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PROBABILISTIC SYNDROME DECODING IN SELF DIAGNOSABLE DIGITAL SYSTEMS

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Abstract. Self-diagnosis of multi-unit digital systems is reconsidered under the following hypotheses:

1) Faults are not equally probable. Each unit of a system has associated its own probability of failure. Unit malfunctions are assumed to be statistically independent.
2) The outcomes of tests performed between units are not deterministic. They are characterized by their conditional probability for any possible status of testing and tested unit. Attemption is restricted to the case where test results are independent of one another.

Given a set of test results, the problem of finding the most likely set of faulty units (probabilistic one-step diagnosability) is considered here. Moreover, an approach to probabilistic diagnosability with repair is presented. It is shown that there exists a significant class of systems for which this problem is easily solved and a decoding procedure is given whose complexity is O(n) where n is the number of system units.

1. INTRODUCTION

Probabilistic models for self-diagnosable digital systems have been introduced in order to have a more realistic representation of existing computers [1,2]. Their main advantages over well known non-probabilistic models [3,4,5] are the capability of accounting for different subsystem complexities and a further insight into the information carried by test results. The concept of partitioning a system into a number of sub-systems or units is retained while any restriction about the capability of testing and test connections of the units is removed. Moreover, no limitations are made about the number of faulty units which may be present.

When a system is to be diagnosed, units having diagnostic capabilities and appropriate connections perform a test (not necessarily complete) over some other units. A test outcome may be "0" (test passes) or "1" (test fails). The set of test outcomes will be referred in the following as the syndrome of the system. In the hypothesis that failure probabilities of all units are known together with conditional probabilities of test outcomes (test parameters) for any pair of testing-and-tested units, each syndrome can be correctly decoded. More specifically, a syndrome can be analyzed to find the most likely set of faulty units for that syndrome or, alternatively, to search for one or more units having high probability of being in a given status (faulty or fault-free). We refer to the first case as to probabilistic one-step diagnosability (p-one-step-diagnosability) whil the alternate hypothesis will be called probabilistic diagnosabil.ty with repair (p-diagnosability with repair). This diagnostic problem has a simple analytical formalization. Nevertheless, decoding a syndrome is a lengthy and heavy operation in the general case.

By means of a graph theoretic approach solutions for both cases of p-one-step diagnosability and p-diagnosability with repair are easily found for a class of systems whose test connections exhibit some characteristics. It must be observed that many self-diagnostic optimal and near optimal designs constructed in non probabilistic models belong to this class [3,5,6,7]. So, the probabilistic analysis presented here is also intended as a significant tool to verify the validity of such models.

2. A PROBABILISTIC MODEL

When a digital system is to be investigated for self-diagnosis, it is considered as partitioned into a number of subsystems or units. These units are not in general equireliable but have a different probability of failure mainly dependent on their complexity. To account for this, it seems reasonable to associate to each unit u_i a probability of failure $p(u_i)$. It is assumed that unit failures are independent or, equivalently, that the status of any unit u_i does not influence the status of any other unit u_j. In this hypothesis the probability of having a given set F of faulty units (also referred to as <u>fault-pattern</u>) is:

$$P\{F\} = \prod_{u_i \in F} p(u_i) \cdot \prod_{u_i \notin F} [1 - p(u_i)] \qquad (1)$$

In non probabilistic models, when unit u_i test unit u_j, test outcome may be completely unreliable depending on the status of testing and tested unit [3,5] while test result is deterministic if testing unit is fault-free. This is not the case in actual systems since test performed by a fault-free unit will not always be correct if test coverage is incomplete. Conversely, it is possible for a faulty unit to render a correct evaluation if faulty logic is not used in testing phase. These situations can be characterized by test parameters, i.e., the conditional probabilities of test outcomes for all possible status of testing and tested unit. Table I, where unit u_i is supposed to test unit u_j, defines these parameters. For simplicity to each unit u_k it is associated a variable x_k whose value is "0" or "1" as u_k is fault-free or faulty, respectively, and test outcome a_{ij} will be "0" if test passes and "1" otherwise.

There are many reasons for making $p_{ij}=1$. When both units are fault-free a "1" outcome would indicate an incorrect test evaluation procedure; this occurrence, which is discarded in most cases, is easily modelled by test parameter p_{ij} less than 1.

TABLE I

x_i	x_j	test outcome a_{ij}	$p\{a_{ij}\|x_i\ x_j\}$
0	0	0	p_{ij}
		1	$1-p_{ij}$
0	0	0	π_{ij}
		1	$1-\pi_{ij}$
1	0	0	r_{ij}
		1	$1-r_{ij}$
1	1	0	ρ_{ij}
		1	$1-\rho_{ij}$

Test parameter π_{ij} is a measure of test incompleteness, i.e., it accounts for that part of failures which are not discovered by the test.

Parameters r_{ij} and ρ_{ij} represent the information that a test outcome may provide even when testing unit is faulty. The probability of a correct evaluation increases as the testing rate of unit u_i decreases.

