Consiglio Nazionale delle Ricerche

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ISC institute for complex systems

SIGNATURES OF GLASSY PHYSICS NEAR THE SUPERCONDUCTOR-INSULATOR TRANSITION

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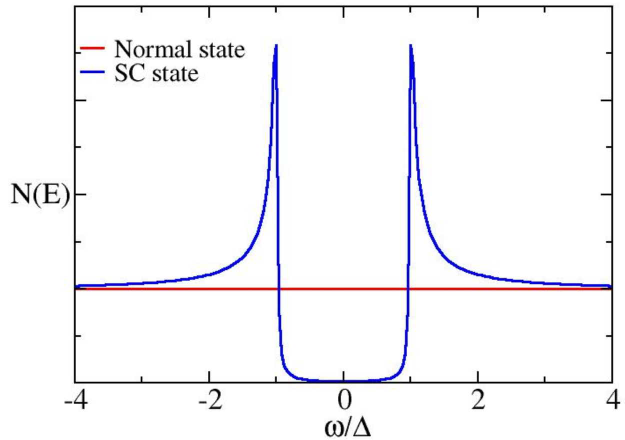
TIDS-15 3 September 2013

Energy scales in a superconductor

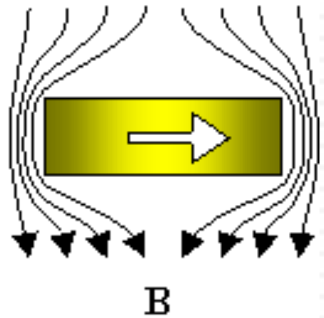
- Binding of electrons in Cooper pairs

$$\psi(x) = \langle c_\uparrow c_\downarrow \rangle = |\Delta(x)| e^{i\theta(x)}$$



  - SC amplitude: gap Δ in quasiparticle excitations
  - SC phase: superfluid diamagnetic response $D_s=n_s/m$

$$J = -\frac{e^2 n_s}{mc} A \quad \Rightarrow \quad \nabla^2 B = \frac{4\pi n_s e^2}{mc^2} B = \frac{1}{\lambda^2} B$$



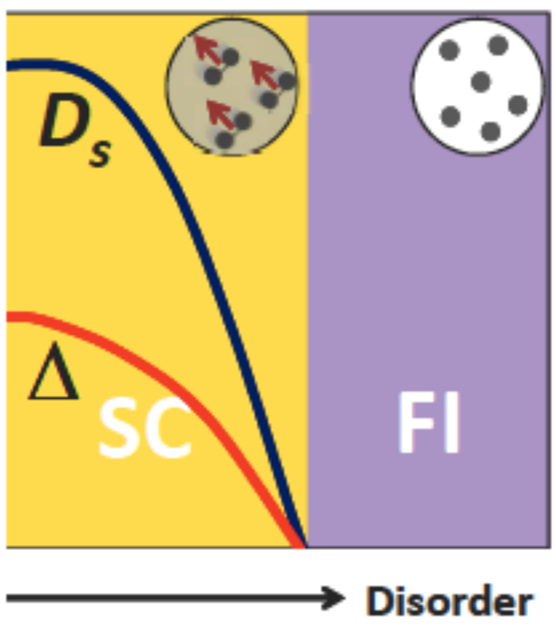
Two energy scales
Superfluid stiffness D_s: phase coherence
SC gap Δ: pairing

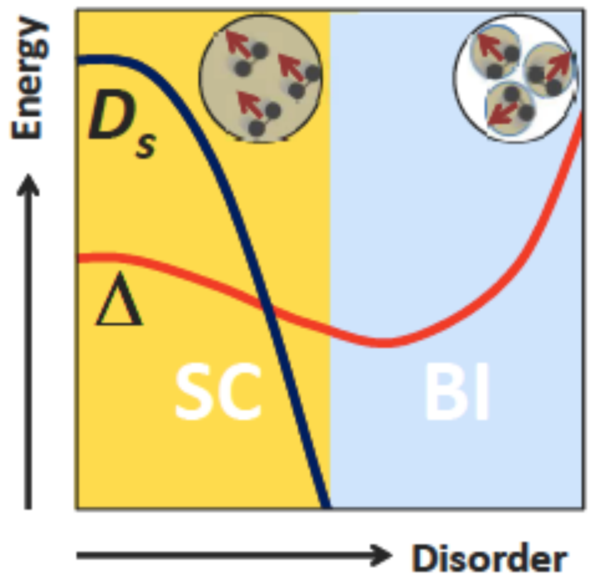
Conventional superconductor: $D_s \gg \Delta$
Disorder can change this picture qualitatively

Disorder and superconductivity

- "Fermionic" vs "bosonic" mechanism

Superconductor-Insulator Transition (SIT)





"Fermionic" mechanism (Finkelstein):
disorder enhances Coulomb repulsion,
pairing strength decreases,
both Tc and Δ go to zero

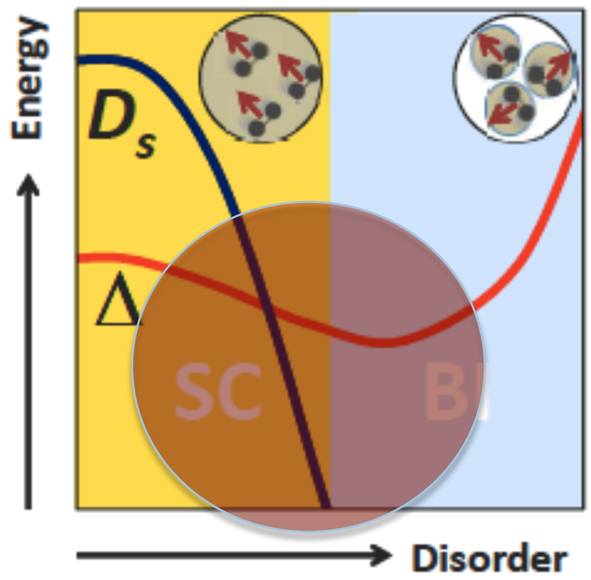
"Bosonic" mechanism (Fisher, Ma & Lee, etc.)
direct localization of Cooper pairs,
finite pairing in the insulating phase

Disorder and superconductivity

How to probe?
New hints from **tunnelling**

Superconductor-Insulator Transition (SIT)

NbN
TiN
InOx



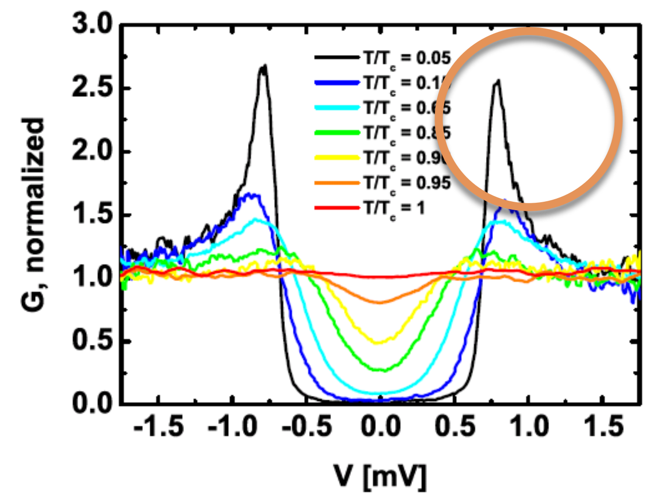
"Bosonic" mechanism (Fisher, Ma & Lee, etc.)
direct localization of Cooper pairs,
finite pairing in the insulating phase

