LONG TERM ORBIT EVOLUTION OF THE UNCONTROLLED SIRIO SATELLITE

A. Agneni (*)
A. Cardillo (+)
A. Foni (+)
C. Ulivieri (*)

Nota interna C84-15

Pisa, Ottobre 1984

(+) CNUCE Istituto del CNR
(*) Dipartimento Aerospaziale, Universita' di Roma "La Sapienza"

ABSTRACT

The aim of this study is to investigate the long term evolution of the orbital inclination of SIRIO satellite. In fact, since 1983, SIRIO orbit inclination can no longer be controlled because of the scarce quantity of fuel remaining on board. While its attitude and longitude are still controlled, the orbit inclination is changing according to the environmental perturbing forces acting on the satellite. To evaluate this long term effect a simplified model is given. A comparison has been made with results obtained using more precise, but time consuming computer programs.

The method presented is applicable to any near-geostationary orbit.

LIST OF CONTENTS.

1. SIRIO mission.

SIRIO is a geostationary spin stabilized experimental satellite. It was successfully launched on 27 August 1977 from Cape Canaveral by means of a Thor Delta vehicle under a NASA-CNR(the Italian National Research Council) contract. Launch and flight support was provided by NASA until the satellite on-station positioning; after that the operative control was transferred to the Telespazio ground station located at Fucino which is still actively monitoring and controlling the satellite.

Main objective of SIRIO mission has been to perform propagation and telecommunication experiments on the 12-17 GHz frequencies; the spacecraft was designed for a minimum lifetime of two years. Its nominal station point at the beginning of the mission was at 15 $\pm$ 1 deg. of longitude west and the orbital inclination was costrained to remain below 0.3 degrees.

Within the above limits, active station-keeping control has been performed through March 1983 (ref. 1-2). At that time the experimental extended campaign was completed and, being the spacecraft still well performing, it was decided to accept a request of the People's Republic of China for performing further transmission experiments between Beijing and Fucino. The on-board remaining fuel was supposed to be sufficient for moving the satellite to the new station point located at 65 deg. of longitude East. The satellite was successfully positioned at the new station point on May 19, 1983.

On 11 July 1983, at completion of a north-south maneuver, the tank system B run out of fuel and since that time no more orbit inclination corrections have been performed. The remaining fuel in the tank system A is being used for the attitude and east-west maneuvers.

2. Considerations on the orbit evolution of uncontrolled satellite SIRIO.

The occurrence of the fuel depletion of one tank system for SIRIO active inclination control, coincided with the beginning of the natural drifting of the orbit plane caused by the only environmental forces acting on the satellite; in fact the orbit inclination variation due to the jets thrusters during east-west and attitude maneuvers, can be considered negligible. At the time of on-board complete fuel depletion (which is foreseen about the 1986), even the S/C longitude will initiate its free drifting. In such condition a question arises : what will happen to the SIRIO orbit in the future?
And then : what can be exected by knowing in advance the trajectory of the uncontrolled satellite?

As better explained in the next paragraphs, the satellite will pass from the same point at almost regular time intervals and by predicting the passage time, the tracking of the satellite makes allowance for evaluating and eventually adjusting the models used for prediction. It should be noted that the satellite at time will be a "passive object" and its tracking should be accomplished by

using methods such as optical devices or radars.
Further helpful use of trajectory prediction is the evaluation of collision probability between satellites controlled or uncontrolled, crossing the same segments of the geostationary orbit.

The selection of the tools for predicting the long term orbit evolution requires further considerations :

- the forces perturbing the state of a geostationary satellite are well known, and therefore it is always possible, in principle, to integrate numerically the spacecraft motion equation respect to time;

- methods for the above mentioned integration are usually used for orbit prediction during the operational life of a satellite. They have been proved to be very precise for short-medium term propagations (1-6 months), but on the other hand they require very expensive computer resources;

- the physics and the geometry of the S/C orbit perturbing forces can be analitically modelled. For effortable models(from the complexity point of view), the required computing resources are not relevant;

- being the first geostationary satellite launched on 1963, no experimental results are available for evaluating the performances of orbital propagators over several tenths of years.

After the above considerations, it has been decided, for studing the SIRIO orbit evolution in the next forty years and more, to realize a quick analytical tool for orbit prediction. Furthermore a comparative analysis follows between the analytically and the numerically obtained results.

2.1 Initial conditions of the study.

The last maneuver for SIRIO orbit inclination correction, was executed on July.11, 1983. The final orbit achieved with the maneuver was:

Osculating Keplerian orbital elements :

SMA	42164.948	Km.
ECC	.000233	
INC	.328	deg.
AN	94.067	"
AP	64.974	"
MA	120.153	"

Epoch : 7/11/1983 19hh 2mm 5ss

Since that time the orbital plane was left drifting according to the environmental perturbations, therefore the above listed parameters were considered as starting point of the present study.

2.2 The orbit perturbations.

Three principal perturbative effects are present in the long term evolution of a geostationary satellite.

Earth's oblateness and gravitation attractions of the Sun and the Moon cause a long periodic motion (about 53 years) of the orbital polar vector.

A longitude drift motion coupled with oscillations in semi-major axis is due to the resonance situation arising from the commensurability of the satellite angular motion with the Earth's rotation rate. The period of this pendulum like oscillation around the nearest stable point, is about 820 days.

At last the solar radiation pressure and, at a lesser degree, the lunar parallatic term, move the eccentricity vector in an almost closed loop over the period of one year.

With a good approximation these effects may be treated independently; in fact the eccentricity of a geostationary orbit is close to zero (circular orbit) and the second harmonic of the terrestrial gravitational field has no secular effect on the eccentricity of a circular orbit. Furthermore only the higher harmonics of the lunar gravitational field, which become more important as the orbital radius is greater than 10 Earth's radii, change the eccentricity of a circular orbit. In the light of these considerations the semi-major axis and the eccentricity can be considered constant in this study, which has the scope of investigating the first effect, i.e. the long term evolution of the orbital pole for a geostationary satellite.