Table I implies that test parameters are independent on the status of system units other than u_i and u_j. This is a reasonable assumption provided that test logic associated with each unit is not shared with other units. This hypothesis can be restated as

$$P\{a_{ij}|x_0 \ldots x_{n-1}\} = P\{a_{ij}|x_i x_j\}$$

where $x_0 \ldots x_{n-1}$ represents the status of all system units.
The probability of occurrence of two (or, in general, more) test outcomes a_{ij}, a_{kh} for a given status of system will take the form

$$P\{a_{ij}\ a_{kh}|x_0 \ldots x_{n-1}\} = P\{a_{ij}|\ a_{kh}\ x_0 \ldots x_{n-1}\} \cdot P\{a_{kh}|x_0 \ldots x_{n-1}\}$$

where $P\{a_{kh}|x_0 \ldots x_{n-1}\}=P\{a_{kh}|x_k x_h\}$ is an appropriate test parameter while $P\{a_{ij}|a_{kh}\ x_0 \ldots x_{n-1}\}$ is in general unknown. From the equality

$$P\{a_{ij}|a_{kh}\ x_0...x_{n-1}\}P\{a_{kh}|x_h x_k\} = P\{a_{kh}|a_{ij}\ x_0...x_{n-1}\}P\{a_{ij}|x_i x_j\}$$

it appears that a_{ij} and a_{kh} are mutually dependent. However, in this paper it will be assumed that test outcomes are independent of one another. This hypothesis, which seems to be legitimate for any pair a_{ij}, a_{kh} with $i,j \neq k,h$, may represent in general a good approximation of the diagnostic behaviour of a system. When a more accurate evaluation is required, conditional probabilities are computer by a system simulation procedure. Preceding assumption allows to compute the probability of a syndrome Σ for a given system status as a simple product of test parameters:

$$P\{\Sigma|x_0...x_{n-1}\} = \prod_{a_{ij}\in\Sigma} P\{a_{ij}|x_0...x_{n-1}\} \qquad (2)$$

This model will be given the graph representation proposed by Preparata et al. To each system S, it is associated a directed graph G(N,A) whose node set N corresponds to the set of system units and an arc exists from node i to node j if and only if unit i tests unit j. Ares an given binary labels coinciding with test outcomes a_{ij}. For brevity, the same notation u_i will be used to refer to the system unit u_i or to the corresponding node of G.

3. THE FUNDAMENTAL DIAGNOSTIC PROBLEM

Let S be a malfunctioning system and F be the set of its faulty units. When a system test is performed, the fundamental diagnostic problem is to detect F from the syndrome or to find, alternatively, a subset $F' \subset F$ of faulty units. If F is detected by one application of the set of tests one-step diagnosability is said to occur, otherwise a diagnosis with repair is performed provided $F' \neq \emptyset$. Both one-step and diagnosability with repair are guaranteed in non-probabilistic models provided the number of faulty units that are present does not exceed a given threshold t which represents the diagnosability (one-step or with repair) of the system. In probabilistic terms this is equivalent to say

$$P\{F' \subseteq F | \Sigma \ (|F| \leqslant t)\} = 1$$

where F is the actual fault pattern, F' is the set of faulty units detected for the given Σ if condition $|F| \leqslant t$ holds and $F' \equiv F$ when one-step diagnosability is performed.

In the probabilistic model no restrictions are made about fault-pattern cardinality and some of limiting assumptions of previously known models are removed. As a consequence, there will be in general more sets of faults F' which are consistent with a syndrome. If test parameters are all different from 1, all thinkable collections of faulty units are consistent with any syndrome since both "0" and "1" outcomes are possible in each case. More generally, consistence can be defined as follow.

Definition 1. A set of faults F' is said to be consistent with a syndrome Σ if

$$P\{F' | \Sigma\} \neq 0$$

4. PROBABILISTIC ONE-STEP DIAGNOSIS

Let F be a fault pattern consistent with syndrome Σ. Probabilistic one-step diagnosis is then obtained according to Definition 2.

Definition 2. For a given syndrome Σ, p-one-step diagnosis is performed by finding the most likely fault pattern F_0 for that syndrome.

The conditional probability of any fault pattern F is expressed by

$$P\{F | \Sigma\} = P\{\Sigma | F\} \frac{P\{F\}}{P\{\Sigma\}}$$

Since syndrome $\sum$ is given, $P\{\sum\}$ is a fixed value in searching for the most likely fault pattern. As a consequence performing p-one-step diagnosis results in maximizing the product $P\{\sum|F\}P\{F\}$. A rough procedure could consist in evaluating this product for each of possible 2^n-1 fault patterns where n is the number of system units. As it will be seen in the following, there exists a class of systems for which this exhaustive search is avoided.

Probabilistic one-step diagnosis can be given an analytical approach. In fact, let unit u_i test unit u_j and a_{ij} be the corresponding test outcome. If x_i and x_j represent the status of testing and tested units according to conventions stated above, conditional probability $P\{a_{ij}|x_i x_j\}$ is

$$P\{a_{ij}|x_i x_j\} = a_{ij}+(-1)^{a_{ij}}[p_{ij}(1-x_i)(1-x_j)+\pi_{ij}(1-x_i)x_j+ + r_{ij}x_i(1-x_j)+\rho_{ij}x_i x_j] \quad (3)$$

It is easily seen that equality (3) coincides with Table I for any possible value of x_i and x_j. Since the validity of equality (2) is assumed in this paper, the probability $P\{\sum|F\}$ for a given fault pattern F is

$$P\{\sum|F\} = \prod_{a_{ij}\in\sum} \{a_{ij}+(-1)^{a_{ij}}[p_{ij}(1-x_i)(1-x_j)+\pi_{ij}(1-x_i)x_j+ +\tau_{ij}x_i(1-x_j)+\rho_{ij}x_i x_j]\} \quad (3')$$