Tunneling investigation of the SIT

- *Pseudogap* and suppression of coherence peaks

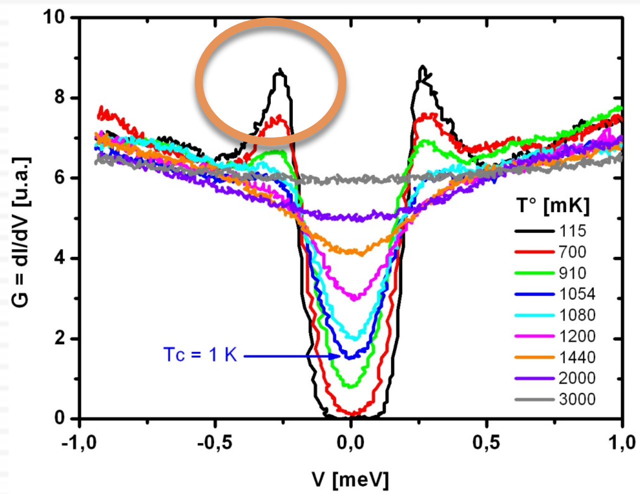
TiN, Tc=4.7 K

W. Escoffier, et al., *Phys. Rev. Lett.* **93**, 217005, (2004)



TiN near SIT, $T_c = 1.0$ K

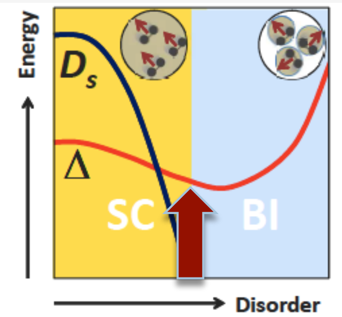
B. Sacépé *et al.* Nature Communications, 1:140 (2010)



Increasing disorder

Spectral gap ⟷ pairing

Peak height ⟷ phase-coherent SC

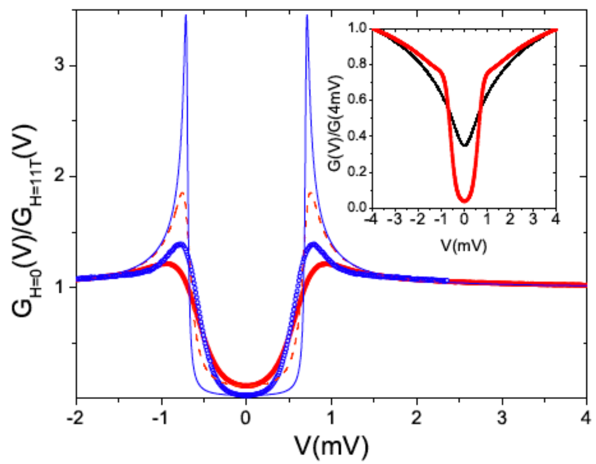


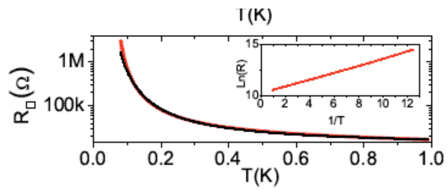
Tunneling investigation of the SIT

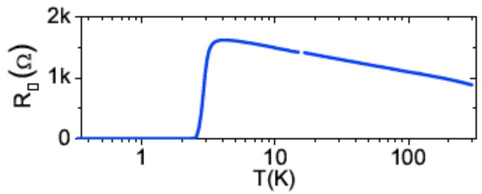
- *Pseudogap* and suppression of coherence peaks
- Persistence of the gap in the insulating side

D. Sherman et al. PRL 108, 177006 (2012)

InO_x







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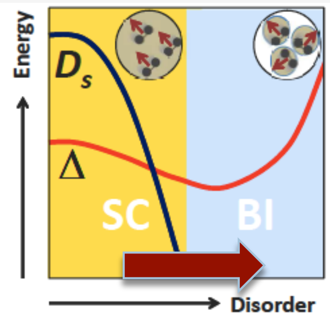
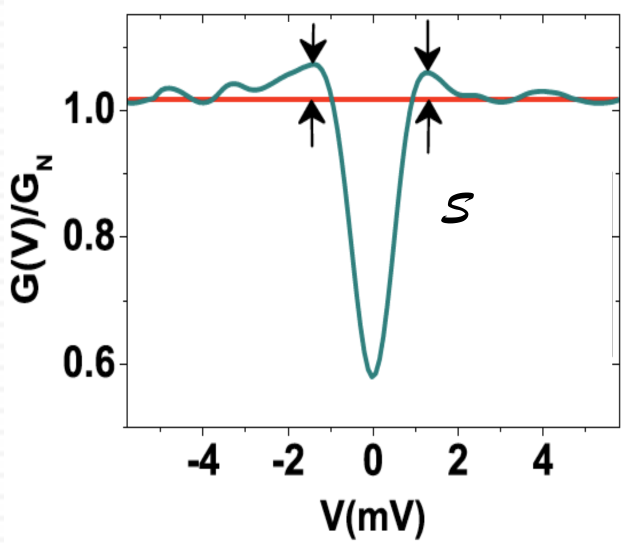
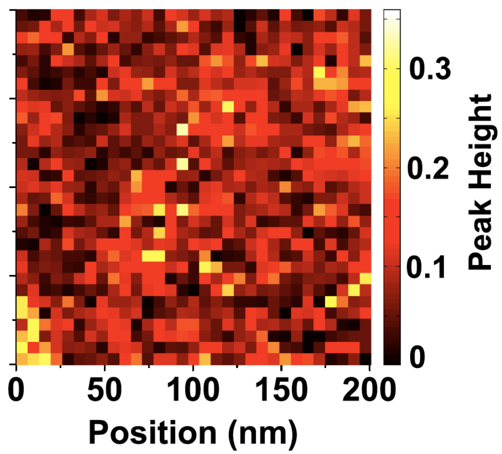


Tunneling investigation of the SIT

- Intrinsic *inhomogeneity* in the peak height below Tc, with anomalous probability distribution

