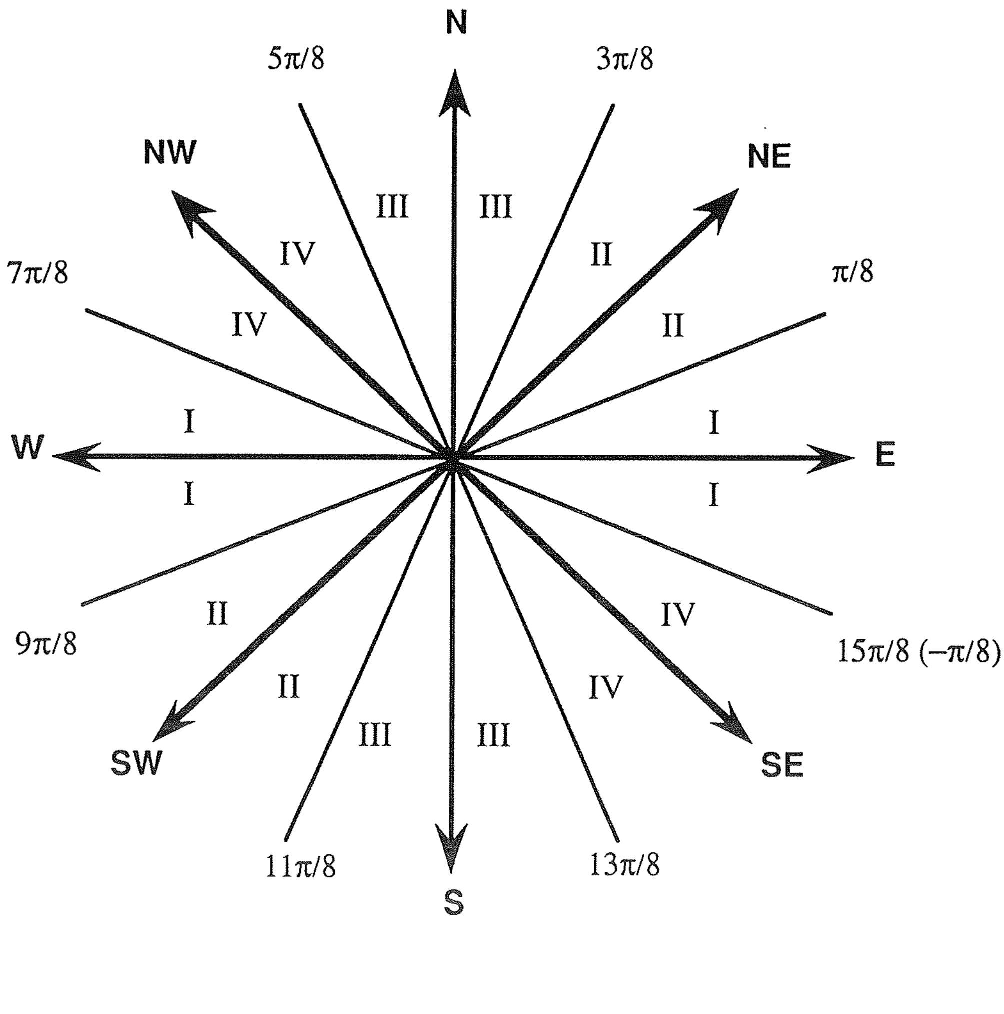

Fig.4-2: The categories of the side of the polygon and the corresponding search directions

The search length selection is one of key issues in searching fo control points {p}: if the length is too long, a control point can be foun pertaining to another feature; conversely, if the length is too short, th control point can be found in uncorrected position. Generally the searc length should be adaptively determined in terms of the complexity o the real features: in our approach the search length has been defined a the mean value of the length of the lines that have been determined b user in inserting the control points {p'}.

Table I: Categories of the sides and corresponding search directions

T	I $[-\pi/8,\ \pi/8)$ $[7\pi/8,\ 9\pi/8)$	II $[\pi/8,\ 3\pi/8)$ $[9\pi/8, 11\pi/8)$	III $[3\pi/8,\ 5\pi/8)$ $[11\pi/8, 13\pi/8)$	IV $5\pi/8,\ 7\pi/8)$ $[13\pi/8, 15\pi/8$
<0	N	SW	E	SE
=0	N & S	NW & SE	E & W	SE & SW
>0	S	SE	W	SW

(3) Finally, the feature curve F is obtained by applying the B Spline curves to the sequence of control points {p}. Fourth-order B Spline curves have been used (k=4), so that the feature curve F i composed of several cubic curve segments with second-order continuity The derivatives of the feature curve F at any point can be obtained b the control points {p} at the same time, so that the analysis of th extracted feature become easy and efficient.

Each real feature curve is obtained by applying the describe procedure to each local feature, an example is shown in Fig.4-3.