3. The model description.

3.1 Equation of motion and disturbing function.

The temporal evolution of the orbital plane of an uncontrolled circular geostationary satellite is governed by the Lagrange equations, which take the vectorial form (ref. 3):

$$\dot{\vec{K}} = \vec{K} \times \vec{\nabla}_{\vec{K}} R^{*}(\vec{K}) \qquad (1)$$

where $\vec{K}$ is the unit vector associated with the orbital pole (along the angular momentum of the satellite), $R^{*}(\vec{K})$ is a function related to disturbing (or perturbing) function R according to the relationship

$$R^{*} = R/(na^{2})$$

(n , the satellite mean motion, and a , its semimajor axis are constant in the long period motion) and the vectorial operator $\vec{\nabla}_{\vec{K}}$ represents the differentiation with respect to vector $\vec{K}$, (ref. 4).

Using this vectorial approach it is simple to express the disturbing function, which includes the second zonal harmonic of the Earth's gravitational field and the lowest order term of the gravitational attraction of the Sun and Moon, in terms of $\vec{K}$ and then to esplicitate $\vec{\nabla}_{\vec{K}} R^{*}$ in (1).

The perturbing effect on a terrestrial satellite P due to the gravitational effect of a third body m_j (fig. 1) is given by :

$$R_j = \mu_j \left(\frac{1}{|\vec{r} - \vec{r}_j|} - \frac{\vec{r} \cdot \vec{r}_j}{r_j^3} \right) \tag{2}$$

where $\vec{r}$, $\vec{r}_j$ indicate the positions of the satellite and the disturbing body relative to the center of the Earth O and μ_j is the body gravitational constant.

From fig.1 it results that:

$$|\vec{r} - \vec{r}_j| = (r^2 + r_j^2 - 2 r r_j \cos \xi_j)^{\frac{1}{2}} \tag{3}$$

Since $r \ll r_j$ we can retain only the lowest-order terms of the expansion of $|r - r_j|^{-1}$ in a series of Legendre polynomials and obtain, subtracting the term $\frac{\mu_j}{r_j}$ the perturbing function

$$R_j = \frac{1}{2} n_j^2 r^2 (3 \cos^2 \xi_j - 1) \tag{4}$$

where $n_j = \left(\frac{\mu_j}{r_j^3} \right)^{\frac{1}{2}}$ is the mean motion of the disturbing body with respect to the Earth.

Since the predicted long period motion of the orbital plane should be reasonably long, compared with the periods of the disturbing bodies (Sun and Moon), it is permissible to average R over the mean anomalies of the satellite and of the bodies. In this approach the satellite-daily, lunar-monthly and solar-yearly fluctuations will be omitted.

If γ_j and Ω_j indicate the position of the satellite orbital pole respect to the orbital plane of the j-th perturbing body (fig.2), we have from the spherical triangle $N_j P P_j$

$$\cos \xi_j = \cos v \cos(v_j - \Omega_j) + \sin v \sin(v_j - \Omega_j) \cos \gamma_j \qquad (5)$$

where v and v_j are anomalies measured for the satellite from the ascending nodal crossing N_j and for the body from the X_j-axis.

Squaring (5) and averaging over v and v_j it results

$$\overline{\overline{\cos^2 \xi_j}} = \frac{1}{4}\left[1 + (\vec{K}\cdot\vec{Z}_j)^2\right] \qquad (6)$$

and, being r=a= constant , eq. (4) becomes

$$R_j = \frac{1}{8} n_j^2 a^2 \left[3(\vec{K}\cdot\vec{Z}_j)^2 - 1\right]$$

Including the effect of the second harmonic in the Earth's gravitational field, the complete double-averaged disturbing function may be written

$$R = na^2\left[\frac{1}{2}\sum_{0}^{2} {}_j\, \omega_j (\vec{K}\cdot\vec{Z}_j)^2 - \sum_{0}^{2} {}_j\, \frac{\omega_j}{6}\right] \qquad (7)$$

where $\vec{Z}_0$, $\vec{Z}_1$ and $\vec{Z}_2$ are the unit vectors normal to the equatorial plane and to the solar and lunar orbital plane respectively and

$$\omega_0 = \frac{3}{2} n \frac{J_2 R_{\oplus}^2}{a^2} \quad ; \quad \omega_j = \frac{3}{4}\frac{n_j^2}{n} \quad j = 1,2$$

where J_2 is the second zonal harmonic in the Earth's gravitational field and $R_{\oplus}$ is the mean equatorial radius.

For a geostationary satellite it results ω_0=4.89 , ω_1=0.738, ω_2=1.611 degs/year .

Substituting the obtained double averaged expression (7) in (1) we obtain:

$$\dot{\vec{K}} = \vec{K} \times \sum_{0j}^{2} \omega_j (\vec{K} \cdot \vec{Z}_j) \vec{Z}_j \qquad (8)$$

If only one of the term of the right-hand side of (8) were present, the unit vector $\vec{K}$ would simply regress around the corresponding $\vec{Z}_j$ at a constant rate $\omega_j (\vec{K} \cdot \vec{z}_j)$. Then the long term motion of the orbital pole of a geostationary satellite is a combination of regressions around three different axes. Whereas the solar apparent orbit describes a fixed plane (ecliptic plane), the Moon's orbital pole is itself regressing around the ecliptic with a period of 18.61 years at an almost constant inclination of i_m=5.145 deg. to the ecliptic. The longitude of the mean ascending node of the lunar orbit in the ecliptic from the mean equinox of date is given by:

$$\Omega_m = 259.183275 - 0.05295392 \, D \quad \text{(degrees)} \qquad (9)$$

being D the number of ephemeris days since 1900 January 0,12h Ephemeris Time (ref. 5).

3.2 Solutions for fixed positions of the lunar pole.

When the lunar pole is assumed fixed in the space, the simultaneous precessions are about fixed axes and eq.(8) can be solved exactly. Two integrals can be found for it; the first one gives the known condition that $\vec{K}$ is a unit vector whilst the second implies that R^* is a constant of the motion. In fact, if we consider the infinitesimal variation of R^* with respect to time, it results:

$$\frac{dR^*}{dt} = \vec{\nabla}_{\vec{K}} R^* \cdot \dot{\vec{K}} \tag{10}$$

being $R^* = R^*(\vec{K})$.

