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Collective Oscillations of Two Colliding Bose-Einstein Condensates

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Two ^{87}Rb condensates ($F = 2$, $m_f = 2$, and $m_f = 1$) are produced in highly displaced harmonic traps and the collective dynamical behavior is investigated. The mutual interaction between the two condensates is evidenced in the center-of-mass oscillations as a frequency shift of 6.4(3)%. Calculations based on a mean-field theory well describe the observed effects of periodical collisions both on the center-of-mass motion and on the shape oscillations.

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Since the first realization of Bose-Einstein condensation with dilute trapped gases [1], systems of condensates in different internal states have deserved attention as mixtures of quantum fluids. In this context, the important issue of the interaction between two distinct condensates was early addressed at JILA [2] with the production of two ^{87}Rb condensates in the hyperfine levels $|F = 2, m_f = 2\rangle$ and $|1, -1\rangle$ in a Ioffe-type trap. Subsequent experiments of the JILA group have focused on the dynamics of two condensates in the states $|2, 1\rangle$ and $|1, -1\rangle$ having nearly the same magnetic moment, confined by a time-orbiting potential (TOP) trap. In these experiments [3–5] the authors have investigated the effects of the mutual interaction in a situation of almost complete spatial overlap of the two condensates. The resulting dynamics reveals a complex structure, and it is characterized by a strong damping of the relative motion of the two condensates [3]. More recently another group [6] has experimentally investigated a mixture of ^{87}Rb condensates in different m_f states in a TOP trap, but no effects of the mutual interaction have been observed.

These experiments have raised several interesting questions from the theoretical point of view, such as the origin of the above mentioned damping [7], the phase coherence properties of the two condensates [8], and the nature of Rabi oscillations in the presence of an external coupling [9], thus providing a challenge to explore new dynamical regimes in different experimental configurations.

In this work we demonstrate an experimental method for a sensitive and precise investigation of the interaction between two condensates. These are made to collide after periods of spatially separated evolution and we get quantitative information from the resulting collective dynamics. Following the scheme originally introduced by Ketterle and co-workers [10] to create an atom laser out-coupler, we use a radio-frequency (rf) pulse to produce two ^{87}Rb condensates in the states $|2, 2\rangle \equiv |2\rangle$ and $|2, 1\rangle \equiv |1\rangle$. Because of the different magnetic moments and the effect of gravity, they are trapped in two potentials whose minima are displaced along the vertical y axis by a distance much larger than the initial size of each condensate. As a consequence the $|1\rangle$ condensate, initially created in the equilibrium position of $|2\rangle$, undergoes large center-of-mass oscillations, in a regime very different from that explored in [3] and analyzed in [7], where the two condensates sit in overlapping traps. The fact that the two condensates periodically collide opens the possibility of detecting even small interactions through changes in frequency and amplitude of the oscillations. Indeed, the periodic collisions of the $|1\rangle$ condensate with the $|2\rangle$, initially remaining almost at rest, strongly affect the collective excitations of both condensates: (i) the center-of-mass oscillation frequency of the $|1\rangle$ condensate is shifted upwards and (ii) the shape oscillations of the $|2\rangle$ condensate, triggered by the sudden transfer to the $|1\rangle$ state, are significantly enhanced.

The complex dynamics is quantitatively analyzed and found in agreement with the theoretical predictions derived

by the numerical solution of two coupled Gross-Pitaevskii (GP) equations at zero temperature.

