



Traffic-Aware Analysis of the Energy–Availability Balance in Railway Switch Heater Anti-icing Policies

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Abstract. Balancing dependability and energy consumption is a critical challenge in modern railway systems, particularly for switch anti-icing mechanisms, where heating policies are used to prevent ice formation and ensure operational reliability while limiting energy use. In this context, we present an enhanced stochastic modeling framework for quantitative trade-off analysis of such competing objectives. Building upon prior work, the proposed framework enables a refined analysis of system behavior under varying environmental and operational conditions by introducing three advancements: i) more realistic descriptions of weather conditions, ii) integration of sensor measurement errors besides communication failures and forecast inaccuracies, and iii) novel modeling of heating control policies driven by actual train traffic patterns. We apply the framework to a comprehensive set of weather profiles and railway traffic scenarios to evaluate the impact of different heating strategies on reliability and energy efficiency. Results demonstrate how incorporating real train schedules into heating management can significantly influence the trade-off between energy consumption and dependability, offering actionable insights to decision-makers. The proposed framework provides a practical decision-support tool to guide the design of energy-efficient, reliable anti-icing systems that comply with operational and regulatory constraints.

Keywords: railway traffic · availability · energy management ·
railway switch heating system · reliability · stochastic activity networks

1 Introduction

Trade-off analysis is relevant in many application contexts, since often contrasting but equally important requirements need to be accommodated when developing a system that provides specific functionalities. Among such trade-offs, the ones involving dependability properties (such as availability or reliability) and energy consumption are receiving increased attention, given their criticality

in many sectors, including the railway domain [3, 34]. A well-studied criticality from the perspective of both dependability and energy consumption in the railway domain concerns the switch heater anti-icing system, which consists of the pre-emptive activation of switch heating to prevent ice formation on railway switches, based on available weather data or forecasts.

In previous studies, we developed a modeling framework for the quantitative trade-off analysis of dependability and energy consumption. This framework identifies and exploits appropriate formalisms and modeling mechanisms to address aspects of the target system impacting on the assessed indicators in a progressively more realistic manner (cf., e.g., [11, 16, 18]). The aim of this line of research is the definition and development of a powerful support system that delivers practical insights to effectively steer future policy decisions on anti-icing in railway switch heater systems, optimizing both energy consumption and operational reliability while adhering to the standards set by regulatory agencies and other stakeholders.

In this paper, we propose a further enhancement to the most recent solution, presented in [16]. It consists of three novel additions: i) incorporating a more accurate representation of realistic weather conditions, ii) accounting for measurement errors from sensor data, in addition to failures of communication channels and forecast errors, and iii) representing the switch heating management based on actual train traffic.

The resulting extended modeling framework is exercised across a variety of weather profiles and traffic scenarios, to show how it works in practice and the kind of support it provides in setting up a suitable anti-icing policy. Results demonstrate that considering real train schedules for heating management, further enriched with information on measurement errors from sensor data, can significantly influence the balance between energy consumption and dependability, offering actionable insights for decision-makers.

Outline. The remainder of the paper is organized as follows. Related work is briefly surveyed in Sect. 2. Section 3 introduces the logical structure of the system under scrutiny, and the representation of weather conditions and measurement errors. Section 4 describes the model adopted to accommodate different train traffic schedules. Section 5 provides an overview of the energy management policies considered. In Sect. 6, the developed stochastic modeling framework is presented. Section 7 illustrates the measures of interest for energy efficiency and dependability. Section 8 presents the use case and parameter settings. Section 9 discusses the simulation results. The paper is concluded with a summary and future work in Sect. 10. Section A contains a glossary of the main symbols used in this paper.

2 Related Work

The railway domain is a prominent application area for formal methods and tools [9, 23], supporting both qualitative and quantitative analysis of stochastic models and evaluation of dependability attributes such as availability, reliability, and safety [12, 22].

Energy efficiency in railway systems has been widely addressed [26, 35], with studies focusing on policies and models for improving component performance. The authors of [30] suggest enhancements to the physical technology of the heating components, while in [24] shows that contactless heating is more efficient than conventional systems due to thermal insulation from the track. Study [29] reports degraded performance of conductive heaters under wet cold conditions, with energy consumption driven by environmental and temperature setpoints, and advocates periodic, controller-based heating over continuous operation for improved efficiency. Studies [24, 30] adopt approaches different from this work, while [29] proposes a periodic heating strategy similar to the one considered here, but with a primary focus on energy efficiency.

Studies such as [4–6, 14, 16–18, 25, 28], including those performed by a subset of the authors of this paper, analyze energy consumption under different supply policies in smart city contexts, including railways and buildings. In railway anti-icing policies, research has mainly focused on developing control strategies and evaluation frameworks based on assumptions on system behavior, faults, and environmental conditions, with particular attention to the trade-off between energy consumption, availability, and reliability under on/off heating policies. These studies employ modeling formalisms such as timed/hybrid automata [1, 27], hybrid Petri nets [21], and Stochastic Activity Networks (SANs) [33], along with tools like Möbius [19] and Uppaal (SMC) [10, 20] for the model construction, evaluation, and verification of switch heating control systems.

Among the most recent contributions in this category, the authors of [5] considered trade-offs between reliability and energy efficiency under simplistic assumptions on weather representation (characterized by the temperature only). In [17], the authors extended previous work by introducing a more accurate evaluation model that incorporates temperature, humidity, and dew point effects, enabling a more realistic weather representation, though limited to specific weather profiles and primarily focused on energy consumption. Recently, in [16], the authors proposed a modeling framework that integrates problem elements introduced progressively as a systematic approach to address system complexity, enabling effective consolidation of individual findings within a unified structure. In particular, they introduced a systematic classification of heating policies, including new strategies for normal and failure conditions, an enhanced model-based framework with realistic weather representation for lifecycle evaluation, and a comprehensive analysis supporting the selection of policies balancing energy consumption and system availability/reliability.

3 Logical View of the Railway Switch Anti-icing System

The architecture of the addressed anti-icing system is logically structured in four parts [17, 18], as shown in Fig. 1:

- i) n heaters SH_{ij} , one for each railway switch RS_{ij} , partitioned into m groups, each managed by a coordinator, with $n = n_1 + \dots + n_m$,
- ii) the railway switch heating control system, that decides when to turn the heaters on or off to prevent ice formation on railway switches,
- iii) the weather conditions at railway switches, used by the control system to decide on control actions, and
- iv) the weather forecast service that supports the control system to decide on control actions.

The heating control system is organized in three levels:

- i) at the higher level, m global coordinators $C_1, \dots, C_m$,
- ii) at the lower level, for each C_i , n_i local controllers $L_{i1}, \dots, L_{in_i}$, deployed to manage the corresponding group of n_i switch heaters (i.e., power on/off) under coordinator C_i , with one sensor S_{ij} per heater, and
- iii) in between these levels, n communication channels CH_{ij} , connecting each L_{ij} to the corresponding C_i , according to a given topology.

In practice, various communication technologies are possible. This study adopts Power-Line Communication (PLC), leveraging the same power lines used to supply the heaters.

3.1 Weather Aspects Related to Control Actions

Weather data is essential to decide when to turn the heating system on or off to ensure safe operation of the switches while saving energy. In particular, air temperature and *dew point* (calculated as a function of temperature and humidity) have a strong impact on the formation of ice and snow. Typically, sensors dedicated to measuring air temperature are placed near each switch. Instead, humidity sensors are not common nowadays in railway systems, and humidity data is usually made available by external systems, such as weather stations.

We consider energy management policies that rely on weather data from both weather forecast stations (for temperature and dew point) and sensors deployed within the railway infrastructure (for the temperature of each switch).

In the stochastic model we use to represent its evolution, the weather conditions are described at time W_k by a stochastic process composed of an $(n + 2)$ -tuple of random variables:

$$(T(W_k), T^d(W_k), T_{11}(W_k), \dots, T_{1n_1}(W_k), \dots, T_{m1}(W_k), \dots, T_{mn_m}(W_k)).$$

Here, W_k represents the time the weather conditions change, $T(t)$ and $T^d(t)$ represent the temperature and the dew point at time t , respectively, and $T_{ij}(t)$ the temperature at time t with respect to the position of the railway switch RS_{ij} .