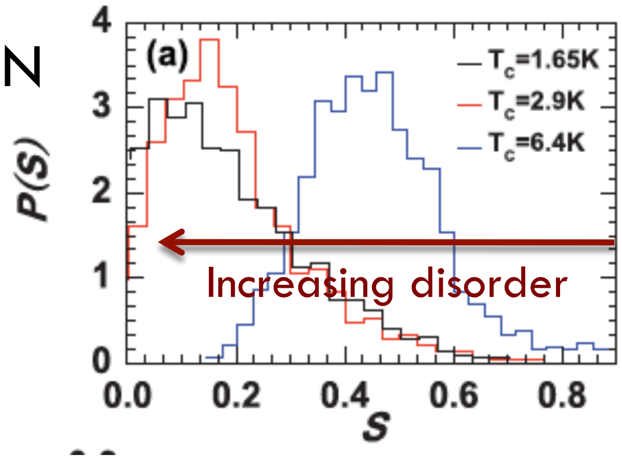
NbN Tc=2.9 K

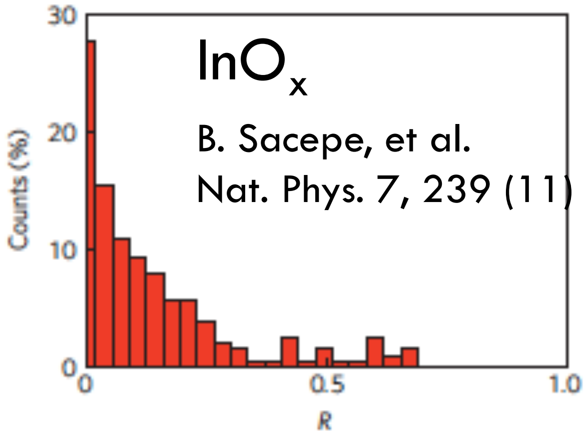




G. Lemarie et al. PRB 87, 184509 (13)

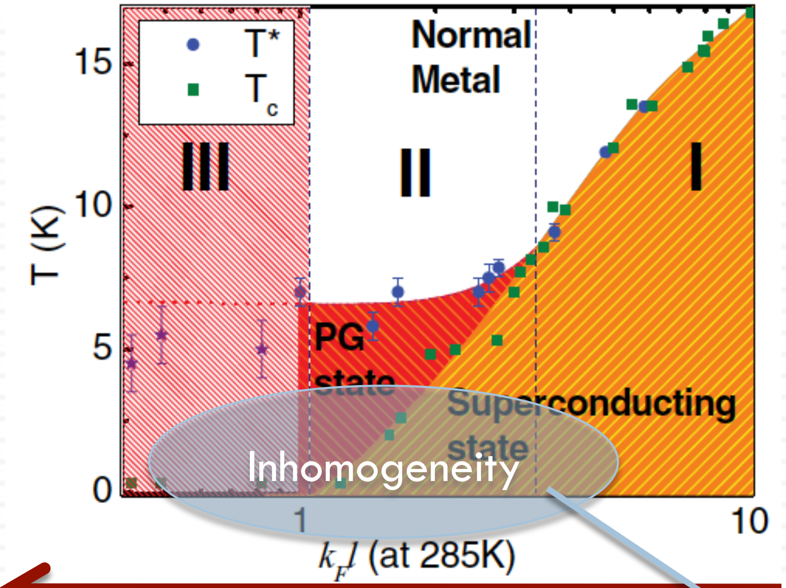
NbN



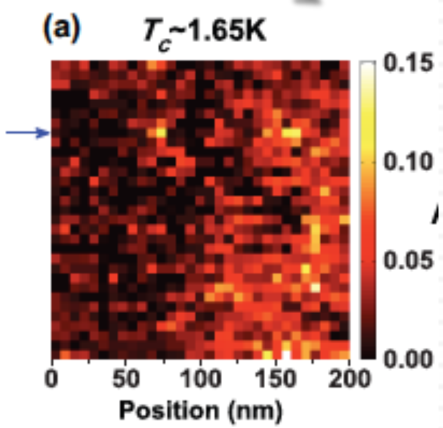


B. Sacepe, et al.
Nat. Phys. 7, 239 (11)

NbN films
M. Chand et al
PRB 2012

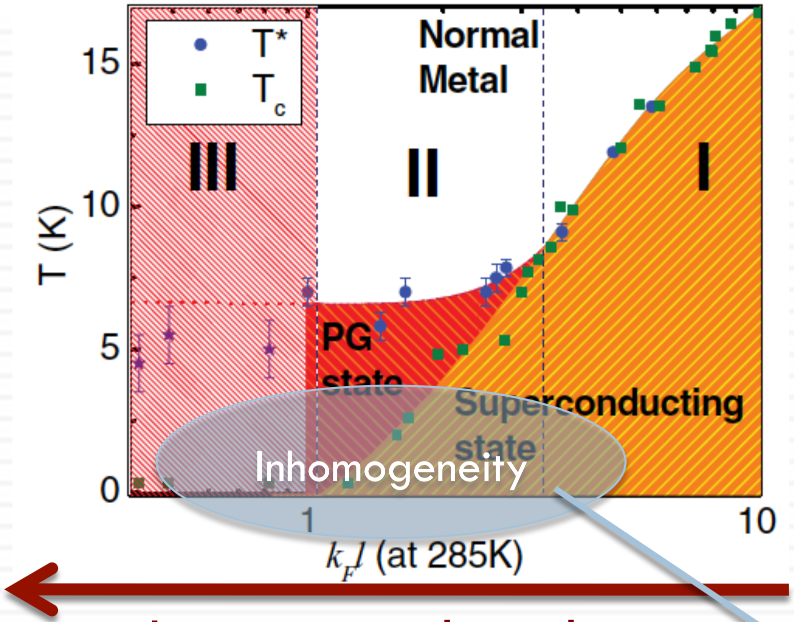


Increasing disorder



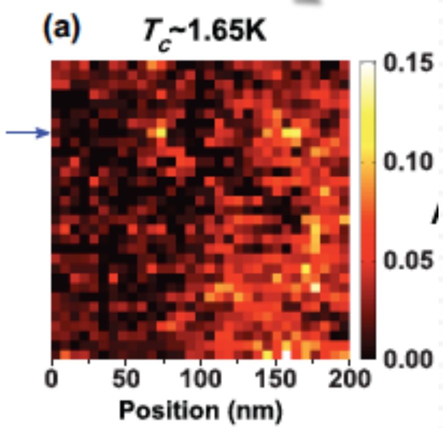
Inhomogeneity and glassy physics

NbN films
M. Chand et al
PRB 2012



Increasing disorder

The emergent mesoscopic inhomogeneity is a signature of **"glassy"** superconductivity



Recent advances on the theoretical approach to the SIT

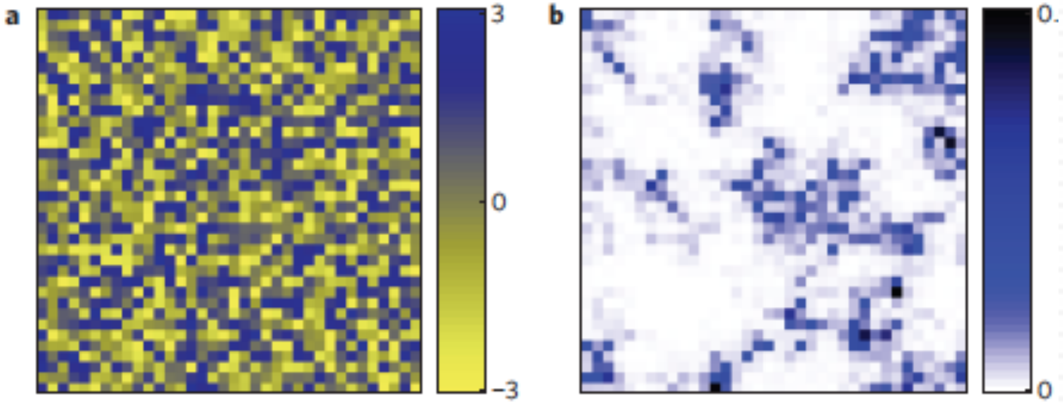
The fermionic model

- Attractive Hubbard model with on-site disorder

$$V_i \in [-V_0, V_0] \qquad H = -t \sum_{<ij>,\sigma} (c^\dagger_{i\sigma} c_{j\sigma} + h.c.) + \sum_{i,\sigma} (V_i - \mu) n_{i\sigma} - |U| \sum_i n_{i\uparrow} n_{i\downarrow}. \qquad \Delta(\mathbf{r}_i) = -|U| \langle c_{i\downarrow} c_{i\uparrow} \rangle$$

- Mean-field solution (Bogoliubov-de-Gennes) (+ MC)
  - formation of SC domains on the scale of the correlation length

distribution of V_i



distribution of Δ_i

Ghosal et al. PRB 2001; Dubi et al. Nature 2008; Bouadim, et al. Nat. Phys. 2011

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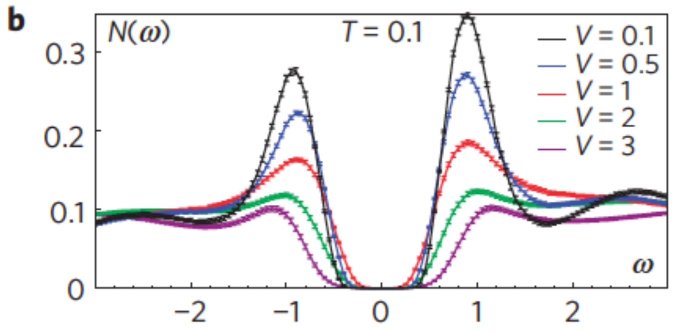
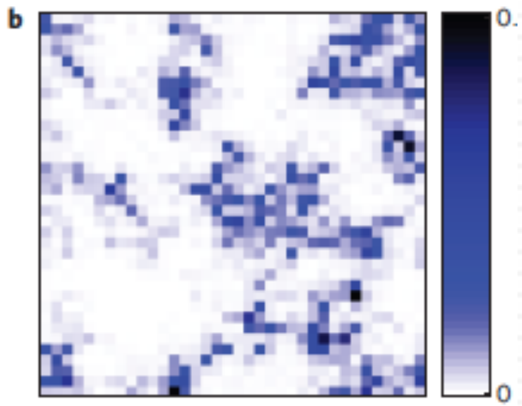
Persistence of large spectral gap but reduced coherence peaks (-> Δ_i)