We prepare a condensate of typically 1.5×10^5 ^{87}Rb atoms in the $F = 2, m_f = 2$ hyperfine level ($|2\rangle$), confined in a 4-coils Ioffe-Pritchard trap elongated along the z symmetry axis [11]. The axial and radial frequencies for the $|2\rangle$ state, measured after inducing center-of-mass oscillations of the condensate, are $\omega_{z2} = 2\pi \times 12.6(2)$ Hz and $\omega_{\perp 2} = 2\pi \times 164.5(5)$ Hz, respectively, with a magnetic field minimum of 1.75 Gauss. By applying a rf pulse, the initial $|2\rangle$ condensate is put into a coherent superposition of different Zeeman $|m_f\rangle$ sublevels of the $F = 2$ state, which then move apart: $|2\rangle$ and $|1\rangle$ are low-field seeking states and stay trapped, $|0\rangle$ is untrapped and falls freely under gravity, while $|-1\rangle$ and $|-2\rangle$ are high-field seeking states repelled from the trap. All the condensates in different Zeeman states are simultaneously imaged by absorption with a $150 \mu\text{s}$ pulse of light resonant on the $F = 2 \rightarrow F' = 3$ transition, shone 30 ms after the switching off of the trap. By fixing the duration and varying the amplitude of the rf field, we control the relative population in different Zeeman sublevels [12]. A 10 cycles rf pulse at 1.24 MHz with an amplitude $B = 10$ mG quickly transfers $\sim 13\%$ of the atoms to the $|1\rangle$ state without populating the $|0\rangle$, $|-1\rangle$ and $|-2\rangle$ states. The $|1\rangle$ condensate experiences a trapping potential with lower axial and radial frequencies ($\omega_1 = \omega_2/\sqrt{2}$) and displaced along the vertical y axis by $y_0 = g/\omega_{\perp 2}^2 \approx 9.2 \mu\text{m}$. After the rf pulse, the $|1\rangle$ condensate moves apart from $|2\rangle$ and begins to oscillate around its equilibrium position. Because of the mutual repulsive interaction, the latter starts oscillating, too, though with a much smaller amplitude. The resulting periodic superimposition modifies the effective potential, which is the sum of the external potential (magnetic and gravitational) and the mean-field one. We have studied the dynamics of the $|1\rangle$ condensate in the presence (“interacting” case) and in the absence (“noninteracting” case) of $|2\rangle$, by varying the permanence time in the trap. We restrict ourselves to permanence times so short, i.e., less than 40 ms, that we can neglect both atom losses due to the condensate finite lifetime, $0.7(1)$ s, and the heating [$dT/dt = 0.11(2) \mu\text{K/s}$]. For the noninteracting case, we have used a stronger rf pulse ($B = 20$ MG) that completely empties the $|2\rangle$ state coupling all the atoms in $|1\rangle$ and in the other untrapped Zeeman states, which rapidly leave the trap.

We start considering the case of the $|1\rangle$ condensate in the absence of the $|2\rangle$ condensate (non-interacting case). In Fig. 1a we plot the center-of-mass oscillations as a function of the trapped evolution time in units of $\omega_{\perp 2}^{-1} \approx 0.967$ ms, after 30 ms of ballistic expansion. The center of mass undergoes sinusoidal oscillation with a measured frequency of $\omega_{\perp 1} = 2\pi \times 116.4(3)$ Hz. This value is, as expected, a factor of $\sqrt{2}$ smaller than $\omega_{\perp 2}$. Furthermore, the experimental data give no evidence of damping within the maximum monitored trapping time (26 ms). Figure 1b

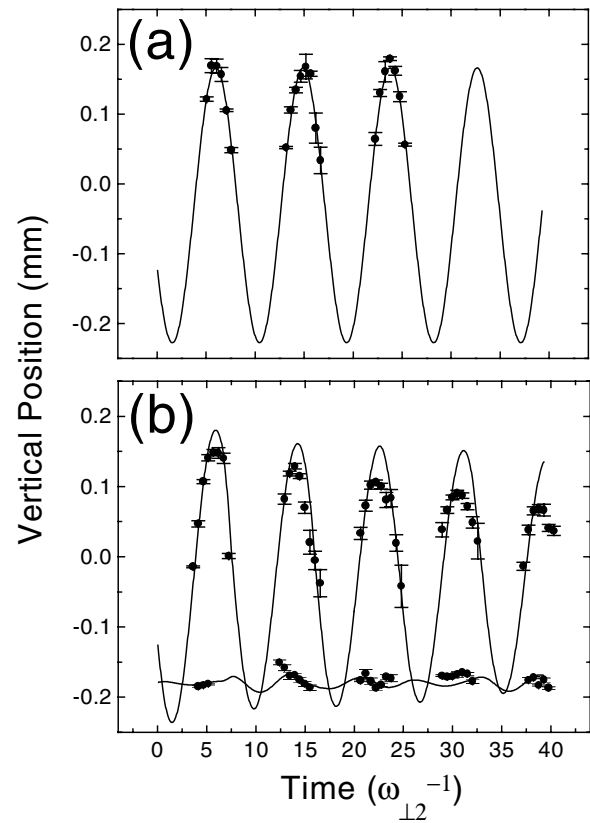


FIG. 1. Center-of-mass oscillations as a function of the trapped evolution time (rf pulse at $t = 0$) after 30 ms of ballistic expansion for (a) the noninteracting $|1\rangle$ condensate and (b) the two interacting condensates. The experimental points are compared with the relative theoretical predictions (discussed in the text). Each data point is the average of typically five measurements.

shows the center-of-mass evolution of the two condensates in the interacting case. The center of mass of the $|1\rangle$ condensate exhibits a substantially different behavior, which allows a clear quantitative analysis. The oscillation frequency is up-shifted to $\omega_{\perp 1} = 2\pi \times 123.9(3)$ Hz, i.e., a 6.4(3)% change with respect to the non-interacting case. This can be understood considering that the mean-field repulsion of the second condensate produces an anharmonic correction to the effective potential experienced by the $|1\rangle$ condensate. Furthermore, the oscillations appear now damped with an exponential decay time of about 60 ms. Indeed, each time the two condensates superimpose (about every 8 ms), there is an energy transfer from the $|1\rangle$ condensate toward the $|2\rangle$ condensate. As a consequence we expect, and we do observe, effects on both the center-of-mass motion and the collective shape-oscillations of the $|2\rangle$ condensate.