Likewise, from equality (1 it follows that $P\{F\}$ can be expressed as

$$P\{F\} = \prod_{u_i\in F} p(u_i) \cdot \prod_{u_i\notin F} [1-p(u_i)]$$

or, equivalently

$$P\{F\} = \prod_{i=0}^{n} [(1-x_i)-(-1)^{x_i}p(u_i)] \quad (4)$$

Combining equalities (3') and (4), the problem of p-one-step diagnosis reduces to :

$$\text{maximize} \prod_{a_{ij} \in \Sigma} \{a_{ij} + (-1)^{a_{ij}} [p_{ij}(1-x_i)(1-x_j) + \pi_{ij}(1-x_i)x_j + \tau_{ij} x_i (1-x_j) + \rho_{ij} x_i x_j]\} \cdot \prod_{i=0}^{n} [(1-x_i) - (-1)^{x_i} p(u_i)] \tag{5}$$

where $x_0, x_1, \ldots, x_{n-1}$ are the unknowns. It has been shown [8] that this is a 0-1 integer programming problem whose solution is difficult to find and may not be unique. Next two Sections will show that a simple procedure exists allowing p-one-step diagnosability in a wide class of diagnostic systems.

5. A GRAPH THEORETIC PROCEDURE FOR P-ONE-STEP DIAGNOSIS

Consider a system S of n units $u_0, u_1, \ldots, u_{n-1}$ whose diagnostic graph G(N,A) is the simple loop shown in Figure 1 and let $\Sigma = \{a_{i,i+1}\}$, $i=0,\ldots,n-1$ be the syndrome resulting from an application of the set of tests.

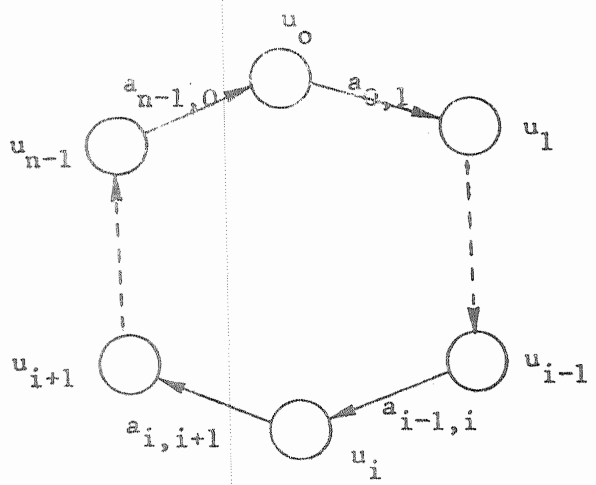

Figure 1

Observing that each unit u_i may represent a fault-free or faulty unit, from G(N ,A) an expanded graph G'(N',A') is derived as follows:

1) To each node $u_i \in N$ in G there correspond two nodes u_{i0} (fault-free) and u_{i1} (faulty) in G', $u_{i0}, u_{i1} \in N'$.

2) For each arc $(u_i, u_{i+1}) \in A$ there are four corresponding arcs $(u_{i0}, u_{i+1,0}), (u_{i0}, u_{i+1,1}), (u_{i1}, u_{i+1,0}), (u_{i1}, u_{i+1,1}) \in A'$.

3) Each arc $(u_{ih}, u_{i+1,k})$, $h,k \in \{0,1\}$, is given a compound weight $w(u_{ih}, u_{i+1,k}) = v(u_{i+1,k}) \cdot v(u_{ih}, u_{i+1,k})$ where:

   i) $v(u_{i+1,k})$ is the probability associated with the status represented by $u_{i+1,k}$, i.e., $v(u_{i+1,k}) = [(1-k)-(1-)^k p(u_{i+1})]$

   ii) $v(u_{ih}, u_{i+1,k})$ is the probability of test outcome $a_{i,i+1}$ for $x_i = h$ and $x_{i+1} = k$. For simplicity, all possible values of $v(u_{ih}, u_{i+1,k})$ are reported in Table II.

TABLE II

x_i	x_{i+1}	$P\{a_{i,i+1}=0 \mid x_i x_{i+1}\}$	$P\{a_{i,i+1}=1 \mid x_i x_{i+1}\}$
0	0	$p_{i,i+1}$	$1-p_{i,i+1}$
0	1	$\pi_{i,i+1}$	$1-\pi_{i,i+1}$
1	0	$r_{i,i+1}$	$1-r_{i,i+1}$
1	1	$\rho_{i,i+1}$	$1-\rho_{i,i+1}$

From arguments of preceding Section, it is clear that the problem of finding the fault pattern F_0 of highest probability for the given syndrome is equivalent to finding the cycle $\mathcal{C}_\Sigma = [u_{0,i}, u_{1,j}, \ldots, u_{0,i}]$ $i,j \ldots \in \{0,1\}$ which maximizes the product

$$P_\Sigma = \prod_{i=0}^{n-1} w(u_{i,h}, u_{i+1,k}) \qquad h,k \in \{0,1\}$$

In fact, for any $u_{i,h} \in \mathcal{C}_\Sigma$, status of corresponding unit u_i in S is derived by letting $x_i = h$, $h \in \{0,1\}$, $i = 0,1,\ldots,n-1$ and:

$$P_\Sigma \;=\; P\{\Sigma|F_0\}P\{F_0\} = P\{F_0|\Sigma\}\cdot P\{\Sigma\}$$

or, also

$$P\{F_0|\Sigma\} = P_\Sigma \,/P\{\Sigma\}.$$

To derive P_Σ and the corresponding $\mathcal{C}_\Sigma$ the following procedure is suggested.