Increasing disorder



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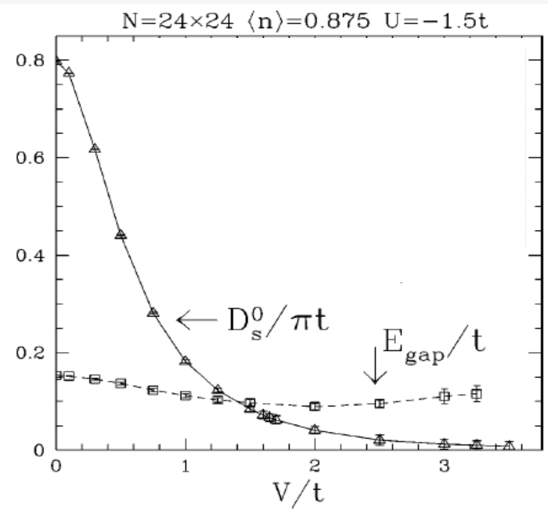
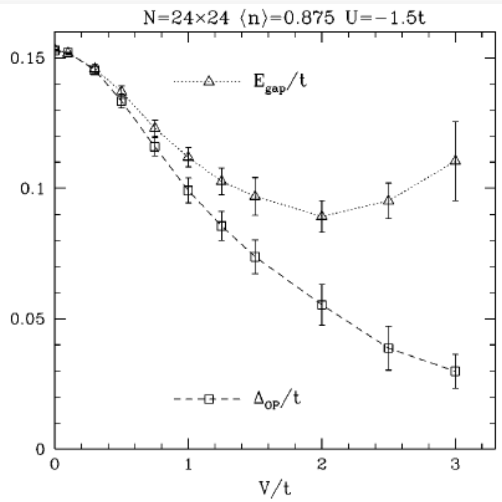
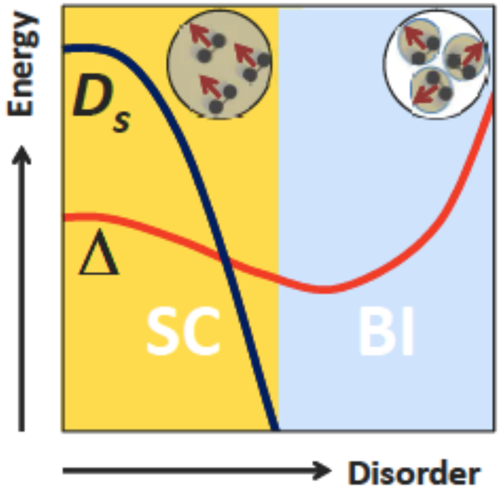
- Mean-field solution (Bogoliubov-de-Gennes) (+ MC)
  - formation of SC domains on the scale of the correlation length
  - Suppression of the superfluid stiffness and local order parameter with persistence of spectral gap







Ghosal et al. PRB 2001; Dubi et al. Nature 2008; Bouadim, et al. Nat. Phys. 2011

The bosonic model

- Pairing the time-reversed eigenstates ϕ_α of H_0

$$H = \sum_{\alpha,\sigma} \xi_\alpha c^\dagger_{\alpha\sigma} c_{\alpha\sigma} - U \sum_{\alpha\beta} M_{\alpha\beta} c^\dagger_{\alpha\uparrow} c^\dagger_{\alpha\downarrow} c_{\beta\downarrow} c_{\beta\uparrow} \qquad M_{\alpha\beta} = \sum_i |\phi_\alpha(\mathbf{r}_i)|^2 |\phi_\beta(\mathbf{r}_i)|^2$$

- Pseudospin Hamiltonian (Anderson, Ma & Lee)

$$H_{PS} = -\sum_\alpha 2\xi'_\alpha S^z_\alpha - \frac{U}{2} \sum_{\alpha\beta} M_{\alpha\beta} \left(S^+_\alpha S^-_\beta + S^-_\alpha S^+_\beta \right)$$

$$S^z_\alpha = \frac{1}{2} \left(\sum_\sigma c^\dagger_{\alpha\sigma} c_{\alpha\sigma} - 1 \right)$$

$$S^+_\alpha = c^\dagger_{\alpha\uparrow} c^\dagger_{\alpha\downarrow}$$

$S^z_\alpha = +\frac{1}{2}$ Cooper pair on site i

$S^z_\alpha = -\frac{1}{2}$ Empty site i

$\Delta_i \propto \langle S^x_i \rangle$ SC: spins in the plane

Strong disorder: : $M_{\alpha\beta}$ short-ranged
Competition between pair hopping and localization

The bosonic model

- Pairing the time-reversed eigenstates ϕ_α of H_0

$$H = \sum_{\alpha,\sigma} \xi_\alpha c^\dagger_{\alpha\sigma} c_{\alpha\sigma} - U \sum_{\alpha\beta} M_{\alpha\beta} c^\dagger_{\alpha\uparrow} c^\dagger_{\alpha\downarrow} c_{\beta\downarrow} c_{\beta\uparrow}$$

$$M_{\alpha\beta} = \sum_i |\phi_\alpha(\mathbf{r}_i)|^2 |\phi_\beta(\mathbf{r}_i)|^2$$

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$$S^z_\alpha = \frac{1}{2} \left(\sum_\sigma c^\dagger_{\alpha\sigma} c_{\alpha\sigma} - 1 \right)$$

$$S^+_\alpha = c^\dagger_{\alpha\uparrow} c^\dagger_{\alpha\downarrow}$$

- Ising model in transverse random field

$g \sim t^2/UV$

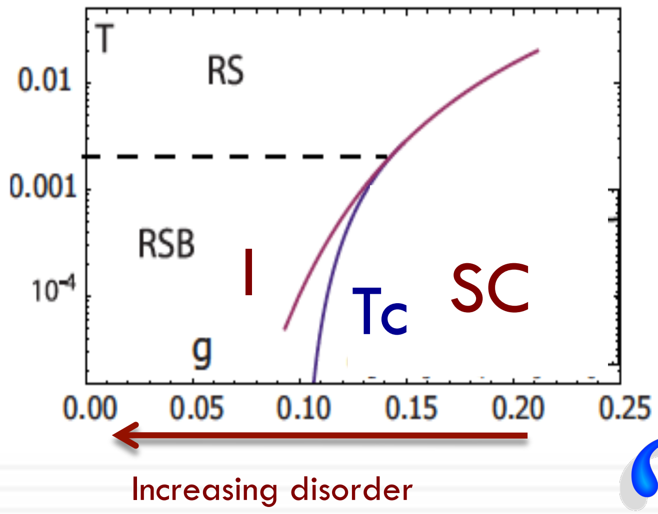
$$H_I = -\sum_i \xi_i \sigma^z_i - g \sum_{\langle ij \rangle} \sigma^x_i \sigma^x_j$$

$\langle \sigma^x_i \rangle = \Delta_i$

- New results from **Cavity Approach on Bethe Lattice** via the mapping into the **Directed Polymer** problem (Ioffe&Mezard,2010)

RSB and its physical consequences

Ioffe and Mezard, PRL 2010, Feigel'man, Ioffe, Mezard PRB 2010

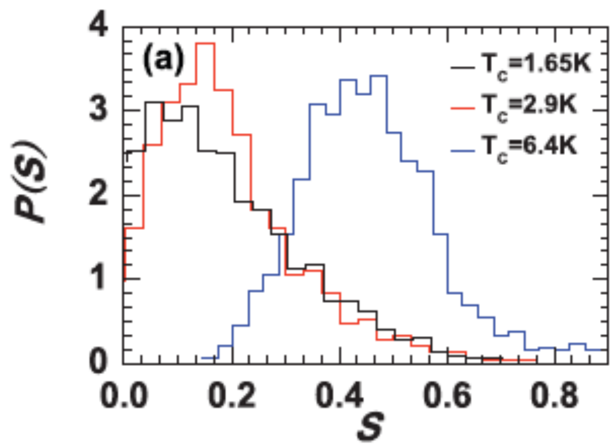


- The SIT occurs at a finite g_c
- Low-T phase: Replica Symmetry Breaking (RSB)
- Universal power-law behavior of the local order parameter??

$$P(S) = \frac{S_0^m}{S^{1+m}}$$

S plays the role of the local SC order parameter, i.e. peak height

m=1-eg_c disorder independent



Not "universal" since finite-dimensional (glassy) effects are relevant

G. Lemarie et al. PRB 87, 184509 (13)

RSB and its physical consequences

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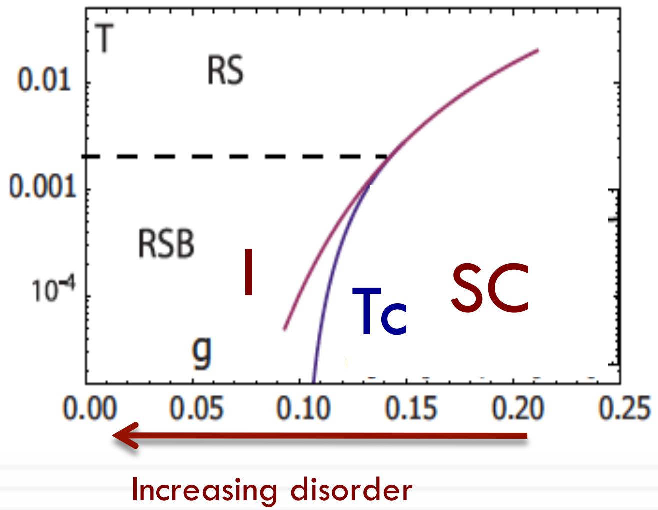


- The SIT occurs at a finite g_c
- Low-T phase: Replica Symmetry Breaking (RSB)
- Universal power-law behavior of the local order parameter

Is there any "glassy" physics emerging in **transport** properties?

