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Improved tracking and docking of Industrial Mobile Robots through UKF vision based kinematics calibration

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ABSTRACT Performing an open-loop movement, or docking, for an industrial mobile robot (IMR), is a common necessary procedure when relying on environmental sensors is not possible. This procedure precision and outcome, solely depend on the IMR forward kinematic and odometry correctness, which is tied to the kinematics parameters, depending on the IMR kind. Calibrating the kinematic parameters of an IMR is a time consuming and mandatory procedure, since the mechanical tolerances and the assembly procedure may introduce a large variation from the nominal parameters. Furthermore, calibration inaccuracies might introduce severe inconsistencies in tasks such as localization, mapping, and navigation in general. In this work, we focus on the so-called kinematic parameter calibration. We propose the use of the unscented Kalman filter to perform a calibration procedure of the geometrical kinematic parameters of a mobile platform. The mobile platform is externally tracked during the calibration phase, using a fixed temporary external sensor that retrieves the position of a visual tag fixed to the platform. The unscented Kalman filter, using the calibration phase collected data, estimates the enlarged system state, which is comprised of the parameters that have to be estimated, the platform odometry and the visual tag position. The method can either be used online, to identify parameters and monitor their value while the system is operating, or offline, on logged data. We validate this method on two different devices, a 4 mecanum-wheel IMR, and a Turtlebot 3, using a camera to track the movement through a reference chessboard, for then comparing the original path to its corrected version.

INDEX TERMS Mobile robot calibration, Unscented Kalman filter

I. INTRODUCTION

Generally, industrial mechanical systems need to have a good parameter calibration to perform accordingly to the standard, hence, a calibration procedure is needed. This calibration procedure, or parameter estimation, is obtainable by means of general purpose algorithms, or specifically tailored methods for particular mechanical systems. Concerning IMRs, the value of the kinematic parameters incorporated in the model, hugely modify the performance of the system in all of its uses, from the mapping phase, where the IMR position infers the map conception, in navigation, where the planning algorithms use the IMR kinematics to devise a path and control the robot movements, and in localization, where the relative position and velocity are used to estimate the displacement progression in an environment. The overall impact of the IMR odometry correctness arises in specific cases such as docking, in which we cannot rely on localization algorithms (e.g. highly de-structured environments, dynamic environments or lacking of localization sensors), and high precision relative displacements are performed in open loop on the IMR kinematic model. Generally, the constant parameters that are taken into account in an IMR kinematic are the wheels radius, which is usually the same for all the wheels, and a geometric length that depends of the IMR structure (e.g omnidirectional mobile platform, car-like, three wheels). Several works related to the general subject of odometry calibration are present in the literature, some focusing on the intrinsic kinematic parameter calibration, some on the calibration of extrinsic parameters related to mounted sensors, or both problems simultaneously. The majority of the works present are designed for differential drive IMRs, while few are specific to omnidirectional mobile platforms, furthermore, most of the methods are offline and based on the repetition of fixed calibration paths. Offline calibration is more commonly employed in standard robotics, but is not suitable to systems where the kinematic parameters change

37 significantly over time (inflatable wheels, robots with great 93
38 dynamic loads, re-configurable mobile platforms), while on- 94
39 line calibration requires real time capability in the measure 95
40 system and in the computation, which might be hard to 96
41 accomplish for complex vision related measure systems. 97