A quantitative description of these experimental features requires the solution of two coupled Gross-Pitaevskii equations. Neglecting the interaction with the thermal cloud, which is so dilute that it cannot be detected in our experiment, the two condensates evolve according to

$$i\hbar \frac{\partial \Psi_i}{\partial t} = \left[-\frac{\hbar^2 \nabla^2}{2m} + V_i + \sum_{j=1,2} \frac{4\pi \hbar^2 a_{ij}}{m} |\Psi_j|^2 \right] \Psi_i, \quad (1)$$

where $i = 1, 2$, and V_i are the trapping potentials:

$$V_1(x, y, z) = \frac{m}{2} \omega_{\perp 1}^2 [(x^2 + y^2) + \lambda^2 z^2], \quad (2)$$

$$V_2(x, y, z) = \frac{m}{2} \omega_{\perp 2}^2 \{x^2 + (y - y_0)^2\} + \lambda^2 z^2, \quad (3)$$

and the asymmetry parameter is $\lambda \equiv \omega_z / \omega_{\perp} \approx 0.0766$ for both traps. For the ^{87}Rb scattering lengths we use $a_{22} = a_{12} = 98a_0$ and $a_{11} = 94.8a_0$ [13].

Our experimental configuration allows us to simplify these equations by using the fact that we are in the Thomas-Fermi (TF) regime due to the large number of atoms, and that the system is strongly elongated along the z axis, $\lambda \ll 1$ [14]. For an elliptic trap, the low-frequency excitations with $m = 0$ are linear superpositions of the monopole ($n = 1, l = 0, m = 0$) and quadrupole ($n = 0, l = 2, m = 0$) modes [15,16]. The dispersion laws for the two modes, at leading order in λ , are given by

$$\omega_{\text{high}} \approx 2\omega_{\perp}; \quad \omega_{\text{low}} \approx \sqrt{\frac{5}{2}} \lambda \omega_{\perp}. \quad (4)$$

In this limit the two frequencies are quite different, and the axial and radial collective excitations are almost decoupled. The radial width is characterized by small-amplitude oscillations with frequency ω_{low} modulated by a large-amplitude oscillation with frequency ω_{high} , and vice versa for the axial width [17]. Therefore, since the interactions mostly affect the radial motion, we assume the axial dynamics to be still characterized by the low-frequency oscillations of the TF regime. Then, we study the trapped dynamics in the x, y plane by solving the GP equation (1), by approximating our system as a uniform cylinder [18].

We start from an initial configuration corresponding to the stationary ground state of Eq. (1), with all the N trapped atoms in the $|2\rangle$ condensate. Afterwards, at $t = 0$, N_1 atoms are instantaneously transferred from the $|2\rangle$ to the $|1\rangle$ state, the former remaining with $N_2 = N - N_1$ atoms. Here we consider $N_1 = 0.13N$ for the interacting case, and $N_1 = N$ for the noninteracting one.

The theoretical curves in Fig. 1 show an up-shift of the center-of-mass oscillation frequency for the $|1\rangle$ condensate occurring in the interacting case, as experimentally observed. This shift is of 5.4%, in qualitative agreement with that measured. Furthermore, in the presence of interactions the model predicts a damping, which is not due to any dissipative process (the total energy is conserved), but to a transfer of energy from the center-of-mass oscillations of the $|1\rangle$ condensate to the other degrees of freedom of the system [21]. Still, this damping time is nearly a factor of 2 longer than that experimentally observed.

To understand the origin of the discrepancies between theory and experiment, it is worth discussing the main approximations of our model. First, since the model is basically two dimensional, the energy transfer in the axial direction is overlooked. Second, we completely disregard the interaction between the two condensates during the expansion [22]. Indeed, in our experiment, during the switching off of the trap the two clouds acquire different velocities and cross each other in the fall, but we expect that they are so dilute that we can neglect their mutual mean-field repulsion. Finally, our model does not take into account the elastic scattering, occurring when the relative velocity of the two condensates exceeds the sound velocity [23]. This effect, whose description lies beyond the mean-field approximation, represents an important channel of both atoms and energy losses which plays a significant role, for example, in the four-wave mixing experiments with Bose condensates [24,25].