Can be understand it theoretically within purely fermionic models?

Below Tc: superfluid response and low-frequency optical conductivity

Diamagnetism in a SC

- SC: purely diamagnetic response D_s to an applied **transverse** field

$$J = -\frac{e^2 n_s}{mc} A = -\frac{e^2}{c} D_s A = \langle j_P \rangle - \frac{e^2 n}{mc} A$$

- Clean SC: $j_P \approx 0$ at T=0, so that $n_s \approx n$;

Diamagnetism in a SC

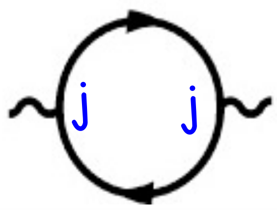
- SC: purely diamagnetic response D_s to an applied **transverse** field

$$J = -\frac{e^2 n_s}{mc} A = -\frac{e^2}{c} D_s A = \langle j_P \rangle - \frac{e^2 n}{mc} A$$

- Clean SC: $j_P \approx 0$ at T=0, so that $n_s \approx n$;
- Dirty SC: $j_P \neq 0$ at T=0, so $n_s \ll n$;

- Standard (dirty) BCS: compute the current-current correlation function (no vertex corrections)

$$\langle j_P \rangle \Rightarrow \langle T j_P^a j_P^b \rangle = K_{BCS}^{ab}(r, r')$$

K_{BCS}= 

BCS approach

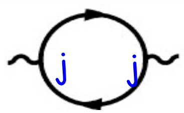
- Attractive Hubbard model with on-site disorder

$$H = -t \sum_{<ij>,\sigma} (c_{i\sigma}^\dagger c_{j\sigma} + h.c.) + \sum_{i,\sigma} (V_i - \mu) n_{i\sigma} - |U| \sum_i n_{i\uparrow} n_{i\downarrow}. \quad \Delta_i \equiv |U| \langle c_{i\downarrow} c_{i\uparrow} \rangle$$

- Current-current correlation function with the BdG Δ_i solutions computed at zero applied field A

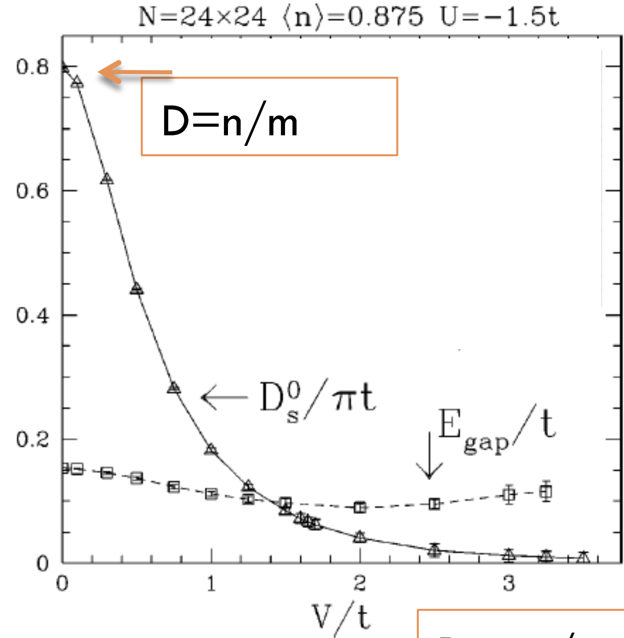
$$J = -\frac{e^2 n_s}{mc} A = -\frac{e^2}{c} D_s A = \langle j_P \rangle - \frac{e^2 n}{mc} A$$

$$\langle j_P \rangle \Rightarrow \langle T j_P^a j_P^b \rangle = K_{BCS}^{ab}(r,r')$$

$K_{BCS}=$ 

Disorder-induced quasiparticle paramagnetism

What about **phase fluctuations?**



Ghosal et al. PRB 2001

D<<n/m

Phase fluctuations in (dirty) SC

- Effective action for phase fluctuations

$$S_g \simeq \frac{1}{8\pi}\int d\mathbf{r} D_s \left(\nabla\theta\right)^2$$

- Minimal-coupling substitution $\nabla\theta \to \nabla\theta - 2e\mathbf{A}$

$$S_g = \frac{1}{8\pi}\int d\mathbf{r} D_s(\nabla\theta - 2e\mathbf{A})^2 \Rightarrow \int D_s\nabla\theta \cdot \mathbf{A} = -\int D_s\theta(\nabla \cdot \mathbf{A})$$

- In the **clean** case θ couples only to the **longitudinal** *A* component
- However in the presence of disorder

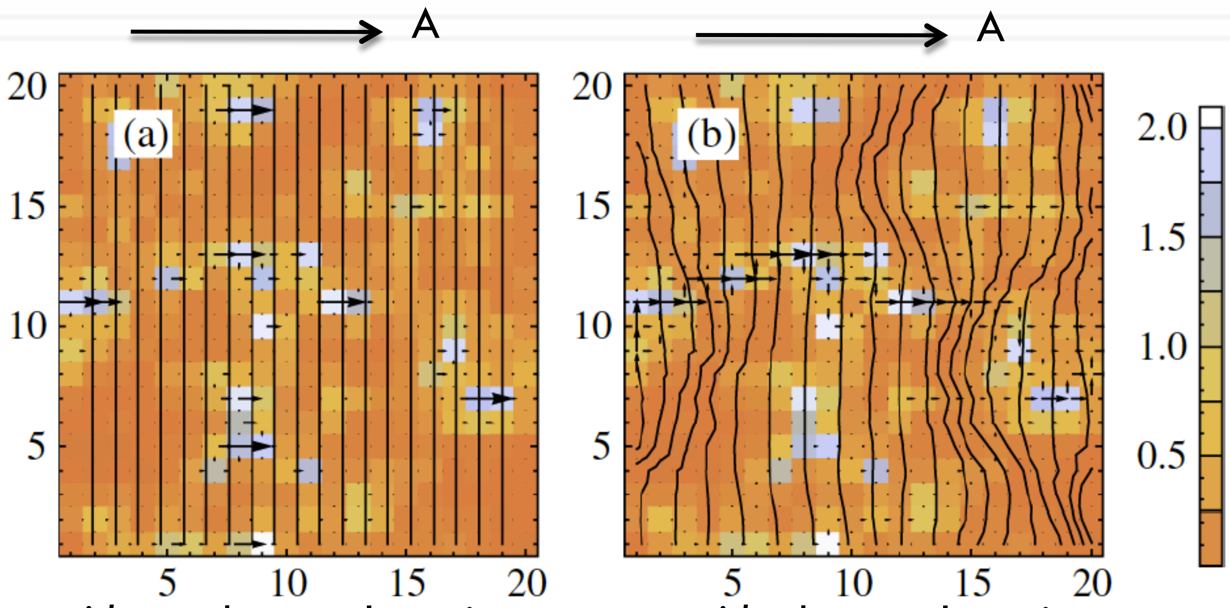
$$S_g \sim \frac{1}{8\pi}\int d\mathbf{r} D_s(\mathbf{r})(\nabla\theta - 2e\mathbf{A})^2$$

so that θ couples also to the transverse *A* component: thus phase fluctuations (i.e. vertex corrections) are relevant also to compute the superfluid stiffness

Current patterns

$$H = -t \sum_{<ij>,\sigma} (c^\dagger_{i\sigma} c_{j\sigma} + h.c.) + \sum_{i,\sigma} (V_i - \mu) n_{i\sigma} - |U| \sum_i n_{i\uparrow} n_{i\downarrow} . \quad \Delta_i \equiv |U| \langle c_{i\downarrow} c_{i\uparrow} \rangle = |\Delta_i| e^{i\theta_i}$$