Remembering the expression (1) for $\dot{\vec{K}}$, we have:

$$\frac{dR^*}{dt} = \vec{\nabla}_{\vec{K}} R^* \cdot \vec{K} \times \vec{\nabla}_{\vec{K}} R^* = 0$$

which, according to eq.(7), leads to:

$$\sum_{0}^{2}{}_{j}\, \omega_j \cdot (\vec{K} \cdot \vec{Z}_j)^2 = constant = L_0 \tag{11}$$

the value of L_0 depending upon the initial conditions of the satellite orbital pole.

Eq.(11), referred to a generical rectangular frame (e.g. the geocentric equatorial frame with Y-axis ≡ vernal equinox), represents a quadric surface, which can be reduced in canonical form by means of a matrix [T] that transforms the matrix [A], associated to the quadric, to diagonal form.

The columns of the orthogonal matrix $[T]$ are the eigenvectors relative to $[A]$. The eigenvalues l_i of $[A]$ are positive, then the quadric surface results an ellipsoid.

In the semplified case of lunar orbit coincident with the ecliptic (M≡E, fig.3), one of the eigenvalues is zero and the ellipsoid degenerates to an elliptic cylinder. In this case the matrix $[T]$ corresponds to a simple rotation δ around the Y-axis (vernal equinox) from ON (fig.3) and its numerical value, 7.38 degrees, is obtained from the relationship:

$$\delta = \frac{1}{2} \, tg^{-1} \left[\frac{(\omega_1 + \omega_2) \sin 2d}{\omega_0 + (\omega_1 + \omega_2) \cos 2d} \right] \qquad (12)$$

where d is the angle between the equatorial plane and the ecliptic. This particular frame is derived following a procedure first introduced by Laplace in order to explain the behaviour of the orbital plane of Iapetus (ref.6) and it will be considered hereinafter as inertial reference frame XYZ (fig.3).

The orbital pole $\vec{K}$ moves on a spherical ellipse, intersection of the ellipsoid (or elliptic cylinder) with the unit sphere and its temporal evolution can be completely defined by only one of the Jacobi's elliptic functions, that are solutions of the vectorial eq.(8).

The dimensions of the spherical ellipse depend upon the initial position of the orbital pole (i.e. orbital inclination i and longitude ascending node Ω, fig.3). In fact its semiaxes can be expressed by the following relationships :

$$S_{MAX} = \left(\frac{l_3 - L_0}{l_3 - l_2} \right)^{1/2} \quad ; \quad S_{MIN} = \left(\frac{l_3 - L_0}{l_3 - l_1} \right)^{1/2}$$

being l_1, l_2, l_3 the eigenvalues of the quadric surface arranged in ascending order and L defined in eq.(11). In our case L can assume values which satisfy the inequality:

$$l_1 < l_2 < L_o < l_3$$

When the initial conditions are such that $L_o = l_3$, the spherical ellipse reduces to the intersection point of the unit sphere with the eigenvector corresponding to l_3.

For any fixed position of the lunar pole, the time required for $\vec{K}$ to perform the motion on the spherical ellipse is given by a complete elliptic integral of the first kind. If we consider the approximation that the lunar orbit lies in the ecliptic, the estimated period for a geosynchronous satellite is about 52.9 years.

3.3 The general solution

For the long term orbital evolution of a satellite it is often assumed that the lunar orbit lies in the ecliptic or that the lunar orbital pole is fixed in a time-averaged position. Because of the significant errors introduced with this assumptions, it is necessary to take into account the lunar pole regression, which leads to a time-varying geometric configuration. In particular the principal axes ξ, η, ζ of the quadric, in its canonical form, rotate around the corresponding principal axes XYZ obtained when the lunar orbital plane coincides with the ecliptic (figs.3-4).

The ζ-axis motion around Z has a period equal to that one of lunar regression around the ecliptic pole at an inclination slightly greater than 1 deg. respect to Z. The

4.The numerical propagation.

In order to performe an accurate orbital element propagation the Ephem program of Goddard Trajectory Determination System (GTDS; version 3.5, May 80) has been used; this version, implemented at CNUCE-Pisa, is currently utilized for SIRIO orbit determination and propagation (ref. 7).

In the following we provide the characteristics of the coordinate systems chosen for the input, the output and the motion integration; we also describe briefly the force model and the orbital propagator.

COORDINATE REFERENCE SYSTEMS		
	INPUT/OUTPUT	MOTION EQUAT. INTEGRATION
ORBITAL ELEMENTS	KEPLERIAN	TRUE OF DATE
COORDINATE SYSTEM	RECTANG. CART. POS. AND VELOC.	INERTIAL-MEAN EQUAT. 1950.0
CENTRAL BODY	EARTH	EARTH

```
*-----------------------------------------------------------------*
I                         FORCE MODEL                              I
I------------------------------------------------------------------I
I                                                                  I
I 1) EARTH GRAVITY FIELD MONOPOLE TERM                             I
I                                                                  I
I 2) NO-SPHERICAL POTENTIAL OF THE EARTH: GEM21                    I
I   (MAX ORDER AND DEGREE = 5)                                     I
I                                                                  I
I 3) SUN AND MOON GRAVITATIONAL PERTURBATION                        I
I                                                                  I
I 4) SOLAR RADIATION PRESSURE (SPHERICAL HYPOTESIS; CROSS           I
I    SECTION = .1479E-5 KMSQ; MASS = 212.5 KG)                     I
*-----------------------------------------------------------------*
```

```
*-----------------------------------------------------------------*
I                          PROPAGATOR                              I
I------------------------------------------------------------------I
I                                                                  I
I ORBIT GENERETOR TYPE        NUMERICAL INTEGRATION OF COWELL      I
I                             EQUATION OF MOTION                   I
I                                                                  I
I NUMERICAL INTEGRATOR        12-TH ORDER SUMMED COWELL/ADAMS      I
I TYPE FOR STATE PROPA-       PREDICT-PARTIAL CORRECT              I
I GATION                                                           I
I                                                                  I
I INTEGRATION STEP MODE       FIXED                                I
I                                                                  I
I INTEGRATION STEP SIZE       800 SEC.                             I
*-----------------------------------------------------------------*
```

5. Comparison and conclusions.

The 40-year orbital pole evolution for SIRIO satellite, obtained by the simplified model, has been plotted along with the corresponding accurate numerical results (figs. 9-10). The achieved accuracy with the simplified model proposed in this paper is easily retrieved from these comparative curves. In fact, it is possible to evaluate a maximum deviation of 0.2 degrees for the orbital inclination (fig. 9) whilst the maximum difference for the longitude of ascending node is limited within 2 degrees (fig. 10).