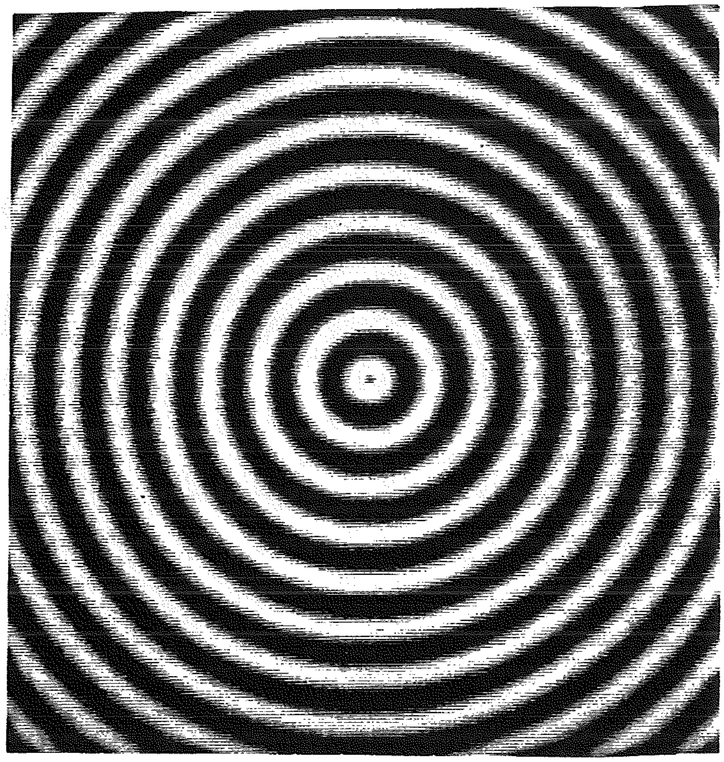

Fig.4-3: (a) Test image.

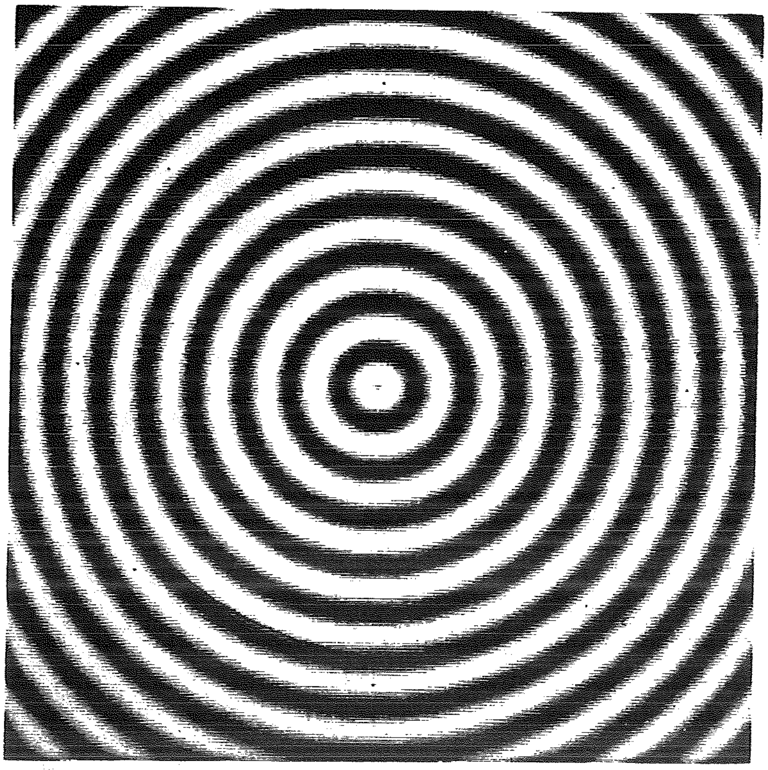

Fig.4-3(b): Insertion of the control points {p'} and creation of the control curve C.