- Current in the presence of a finite transverse A, by allowing for the local phases θ_i of the BdG solutions to relax to the applied field A



without phase relaxation — *with* phase relaxation

map: local Δ_i
lines: constant phase
$\nabla\theta_i - 2A$
arrows: local current

G. Seibold, L.Benfatto, J. Lorenzana and C. Castellani
PRL 108, 207004 (2012)

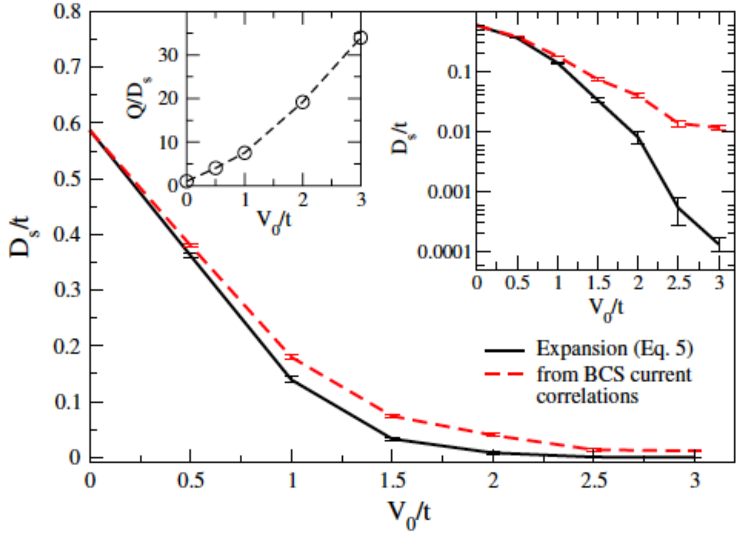
Unidimensional patterns for the current: glassy-like behavior

Superfluid stiffness beyond dirty-BCS

- Superfluid stiffness: compute the second-order derivative of the ground-state energy

$$D_s = \frac{1}{L^2}\frac{\partial^2 E(A)}{\partial A^2}.$$

by including phase relaxation in E(A)



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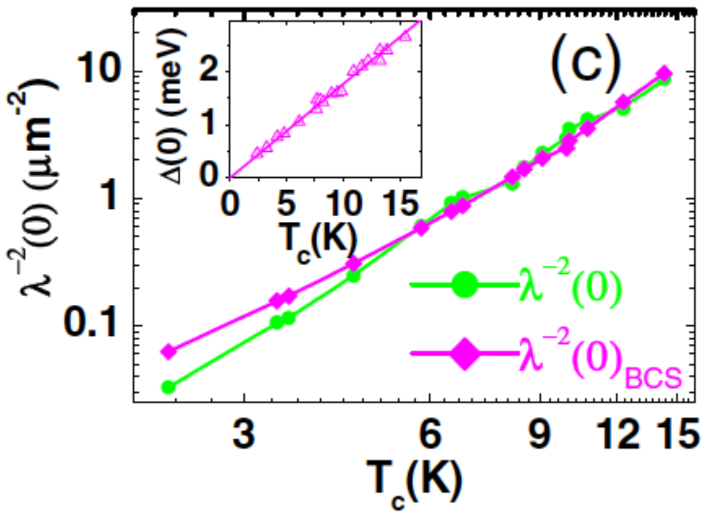


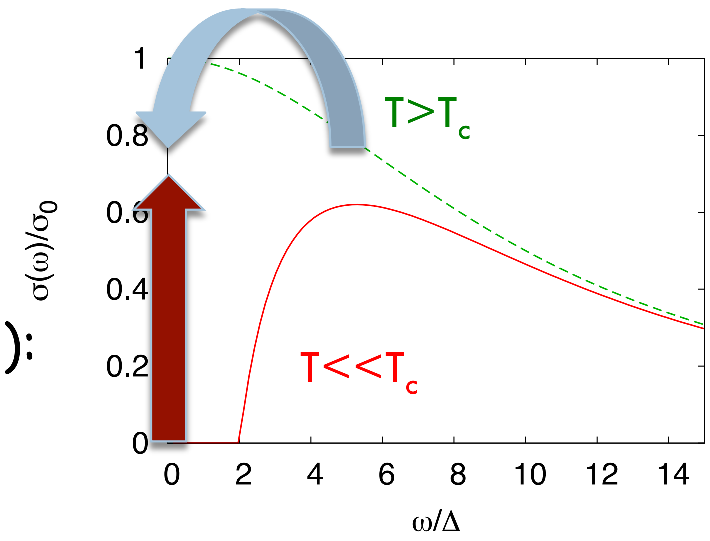
NbN films,
M. Mondal et al
PRL 2011

Stiffness, phase fluctuations and $\sigma_1(\omega)$

- Optical sum rule:

$$\int_{-\infty}^{\infty} d\omega \sigma_1(\omega) = \frac{\pi e^2 n}{m}$$

- Dirty-BCS approach (Mattis-Bardeen): only a fraction of the total spectral weight condenses into the superfluid response at ω=0



$$\sigma_1(\omega) = \frac{\pi e^2 n_s}{m}\delta(\omega) + \sigma_{1,reg}(\omega)$$

$$\sigma_2(\omega) = \frac{n_s e^2}{m\omega}$$

Stiffness, phase fluctuations and $\sigma_1(\omega)$

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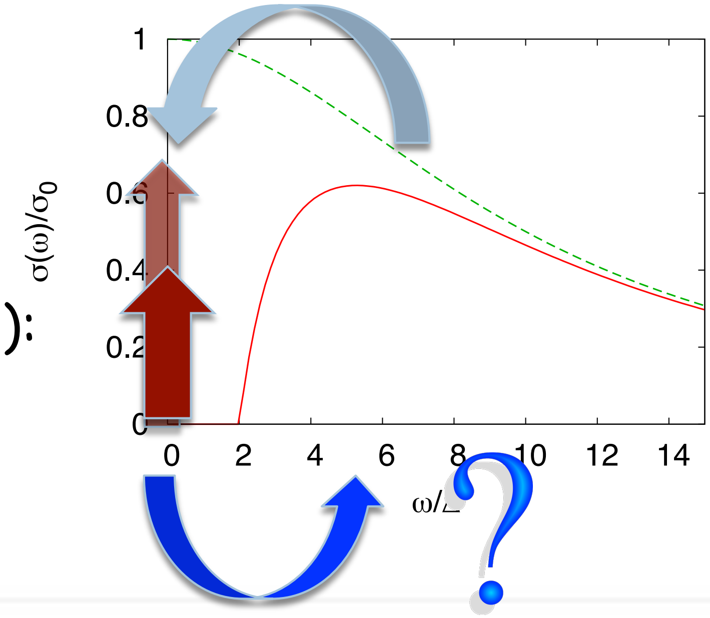


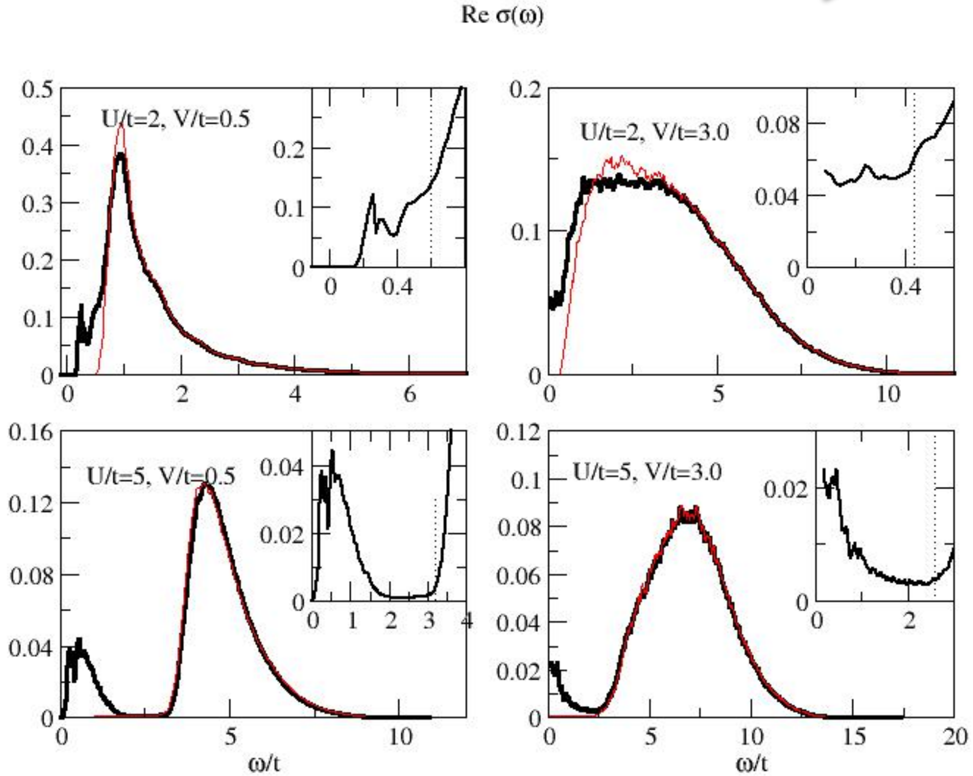
Phase fluctuations reduce n_s,
so some spectral weight moves back
to finite frequency
Where is it found?