An enlargment of the temporal i evolution, limited to the first five years, allows to assess in the GTDS curve the small perturbing solar-yearly fluctuation which has a period of six months (fig. 11). This effect cannot be present in the simplified model that assumes an averaged disturbing function (sec. 3.1).

6. References.

1) A. Foni, E. Letico, G. Pasquinelli : SIRIO in-orbit control, Proc. of the Symphonie International Symposium. Berlin 1980, pg. 567.

2) A. Cardillo, N. Celandroni, A. Foni, S. Trumpy : Evolution of the SIRIO in-orbit control strategy, Alta Frequenza vol XLIX - n.5, pg. 319.

3) R.R. Allan and G.E. Cook : The long-period motion of the plane of a distant circular orbit. Proc. of the Royal Society, A, vol 280, 1964, pp.97-109

4) T.R. Kane and P.W. Likins : Gravitational forces and moments on spacecraft, NASA CR-2618, October 1975.

5) J.R. Wertz Ed.: Spacecraft attitude determination and control, D. Reidel Publ. Co., Dordrecht, Holland, 1980.

6) P.S. Laplace : Mecanique Celeste, Tome IV, Livre VIII, Cap. XVI, pp. 173-189 , 1805.

7) W.E. Wagner and C. Velez, (editors) : Goddard Trajectory Determination System Mathematical Specifications, CSFC/NASA Report X-552-72-244, March 1972.

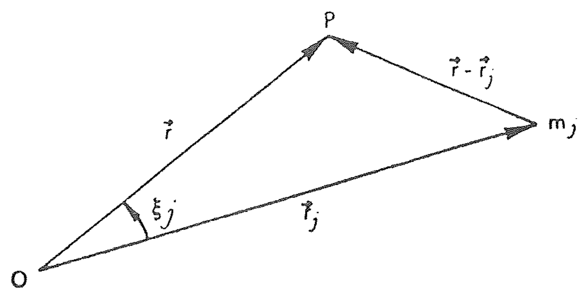

fig.1

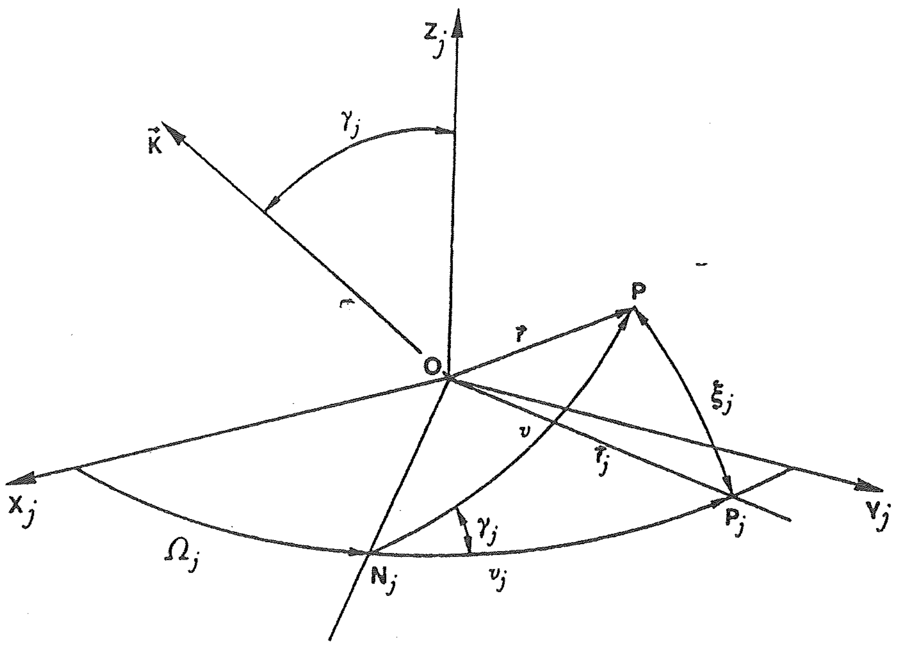

fig.2 Geometrical position of the satellite and of a generical perturbing body.