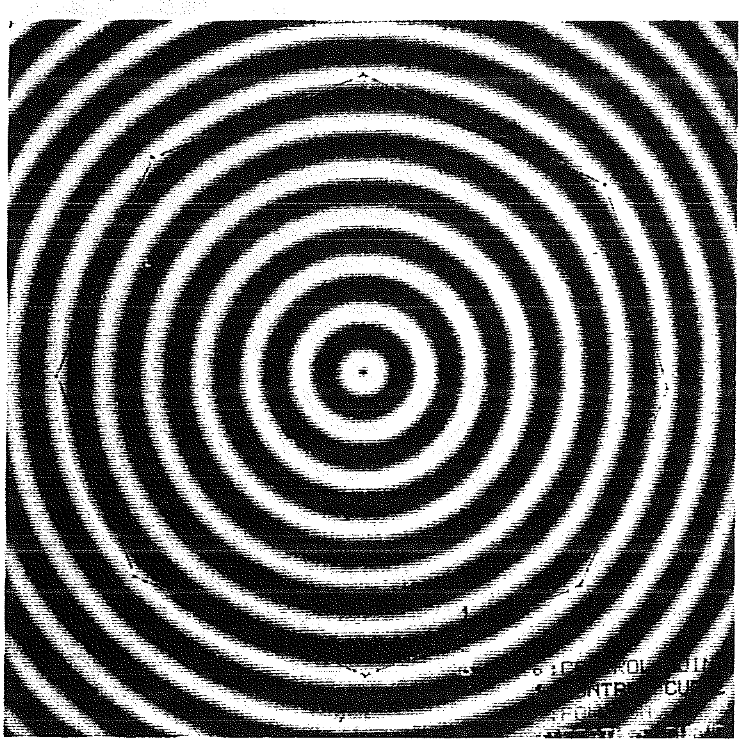

Fig.4-3: (c) The convex hull property of the B-Spline curves and searching of the control points {p}.

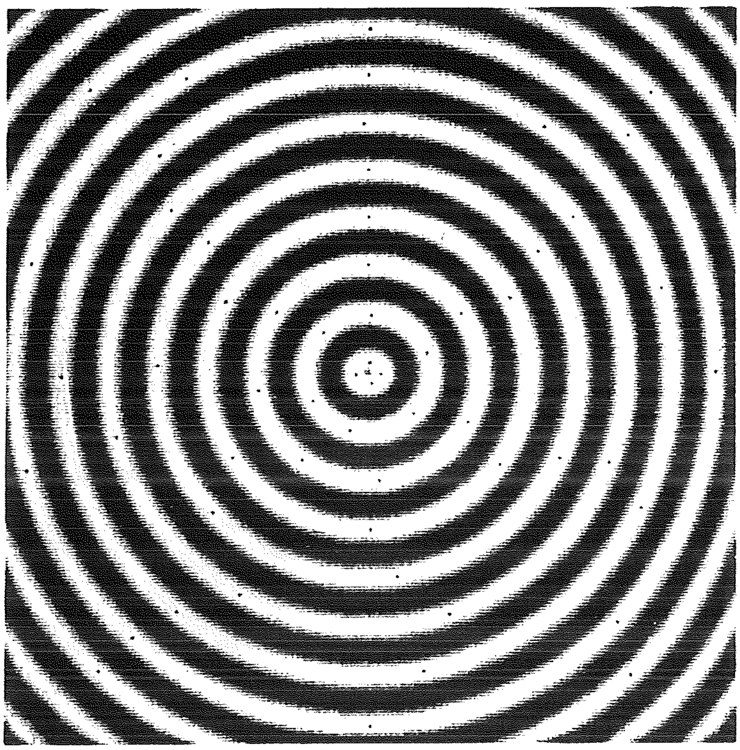

Fig.4-3(d): Feature curves F(x,y).

5. Examples

The procedure has been applyed to real images that are normally complicated, and degraded due to noise or lack of contrast with respect to the test image. An example is given in Fig.5-1.

The fluorangiographic image contains arteries, veins and retinal tissue, which can be assumed as image features. These features interfere with each other, i.e. intersect, overlap, or pass very closely, and are degraded due to noise and low contrast. Despite of these problems, good results have been obtained (see Fig.5-1c).

An example of type II analysis is also reported.