42 Works dealing with the topic of parameter estimation and 98
43 calibration are vastly present in literature, for generic robotic 99
44 chains, many classical methods have been devised, from a 100
45 classical Denavit – Hartenberg approach [1], to rejecting- 101
46 sampling methods [2], virtually closed kinematic chains [3], 102
47 and Kalman filtering based methods, such as in [4], [5], 103
48 where Kalman theory is used alongside a particle filter to
49 estimate serial manipulator parameters. Recent works [6]– 104
50 [8] apply visual based measurement system for the estima- 105
51 tion of manipulator parameters, [9] deals with a complete
52 $AXB = YCZ$ problem, while deep learning based methods
53 have also been used extensively on the subject [10], [11].
54 For an extensive survey on the overall topic, refer to [12].
55 Despite IMRs can be treated as a generic kinematic chains,
56 many works focus on the different kinds existing. On the sub-
57 ject, systematic odometry errors have been initially studied
58 by [13]–[18], while more recent works on the subject use 106
59 iterative learning approaches [19], least squares estimation 107
60 [20], and 'Effective Kinematic Parameters' [21]. All the 108
61 mentioned methods, follow a trajectory based optimization, 109
62 where the aim is to minimize the positional error on a 110
63 given goal path that is repeated constantly, and where the 111
64 information needed are related only to the end points. In such 112
65 works, the calibration performance is affected by the path 113
66 parameters, hence the number of trials and the length of the
67 paths [22], while the paths have to be manually designed and
68 the measurements of the final error carried out manually, with
69 the possibility to bring inaccuracies and measurement errors.
70 Many works take in consideration a possible kinematic flaw,
71 such as an unequal wheel diameter [23]–[25], generic opti-
72 mization methods tackle the model inaccuracies by means
73 of corrective coefficients [26], [27], while dynamic wheel
74 model and lateral dynamics and taken into account in [28].
75 Specific works have also been carried out regarding the type
76 of IMR: car-like robot [29]–[31], differential drive [22], [32]
77 and omnidirectional robots [25], [27], [33]–[35]. Many meth-
78 ods based on Kalman filtering algorithms have been used,
79 where the localization and calibration are solved as a single 114
80 combined task, using the available sensors on the IMR [36]– 115
81 [38]. The advantage in using Kalman filtering techniques is 116
82 that the calibration can be depleted online, while the mobile 117
83 platform performs its tasks. For an extensive survey on IMRs 118
84 odometry calibration methods, refer to [39]. The contribution 119
85 of this paper is to propose the usage of an external sensor 120
86 system to track the IMR state, using a visual tag fixed on it, 121
87 while reading the values of the wheels encoders, in order to 122
88 perform a model parameter identification using the collected 123
89 data, by means of an unscented Kalman filter. The usage of an 124
90 external visual sensor as a tracking method has many advan- 125
91 tages on the calibration system, nowadays industrial cameras 126
92 reach a high level of fidelity, given by the combination of high 127

resolution, advanced calibration techniques, and high frame
rate, which allows a smooth tracking of the IMR movement.
Furthermore, this method doesn't require the design of a spe-
cific path, nor requires hardware changes to the IMR, making
it easily integrable in every scenario. In order to identify the
parameters, a system model that includes the mobile platform
position, velocity, and the relative tag position is needed,
where the unknown relative position between the reference
tag and the mobile platform reference system corresponds to
an hand-eye problem, which parameters will be included in
the system model and estimated.

II. BACKGROUND

A. MOBILE PLATFORM KINEMATIC AND ODOMETRY

Consider a generic IMR as shown in Fig 1, where W is the
globally fixed world frame, MR is the frame fixed to the
IMR, and

$$T_{MR}^W = \begin{pmatrix} R_{MR}^W & \mathbf{t}_{MR}^W \\ 0 & 1 \end{pmatrix} \quad (1)$$

is the transformation matrix, composed by the rotational
 R_{MR}^W and translational $\mathbf{t}_{MR}^W = [p_x \ p_y \ p_z]^T$ parts, from the
 MR frame to the W . The robot position in the world is de-
scribed by its translational and rotational components, hence
the six element vector $[p_x, p_y, p_z, \phi_1, \phi_2, \phi_3] = [\mathbf{p}_W, \boldsymbol{\phi}_W]$,
prompt by the requirement of describing also rough and
uneven terrains, uneven wheels diameters, and other environ-
ment irregularities.

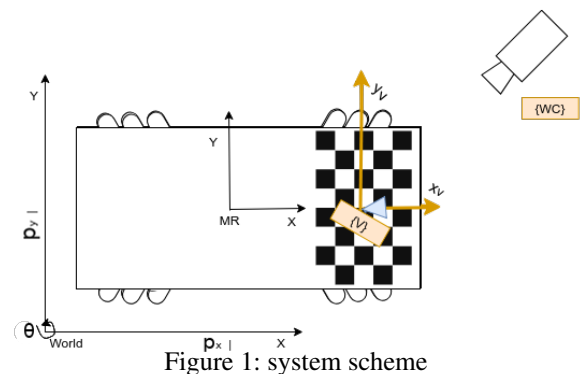


Figure 1: system scheme

The IMR model generally includes the j -th wheel radius
 r_j , with $\mathbf{r} = [r_0, \dots, r_j]$, a geometric distance depending on
the robot kind l , a general purpose parameter vector \mathbf{q} , and
the wheels rotational speed $\boldsymbol{\theta}$, composed by a j number of
elements $\boldsymbol{\theta} = [\theta_0, \dots, \theta_j]$, computed by the wheels encoder
sensors.