Sub-gap contribution of phase fluctuations

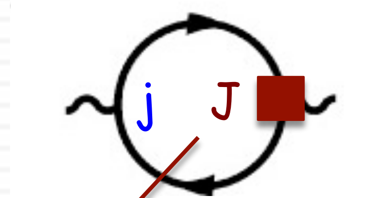
- Optical conductivity with vertex corrections: ***in-gap*** spectral weight due to phase fluctuations

Increasing disorder

Increasing SC coupling



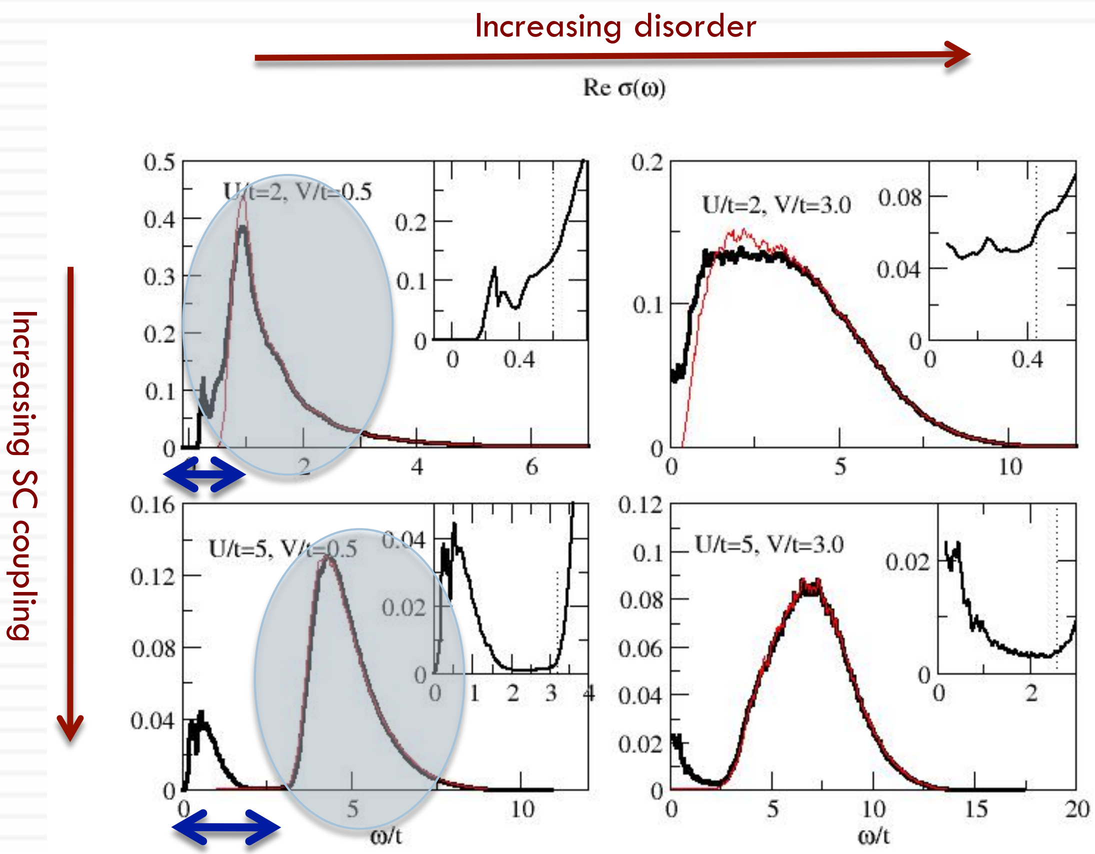
$K_{full}=$

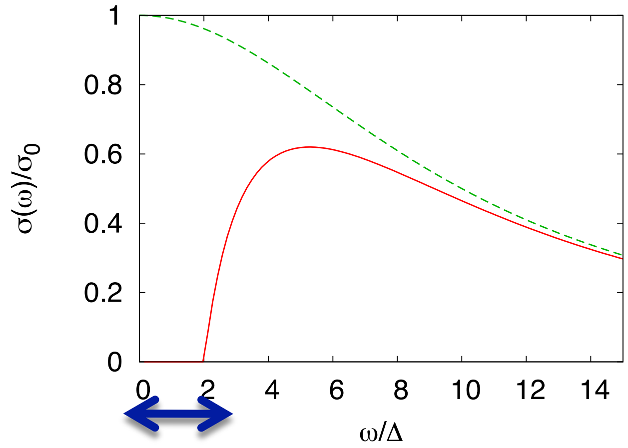


J: collective modes (**phase**, amplitude, charge)

Sub-gap contribution of phase fluctuations

- Optical conductivity with vertex corrections: ***in-gap*** spectral weight due to phase fluctuations



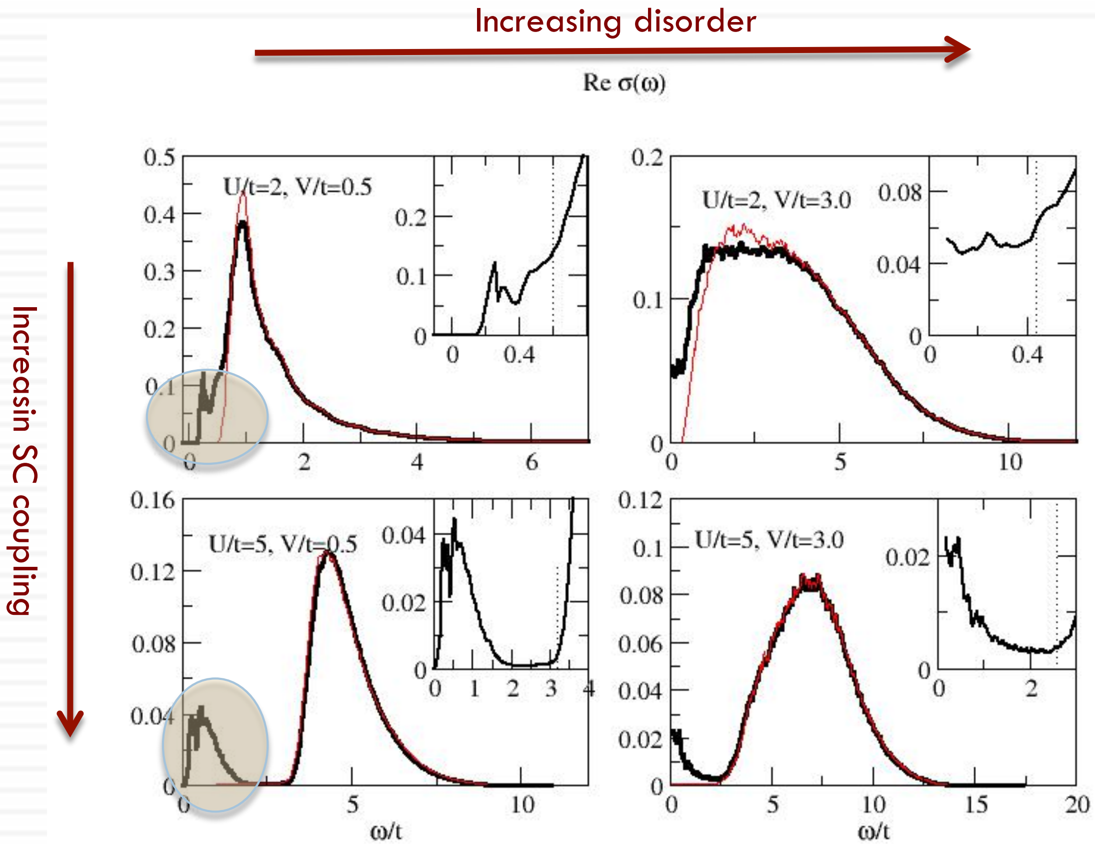


Standard Mattis-Bardeen

Spectral gap Δ, i.e. the same found in the DOS and measured by STM

Sub-gap contribution of phase fluctuations

- Optical conductivity with vertex corrections: ***in-gap*** spectral weight due to phase fluctuations



Collective-modes contribution at low energies

"Smaller" optical gap than STM gap?