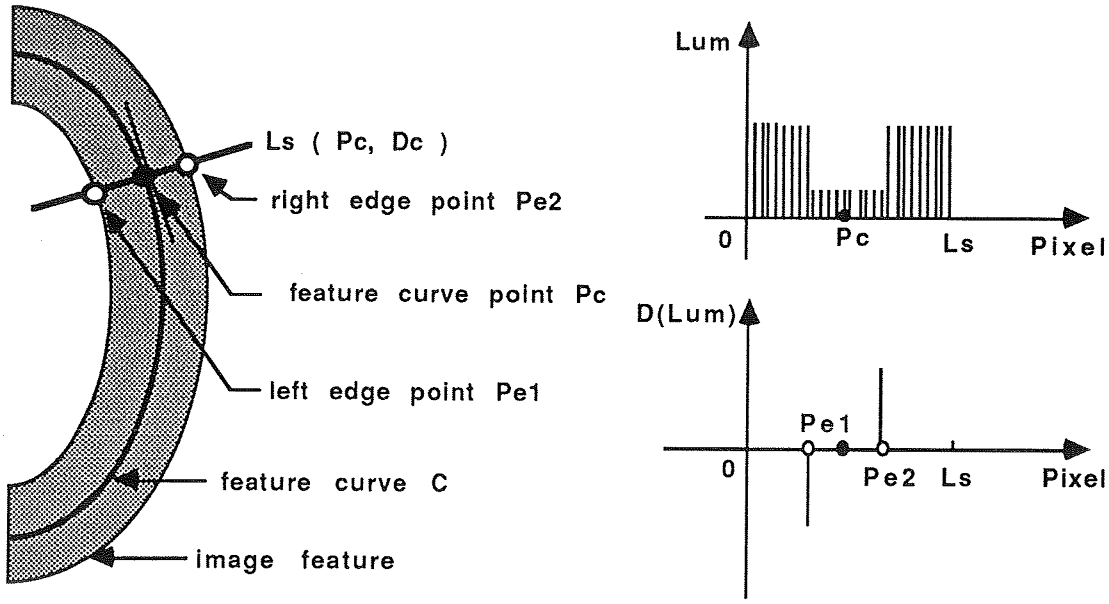

Fig.5-2: Width and edge points of the feature

The width of a selected feature has been measured based on the extracted feature curve. As shown in Fig.5-2, for each point on the feature curve $p_c(x_c, y_c)$, the width of the feature S_c is defined as:

$$S_c = \sqrt{(x_{e1}-x_{e2})^2 + (y_{e1}-y_{e2})^2} \qquad (5\text{-}1)$$

where $p_{e1}(x_{e1}, y_{e1})$ and $p_{e2}(x_{e2}, y_{e2})$ are the edge points of the feature on the line segment L_s. L_s is in turn defined by the point $p_c(x_c, y_c)$ and the slope D_c of the curve at the point P_c; we have

$$y = y_c + D_c \cdot (x - x_c) \qquad (5\text{-}2)$$

The edge points p_{e1} and p_{e2} are defined as the extreme points on L_s in terms of the difference of the luminance values of two adjacent pixels.

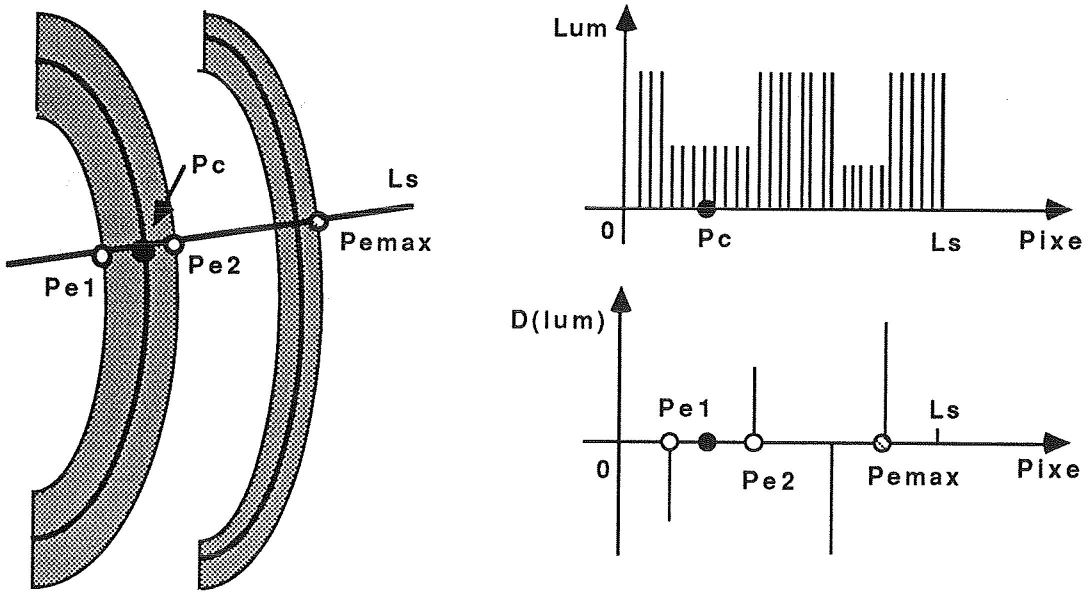

Fig.5-3(a): The analysis of the error E_{max}.