Depending on the specifics, the wheels radius might be
assumed equal, or different for each wheel. Due to the digital
nature of IMR controllers and encoders, the system will be
described with a discrete time model. The IMR velocities
 \mathbf{v} and $\boldsymbol{\omega}$ at time t are expressed with respect to the MR
frame as the vector $\mathbf{v}_{MR}^t = [v_x^t, v_y^t, v_z^t]^T$ and $\boldsymbol{\omega}_{MR}^t =$
 $[\omega_1^t, \omega_2^t, \omega_3^t]^T$, as computed by the forward kinematic of the
mobile robot :

$$\begin{bmatrix} \mathbf{v}_{MR}^t \\ \omega_{MR}^t \end{bmatrix} = \mathbf{f}(\dot{\theta}^t, \mathbf{r}, l, \mathbf{q}) \quad (2)$$

where the function \mathbf{f} depends on the mobile platform type (e.g. for car-like robots $v_y^t = 0$), while the same velocities referred to the W are:

$$\begin{bmatrix} \mathbf{v}_W^t \\ \omega_W^t \end{bmatrix} = \begin{pmatrix} R_{MR}^W & [\mathbf{t}]_X R_{MR}^W \\ 0 & R_{MR}^W \end{pmatrix} \begin{bmatrix} \mathbf{v}_{MR}^t \\ \omega_{MR}^t \end{bmatrix} \quad (3)$$

where $[\cdot]_X$ is the skew operator.

Referred to the frame W at time $t + 1$, the position and the orientation (odometry) of the IMR is then computed as the discrete-time integral given dt as the update time, using position, orientation, linear velocity and angular velocity at time t (passing from angular velocity to Cardan angles is done accordingly to [40]).

B. UNSCENTED KALMAN FILTER

Consider a discrete time nonlinear dynamical system:

$$\zeta^{t+1} = F(\zeta^t, u^t, v^t, e) \quad (4)$$

$$y^t = G(\zeta^t, n^t, b) \quad (5)$$

where t is the time index, ζ is the state vector, u is the system input, y is the system output, and e and b are generic constant parameters. The model includes v and n which are respectively the state noise, in order to include model uncertainties, and the measurement noise, both are zero-mean white noises with known covariance matrix. Both the system dynamic model F and the output function G , are known, and include some unknown constant parameters that have to be estimated. In order to use the Kalman filtering theory to estimate the constant parameters, they have to be given a fake dynamic, and to be included in the state vector:

$$x^{t+1} = \begin{bmatrix} \zeta^{t+1} \\ e^{t+1} \\ b^{t+1} \end{bmatrix} = \begin{bmatrix} F(\zeta^t, u^t, v^t) \\ e^t \\ b^t \end{bmatrix} \quad (6)$$

where x is the enlarged state, and the parameters e and b , that have to be estimated, do not have an evolution because their value at instant $t + 1$ is the same as at instant t .

Kalman filtering theory can be optimally used in linear dynamical systems to estimate the state value, while, in case of a nonlinear system, variants of the original filter exist, such as the extended Kalman filter (EKF) or the Unscented Kalman filter (UKF). Considering the strong non-linearity of the system, and the additional complexity brought by the hand-eye problem, an UKF is employed, due to its increased reliability in non-linear system identification [41], and robustness to parameter initial value [42]. While the inaccuracies of the EKF, come from the linearization of the dynamic model, the UKF keeps the original nonlinear dynamic model, using the unscented transform methodology [43] to propagate the state uncertainty¹. Following [41], denote $x \in \mathbb{R}^L$, \bar{x} , P_x as