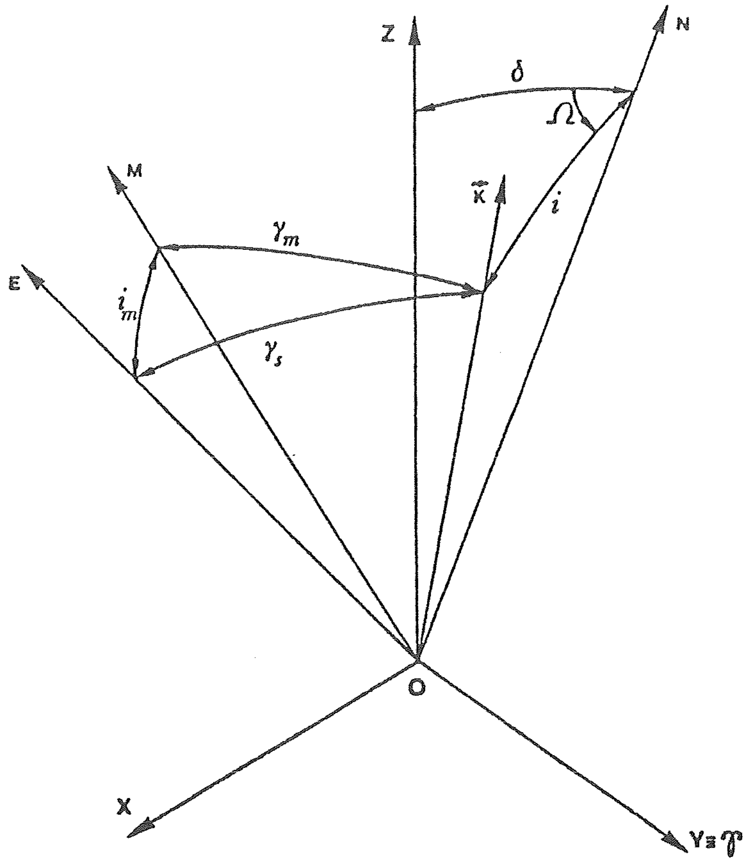

fig.3 Definition of the inertial reference frame X Y Z and position of the considered orbital poles. The angles are so defined: $i \equiv \gamma_0$, $\gamma_s \equiv \gamma_1$, $\gamma_m = \gamma_2$; the unit vectors relative to the directions ON, OE, and OM are $\vec{Z}_0$, $\vec{Z}_1$ and $\vec{Z}_2$ respectively.

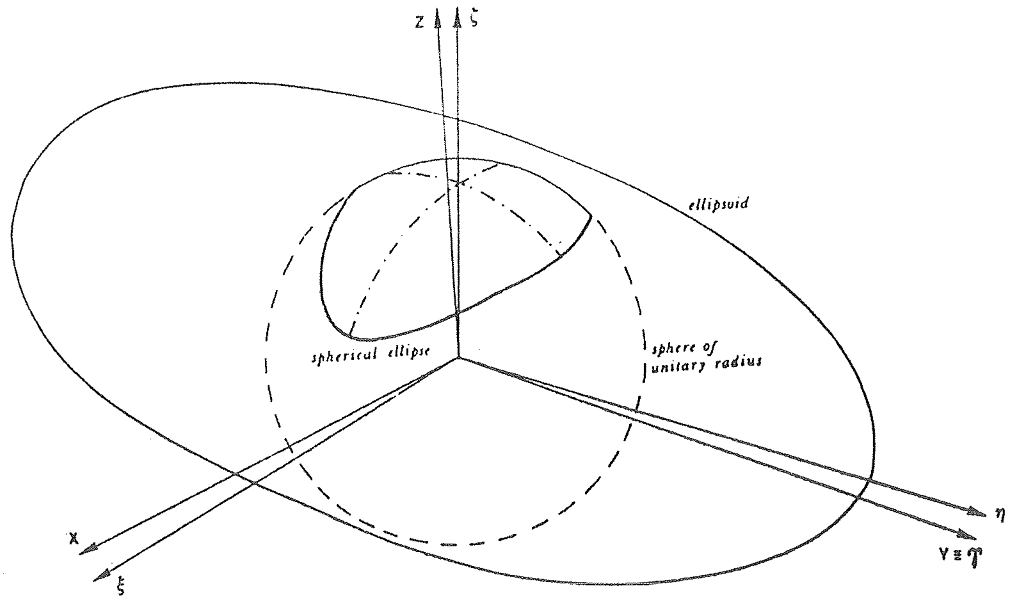

fig.4 Intersection of the ellipsoid with the unit sphere (spherical ellipse). Representation of the principal axes and of the reference frame X Y Z.

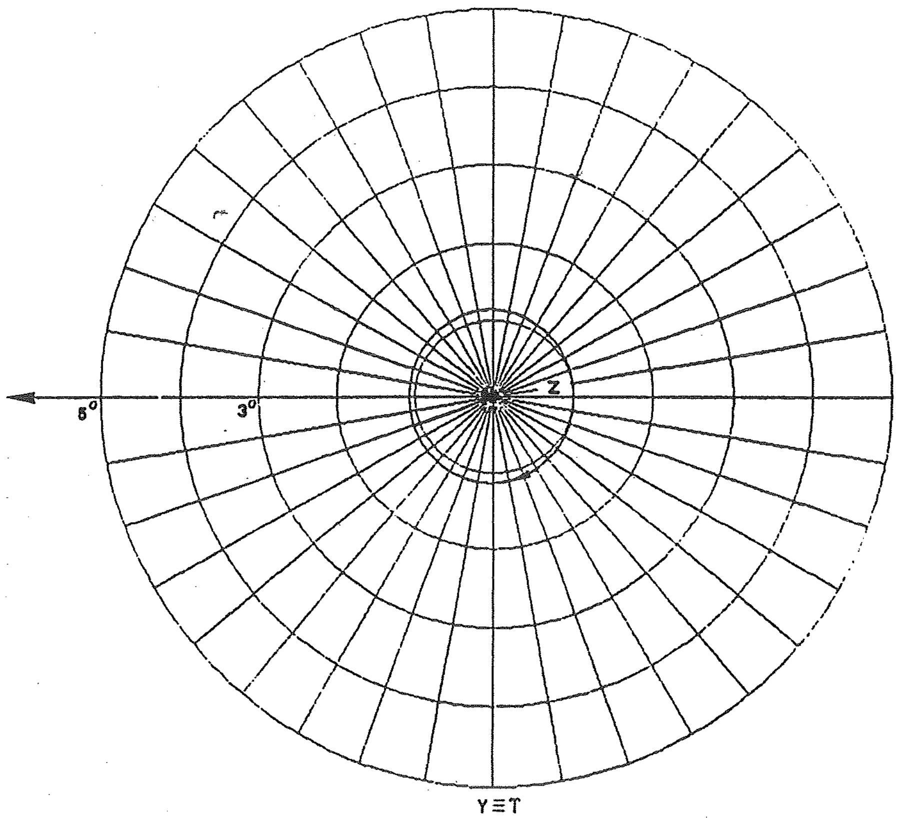

fig.5 Long term evolution of axis around Z.