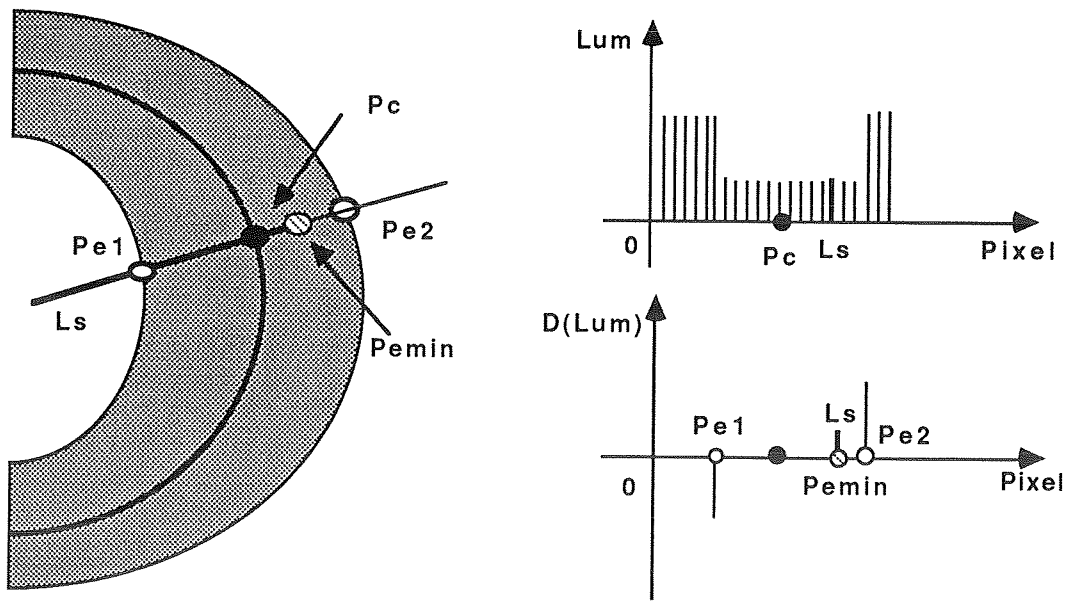

Fig.5-3(b): The analysis of the error E_{min}.

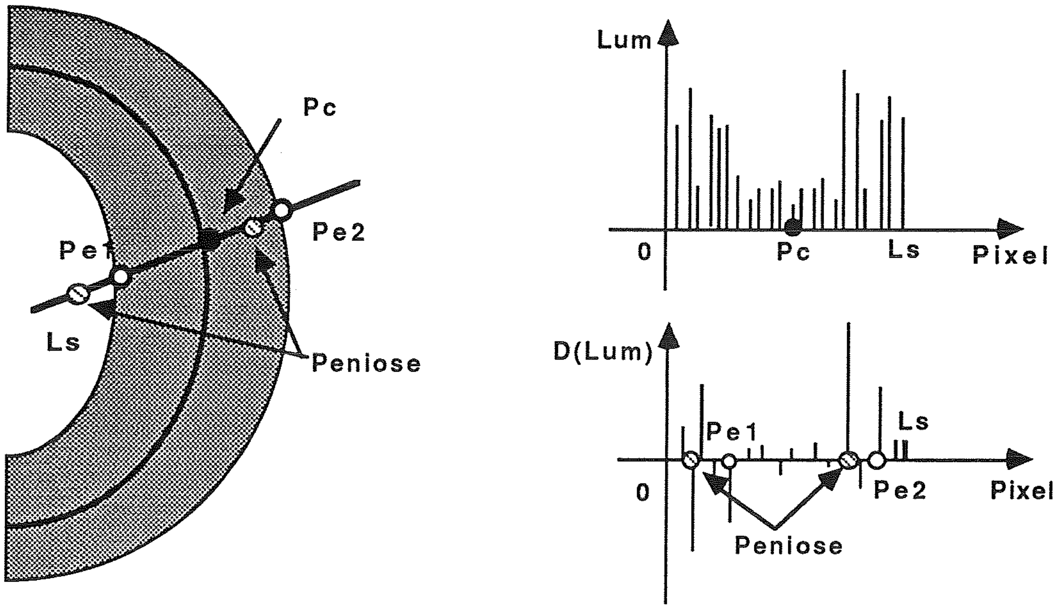

Fig.5-3(c): The analysis of the error E_{noise}.

The length of Ls is determined by the user in an interactive way depending on the characteristics of the image. But because of the presence of noise and the complexity of the image, i.e. intersection or overlapping of feature curves, three kinds of error may occur.

(1) Maximum value error (Emax), which occurs when the length Ls is too large, or the feature overlaps or passes very closely with another one. The computed width affected by this kind of error, S_{Emax}, is greater than the real width of the feature at the given point, as shown in Fig.5-3a.

(2) Minimum value error (Emin), which occurs when the length of Ls is too short, or the feature intersect with another one. The corresponding width S_{Emin} is shorter than the real width of the feature at the given point, as shown in Fig.5-3b;

(3) Noise error (Enoise), which occurs due to the existence of noise, as shown in Fig.5-3c.

In order to reduce the influence of noise, the width of the feature is computed in the following way, as shown in Fig.5-4.