¹The unscented transformation is a method to compute the evolution of a random variable through a nonlinear function.

an L dimensional random variable, its mean value, and its co-variance matrix respectively, and denote $y = g(x)$ as a generic nonlinear map. The distribution of y , its mean value \bar{y} and its co-variance P_y can be approximated using a number $2L + 1$ of sigma row vectors χ_i , which form a matrix χ , and associated weights W_i as following

$$y_i \equiv g(\chi_i) \quad i = 0, \dots, 2L \quad (7)$$

$$\bar{y} \approx \sum_{i=0}^{2L} W_i^m y_i \quad (8)$$

$$P_y \approx \sum_{i=0}^{2L} W_i^c (y_i - \bar{y})(y_i - \bar{y})^T. \quad (9)$$

The set of vectors χ_i and weights W_i are computed as

$$\begin{aligned} \chi_0 &= \bar{x}, \\ \chi_i &= \begin{cases} \bar{x} + (\sqrt{(L+\lambda)P_x})_i & \text{if } i = 1, \dots, L \\ \bar{x} - (\sqrt{(L+\lambda)P_x})_{i-L} & \text{if } i = L+1, \dots, 2L \end{cases} \\ W_0^m &= \lambda / (L + \lambda) \\ W_0^c &= \lambda / (L + \lambda) + (1 - \mu^2 + \nu) \\ W_i^m &= W_i^c = 1 / (2(L + \lambda)). \end{aligned} \quad (10)$$

where $(\cdot)_i$ extracts the i -th row of the resulting matrix, and $\lambda = \mu^2(L + \kappa) - L$ is the gamma vectors scaling parameter, with μ , ν and κ tuned accordingly to the process.

The UKF, extends the unscented transform, including it into the Kalman filter iterative estimation, the UKF steps are reported in algorithm 1, for an extensive survey, refer to [41].

III. METHODOLOGY

The system is comprised by an IMR endowed with wheel encoders, a visual tag fixed on the platform, and an external tracking camera to estimate the position of the visual tag, as shown in Fig 1. The system model includes the IMR kinematic and odometry, the tag frame position, the fixed transformation between the IMR frame and the tag frame, and the geometrical parameters related to the kinematic formulation.

Four reference systems are defined prior to the model, the frame W , a fixed global frame to which the mobile platforms position are referred, equal to MR , the mobile platform frame, at the starting time, and VW , the visual world fixed reference system, equal to V , the tag frame, at starting time, refer to Fig 2 for a system representation.

The tag position results in

$$T_{V|t}^{WV} = H_{MR}^V T_{MR|t}^W H_V^{MR} \quad (11)$$

$$(12)$$

where H_{MR}^V is the constant transformation matrix between the MR frame and the tag frame V , and $T_{MR|t}^W$ is the odometry transformation.

In order to properly include the H_{MR}^V and T_V^{WV} matrix in the model, its formulation from the Euler xyz angles is used :

Algorithm 1: UKF

- 1: $x^a \leftarrow [x^T v^T n^T]^T$
- 2: $\chi^a \leftarrow [(\chi^x)^T (\chi^v)^T (\chi^n)^T]^T$
- 3: x^a is the concatenation of original state and noise variables
- 4: P_v is the process noise co-variance matrix
- 5: P_n is the measurement noise co-variance matrix
- 6: x_k^- is the optimal prediction of such variable.

7: procedure UKF BASED CALIBRATION

- 8: $\hat{x}_0 \leftarrow E[x_0]$
- 9: $P_0 \leftarrow E[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T]$
- 10: $\hat{x}_0^a \leftarrow E[x^a] \leftarrow [\hat{x}_0^T 0 0]^T$
- 11: $P_0^a \leftarrow E[(x_0^a - \hat{x}_0^a), (x_0^a - \hat{x}_0^a)^T] = \begin{bmatrix} P_0 & 0 & 0 \\ 0 & P_v & 0 \\ 0 & 0 & P_n \end{bmatrix}$