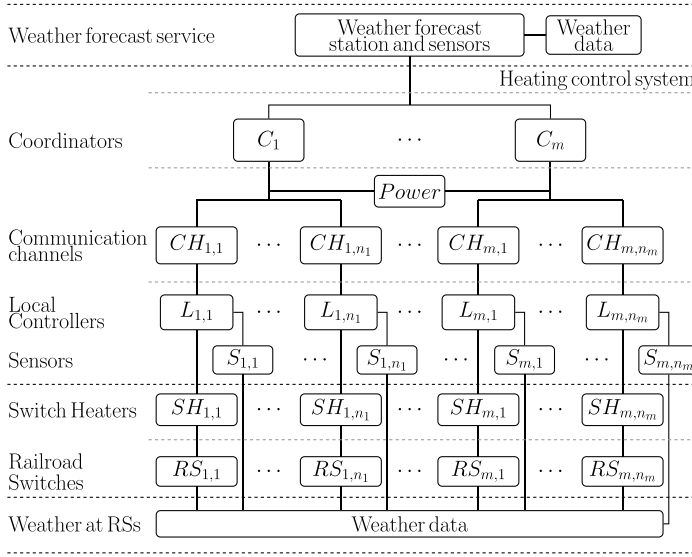


Fig. 1. Logical architecture of the considered anti-icing system.

the temperature sensor is attached to. These random variables are piece-wise constant over time and change value after a random number X_k of time units.

A wide and general variety of weather profiles, characterizing different regions where railway switches are installed, is addressed through four key parameters:

μ_T is the temperature assigned to $T(0)$; the temperature for $T(t)$ is expected to be around this value.

σ_T represents how much $T(t)$ varies from its expected value, determining the maximum and minimum values that $T(t)$ can assume, thus impacting the variance of $T(t)$.

μ_Δ is the difference between $T(0)$ and $T^d(0)$, which represents the expected spread between $T(t)$ and $T^d(t)$.

σ_d represents the variation of $T^d(t)$ from the expected value and has an impact on the variance.

A variety of weather patterns can be represented by different combinations of values for $(\mu_T, \sigma_T, \mu_\Delta, \sigma_d)$. In particular, weather patterns characterized by a high risk of switch freezing are those with $\mu_T \leq 0$ and low values for μ_Δ . However, also of interest are those with high values for μ_Δ, σ_d , and σ_T , for which the actual spread values due to the high variance for $T(W_k)$ and $T^d(W_k)$ may be close to 0. The impact of these parameters must be evaluated in detail.

3.2 Measurement Errors from Sensor Data

We consider measurement errors $\varepsilon_{ij}^s(t_k)$ from sensor data depending on the relative humidity, defined as

$$\varepsilon_{ij}^s(t_k) = \pm \varepsilon_s \cdot w_s \frac{\max\{H_{ij}(t_k) - H^s, 0\}}{1 - H^s}, \quad (1)$$

where ε_s is the maximum non-negative sensor error, influenced only by sensor characteristics, and $w_s \geq 1$ represents the impact of the relative humidity on the sensor error, which is significant only for relative humidity values above H^s .

The measured temperatures $\hat{T}_{ij}(t)$ and dew points $\hat{T}^d(t)$, for each time t , are given by

$$\hat{T}_{ij}(t) = T_{ij}(t) + \varepsilon_{ij}^{s,T}(t_k) \text{ and} \quad (2)$$

$$\hat{T}^d(t) = T^d(t) + \varepsilon_{ij}^{s,d}(t_k), \quad (3)$$

where $\varepsilon_{ij}^{s,T}(t_k)$ and $\varepsilon_{ij}^{s,d}(t_k)$ denote the measurement errors associated with the temperature and dew point sensors, respectively.

Expressions for $\varepsilon_{ij}^{s,T}(t_k)$ and $\varepsilon_{ij}^{s,d}(t_k)$ are obtained from (1) by replacing $\pm \varepsilon_s$ with ε_s^T and ε_s^d , respectively, and w_s with w_s^T and w_s^d , respectively. Errors in temperature and dew point measurements are outside the scope of our work. For the sake of simplicity, we consider $|\varepsilon_s^T| = |\varepsilon_s^d| = \varepsilon_s \geq 0$, and $|w_s^T| = |w_s^d| = w_s \geq 0$. Energy management policies are based on (2) and (3). Thus, the turning on and off of the heaters depends on the following expressions:

$$T_{ij}(t) - T^d(t) \leq \varepsilon_{ij}^{s,d}(t_k) - \varepsilon_{ij}^{s,T}(t_k) \text{ and} \quad (4)$$

$$T_{ij}(t) \leq -\varepsilon_{ij}^{s,T}(t_k). \quad (5)$$

We assume that the errors $\varepsilon_{ij}^{s,T}(t_k)$ and $\varepsilon_{ij}^{s,d}(t_k)$ do not offset each other and all the measurement errors are between $-|\varepsilon_{ij}^{s,T}(t_k)|$ and $+|\varepsilon_{ij}^{s,T}(t_k)|$. It is easy to show that, with the two settings $\varepsilon_s^T = -\varepsilon_s^d = -\varepsilon_s$ and $\varepsilon_s^T = -\varepsilon_s^d = \varepsilon_s$, the two expressions $\varepsilon_{ij}^{s,d}(t_k) - \varepsilon_{ij}^{s,T}(t_k)$ and $-\varepsilon_{ij}^{s,T}(t_k)$ in (4) and (5) take the maximum and minimum values, corresponding to maximum or minimum energy consumption (minimum and maximum dependability), respectively.

4 Switch Heating Management Based on Train Traffic

The proposed approach is described in two phases. First, the train schedule and the effects of train delays are modeled to identify the time intervals during which the switches must be available. Next, these intervals are converted into heating control periods and used to define the corresponding level of heating activity.

4.1 Train Schedule and Delay Model

To further save on energy consumption, the actual train movement is accounted for, such that the heating of railway switches is needed only in time intervals when trains are actually scheduled. In the analysis framework, this is addressed by alternating, for each switch, randomly chosen time intervals during which the heating control system is active (turning on or off the heater) with periods where it is not active (and the heater is always off).

Instead of representing the movement of individual trains along the switches, an abstraction of the time intervals when switch availability is required is represented based on the train schedule.

To accommodate different train traffic schedules, corresponding to different alternations of active and inactive control periods, a parameter α is introduced to represent the percentage of time during which the heater control is active within the analysis time interval $[0, t]$. Through this parameter α , it is possible to represent a variety of different scenarios, such as a few long periods of active control alternating with short periods of inactive control, or many short periods of active control alternated with short periods of inactive control. When it reaches 100%, the control is always active.

Formally, for each railway switch RS_{ij} , the time schedule at which trains are expected to arrive at RS_{ij} , is defined by the sequence of deterministic instants

$$\theta_0^{ij}, \dots, \theta_{n_t^{ij}-1}^{ij}, \quad (6)$$

with $\theta_k^{ij} \leq \theta_{k+1}^{ij}$, $\theta_0^{ij} = 0$ and $\theta_{n_t^{ij}-1}^{ij} = t$.

As a simple example, consider a switch for which the heating control system must be active during the time intervals $[420, 600]$ (from 7:00 a.m. to 10:00 a.m.) and $[1080, 1260]$ (from 6:00 p.m. to 9:00 p.m.) to accommodate scheduled train passages. In this case, the programmed sequence is given by 0, 420, 600, 1080, 1260, 1440. If the first train is delayed by 10 minutes, the corresponding active control interval is extended to $[420, 610]$. This example illustrates the role of the scheduled times θ_k^{ij} , as well as that of the actual times Θ_k^{ij} and the delay variable $X_k^{d,ij}$, which are defined below.

The actual time schedule at which trains arrive at RS_{ij} , including delays, is given by the sequence of instants

$$\Theta_0^{ij}, \dots, \Theta_{n_t^{ij}-1}^{ij}, \quad (7)$$

From this sequence, the k th random time interval $[\Theta_{2k-1}^{ij}, \Theta_{2k}^{ij}]$ for which heating control system is active, with $1 \leq k \leq \lceil n_t^{ij}/2 \rceil - 1$, is given by

$$[\Theta_{2k-1}^{ij}, \Theta_{2k}^{ij}] = [\theta_{2k-1}^{ij}, \theta_{2k}^{ij} + \min\{X_k^{d,ij}, \theta_{2k+1}^{ij} - \theta_{2k}^{ij}\}], \quad (8)$$

where $X_k^{d,ij}$ is the random variable representing by how much the deterministic scheduled interval $[\theta_{2k-1}^{ij}, \theta_{2k}^{ij}]$ is expanded (also up to, but not beyond, the next

active control interval) due to the delay of the trains. It is assumed that when $X_k^{d,ij} > \theta_{2k+1}^{ij} - \theta_{2k}^{ij}$, then the delay of the trains impacts the next scheduled interval, such that the mean of $X_{k+1}^{d,ij}$ is doubled.