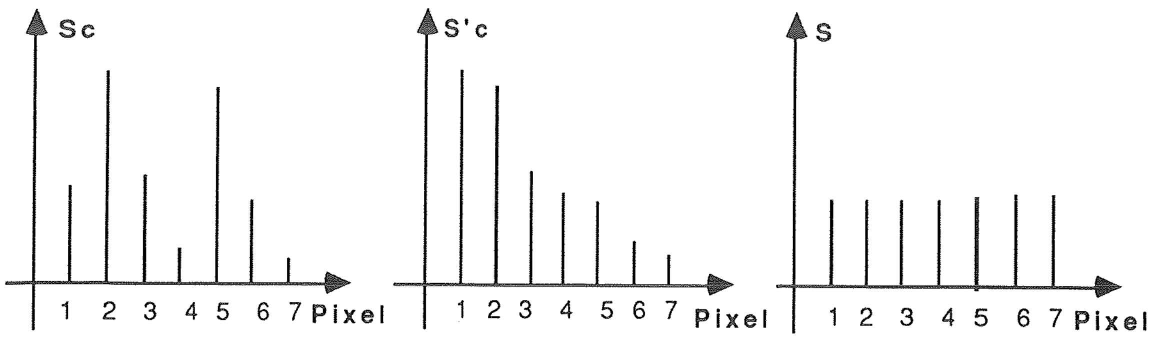

Fig.5-4: The error smoothing.

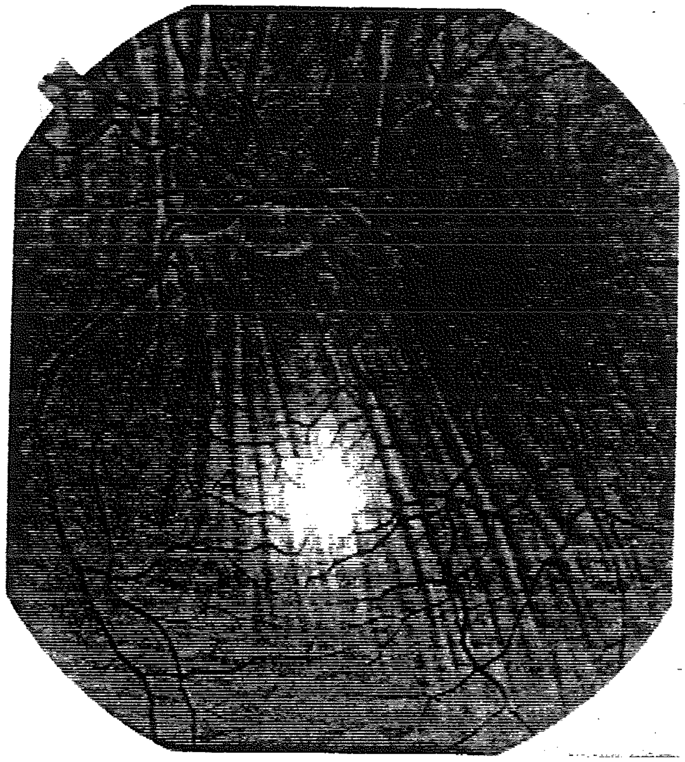

Fig.5-1(a): Input image.

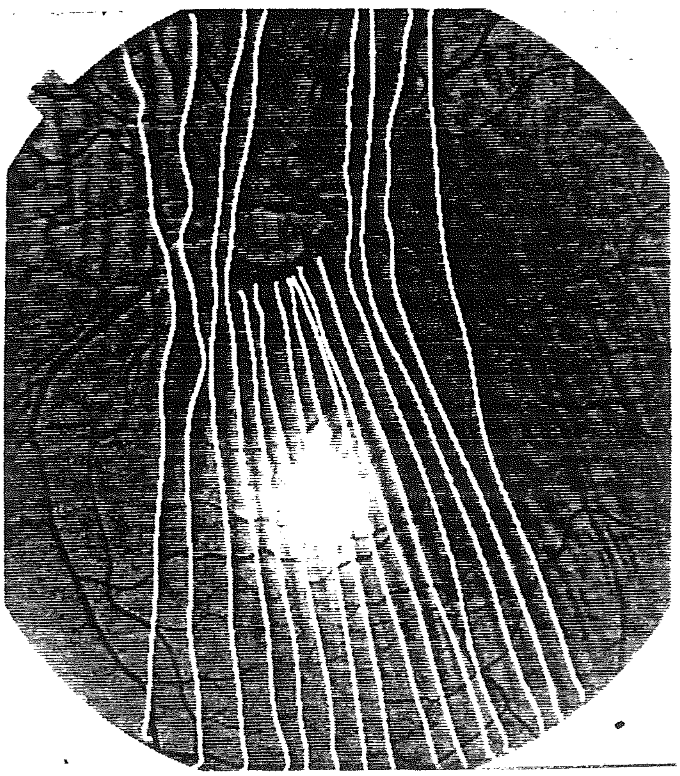

Fig.5-1(b): The retinal tissue features of the image.