for every step k do

- 12: compute sigma points:
- 13: $\chi_{k-1}^a \leftarrow [\hat{x}_{k-1}^a \quad \hat{x}_{k-1}^a \pm \sqrt{(L + \lambda)P_{k-1}^a}]$
- 14: time update:
- 15: $\chi_{k|k-1}^x \leftarrow F[\chi_{k-1}^x, \chi_{k-1}^v]$
- 16: $\hat{x}_k^- \leftarrow \sum_{i=0}^{2L} W_i^m \chi_{i,k|k-1}^x$
- 17: $P_k^- \leftarrow \sum_{i=0}^{2L} W_i^c [\chi_{i,k|k-1}^x - \hat{x}_k^-][\chi_{i,k|k-1}^x - \hat{x}_k^-]^T$
- 18: $y_{k|k-1} \leftarrow H[\chi_{k|k-1}^x, \chi_{k-1}^n]$
- 19: $\hat{y}_k^- \leftarrow \sum_{i=0}^{2L} W_i^m y_{i,k|k-1}$
- 20: Measurement update:
- 21: $P_{\hat{y}_k, \hat{y}_k} \leftarrow \sum_{i=0}^{2L} W_i^c [y_{i,k|k-1} - \hat{y}_k^-][y_{i,k|k-1} - \hat{y}_k^-]^T$
- 22: $P_{x_k, y_k} \leftarrow \sum_{i=0}^{2L} W_i^c [\chi_{i,k|k-1}^x - \hat{x}_k^-][y_{i,k|k-1} - \hat{y}_k^-]^T$
- 23: $\kappa \leftarrow P_{x_k, y_k} P_{\hat{y}_k, \hat{y}_k}^{-1}$
- 24: $\hat{x}_k \leftarrow \hat{x}_k^- + \kappa(y_k - \hat{y}_k^-)$
- 25: $P_x \leftarrow P_k^- - \kappa P_{x_k, y_k} \kappa^T$
- 26: **end procedure**

Hence, defining $\tilde{\mathbf{h}}$ and $\tilde{\mathbf{c}}$ as

$$\tilde{\mathbf{h}}^t = [h_x^t, h_y^t, h_z^t, \alpha^t, \beta^t, \gamma^t]^T \quad (15)$$

$$\tilde{\mathbf{c}}^t = [c_x^t, c_y^t, c_z^t, \eta^t, \epsilon^t, \delta^t]^T \quad (16)$$

the state of the system becomes :

$$\begin{bmatrix} \mathbf{v}_W & \mathbf{p}_W & \tilde{\mathbf{c}} & \tilde{\mathbf{h}} & \mathbf{r} & \mathbf{l} & \mathbf{q} \end{bmatrix}^T \quad (17)$$

where the last elements are the constant parameters that have to be estimated:

$$\begin{bmatrix} \vdots \\ \tilde{\mathbf{h}}^{t+1} \\ \mathbf{r}^{t+1} \\ \mathbf{l}^{t+1} \\ \mathbf{q}^{t+1} \end{bmatrix} = \begin{bmatrix} \vdots \\ \tilde{\mathbf{h}}^t \\ \mathbf{r}^t \\ \mathbf{l}^t \\ \mathbf{q}^t \end{bmatrix} \quad (18)$$

The measurable output y^t of the system is the visual tag position, computed by an external tracking system as shown in Fig. 3. In order to retrieve the position of the mobile tag frame by means of the external sensor, the vision tag displacement at time t is always referred to the position at instant 0, using the following: $T_{V|t}^{WV} = T_{WC}^{WV|t=0} T_{V|t}^{WC}$. The output y^t is hence expressed as

$$y^t = \tilde{\mathbf{c}}^t \quad (19)$$

The parameters are then estimated online, or offline, using the output readings and encoders value to evolve the model dynamic.

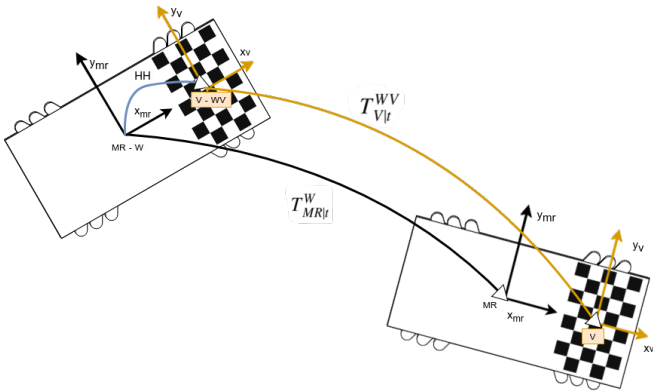


Figure 2: Coordinates systems in two different instant times.