We consider now the $|2\rangle$ condensate. In the interacting case, both small center-of-mass oscillations and significant features for the aspect ratio oscillations emerge from our model. The latter are initially induced by the sudden change in the internal energy, consequent on the transfer of N_1 atoms from the $|2\rangle$ to the $|1\rangle$ level [10,26]. In Fig. 2 we compare the theoretical evolution of the $|2\rangle$ condensate aspect ratio, i.e., the radial to axial width ratio, in the absence and in the presence of collisions. In the former case, a faster (radial) oscillation superimposes to a slow (axial) one as a consequence of the decoupling between the two oscillation modes. The frequencies ω_{high} and ω_{low} were separately measured by means of resonant modulation of the trapping magnetic field [11].

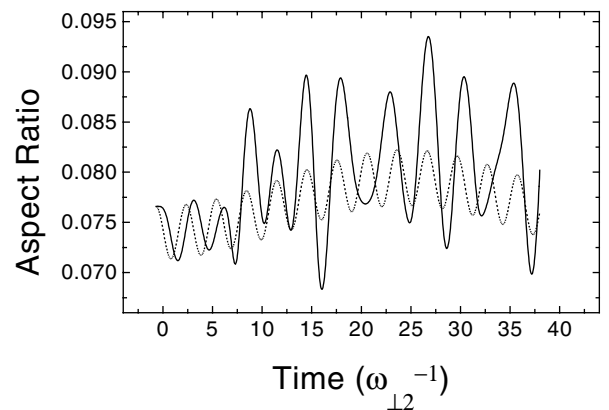


FIG. 2. Evolution of the aspect ratio for the $|2\rangle$ condensate in the trap obtained solving the GP equations for the interacting (solid line) and noninteracting cases (dashed line). In the interacting case the evolution of the aspect ratio is not a simple superposition of the two oscillations with frequency ω_{low} and ω_{high} , as in the noninteracting case. We note also that the interaction between the two condensates produces an initial delay for the onset of collective excitations, since the two condensates are created with the same density profile, and the scattering lengths a_{22} and a_{12} have the same value.

The noninteracting behavior predicted in Fig. 2 should of course yield when there is only one trapped state. For instance, this is the case of an atom laser outcoupled in a single-step transition, as for sodium [10], or for rubidium in $F = 1$ state [27]. In the interacting case, we see from Fig. 2 that a significantly different behavior is expected. Before the first collision at $t \approx 8$ ms, the oscillations are rather small since they are determined only by the sudden change in the number of atoms. Instead, at longer times, the changes in the aspect ratio become more pronounced due to the energy transfer during collisions between the two condensates. Once the ballistic expansion is taken into account, the simulation results are compared to the experimental data in Fig. 3. The agreement is only of a qualitative character. We attribute the discrepancy to the approximations we used in our simplified analysis. Nevertheless, we believe that our model provides a useful physical insight of the relevant aspects of the problem.

In conclusion, we have developed a powerful tool for the investigation of the interactions between condensates and we have demonstrated how they can quantitatively affect frequency, amplitude, and shape of oscillations. In particular, the frequency shift measurement gives access to the number of atoms in the parent condensate N_2 or, alternatively, to the a_{21} scattering length. This opportunity seems particularly interesting for low N_2 , as we have found that the frequency shift scales roughly as $\log(N_2 a_{12})$ [14].

The agreement with the model, given its simplicity and lack of free parameters, is generally satisfactory. Furthermore, the discrepancy observed for the damping of center-of-mass oscillations suggests the investigation of the relaxation beyond the mean-field theory (namely, by including the elastic scattering [25]) as an interesting subject for future studies. The experimental perspectives of this system of two periodically overlapping condensates lead to the investigation of the interference between condensates

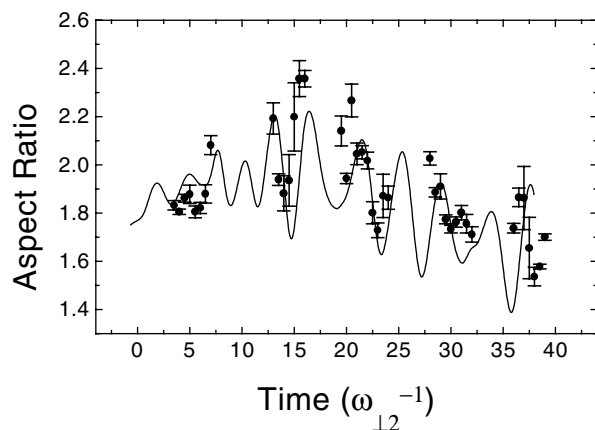


FIG. 3. Aspect ratio for the $|2\rangle$ condensate, as a function of the trapped evolution time, after 30 ms of ballistic expansion. The experimental points are compared with the theoretical predictions for the case of two interacting condensates.

spatially separated during their evolution and, eventually, to studies of Josephson effects in two weakly linked condensates [28].

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