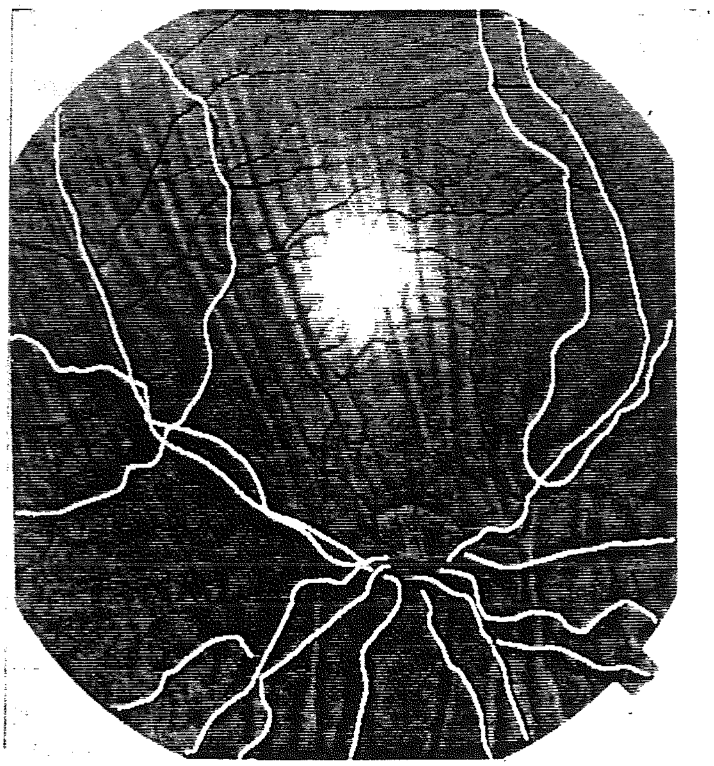

Fig.5-1(c): The artery and vein features of the image.

Let $\{S_{01}, S_{02}, S_{03}, S_{04}, S_{05}, S_{06}, S_{07}\}$ be a set of seven neighbouring widths and $\{S'_{01}, S'_{02}, S'_{03}, S'_{04}, S'_{05}, S'_{06}, S'_{07}\}$ be its positive sort result, thus the two maximum widths S'_{01}, S'_{02} and the two minimum widths S'_{06}, S'_{07} are discarded; and the statistical result of the feature width is defined on the remaining three values as:

$$S=(S'_{03}+S'_{04}+S'_{05})/3 \qquad (5\text{-}3)$$

For a given feature the statistics widths are organized in a width vector $\{S_1, S_2, , S_N\}$; we can thus define:

$$S_{max} = \max \{S_1, S_2, .., S_N\} \qquad (5\text{-}4a)$$

$$S_{min} = \min \{S_1, S_2, .., S_N\} \qquad (5\text{-}4b)$$

$$S_{mean} = \frac{1}{N} \sum_{i=1}^{N} S_i \qquad (5\text{-}4c)$$

A feature can also be characterised by the parameter G:

$$G = S_s / S_{mean} \qquad (5\text{-}4d)$$

where $S_s = \sqrt{\frac{1}{N^2} \sum_{i=1}^{N} (S_i - S_{mean})^2}$ is the variance of S.

If G~0, it means that the width of the feature is almost constant.

The above mentioned procedure has been used to analyze th width of veins. For a normal vein, the width is almost constant, i.e G~0; else pathologies may occur in the positions of S_{max} or S_{min} value (see Fig.5-5).

Fig.5-5(a): Example of type II analysis: input image.

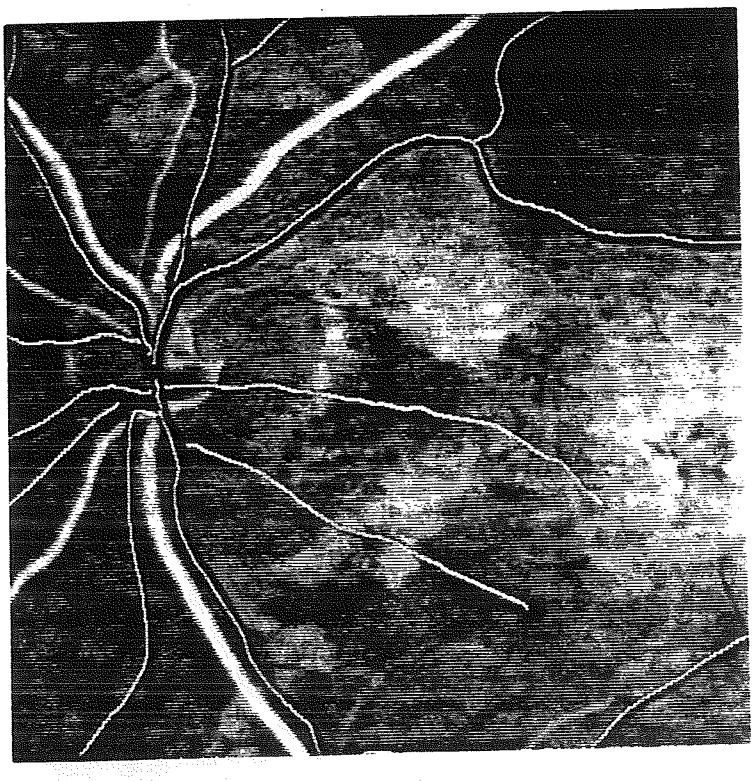

Fig.5-5(b): The vein features of the image.