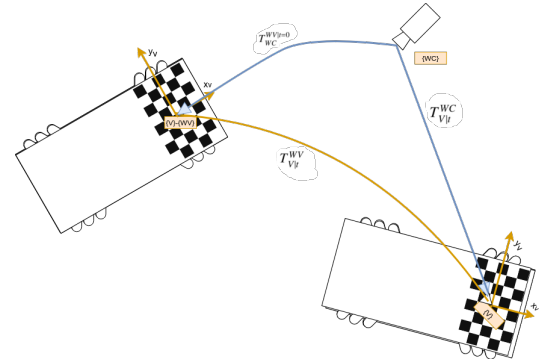


Figure 3: Reference tag measure

$$H_{MR}^V = \left(\begin{array}{ccc|c} R_z(\gamma)R_y(\beta)R_x(\alpha) & h_x & & \\ & h_y & & \\ & h_z & & \\ \hline 0 & 0 & 0 & 1 \end{array} \right) \quad (13)$$

$$T_V^{WV} = \left(\begin{array}{ccc|c} R_z(\delta)R_y(\epsilon)R_x(\eta) & c_x & & \\ & c_y & & \\ & c_z & & \\ \hline 0 & 0 & 0 & 1 \end{array} \right) \quad (14)$$

IV. TEST CASES

A. DESIGN OF EXPERIMENTS

Tests have been performed in a 2D planar environment, with two different IMRs, one omnidirectional and one with differential drive, in order to assess its effectiveness in two different scenarios. As a reference, the omniwheel platform is shown in Fig. 4, while the differential drive Turtlebot 3 waffle is shown in Fig. 5. The parameters to be estimated

175 are the radius of the platform wheels, considered to be equal 203
 176 for wheels of the same platform, and a geometric parameter, 204
 177 which is l_{xy} for the omniwheel platform, and the parameter 205
 178 l_d for the differential drive. The parameters are initialized 206
 179 with the CAD values for the omniwheel internally developed 207
 180 platform, and with data-sheet values for the differential drive 208
 181 commercial platform. The platforms have been teleoperated, 209
 182 using a joystick as a remote manual speed controller, along 210
 183 10 different paths, each with an average duration of 50 211
 184 seconds, as a reference, the nominal paths performed by the 212
 185 omnidirectional robot are shown in Fig. 6, we refer to the 213
 186 nominal path as the one measured by the external camera.