Procedure 1.

a) Let $k=0$

b) Associate to each node $u_{i,j} \in N'$, $i\in\{0,\dots,n-1\}$, $j\in\{0,1\}$, the weight $\lambda(u_{i,j})$ with

$$\lambda(u_{i,j}) = \begin{cases} w(u_{0,k},u_{1,j}) & \text{for } i=1;\ j\in\{0,1\} \\ 0 & \text{for } i\in\{2,\dots,n-1\};\ j\in\{0,1\} \end{cases}$$

c) Let $i=1$

d) $i=i+1$

$$\lambda(u_{i,j})=\max[\lambda(u_{i-1,0})w(u_{i-1,0},u_{i,j});\lambda(u_{i-1,1})w(u_{i-1,1},u_{i,j})];\ j\in\{0,1\}$$

e) If $i<n-1$ repeat step d) else

$$\lambda_k = \max[\lambda(u_{n-1,0})\cdot w(u_{n-1,0},u_{0,k});\lambda(u_{n-1,1})w(u_{n-1,1},u_{0,k})]$$

f) $k=k+1$; if $k<2$ repeat step d) else

$$P_\Sigma = \max[\lambda_0,\lambda_1]$$

Cycle $\mathcal{C}_\Sigma$ is determined as follows:

a') If $P_\Sigma=\lambda_0$, then $x_0=0$, else $x_0=1$

b') Let $\lambda(u_{n,x_0})=P_\Sigma$; $x_n=x_0$; $j=0$

c') $j=j+1$;

if $\lambda(u_{n-j,0})=\dfrac{\lambda(u_{n-j+1,x_{n-j+1}})}{w(u_{n-j,0},u_{n-j+1,x_{n-j+1}})}$ then $x_{n-j}=0$, else $x_{n-j}=1$;

d') if $j<n-1$ repeat step c').

It is easy to see that Procedure 1 requires 3n-3 comparisons, (n-2) multiplications and (n-1) divisions. As a consequence, the complexity of syndrome decoding procedure is O(n) where n is the number of system units. An example will clarify the application of Procedure 1.

Example 1. Consider the simple loop diagnostic graph G(N,A) of Figure 2 with the syndrome Σ reported on the arc labels. The extended graph

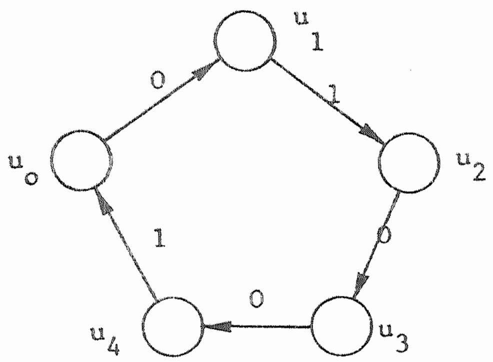

Figure 2

G'(N',A') is then derived following preceding specifications. Graph G' is shown in Figure 3: nodes $u_{n,0}$ and $u_{n,1}$ coincide with $u_{0,0}$ and $u_{0,1}$, respectively.

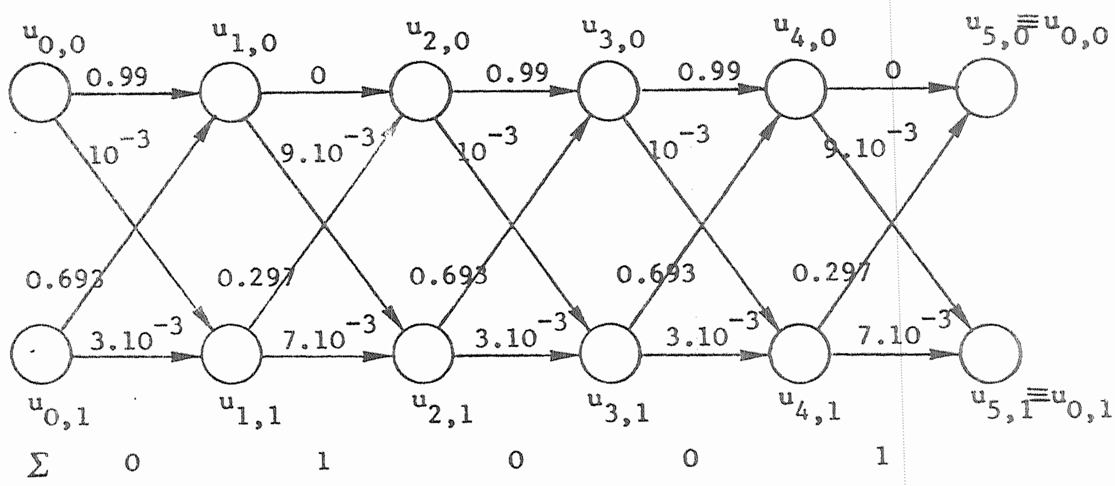

Figure 3

To perform p-one-step diagnosis assume that failure probabilities of the units are $p(u_i)=0.01$ for any $i \in \{0,...,n-1\}$ and test parameters take the values:

$p_{ij} = 1$
$\pi_{ij} = 0.1$
$r_{ij} = 0.7$
$\rho_{ij} = 0.3$

for any pair $i,j \in \{0,...,n-1\}$. Computing arc weights $w(u_{i,j},u_{i+1,h})$ results in the arc labels of Figure 3. Arcs $(u_{1,0},u_{2,0})$ and $(u_{4,0},u_{5,0})$ whose corresponding weights are zero, can be omitted in searching for the path which maximizes the product of arc weights (maximal path). Note that when an arc $(u_{i,h},u_{i+1,k})$ is labeled with zero, the corresponding probability of having test outcome $a_{i,i+1}$ as specified by the syndrome is zero for $x_i=h$ and $x_{i+1}=k$.
Procedure 1 starts by letting k=0. This corresponds to finding maximal path from $u_{0,0}$ to $u_{n,0} \equiv u_{0,0}$ i.e., in the hypothesis that unit u_0 is fault-free. Graph $G'(N',A')$ reduces to the subgraph G'_0 of Figure 4, where nodes $u_{0,1}, u_{n,1}$ and arcs $(u_{1,0},u_{2,0}),(u_{4,0},u_{5,0})$ are omitted.

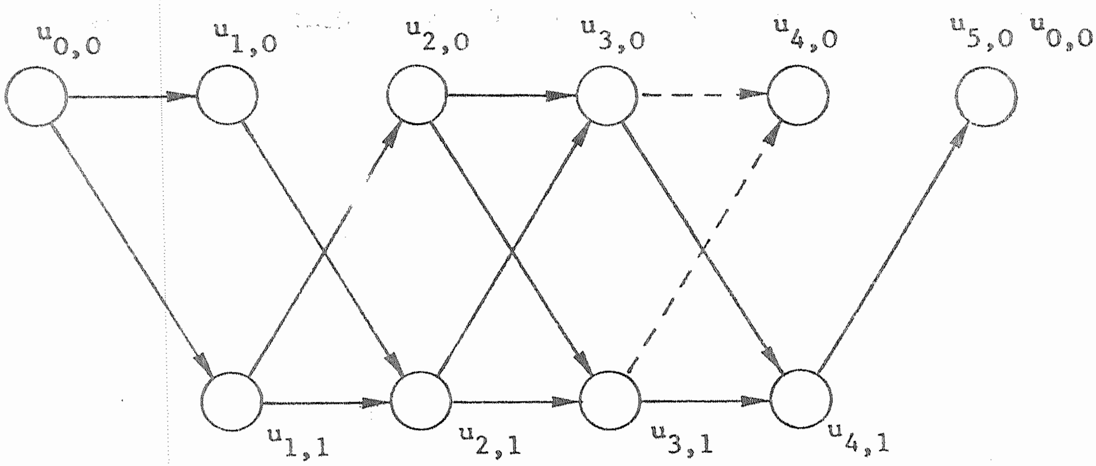

Figure 4

Node $u_{4,0}$ has no outcoming arcs. So, it can be omitted with its incoming arcs since no path from $u_{0,0}$ can reach $u_{5,0}$ through node $u_{4,0}$. Computing node weights λ's gives:

$\lambda(u_{1,0}) = 0.99$ $\lambda(u_{1,1}) = 10^{-3}$

$\lambda(u_{2,0}) = 0.297.10^{-3}$ $\lambda(u_{2,1}) = 8.91.10^{-3}$

$\lambda(u_{3,0}) = 6.17463.10^{-3}$ $\lambda(u_{3,1}) = 26.73.10^{-6}$

$\lambda(u_{4,0}) =$ (omitted) $\lambda(u_{4,1}) = 6.17463.10^{-6}$

$\lambda_0 = 1.8338651.10^{-6}$

Procedure is repeated starting from $u_{0,1}$ (k=1). The extended graph $G'(N',A')$ reduces to the subgraph G_1' of Figure 5.

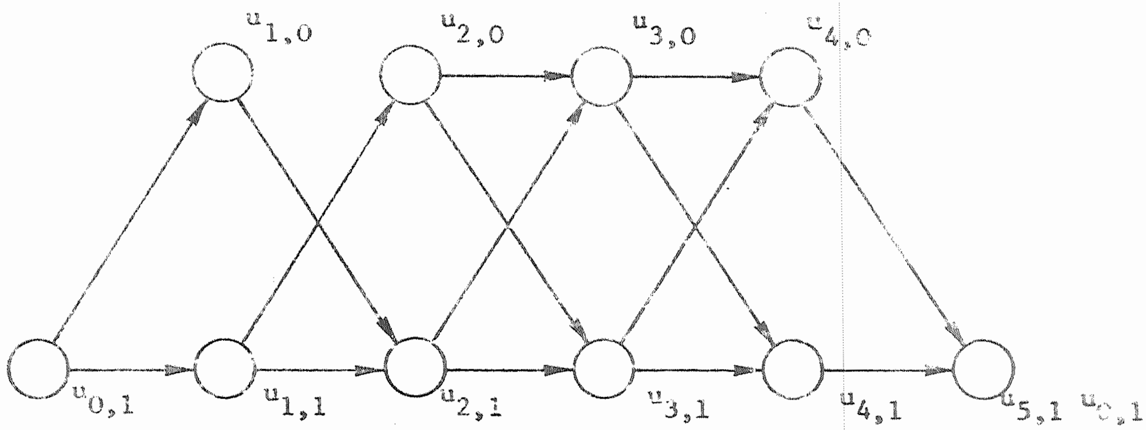

Figure 5

To find λ_1, node weights are sequentially computed in G_1':

$\lambda(u_{1,0}) = 0.693$ $\lambda(u_{1,1}) = 3.10^{-3}$

$\lambda(u_{2,0}) = 0.891.10^{-3}$ $\lambda(u_{2,1}) = 6.237.10^{-3}$

$\lambda(u_{3,0}) = 4.322241.10^{-3}$ $\lambda(u_{3,1}) = 18.711.10^{-6}$

$\lambda(u_{4,0}) = 4.2790185.10^{-3}$ $\lambda(u_{4,1}) = 4.322241.10^{-6}$

$\lambda_1 = 38.511166.10^{-6}$

and $P_\Sigma = \max[\lambda_0, \lambda_1] = \lambda_1$ is determined.