Depending on the topology of the specific railway switch network at the station, a train delay that affects a scheduled interval can propagate to intervals scheduled for other switches at successive instants in time. This propagation path is accounted for in the analysis by building a topology of interactions, in a similar manner as performed for the propagation of the failures in the PLC communication network, and described in detail in [17]. In particular, the considered topology is defined by associating each switch RS_{ij} with all switches that can be reached by trains starting from that switch, as shown in Fig. 2.

Thus, for example, a delay can propagate from $RS_{1,1}$ to $RS_{1,2}$ and $RS_{1,3}$, or from $RS_{1,3}$ to $RS_{1,4}$, and so on. It is assumed that the delay of the trains on a switch propagates to the other switches with probability p^{dprop} , and when it propagates to a switch RS_{ij} , the mean of the corresponding $X_k^{d,ij}$ doubles for each directly connected switch from which the delay started. For simplicity, the topology considered for the other switches not shown in Fig. 2 is the same as that obtained by grouping the elements into groups of 6 and considering for each group the same topology shown in Fig. 2 (and linking the last element of each group, e.g., $RS_{1,6}$, with the third in the next group, e.g., $RS_{1,9}$).

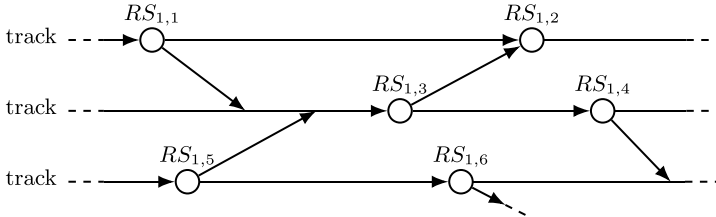


Fig. 2. Logical scheme of the dependencies among the railway switches RS_{ij} with respect to the delay of the trains.

4.2 Heating Control Schedule

The train schedule model presented earlier defines the intervals during which the switches must be available. We now show how these intervals are translated into heating control periods.

To define the sequence in (6) as a function of the percentage α , we introduce the corresponding basic schedule given by the sequence of deterministic instants

$$\bar{\theta}_0^{ij}, \dots, \bar{\theta}_{n_t^{ij}-1}^{ij}, \quad (9)$$

with $\bar{\theta}_k^{ij} \leq \bar{\theta}_{k+1}^{ij}$, $\bar{\theta}_0^{ij} = 0$ and $\bar{\theta}_{n_t^{ij}}^{ij} = t$.

Also, let $\bar{\alpha}$ be the percentage of time the heating control system is active based on the basic schedule (9), defined as

$$\bar{\alpha} = \frac{\sum_{k,i,j} \bar{\Phi}_k^{ij}}{nt}, \quad (10)$$

where $\bar{\Phi}_k^{ij}$ is the deterministic duration of the k th scheduled active control interval based on (9), defined as

$$\bar{\Phi}_k^{ij} = \bar{\theta}_{2k}^{ij} - \bar{\theta}_{2k-1}^{ij}. \quad (11)$$

In the above numerical example for $n = 1$, the heating control system is active for a total of 360 minutes over a 24-hour period, yielding $\bar{\alpha} = 0.25$. Values of α smaller or larger than $\bar{\alpha}$ correspond to schedules requiring proportionally less or more heating activity.

For each scheduled active control interval, δ_a is the length of the initial buffer period reserved for recovery from freezing. It is defined as the sum of the time between two consecutive control actions δ_c and the maximum recovery time from ice μ_{ice} , i.e., $\delta_a = \delta_c + \mu_{ice}$. This is because the related switch could be frozen when the heating control system is activated and the first control action could happen after δ_c time units. It is assumed that recovery from ice occurs only when the heater control is active. So the freezing of the switch occurring in the inactive control interval $[\theta_{2k}^{ij}, \theta_{2k+1}^{ij}]$ is removed in the next interval $[\theta_{2(k+1)-1}^{ij}, \theta_{2(k+1)}^{ij}]$.

Formally, each scheduled active control interval $[\bar{\theta}_{2k-1}^{ij}, \bar{\theta}_{2k}^{ij}]$ consists of two consecutive time periods: a buffer period $[\bar{\theta}_{2k-1}^{ij}, \bar{\theta}_{2k-1}^{ij} + \delta_a]$ (aimed to remove and prevent freezing before train passage) and a train passage period $[\bar{\theta}_{2k-1}^{ij} + \delta_a, \bar{\theta}_{2k}^{ij}]$.

The maximum recovery time from ice is defined as

$$\mu_{ice} = \bar{\mu}_{ice} \frac{k_{ice}}{k_W}, \quad (12)$$

where $k_{ice} > 0$ is the scaling factor for ice quantity (which represents different amounts of ice to be melted), $k_W > 0$ is the scaling factor for heating power (representing heaters of different power ratings), and $\bar{\mu}_{ice}$ is the maximum base recovery time for $k_{ice} = 1$ and $k_W = 1$, with

$$\frac{k_{ice}}{k_W} < \frac{\bar{\Phi}_k^{ij} - \delta_c}{\bar{\mu}_{ice}}. \quad (13)$$

By skipping some of the intermediate formulations, the width of the k th scheduled active control interval based on (6) is defined by

$$\Phi_k^{ij} = \theta_{2k}^{ij} - \theta_{2k-1}^{ij} \quad (14)$$

$$= \begin{cases} \delta_a & \text{if } \alpha \leq \frac{\delta_a \bar{\alpha}}{\bar{\Phi}_k^{ij}}, \\ \frac{\alpha}{\bar{\alpha}} \bar{\Phi}_k^{ij} & \text{if } \frac{\delta_a \bar{\alpha}}{\bar{\Phi}_k^{ij}} < \alpha \leq \min \{\alpha_k^a, \alpha_k^b\}, \\ \left(1 + \frac{\bar{\theta}_{2k-1}^{ij} - \bar{\theta}_{2k-2}^{ij}}{\bar{\Phi}_{k-1}^{ij} + \bar{\Phi}_k^{ij}} + \frac{\alpha - \bar{\alpha}}{2\bar{\alpha}}\right) \bar{\Phi}_k^{ij} & \text{if } \alpha_k^a < \alpha < \alpha_k^b, \\ \left(1 + \frac{\alpha - \bar{\alpha}}{2\bar{\alpha}} + \frac{\bar{\theta}_{2k+1}^{ij} - \bar{\theta}_{2k}^{ij}}{\bar{\Phi}_k^{ij} + \bar{\Phi}_{k+1}^{ij}}\right) \bar{\Phi}_k^{ij} & \text{if } \alpha_k^b < \alpha < \alpha_k^a, \\ \left(1 + \frac{\bar{\theta}_{2k-1}^{ij} - \bar{\theta}_{2k-2}^{ij}}{\bar{\Phi}_{k-1}^{ij} + \bar{\Phi}_k^{ij}} + \frac{\bar{\theta}_{2k+1}^{ij} - \bar{\theta}_{2k}^{ij}}{\bar{\Phi}_k^{ij} + \bar{\Phi}_{k+1}^{ij}}\right) \bar{\Phi}_k^{ij} & \text{if } \alpha \geq \max \{\alpha_k^a, \alpha_k^b\}. \end{cases} \quad (15)$$

5 Overview of the Energy Management Policies

Following the hierarchical structure of the anti-icing system as shown in Fig. 1, and considering the possible failure of the communication channel between the coordinator and the controller local to the railway switch, energy management policies have been developed for both the case when the communication channel is working well (global policies) and the case when it fails (local policies).

The logic to decide whether to turn on or off the switch heater following the global anti-icing policies is mainly of two types. A basic policy, named P_{bas}^G , reflects the current practice and is based on comparing the actual (measured) temperature and dew point with a fixed threshold τ . The other policy, named P_{pre}^G , exploits the temperature and dew point prediction.