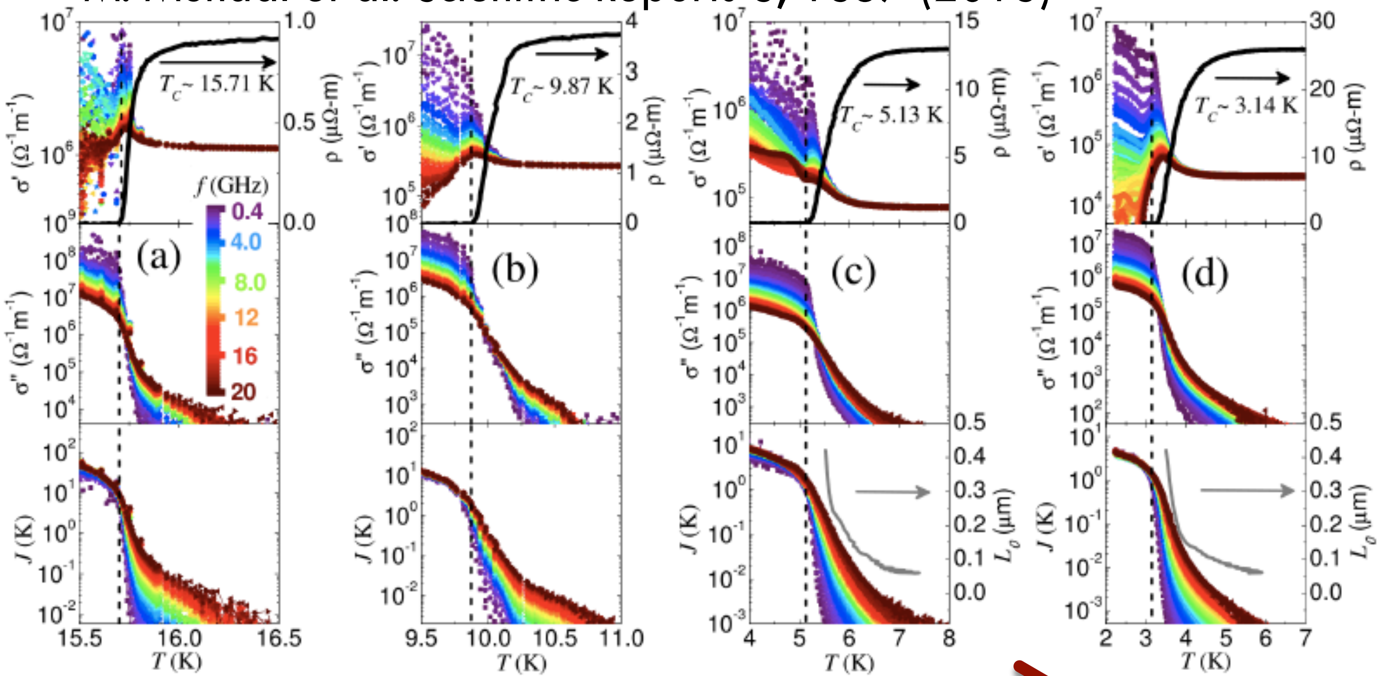
see talk by D. Sherman

Above Tc: enhanced low-frequency SC fluctuations near the SIT

Fluctuation conductivity above Tc

- Finite-frequency response above Tc measures the scale of dynamical SC fluctuations

M. Mondal et al. Scientific Reports 3, 1357 (2013)



0.4-20 GHz

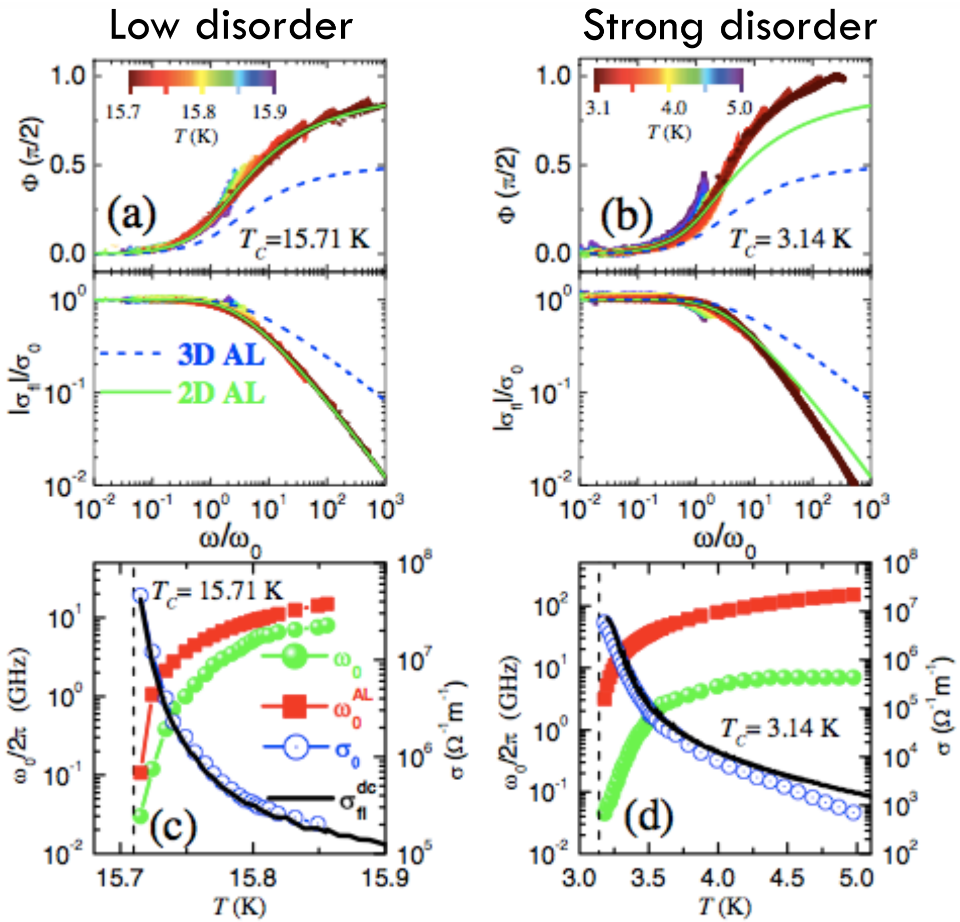
$$J_s(\omega) = \omega\sigma_2(\omega)$$

Increasing disorder

"Persistent" SC fluctuations in the PG state induced by strong disorder

Fluctuation conductivity above Tc

- Finite-frequency response above Tc measures the scale of dynamical SC fluctuations



$$\frac{\sigma_{fl}(\omega,T)}{\sigma_0} = S\left(\frac{\omega}{\omega_0(T)}\right)$$

Standard Aslamazov-Larkin fluctuations:

$$\omega_0^{AL}(T) \sim T_c \ln\left(T/T_c\right)$$

$\omega_0 << \omega_0^{AL}$
"slow dynamics"
(glassy??)
with respect to conventional SC

Conclusions and perspectives

- Disorder-induced phase-fluctuation effects have non-trivial consequences on the transport properties

- Theoretical and experimental suggestions of some emerging glassiness near the SIT. Several open problems (e.g. connection with the Directed Polymer physics is still not fully understood)

  M. Mondal et al., 106, 047001 (2011); M. Chand et al., PRB 85, 014508 (2012;
  G. Seibold et al. PRL 108, 207004 (2012); M. Mondal et al. Scien. Rep. 3, 1357 (2013);
  G.Lemarie et al. PRB 87, 184509 (2013).

- Quasi-2D films: robustness of Beresinkii-Kosterlitz-Thouless physics near the SIT, but role of inhomogeneity and vortex-core energy to be taken into account, more to be done…

  L. Benfatto, C. Castellani and T. Giamarchi, review paper on the book
  "40 years of Beresinskii-Kosterlitz-Thouless theory, arXiv:1201.2307
  Jie Yong, T. Lemberger, L. Benfatto, K. Ilin, M. Siegel, PRB 87, 184505 (2013).