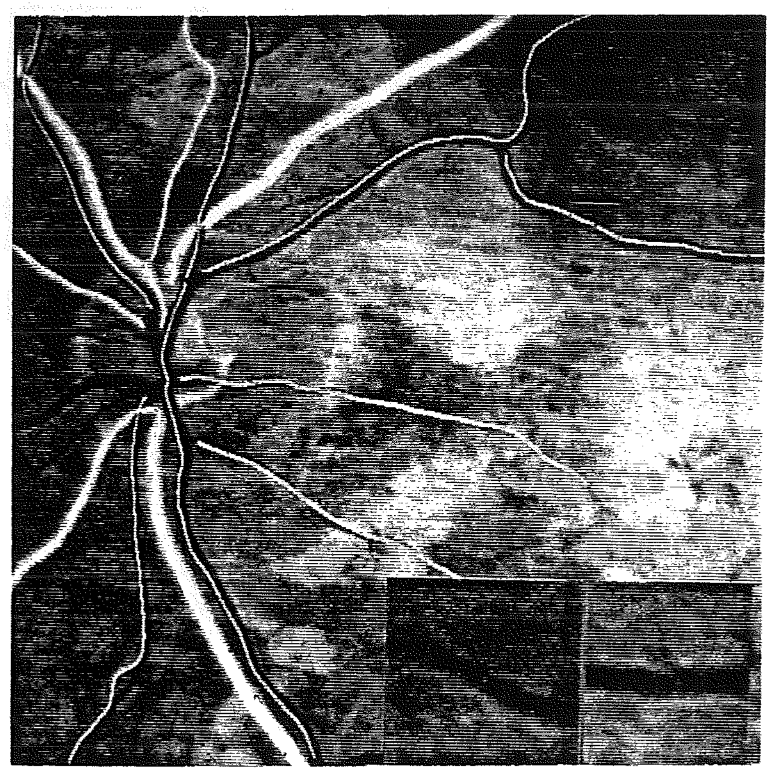

Fig.5-5(c): The width of selected vein.

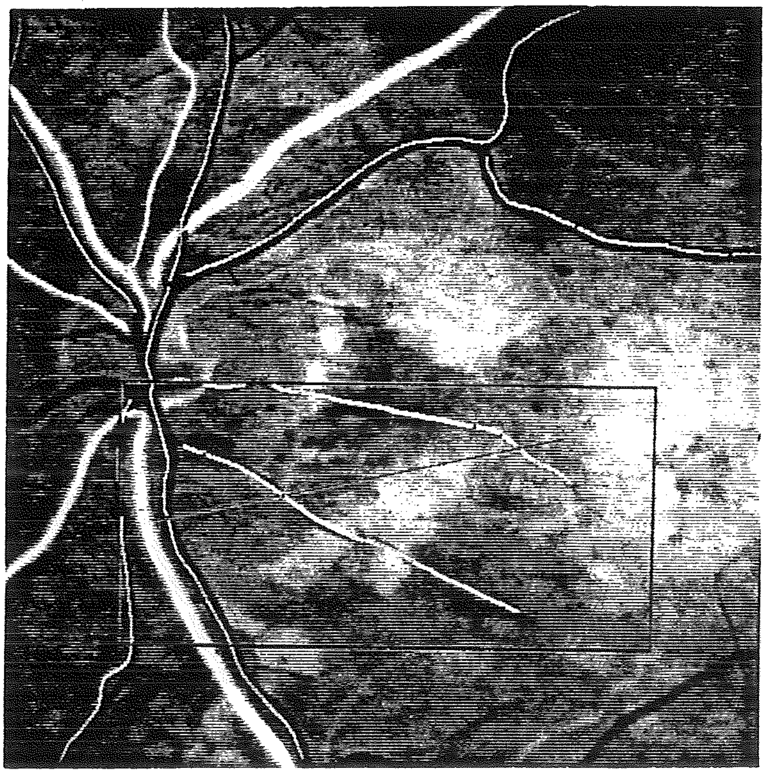

Fig.5-5(d): The distance between the feature.

6. Conclusions

An interactive method of image feature extraction based on B-Spline curves has been designed and realized. The main advantages of the proposed method are that the real features of an image can be extracted and described by a series of image feature curves, so that the analysis of image features becomes easy and efficient; at the same time a significant data compression is achieved.because each feature is described by a limited set of control points

An image feature extraction procedure based on the proposed method has been realized A software package for image feature extraction and analysis, based on the proposed method, has been realized in C language, and its performances have been tested on test and real images with good results.

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