Similarly, local policies are based on different logics, ranging from using data collected from external systems before the interruption (such as actual and forecasted temperatures and dew points) and from sensors local to switches before and during the interruption (such as actual temperatures), to making a future prediction of missing fresh weather data starting from the last ones obtained.

In this case, the basic policy named P_{bas}^L is based on comparing the actual (measured) temperature with a threshold τ . Other locally operated energy management policies are based on sophisticated logics that aim to compensate for

Table 1. Overall, global and local policies for energy management.

Overall policy	Global policy			Local policy			Literature
	Symbol	Name	Params	Symbol	Name	Params	
$P_{\text{bas,bas}}$	P_{bas}^G	Basic	τ	P_{bas}^L	Basic	τ	[14, 17]
$P_{\text{bas,mem}}$	P_{bas}^G	Basic	τ	P_{mem}^L	Memory based	τ, ξ	[18]
$P_{\text{bas,pre}}$	P_{bas}^G	Basic	τ	$P_{\text{pre-tbats}}^L$	Prediction based (tbats)		[18]
$P_{\text{pre,mpr}}$	P_{pre}^G	Prediction based (SSA)		P_{mpr}^L	Prediction and memory based (SSA)	ξ	[16]
$P_{\varepsilon, m\varepsilon}$	P_{ε}^G	Prediction-error based	$\varepsilon_G, \varepsilon_G^d$	$P_{m\varepsilon}^L$	Prediction-error and memory based	ε_L, ξ	[16]

the unavailability of rich meteorological information gathered by the coordinator. They keep a memory of the last dew point value received and use it directly for a forecasted value (depending on the duration of the channel failure), together with local temperature values. Note that errors introduced by imperfect forecasts are accounted for in the analysis. In particular, to consider measurement errors $\varepsilon_{ij}^s(v_s)$, the actual absolute forecast error ε_f at time v_s is extended by adding $-|\varepsilon_{ij}^s(v_s)|$ or $|\varepsilon_{ij}^s(v_s)|$, when, respectively, $\varepsilon_G = -\varepsilon_f$ or $\varepsilon_G = \varepsilon_f$.

Table 1 shows a summary of the energy management policies accounted for in the analysis framework, along with their acronyms, names, and parameters. During operation of the anti-icing system, global and local policies are expected to alternate at each local switch heater controller, depending on the failure rate of the communication channel connecting that local controller with its coordinator.

6 Stochastic Modeling Framework

A stochastic model-based approach is well-suited to evaluate the efficiency, dependability, and costs (also in terms of energy consumption) of systems subject to stochastic phenomena, such as failure occurrence and weather forecast. This is also the case for the case study of anti-icing system this paper focuses on.

6.1 Formalism and Tool

Several stochastic modeling methods have been proposed in the literature, including [1, 2, 7, 21, 33]. Since our system is characterized by stochastic phenomena that cannot be accurately represented by the exponential distribution, we have resorted to non-Markovian models, in particular the ones defined by SANs [33].

SAN is a formalism widely used for performance, dependability, and performability evaluation of complex systems, given its high expressiveness and the powerful tools for modeling and evaluating them. The SAN formalism is a variant of Stochastic Petri Nets [7], and has similarities with Generalised Stochastic Petri Nets [2]. It is based on the following primitives: plain and extended places (blue and orange circles), timed and instantaneous activities (hollow and solid vertical bars), input and output gates (triangles pointing left or right). Plain and extended places represent, respectively, C++ short types and primitive C++ data types, also including structures and arrays. Input gates enable activities. Input and output gates change the marking when an activity completes. The primitives are defined by C++ expressions or statements.

Möbius [19] is a software tool for modeling the behavior of complex systems. It supports a variety of modeling formalisms, including SANs, and different analytical and simulation solvers. It follows a compositional and modular modeling approach, where atomic models are building blocks that can be composed with the Rep, Join, and the more recently introduced *DARep* operators, to generate an overall composed model.

In particular, the *DARep* approach was proposed in [13,15] to address the non-anonymous replication of a population of system components. It is based on template stochastic models (each one representing a different generic component), dependency-aware State Variables (SVs) with the associated topology, and the compositional operator $\mathcal{D}$. The $\mathcal{D}$ operator automatically generates instances of each template model. Within each instance, $\mathcal{D}$ generates different instances of each dependency-aware SV, following the topology associated with it. Also, $\mathcal{D}$ merges all the occurrences of an instance of dependency-aware SV in different instances of the template model into one SV.

6.2 Overview of the SAN Models

Three atomic templates TM_W , TM_L , and $TM_CTRLSched$, were defined using the SAN formalism. TM_W represents the global weather conditions. TM_L represents the CH_{ij} and L_{ij} components and the weather conditions local to each switch.

$TM_CTRLSched$, depicted in Fig. 3, represents the sequence of scheduled and random time intervals defined by (8) for which control system is active for SH_{ij} . In particular, Fig. 3 depicts: i) the alternating intervals when heating control system is active ($CtrlOn==1$) with periods when it is not active ($CtrlOn==0$), through the activities *AIK* and *COFFI*, ii) the propagation of the delay of the trains to the other switches, through the activity *tD2SW* and the dependency-aware SV $D2SW$, and to the next scheduled interval, through the SV $D2II$, and iii) the impact $X_k^{d,ij}$ on the delay of the trains on the scheduled interval, through the activity *ConD* defined as a function of $D2SW$ and $D2II$.

The overall system model is obtained by generating and composing automatically, through the $\mathcal{D}$ operator, one instance TM_W_1 of TM_W and $TM_CTRLSched$, i.e., one instance TM_L_{ij} and $TM_CTRLSched_{ij}$ for each L_{ij} . Local SVs are used to represent state information local to each template instance, while common and dependency-aware SVs (white circles in Fig. 3) are used to share state information among, respectively, all or subsets (based on the topology) of TM_W_1 , TM_L_{ij} , and $TM_CTRLSched_{ij}$ instances.

The proposed model maintains the same hierarchical structure as those proposed in [14,18], but the $TM_CTRLSched$ model is new and the models TM_W and TM_L are a significant extension and update of the corresponding models proposed in [14,18], based on the new stochastic weather model, and with the sensor measurement errors and trains schedule that have been included in TM_L , where the policies are implemented.

Furthermore, as in [16], real-world data series can be used as trend functions for temperature and dew point, from which $T(W_k)$ and $T^d(W_k)$ are derived, replacing the sinusoidal trends used in [18]. We also introduced data structures and functions to model energy consumption based on the current and target temperatures, allowing the use of real-world datasets such as [29].

Moreover, new and original C++ class templates have been defined to implement each individual local and global policy listed in Table 1 as well as their combinations to obtain the overall policies.

This approach provides great flexibility and modularity in representing the different policies in the single template model TM_L through SAN primitives, including C++ objects defined as different specializations of C++ class templates, and are instantiated at the compilation time of the simulator solver. Thus, there is no need to create distinct template models for each combination of global and local policies.

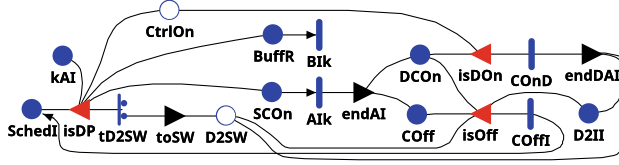


Fig. 3. SAN template model $TM_CTRLScheduled$ for control action scheduling of SH_{ij} .

7 Assessed Indicators

The following indicators are introduced to represent energy efficiency and dependability, as in [16].

The efficiency is addressed in terms of the electrical energy consumed in the interval $[0, t]$ by all the heaters, defined as

$$\mu_E = \mathbb{E} \left[\sum_{i,j} \int_0^t E^{SH}(T_{ij}(u), Z_{ij}^R(u)) Z_{ij}(u) du \right], \quad (16)$$

where random variable $Z_{ij}(u)$ represents the state of SH_{ij} at time u ($Z_{ij}(u) = 1$ if SH_{ij} is on and $Z_{ij}(u) = 0$ otherwise), and $E^{SH}(T_{ij}(u), Z_{ij}^R(u))$ is the energy per time unit (KJ/min) required by each heater SH_{ij} turned on, which depends on the current temperature and on the state $Z_{ij}^R(u)$ of the heater being on (either for ice recovery or to maintain temperature and prevent icing). Note that SH_{ij} is only active during the time intervals $[\Theta_{2k-1}^{ij}, \Theta_{2k}^{ij}]$, when the heating control system is operational, but trains are only expected during the period $[\Theta_{2k-1}^{ij} + \delta_a, \Theta_{2k}^{ij}]$ (after the period reserved for recovery from freezing).