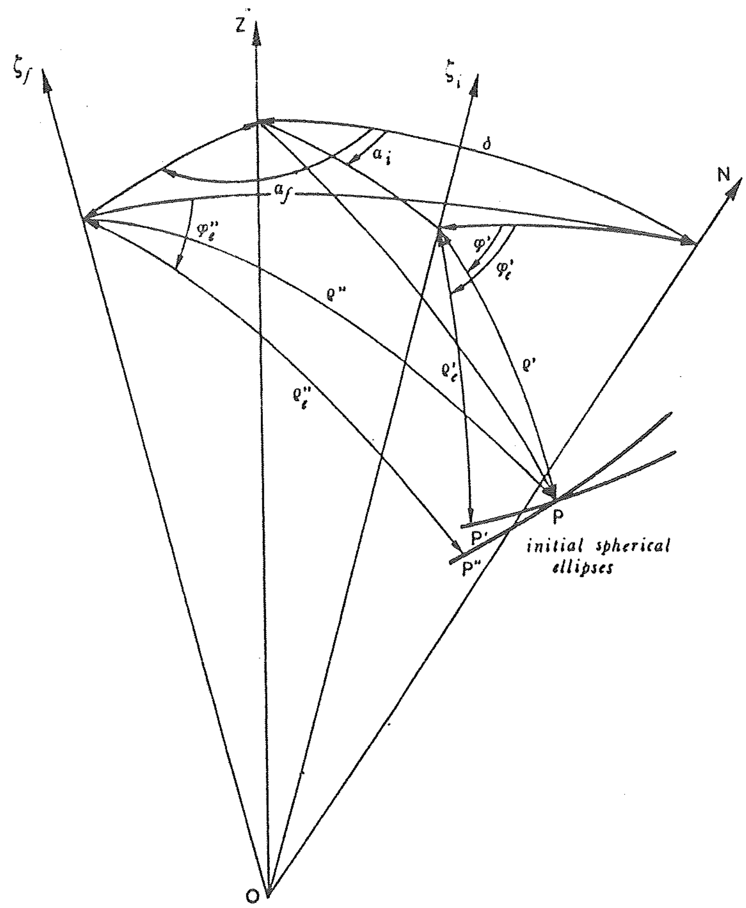

Fig.6 Description of the orbital pole rotation around Z axis in the simplified model.

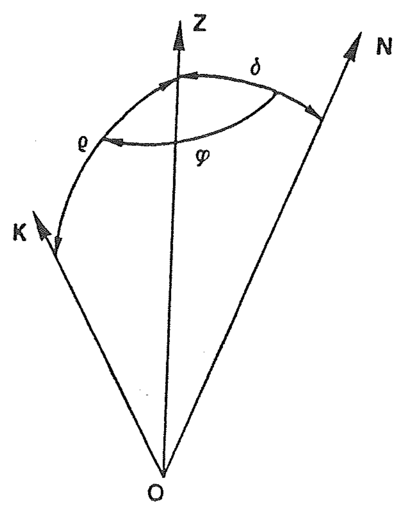

Fig.7 Angles defining the orbital pole position.

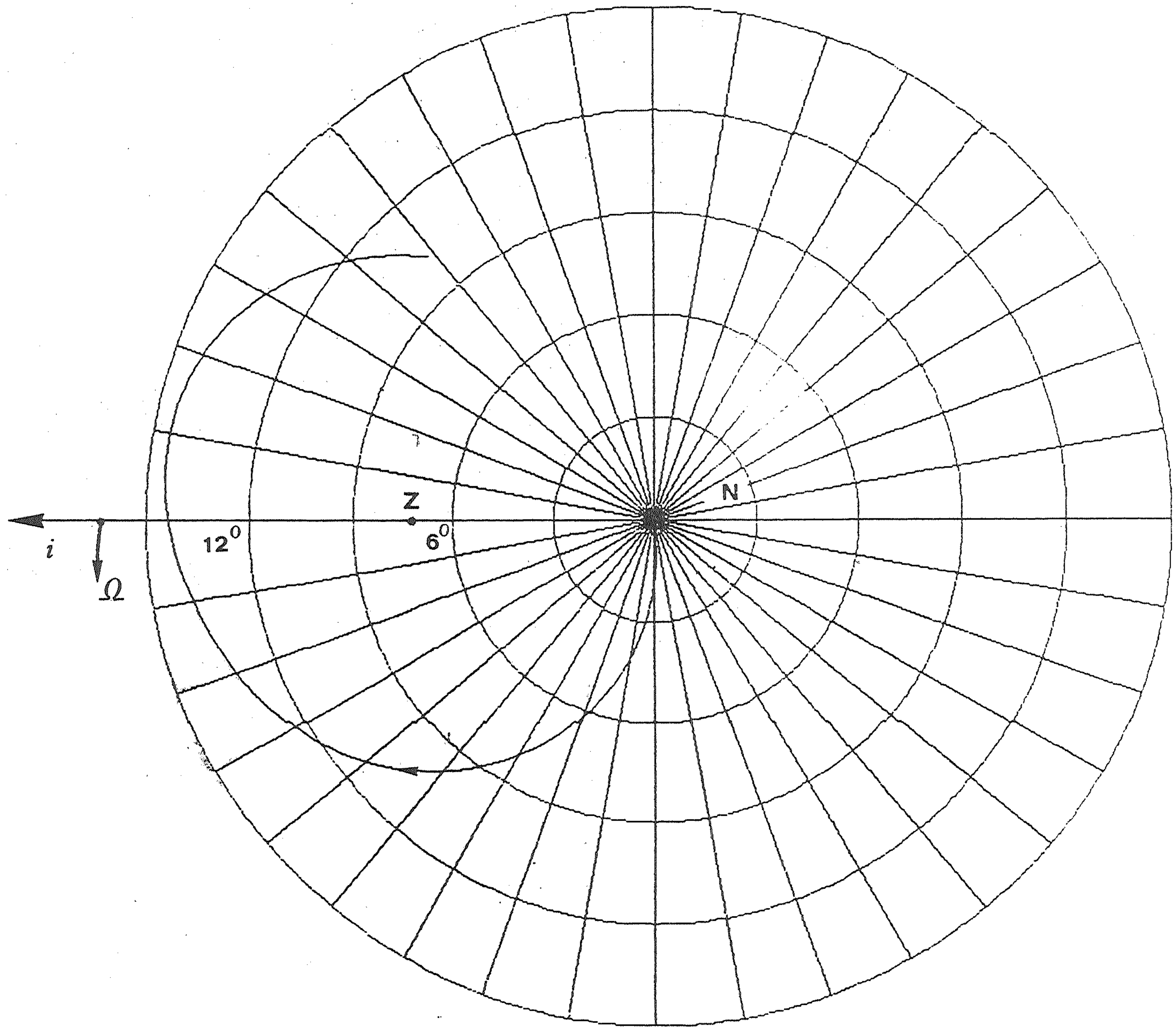

fig.8 Long term evolution of SIRIO satellite in a polar representation.