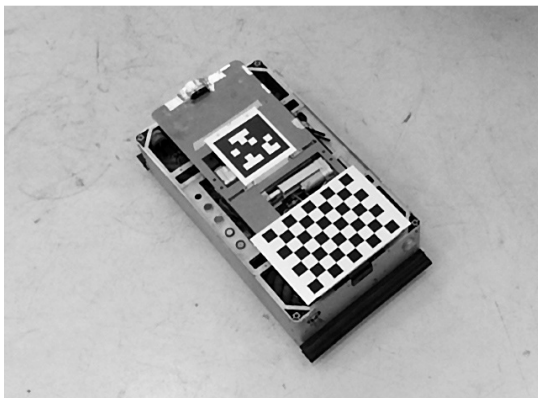


Figure 4: Omniwheel platform during the acquisition

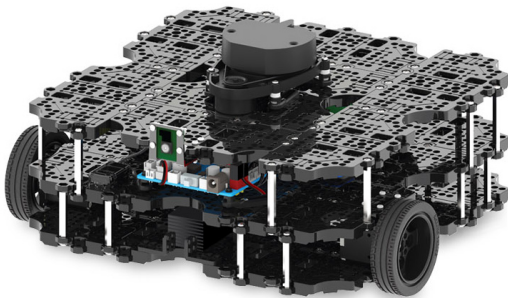


Figure 5: Turtlebot3 waffle

187 In order to ease the estimation, we suppose the tag frame 242
 188 to be planar to the platform xy frame plane (i.e. $\alpha, \beta = 0$), 243
 189 and for both the frames to have the same z component 244
 190 in the world frame (i.e. $h_z = 0$). All the needed data are 245
 191 logged during the runs, and then used offline to evolve 246
 192 the dynamic of the discrete-time system, at every step(time 247
 193 instant), and improve the estimation of the state, repeating 248
 194 the overall estimation procedure for a number of iterations. 249
 195 The data is shuffled between each estimation trial, where 250
 196 half of the data are used to simulate the model and estimate 251
 197 the parameters, while the other half to validate the trial 252
 198 and assess improvement. Parameters relative to the UKF are 253
 199 tuned according to [43], [44], specifically the values relative 254
 200 to the Unscented transform are: number of points(14), $\alpha(0.1)$, 255
 201 $\beta(2.0)$ and $\kappa(-11)$. Regarding the initial covariances of the 256
 202 UKF, the numerous trials we performed offline suggested 257

that the initial values mostly affect the convergence time, and not the actual estimation value. Specifically, the value of the state covariance matrix P is set to $0.01 * I$, where I is the identity matrix, the state noise covariance matrix Q is set to $0.01 * I$, aside from the values corresponding to parameters that have to be estimated, which is set to zero, while the output noise covariance matrix R is $0.0025 * I$, which has been approximately computed using the camera resolution, the camera field of view and the average distance from camera to the tracked object. The identification algorithm is run in two different forms:

- parallelly, performing an estimation for each of 5 estimation data sets, using 10 iterations of the whole data per run, for then taking the average of the values. In order to assess the convergence to common values of the estimations on the data sets.
- on the 5 data sets in series, for 10 iterations, for having a single estimate of the parameters

221 furthermore, the outcomes are compared, to establish the
 222 more appropriate approach to the problem.

223 Each estimation is repeated 50 times, randomly shuffling
 224 the estimation and validation paths in order to have statisti-
 225 cally relevant results.

226 The IMRs controllers are synchronized with the camera
 227 acquisition system using the NTP protocol. Both robots are
 228 moved at a maximum velocity of $0.15 \frac{m}{s}$, while externally
 229 tracked at 30Hz, using a calibrated Kinect V2 camera, hence
 230 allowing a maximum displacement of $0.005m$ per frame.
 231 Specifically, the Kinect V2 camera has a 1920×1080 RGB
 232 resolution, and , most importantly, adopt a global shutter,
 233 which reduces blurs and improves the measure quality. The
 234 wheels encoders are then down-sampled at 30 Hz in order to
 235 have the same time scale. The relative position between the
 236 tag and the camera is computed using the perspective-n-point
 237 algorithm in the OpenCv libraries [45]. For the algorithm
 238 development, Python3 has been used, while the UKF library
 239 is provided by [46]. The estimation algorithm has been run
 240 using an Intel Core i7-7700HQ processor. Given this setup,
 241 the running time of a single estimation instance on the offline

data, for both methods, was of 35 seconds. In order to
 measure the performance improvements, 2 indexes will be
 used, one that takes in account the whole movement along
 the paths, and one which aim is to measure the docking
 precision, that relies only on the final position after a path
 is performed. We define a path $\mathcal{P} = \{\mathbf{p}_0, \dots, \mathbf{p}_{N_p}\}$ as an
 ordered collection of IMR positions \mathbf{p} , where each position
 is composed by two translation(x, y) and one rotation(ω),
 being our experiment setup modelled in a 2D environment.
 Then, we will define a nominal path \mathcal{P}_n , as the one measured
 by the external camera, an original path \mathcal{P}_o , as the one
 computed using the initial parameters values by the IMR
 kinematics, and a corrected \mathcal{P}_c path, as the one re-computed
 with the newly estimated parameters. In order to measure
 the improvement given by the estimation of the parameters,
 we compare the different paths, using the summation of the