The attributes of dependability that are evaluated are unavailability and unreliability. Unavailability is the expected cumulative freezing time for each switch that freezes in time intervals $[\Theta_{2k-1}^{ij} + \delta_a, \Theta_{2k}^{ij}]$ when the heating control system is active during the train passage period, defined as

$$\mu_U = \mathbb{E} \left[\sum_{i,j} \int_0^t U_{ij}(u) du \right], \quad (17)$$

where the random variable $U_{ij}(u)$ models a working ($U_{ij}(u) = 0$) or frozen ($U_{ij}(u) = 1$) RS_{ij} at time $u \in [\Theta_{2k-1}^{ij} + \delta_a, \Theta_{2k}^{ij}]$, while $U_{ij}(u) = 0$ for $u \in [\Theta_{2k-1}^{ij}, \Theta_{2k-1}^{ij} + \delta_a]$ or $u \in [\Theta_{2k}^{ij}, \Theta_{2k+1}^{ij}]$.

Unreliability is the probability of ice formation on at least one switch in the time intervals $[\Theta_{2k-1}^{ij} + \delta_a, \Theta_{2k}^{ij}]$, defined as

$$P_F = \Pr \{ \exists i, j, k \text{ and } u \in [\Theta_{2k-1}^{ij} + \delta_a, \Theta_{2k}^{ij}] \text{ s.t. } U_{ij}(u) = 1 \}. \quad (18)$$

Finally, to explore the scalability of the proposed model to the size n of the railway station network, the simulation performance measure $\mu_{P,n}(k)$ is defined as the total amount of CPU time, in seconds, used by the Möbius simulator to run k independent model simulations (batches). This native support for studies (batches), which allows one to define parameter sets and run experiments systematically (including parallel execution) was reported in a comparison experiment between Möbius and Uppaal SMC [4] as a significant advantage of Möbius over Uppaal SMC [20], where parameter exploration requires external scripting or repeated instantiation of models, since native support for structured batch experimentation is more limited.

Note that the proposed indicators assess policy effectiveness on a system-wide level but can also be adapted for individual heaters or specific groups. In fact, measures for individual heaters have also been defined and evaluated, but are not included here due to space constraints.

8 Use Case and Parameter Settings

The configuration of the railway station adopted as a use case to compare the proposed switch heating policies is that of a typical low-medium sized station already adopted in previous studies [14, 17, 18]. Its logical structure comprises $n = 19$ railway switches, partitioned in $m = 2$ groups denoted as *north* and *south* switches, with size $n_1 = 9$ and $n_2 = 10$, respectively. Each of the two coordinators and its local controllers is connected in series through the same power line, so that the failure of CH_{ij} propagates to the other $CH_{ij+1}, \dots, CH_{in_i}$.

This propagation path is taken into account in the analysis by constructing a topology of interactions, as performed in [17]. The time to the physical fault of CH_{ij} is modeled as a random variable exponentially distributed as in [17]. This means that, without taking humidity into account, channel failure is expected once every two days (only based on the parameter $c = 3.472 \cdot 10^{-4} \text{ min}^{-1}$). In the worst-case scenario, the relative humidity can lead to an increase in the failure rate by half an order of magnitude [14]. The recovery time of CH_{ij} is a uniform random variable that takes values from 45 min to 75 min.

The analysis time window is 1440 min (24 h). Data from the “Edmonton Blatchford AB_3012209”¹ meteorological station in the Alberta region (in the Canadian Prairies), collected on December 20, 2024 (from midnight on the 19th to midnight on the 20th, plus the next 24 values used for forecasting), define the trend functions of $T(W_k)$ and $T^d(W_k)$. We considered a typical icing risk day, with temperatures and dew points varying from -14.6°C and -18.5°C at midnight to -8.1°C and -13.5°C around 3:00 p.m., respectively. Based on the

¹ <https://climate.weather.gc.ca>.

real-world data, we defined and evaluated eight weather profiles, varying the four weather-related parameters: $\mu_T = -15^\circ\text{C}$ and σ_T, μ_Δ , and σ_d are 1°C or 10°C .

Note that these scenarios pose significant concerns for railway operators: prolonged heater usage increases energy consumption, while the risk of switch freezing affects dependability. Actually, the variability considered allows exploring the impact of temperature and dew point across various climates, including extremes (e.g., when the variability is 10°C). In addition, to differentiate the weather between the north and south heaters, we also consider small variations of $T_{ij}(W_k)$ relative to $T(W_k)$ due to sun or shade exposure, as in [18].

Real-world data on energy consumption and de-icing times are also extrapolated from [24, 29] to determine the energy $E^{SH}(T_{ij}(u), Z_{ij}^R(u))$ required by each 1050 W conductive heater when turned on. In particular, the time and the energy consumption required to heat a section of the railway from ambient temperatures of -20°C and -5°C to 5°C , as reported in Table 5 (with no wind) of [29], are used to extrapolate the times and energy consumption required to reach a target temperature of 5°C for each starting temperature between -30°C and 5°C . The same approach is applied to determine the energy required to maintain a target temperature of 5°C with ambient temperatures ranging from -30°C and 5°C . Finally, the energy required to maintain a target temperature at a specified offset above the current rail switch temperature, ensuring that the target temperature remains above the current dew point temperature $T^d(t)$ is calculated as follows. First, the energy required to maintain a target temperature at -20°C or -5°C ambient temperature is determined by extrapolating from the energy values reported in [29, Table 5] for target temperatures of 5°C , 7°C , and 10°C at ambient temperatures of -20°C or -5°C . Using the derived energy values to maintain the target temperatures of -15°C with $T_{ij}(t) = -20^\circ\text{C}$ and 0°C with $T_{ij}(t) = -5^\circ\text{C}$, we extrapolate the energy required to maintain the target temperature of $T_{ij}(t) + x$ for each x and $T_{ij}(t)$. Note that maintaining a temperature above freezing requires substantially less energy than bringing the temperature to the thawing point and keeping it there [29]. Consistent with [24], a maximum defrosting time of $\bar{\mu}_{ice} = 90$ min minutes was adopted, with $k_{ice} = 1$ and $k_W = 1$. We consider $k_{ice} = 0.1, 0.3, 0.5, 0.8, 1$ in the sensitivity analyses.

The time interval X_k for the change in weather is uniformly distributed on [15 min, 30 min]. The time intervals δ_v and δ_c between two consecutive weather data collections and control actions, respectively, are equal to 15 min. The parameter values for the stochastic processes $T(W_k)$, $T^d(W_k)$, and $T_{ij}(W_k)$ are the same as used in [18]. Regarding the setting for policy-specific parameters, τ and ξ take values 1.5°C and 24, respectively; and the error ε_f^{day} takes value 1.5°C d^{-1} . Local weather forecasts are considered less accurate than the global one, with $\varepsilon_L = 1.5\varepsilon_G$. The maximum non-negative sensor error ε_s takes values 0°C by default, or from 0°C to 1°C in the sensitivity analyses. While the impact of the relative humidity on the sensor error w_s takes values 1 by default or from 1 to 2 in the sensitivity analyses. We consider that w_s is significant only for relative humidity values above $H^s = 0.95$.

We recall that, to represent switch heating management based on actual train movements for each switch, random time intervals when the heating control system is active (control active state) alternate with periods when it is not active (control inactive state). Therefore, when no trains are scheduled through a switch, the heating control is kept inactive, and the heater stays off. Whereas, when trains are scheduled to pass through a switch, the heating control is activated and dynamically switches the heater on or off based on the actual or predicted temperature and the dew point, according to the adopted control policy. More precisely, the active heating control intervals consist of two parts. The first part is an initial buffer period designed for defrosting or preventing frost, during which no trains are expected to pass through the switch. The second part occurs when trains are actually scheduled to pass through.