Status variables α_i (i=0,...,4) or, equivalently, maximal cycle $\mathcal{C}_\Sigma$ are determined from P_Σ as described by the last part of Procedure 1

$x_n = x_0 = 1$
$x_{n-1} = x_4 = 0$
$x_{n-2} = x_3 = 0$
$x_{n-3} = x_2 = 1$
$x_{n-4} = x_1 = 0$

It is concluded that the most likely fault pattern F_0 for syndrome Σ is $F_0 \equiv \{u_0, u_2\}$.

The validity of Procedure 1 is immediately extanded to a wider class of diagnostic graphs. In fact, any graph G(N,A) having at least one eulerian cycle, i.e., a cycle that traverses each arc exactly once, can be treated in a manner very similar to that shown for simple loop connections. If $\mathcal{E}$ is an eulerian cycle in G(N,A), an extended graph G'(N',A') can be associated to G provided that each node which is traversed h times in $\mathcal{E}$ is considered as h different pair of nodes in G'. This implies that each repeated node is given the same status as procedure is carried out. A rough method will consist in 2^h iterations of Procedure 1, i.e., computations are repeated for all possible combinations of status of repeated units. Note that assuming u_0 to be coincident with a repeated node reduces to 2^{h-1} the number of iterations; moreover inconsistency conditions also result in a shortening of decoding procedure. To clarify preceding considerations, consider the diagnostic graph G(N,A) of Figure 6.
A possible eulerian cycle in G is $\mathcal{E} = [u_0, u_1, u_2, u_0, u_3, u_4, u_5, u_3, u_6, u_0]$ where nodes u_0 and u_3 are repeated twice. Since u_0 coincides with a repeated node, Procedure 1 is iterated for two possible status conditions of u_3, i.e., fault-free and faulty. Note also that if the diagnostic graph has associated the syndrome of Figure 6 and p_{03}=1, the case where

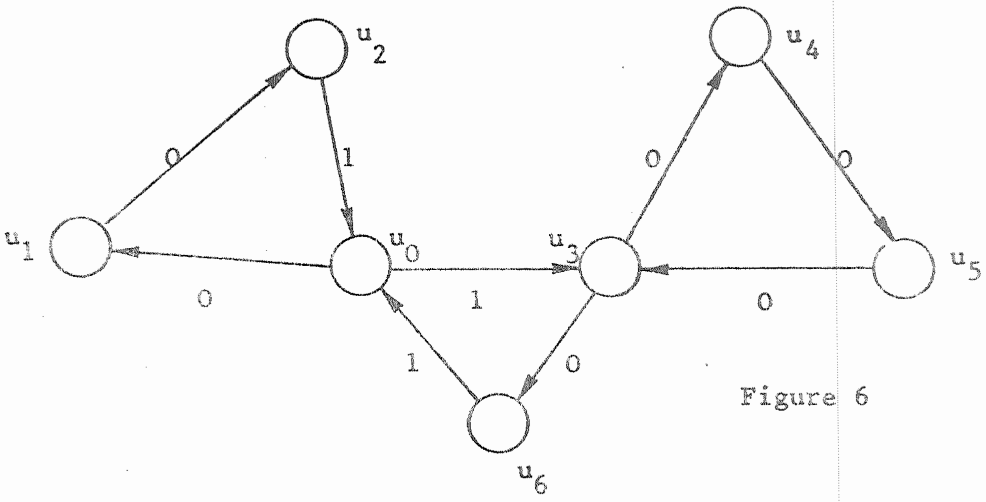

Figure 6

u_0 and u_3 are assumed fault-free can be skipped since $x_0=x_3=0$ is not consistent with $a_{03}=1$.
With a proper choice of unit u_0, the complexity of procedure is increased by a factor 2^{h-1} against Procedure 1, where h is the number of repeated nodes.

Example 2. Consider the diagnostic graph of Figure 7. Here $2^{h-1}=1$ and an application of Procedure 1 is sufficient to identify the most likely fault-pattern F_0 for the given syndrome Σ . Note that the considered diagnostic graph corresponds to a near optimal design in Preparata's model [7] and to an optimal design in the model of Barsi et al. [5]. In the hypothesis that failure probabilities of units and test parameters are those of example 1, the extended graph $G'(N',A')$ is reported in Figure 8 for the eulerian cycle $\mathcal{C}_\Sigma = [u_0,u_1,u_2,u_0,u_3,u_4,u_0,u_5,u_6,u_0]$. Arcs $(u_{1,0},u_{2,0})$, $(u'_{0,0},u_{3,0})$, $(u_{3,0},u_{4,0})$, $(u''_{0,0},u_{5,0})$, $(u_{5,0},u_{6,0})$ and $(u_{6,0},u_{0,0})$ are omitted since the associated weights are zero.