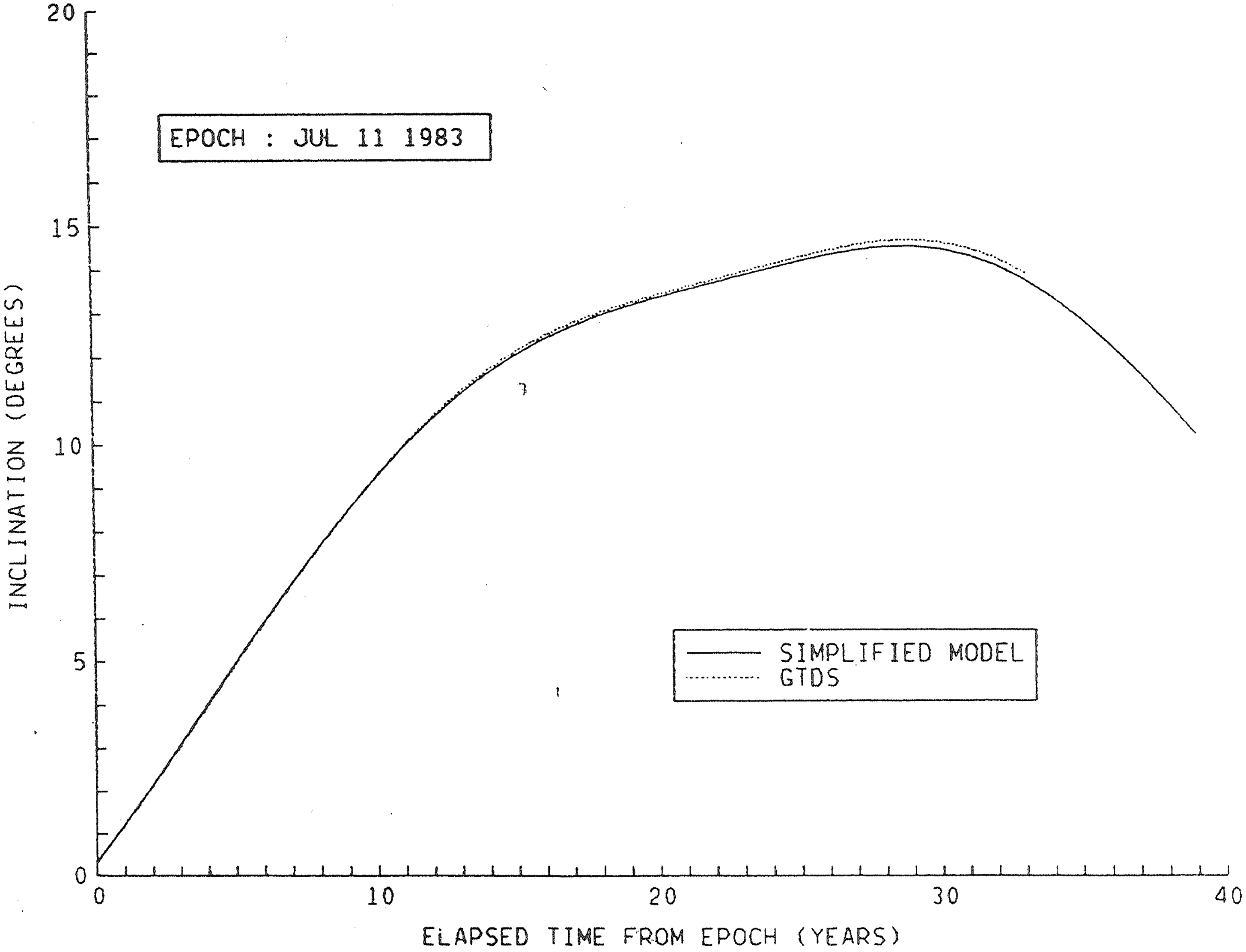

fig.9 Orbital inclination evolution for simplified model and GTDS.