In the conducted analyses, the following four different schedule scenarios of alternating active $\bar{\Phi}_k^{ij}$ and inactive $\bar{\Psi}_k^{ij}$ control periods are considered based on different time patterns $(\bar{\Phi}_k^{ij}, \bar{\Psi}_k^{ij})$:

- (120, 30) in which many 120-minute periods of active state alternate with short 30-minute periods of inactive state,
- (120, 300) in which many 120-minute periods of active state alternate with long 300-minute periods of inactive state,
- (240, 30) in which a few 240-minute periods of active state alternate with short 30-minute periods of inactive state, and
- (240, 300) in which long periods of active state alternate with long 300-minute periods of inactive state.

Regarding the assumed parameter values, note that: i) the modeling framework can be exercised with any admissible value for the model parameters, in accordance with the operator's requirements, who depends on the analysis outcomes

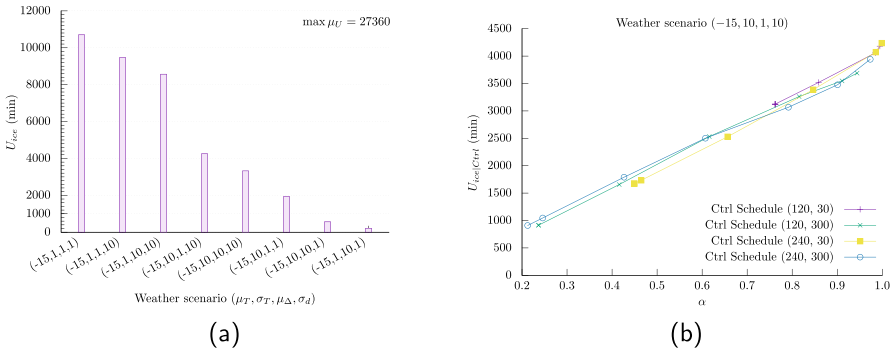


Fig. 4. Total cumulative time U_{ice} during which the switches are at risk of freezing in the absence of heating for different weather scenarios $(\mu_T, \sigma_T, \mu_\Delta, \sigma_\Delta)$ (a), and the corresponding measure $U_{ice|Ctrl}$ given that the heater control is active, as a function of the percentage of time the heating control system is active, for different heating control activation schedules (Ctrl Schedule) and for a specific weather scenario $(-15, 10, 1, 10)$ (b)

to make informed decisions, and ii) the sensitivity analysis by evaluating multiple values for key modeling parameters can be extended to any other beyond those considered in this paper, which were selected among those expecting to have a higher impact on the assessed metrics.

Finally, the evaluation results are obtained using the simulator of Möbius and performing 10000 independent model simulations for each obtained result.

9 Evaluation Results

The analysis results shown in this section focus on unavailability and energy efficiency, with the aim of assessing the impact of different heating control activation schedules for a specific weather scenario of medium severity among the eight possible scenarios. Similar trends have been observed for the unreliability and energy efficiency, therefore omitted in this paper.

In order to identify a suitable weather scenario of medium severity, in Fig. 4a, the eight different weather scenarios are listed based on their severity, represented by the total cumulative time $U_{\text{ice|Ctrl}}$ during which the switches are at risk of freezing in the absence of heating. Figure 4a shows that there are significant differences in the severity of the weather scenario, with values ranging from 214 min for $(-15, 1, 10, 1)$ to 10704 min for $(-15, 1, 1, 1)$.

Figure 4b depicts the total cumulative time $U_{\text{ice|Ctrl}}$ during which the switches are at risk of freezing in the absence of heating, given that the heater control is activated for different heating control activation schedules (Ctrl Schedule), considering the weather scenario $(-15, 10, 1, 10)$ that shows a medium severity level with $U_{\text{ice|Ctrl}} = 4253$ (cf. Figure 4a).

As shown in Fig. 4b, the different heating control schedules primarily result in distinct minimum values for $U_{\text{ice|Ctrl}}$, while the trends and values of $U_{\text{ice|Ctrl}}$ remain similar. The minimum value of $U_{\text{ice|Ctrl}}$ is directly related to the minimum overall percentage α of control activation time for a given schedule. The minimum threshold of α for a given heating control schedule is determined by the initial buffer period δ_a , allocated for recovery from freezing in each different operational interval $[\Theta_{2k-1}^{ij}, \Theta_{2k}^{ij}]$ in the given schedule. In fact, when the heating control system is activated after an inactive period, the related switch may be frozen, and the first control action occurs δ_c time units later. Therefore, the value of δ_c and the number $\lceil n_t^{ij}/2 \rceil - 1$ of operational intervals in the schedule jointly determine the minimum threshold for the overall percentage α .

Figures 5, 6 and 7 depict Energy consumption μ_E and Unavailability μ_U as a function of the percentage of time α the heating control system is active, for different heating control activation schedules (Ctrl Schedule), for a specific weather scenario $(-15, 10, 1, 10)$ and policies $P_{\text{bas,bas}}$, $P_{\text{pre,mpr}}$, and $P_{\varepsilon,\text{m}\varepsilon}$, respectively.

The trends of μ_E and μ_U , as shown in Figs. 5, 6 and 7, are comparable across the three policies when the same heating control activation schedule is used, although the absolute values differ.

Furthermore, the impact of different control schedules on both μ_E and μ_U can be significant, even when the heating control system is active for the same percentage of time.

As with $U_{ice|Ctrl}$ in Fig. 4b, the minimum thresholds for μ_E vary across different heating control schedules, depending on the minimum time the heating control system can be active for a given schedule.

When the control activation time reaches its maximum value ($\alpha = 1.0$), the decreasing trend for schedules (120, 30) and (240, 30) can be explained by the fact that the heating control system is always active, preventing freezing and reducing the energy needed for recovery. Figure 8b, discussed next, supports this finding.

Figure 8 depicts the Energy consumption μ_E (Fig. 8a) and Unavailability μ_U (Fig. 8b) as a function of the percentage of time the heating control system is

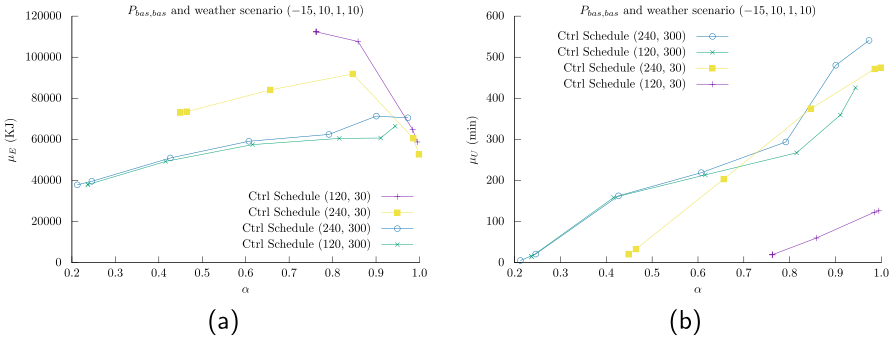


Fig. 5. Energy consumption μ_E (a) and Unavailability μ_U (b) as function of the percentage of time the heating control system is active, for weather scenario $(-15, 10, 1, 10)$ and policy $P_{bas,bas}$, and different heating control activation schedules (Ctrl Schedule).

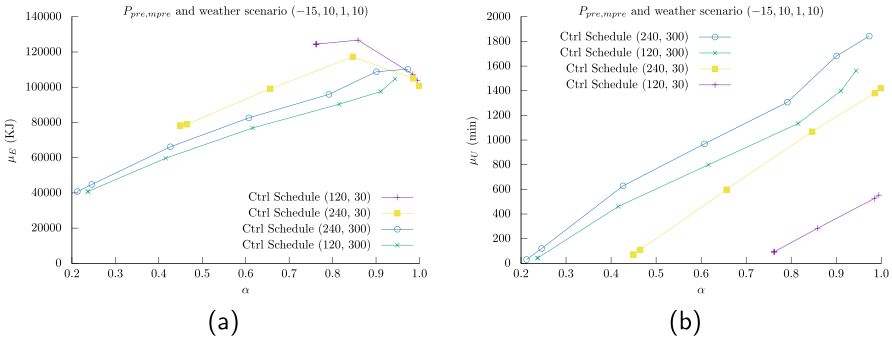


Fig. 6. Energy consumption μ_E (a) and Unavailability μ_U (b) as function of the percentage of time the heating control system is active, for weather scenario $(-15, 10, 1, 10)$ and policy $P_{pre,mpr}$, and different heating control activation schedules (Ctrl Schedule).