Starting from node $u_{0,0}$ implies that nodes $u'_{0,0}$ and $u''_{0,0}$ are automatically included in the cycle and the subgraph G'_0 of Figure 9 is derived where nodes $u_{3,0}$, $u_{5,0}$ and $u_{6,0}$ and their related arcs dont' appear

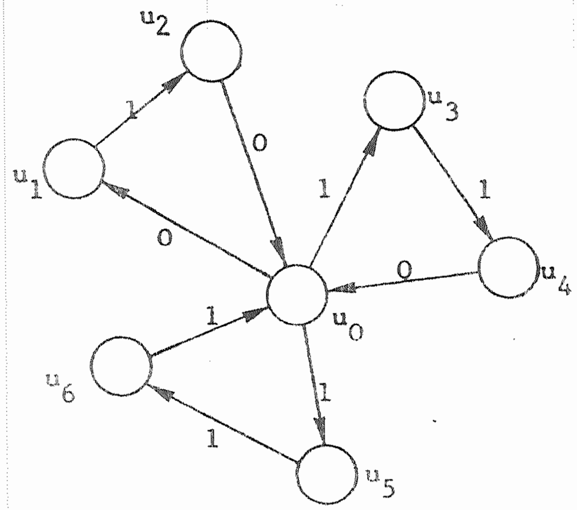

Figure 7

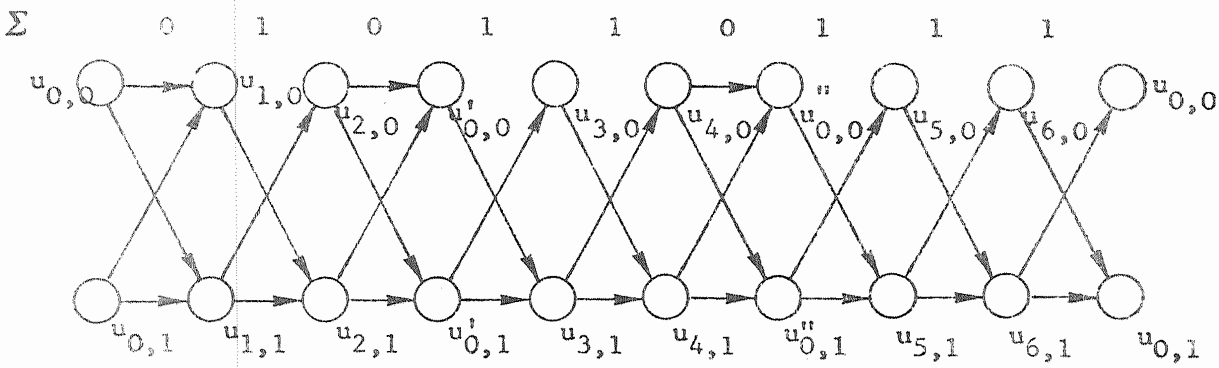

Figure 8

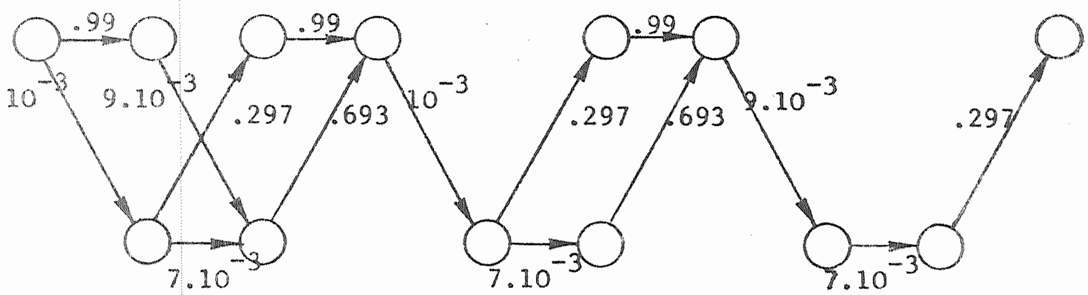

Figure 9

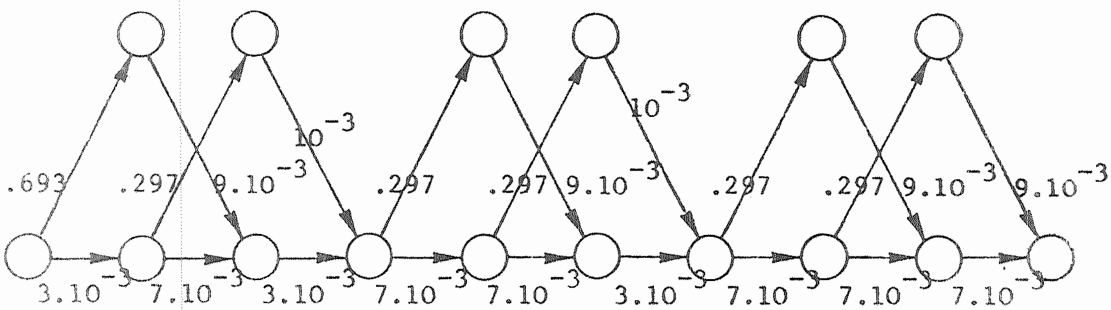

Figure 10

since they would have no incoming or outcoming arcs in G_0'. The weights of the remaining nodes are:

$\lambda(u_{1,0}) = 0.99$ $\lambda(u_{1,1}) = 10^{-3}$
$\lambda(u_{2,0}) = 0.297.10^{-3}$ $\lambda(u_{2,1}) = 8.91.10^{-3}$
$\lambda(u_{0,0}') = 6.17463.10^{-3}$
$\lambda(u_{3,1}) = 55.57167.10^{-6}$
$\lambda(u_{4,0}) = 16.504785.10^{-6}$ $\lambda(u_{4,1}) = 389.00169.10^{-9}$
$\lambda(u_{0,0}'') = 16.339737.10^{-6}$
$\lambda(u_{5,1}) = 147.05763.10^{-9}$
$\lambda(u_{6,1}) = 1.0294034.10^{-9}$
$\lambda_0 = 0.3057328.10^{-9}$

Repeating to find λ_1 it is obtained subgraph G_1' (Figure 10) and the corresponding λ's listed below:

$\lambda(u_{1,0}) = 0.693$ $\lambda(u_{1,1}) = 3.10^{-3}$
$\lambda(u_{2,0}) = 0.891.10^{-3}$ $\lambda(u_{2,1}) = 6.237.10^{-3}$
$\lambda(u_{0,1}') = 18.711.10^{-6}$
$\lambda(u_{3,0}) = 5.557167.10^{-6}$ $\lambda(u_{3,1}) = 130.977.10^{-9}$
$\lambda(u_{4,0}) = 38.900169.10^{-9}$ $\lambda(u_{4,1}) = 50.014503.10^{-9}$
$\lambda(u_{0,1}'') = 150.0435.10^{-12}$
$\lambda(u_{5,0}) = 44.562919.10^{-12}$ $\lambda(u_{5,1}) = 1.0503045.10^{-12}$
$\lambda(u_{6,0}) = 0.31194043.10^{-12}$ $\lambda(u_{6,1}) = 401.06627.10^{-15}$
$\lambda_1 = 2.8074636.10^{-15}$

and $P_\Sigma = \lambda_0 = 0.30573228.10^{-9}$. Procedure 1 is concluded by determining status variables x_i which result $x_0=0$, $x_6=1$, $x_5=1$, $x_4=0$, $x_3=1$, $x_2=1$ and $x_1=0$.

6. AN APPROACH TO P-DIAGNOSABILITY WITH REPAIR

As it has been shown in preceding Section, the probability of the most likely fault pattern is strongly dependent on the status of unit u_0 or, more generally, of any unit u_i, $i \in \{0,\ldots,n-1\}$. Consider, for instance, Example 2 and let $F(x_0=0)$, $F(x_0=1)$ represent the fault patterns of highest probability in the hypothesis of unit u_0 fault-free or faulty, respectively. Then

$$P\{F(x_0=0)|\Sigma\} = \lambda_0/P\{\Sigma\}$$
$$P\{F(x_0=1)|\Sigma\} = \lambda_1/P\{\Sigma\}$$

and

$$P\{F(x_0=0)|\Sigma\}/P\{F(x_0=1)|\Sigma\} = \frac{\lambda_0}{\lambda_1} = 108899 \simeq 10^5$$

More generally, for a syndrome Σ and any unit u_i, $i \in \{0,\ldots,n-1\}$ the fraction $f_i(\Sigma)=P\{F(x_i=1)|\Sigma\}/P\{F(x_i=0)|\Sigma\}$ is a measure of probability of unit u_i being faulty when syndrome Σ is present. Computing $f_i(\Sigma)$ for each unit of system S and letting

$$f(\Sigma) = \max_{i \in \{0,\ldots,n-1\}} \{f_i(\Sigma)\} \qquad (6)$$

enables a definition of probabilistic diagnosis with repair.

Definition 3. For any syndrome Σ, p-diagnosis with repair is performed if $f(\Sigma) > L$, where L is an appropriate threshold.
It is to be observed that in non probabilistic models, diagnosis with repair implies $f(\Sigma)=\infty$ for all Σ's having at least one non-zero test outcome; in fact, under the condition of a number of faulty units not exceeding the diagnosability with repair, there exists at least a unit belonging to all fault pattern which are consistent with Σ .

Similarly, high values of L in p-diagnosis guarantee against erroneous repair action.

Probabilistic diagnosis with repair can be given a different formulation observing that if a unit u_i is found for which $f_i(\Sigma) > L$, diagnosis is guaranteed.

Example 3. Given the diagnostic graph with the syndrome Σ of Figure 11, compute $f_i(\Sigma)$ for i=0,...,3. Unit failure probabilities and test

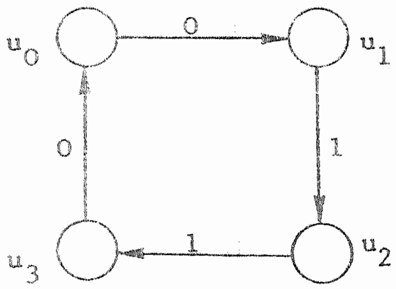

Figure 11

parameters are assumed to be the same of preceding examples.

Repeated applications of Procedure 1 give

$f_0(\Sigma) = 0.7070.10^{-3}$

$f_1(\Sigma) = 0.7935.10^{-3}$

$f_2(\Sigma) = 1.4143.10^{-3}$

$f_3(\Sigma) = 16.50.10^{-3}$

and $f(\Sigma)=f_2(\Sigma)=1.4143.10^3$ i.e., the probability of u_2 being faulty exceeds 10^3 times the probability of the reverse hypothesis.

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