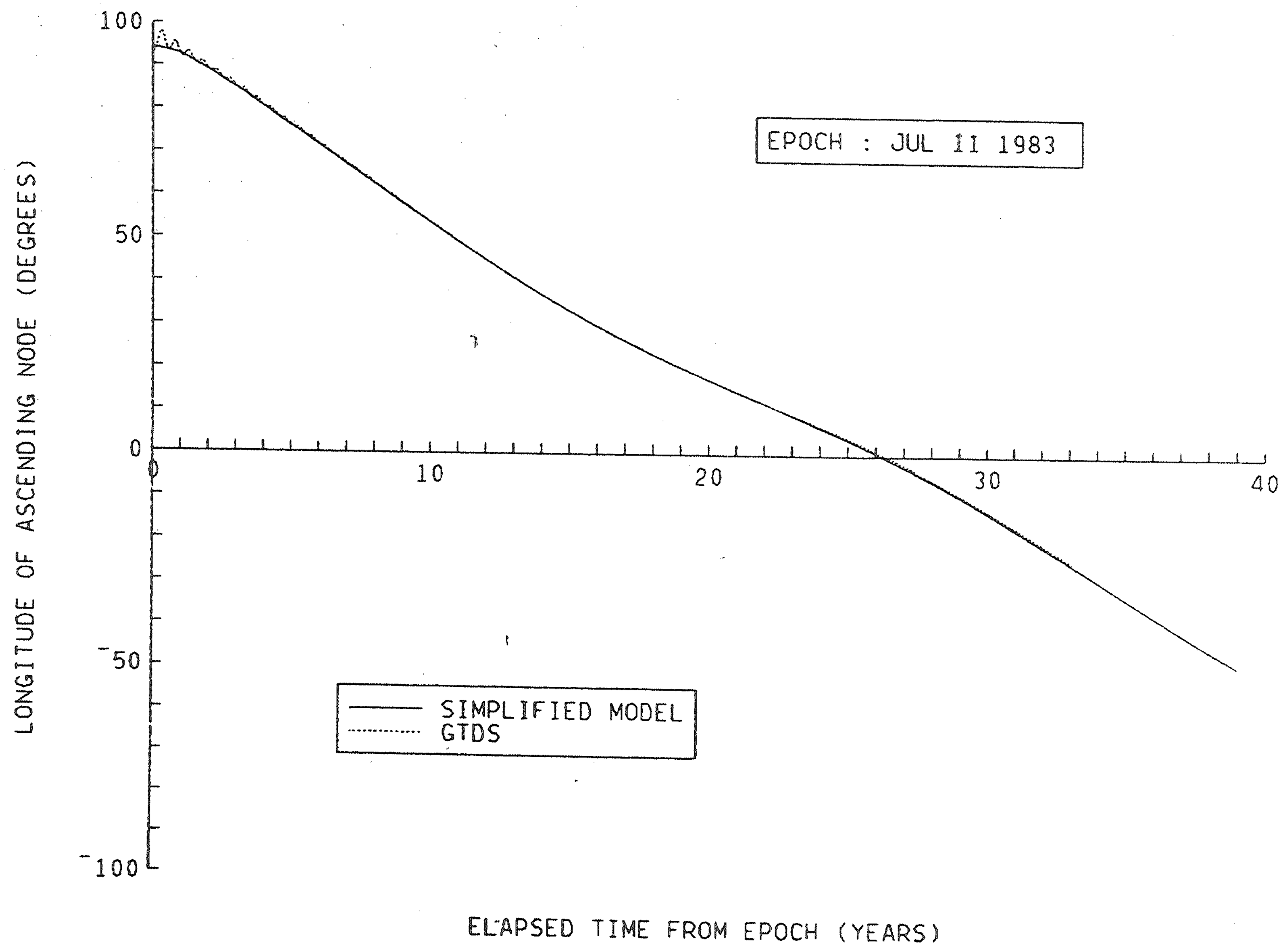

fig.10 Longitude evolution of ascending node for the simplified model and GTDS.

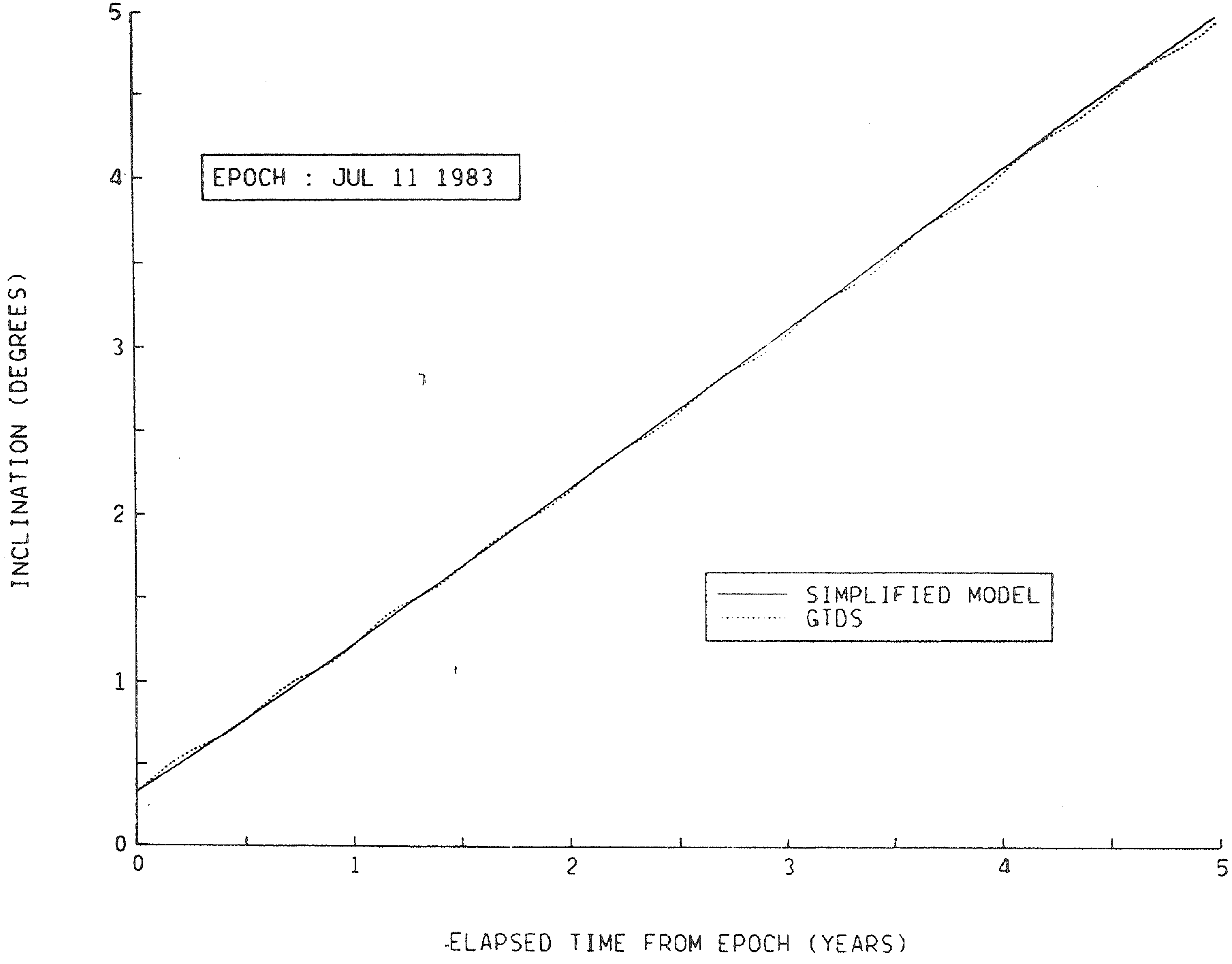

fig.11 Solar-yearly effect assessed by GTDS computations.