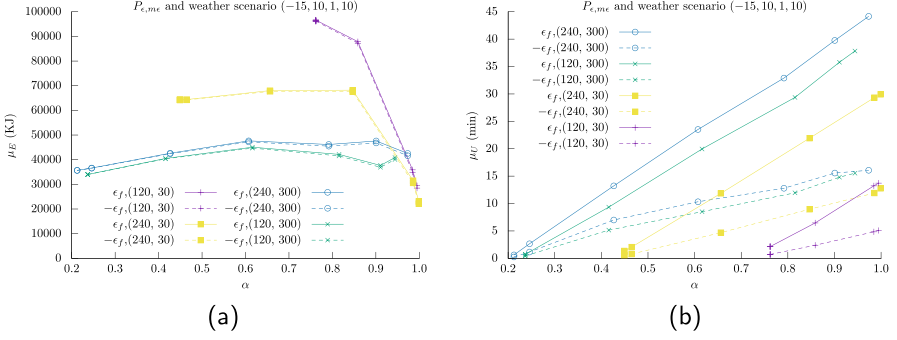


Fig. 7. Energy consumption μ_E (a) and Unavailability μ_U (b) as function of the percentage of time the heating control system is active, for weather scenario $(-15, 10, 1, 10)$ and policy $P_{\epsilon, m\epsilon}$, and different heating control activation schedules (Ctrl Schedule).

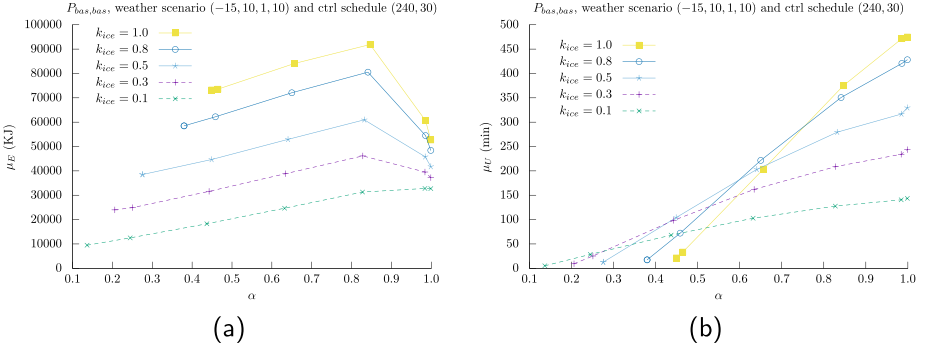


Fig. 8. Energy consumption μ_E (a) and Unavailability μ_U (b) as function of the percentage of time the heating control system is active, for heating control activation schedule (240, 30), weather scenario $(-15, 10, 1, 10)$, policy $P_{bas,bas}$, and different ice quantity k_{ice} .

active, for different ice quantity k_{ice} , for a specific heating control activation schedule (240, 30), weather scenario $(-15, 10, 1, 10)$, and policy $P_{bas,bas}$.

Figures 9a and 9b depict the Energy consumption μ_E and Unavailability μ_U , respectively, as a function of the percentage of time the heating control system is active, for different sensor errors ϵ_s , for a specific heating control activation schedule (240, 30), weather scenario $(-15, 10, 1, 10)$, and policy $P_{bas,bas}$.

Figures 9a and 9b show that sensor errors can lead to significant variations in Energy consumption and Unavailability, which grow with the fraction of time the heating control system is active. For example, at $\alpha = 0.85$, μ_E increases by approximately 2%, 5%, and 17% for sensor errors ϵ_s of 0.2, 0.4, and 1 °C, respectively. And μ_U is also more sensitive to variations in the sensor error. In fact, at $\alpha = 0.85$, μ_U increases by approximately 12%, 31%, and 98% for sensor errors ϵ_s of 0.2, 0.4, and 1 °C, respectively.

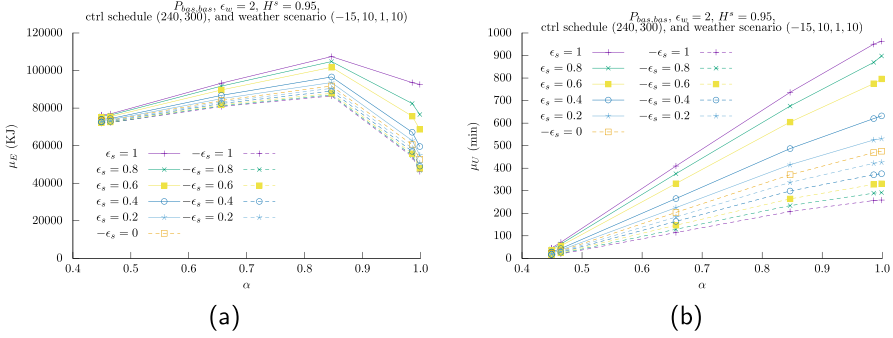


Fig. 9. Energy consumption μ_E (a) and Unavailability μ_U (b) as function of the percentage of time the heating control system is active, for heating control activation schedule (240, 30), weather scenario $(-15, 10, 1, 10)$, policy $P_{bas,bas}$, and different sensor errors ϵ_s .

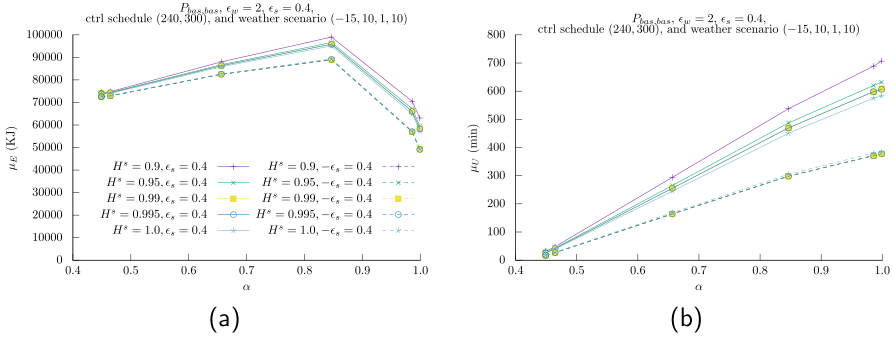


Fig. 10. Energy consumption μ_E (a) and Unavailability μ_U (b) as function of the percentage of time the heating control system is active, for heating control activation schedule (240, 30), weather scenario $(-15, 10, 1, 10)$, policy $P_{bas,bas}$, and different H^s , with $w_s = 2$ and sensor error $\epsilon_s = 0.4$.

Figures 10a and 10b depict the Energy consumption μ_E and Unavailability μ_U , respectively, for varying humidity thresholds H^s affecting sensor performance. They illustrate how the humidity tolerance of the sensors (lower or higher H^s) affects energy consumption and unavailability. For example, at $\alpha = 0.85$, μ_E and μ_U increase by at most 4% and 19%, respectively, when sensors with a humidity tolerance of 90% ($H^s = 0.9$) are used.

From an industrial perspective, the obtained results indicate that the choice of the control schedule can have a major impact on both energy consumption and unavailability, even when the overall control activation percentage remains unchanged.

For example, under policy $P_{\epsilon, m\epsilon}$ (cf. Fig. 7), an unavailability $\mu_U < 15$ minutes can be achieved with energy consumptions ranging from approximately

20 000 kJ (≈ 5.6 kWh) for control schedule (120, 30) with $\alpha = 1$, to approximately 70 000 kJ (≈ 19.4 kWh) for control schedule (240, 30) with $\alpha = 0.7$. This result suggests that a careful selection of the control schedule can substantially reduce energy consumption while maintaining the same availability target.

9.1 Simulation Performance

To show the scalability of the modeling framework to the network size n of railway stations, the results relative to the $\mu_{P,n}(10000)$ indicator are presented in Table 2. They are computed considering the policies $P_{\text{bas,bas}}$, $P_{\text{pre,mpré}}$, and $P_{\varepsilon,\text{m}\varepsilon}$, and the weather scenario $(-15, 10, 1, 10)$, $\tau = 1.5$, $\xi = 24$, $\varepsilon_{\text{f}}^{\text{day}} = 1.5$, and $\alpha = 0.6$. In particular, the ratio between $\mu_{P,n}(10000)$ and $\mu_{P,19}(10000)$ is also shown next to each value of $\mu_{P,n}(10000)$.

First, it can be observed that the values of $\mu_{P,n}(10000)$ for $P_{\text{bas,bas}}$ and $P_{\varepsilon,\text{m}\varepsilon}$ are almost equivalent. But $\mu_{P,n}(10000)$ for $P_{\text{pre,mpré}}$ is significantly greater than $\mu_{P,n}(10000)$ for the other two policies, although the gap tends to decrease as n increases. Specifically, the gap between $P_{\text{pre,mpré}}$ and the other policies goes from about 4 times larger (for small n) to about 1.7 times larger (for large n). This gap is due to the Singular Spectrum Analysis (SSA) time series analysis method adopted in $P_{\text{pre,mpré}}$ to forecast future temperatures and dew points. The ratio $\mu_{P,n}(k)/\mu_{P,19}(k)$ shows that the best scalability is obtained with the policy $P_{\text{pre,mpré}}$. In general, for all the three policies, for increasing n , the relative increase of $\mu_{P,n}(10000)$ tends to be smaller.

From an industrial perspective, the network sizes $n = 19, 38, \dots, 190$ that we considered span a wide range of practical scenarios, from medium-size stations to large railway hubs with approximately 200 switches.² The results show that a complete analysis with 10000 simulation runs can be performed within a few hours even for the largest networks, making the proposed framework suitable for off-line engineering studies aimed at comparing alternative heating-control strategies, weather scenarios, and maintenance policies.

Table 2. Simulation performance $\mu_{P,n}(k)$ (s) and ratio $\mu_{P,n}(k)/\mu_{P,19}(k)$ for different $n = 19, 38, \dots, 190$ and $k = 10000$.

	$n \ n/19$											
	19	1.0	38	2.0	76	4.0	114	6.0	152	8.0	190	10.0
$P_{\text{bas,bas}}$	393	1.0	1078	2.74	3856	9.8	8234	21.0	14 714	37.4	23 311	59.3
$P_{\varepsilon,\text{m}\varepsilon}$	388	1.0	1067	2.75	3867	10.0	8351	21.5	14 636	37.7	22 802	58.8
$P_{\text{pre,mpré}}$	1681	1.0	3644	2.17	8961	5.3	16 466	9.8	26 667	15.9	38 810	23.1

² For example, St. Pölten Hauptbahnhof (Austria) comprises 61 switches [31], whereas Praha-jih Marshalling Yard (Czech Republic) comprises approximately 200 switches [32].

10 Conclusion and Future Work

Focusing on balancing dependability and energy efficiency, we have presented advanced research aimed at the effective management of heater-based anti-icing systems for railway switches, building upon previous studies in multiple ways. The new contributions comprise: i) incorporating a more accurate representation of realistic weather conditions, ii) accounting for measurement errors from sensor data, in addition to failures of communication channels and forecast errors, and iii) representing the switch heating management based on actual train traffic.

Through a thorough analysis in a variety of operational conditions, we demonstrated the impact of these extensions and the benefits of accounting for them in assessing a more accurate balance between energy consumption and dependability indicators. For example, the obtained analysis results show that the impact of different train schedules on both energy consumption and unavailability can be significant, even when the heating control system is active for the same percentage of time (as shown in Figs. 5, 6 and 7). A notable behavior is that when the anti-icing system is always active to prevent freezing, in which case the higher energy costs are partially offset by a reduction in the energy needed for recovery. This recovery is typically required when the system operates only a portion of the time and is exposed to freezing conditions, resulting in periods of unavailability (as shown in Fig. 8b). Furthermore, incorporating sensor error contributions enhanced result accuracy under more realistic conditions. Finally, the results from the simulation performance demonstrate the scalability of the developed assessment framework and its applicability to railway stations of medium to large size (the study analyzed railway stations with up to 190 railway switches).

Future Work. The presented extended analysis framework for the anti-icing railway switch system addresses important open questions raised in prior developments. In addition to further expanding the analysis scenarios to increase knowledge of the impact of the considered phenomena, some open challenges remain. A prominent one concerns the exploration of freezing conditions beyond those considered so far (based on the relationship between temperature and dew point), such as freezing caused by rapid temperature changes or by the interplay between heavy snowfall and the dew point.

Railway transportation is among the most energy-efficient and environmentally friendly means of transportation, which is expected to contribute significantly to the European Green Deal by improved digitalization. According to [8], one of the challenges for further increasing the levels of safety, reliability, and comfort in the railway domain concerns the integration of AI-based systems.

For the specific case of the anti-icing system for railway switches addressed in this study, AI technologies appear promising for supporting heater policies towards improving the balance between energy consumption and dependability, e.g., by providing more accurate forecasts of weather dynamics. Furthermore, AI-based models trained on historical data regarding timetable, delays, and weather

conditions could be used to predict the expected duration of train delays and their propagation across switches. These predictions could be integrated into the proposed framework through the parameters that govern the delay and delay propagation mechanisms.

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Disclosure of Interests. The authors have no competing interests to declare that are relevant to the content of this article.

A List of Main Symbols

α	Percentage of time the heating control system is active
$\bar{\alpha}$	Percentage of time the heating control system is active based on $\bar{\theta}_0^{ij}, \dots, \bar{\theta}_{n_i^{ij}-1}^{ij}$
c	How power line characteristics affect channel failure
CH_{ij}	Communication channel between C_i and L_{ij}
C_i	i th coordinator
δ_a	Minimum width of scheduled active control intervals
δ_c	Time between two consecutive control actions
δ_v	Time between two consecutive weather observations
ε_f	Maximum non-negative forecast error
ε_f^{day}	Maximum non-negative one day forecast error
ε_s	Maximum non-negative sensor error, influenced only by sensor characteristics
$\varepsilon_{ij}^s(t_k)$	Maximum non-negative sensor error at time t_k
H^s	Relative humidity threshold above which sensor error becomes significant
$H_{ij}(t)$	Relative humidity at time t close to SH_{ij}
k_{ice}	Scaling factor for ice quantity
k_W	Scaling factor for heating power
L_{ij}	j th controller coordinated by C_i
μ_E	Expected energy consumption
$\mu_{P,n}(k)$	Total amount of CPU time to run k simulations
μ_{ice}	Maximum recovery time from ice
$\bar{\mu}_{ice}$	Maximum base recovery time from ice for $k_{ice} = 1$ and $k_W = 1$
μ_Δ	Difference $T(0) - T^d(0)$
μ_T	Value assigned to $T(0)$
μ_U	Expected unavailability for switches

n_t^{ij}	Number of time instants θ_k^{ij}
p^{dprop}	Probability that the train delay on a switch propagate to other switches
P_F	Probability of ice formation on at least one switch
$\bar{\Phi}_k^{ij}$	Width of the k th active control interval based on the basic schedule for SH_{ij}
Φ_k^{ij}	Duration of the k th active control interval obtained as a function of α for SH_{ij}
$\bar{\Psi}_k^{ij}$	Width of the k th inactive control interval based on the basic schedule for SH_{ij}
RS_{ij}	Railroad switch heated by SH_{ij}
SH_{ij}	Railroad switch heater controlled by L_{ij}
σ_d	How much $T^d(t)$ varies from its expected value
σ_T	How much $T(W_k)$ varies from μ_T
S_{ij}	j th weather sensor associated to L_{ij}
τ	Temperature threshold
$T^d(t)$	Dew point at time t
$\hat{T}^d(t)$	Measured value for $T^d(t)$
$\bar{\theta}_k^{ij}$	Time instants defining the scheduled active control intervals for SH_{ij}
Θ_k^{ij}	Time instants defining the scheduled active control intervals for SH_{ij} as a function of α extended by the delay of the trains
θ_k^{ij}	Time instants defining the scheduled active control intervals for SH_{ij} as a function of α
t_k	Time of the k th control action
$T(t)$	Temperature at time t
$T_{ij}(t)$	Temperature of SH_{ij} at time t
$\hat{T}_{ij}(t)$	Measured value for $T_{ij}(t)$
v_k	Time of the k th weather observation
W_k	Time of the k th weather change
w_s	Impact of the humidity on the sensor error
w_s^d	Impact of the humidity on the sensor error in dew point measurement
w_s^T	Impact of the humidity on the sensor error in temperature measurement
ξ	Maximum number of control actions for which dew point data are used during channel failure

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