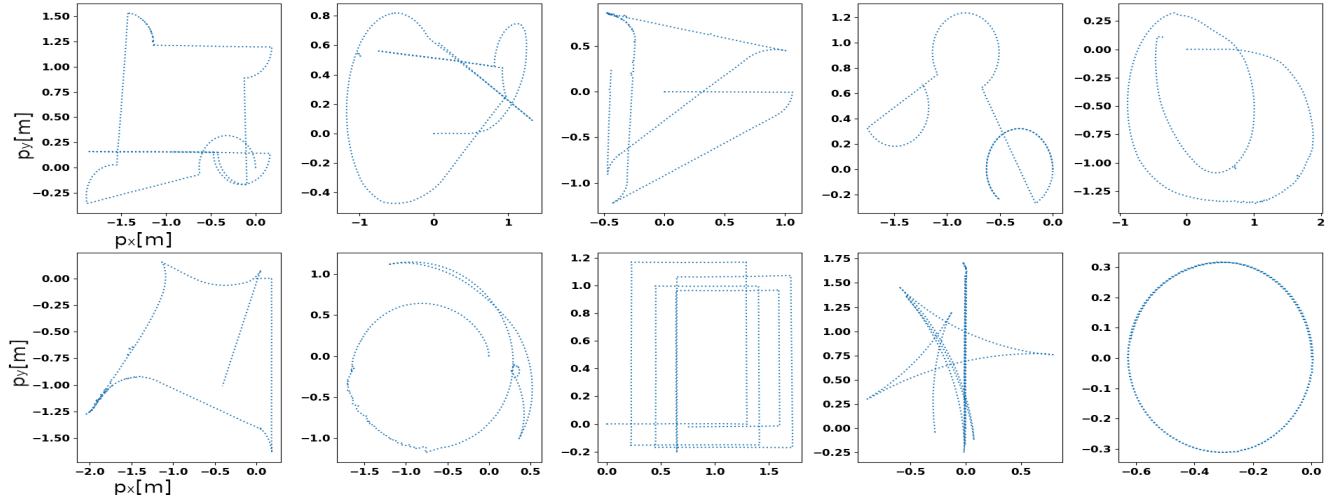


Figure 6: Nominal paths used for the omnidirectional platform

258 euclidean distance between each $j - th$ point $\mathcal{P}(j) = p_j$ in
 259 \mathcal{P}_o and \mathcal{P}_c , with respect to the $j - th$ point in \mathcal{P}_n as:

$$\mathcal{E}_o = \sum_j^{N_p} \|\mathcal{P}_n(j) - \mathcal{P}_o(j)\| \quad \mathcal{E}_c = \sum_j^{N_p} \|\mathcal{P}_n(j) - \mathcal{P}_c(j)\| \quad (20)$$

260 where N_p is the equal number of points of the paths, for then
 261 computing the percentage improvement of the corrected path
 262 as

$$\sigma_c = \frac{\mathcal{E}_o - \mathcal{E}_c}{\mathcal{E}_o} \quad (21)$$

263
 264 **B. TEST CASE A - OMNIWHEEL MOBILE PLATFORM**

265 The test is performed on an omniwheel mobile plat-
 266 form, endowed with 4 mecanum wheels with an axis
 267 of rotation at 45° to the wheel plane and at 45°
 268 to the axle line, each wheel has a high frequency
 269 (1kHz) encoder to compute the speed and position. For
 270 this specific robot, equations 2 and 3 take this form:

$$\begin{bmatrix} v_x^t \\ v_y^t \\ v_z^t \\ \omega_1^t \\ \omega_2^t \\ \omega_3^t \end{bmatrix} = \frac{r}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{1}{l_{xy}} & \frac{1}{l_{xy}} & -\frac{1}{l_{xy}} & \frac{1}{l_{xy}} \end{bmatrix} \begin{bmatrix} \dot{\theta}_0^t \\ \dot{\theta}_1^t \\ \dot{\theta}_2^t \\ \dot{\theta}_3^t \end{bmatrix}$$

271 where l_{xy} is half-distance between front wheels and rear
 272 wheels plus half-distance between left wheels and the right
 273 wheels, refer to [47] for a complete survey on mobile robot
 274 kinematics.
 275

276 Regarding the parallel estimation approach, Fig. 7 shows
 277 the convergence of the radius estimation, while Fig. 8 shows
 278 the relation between the nominal, original and corrected
 279 paths for the test dataset, in the same way, for the series
 280 approach, Fig. 9 shows the convergence of the radius and l_{xy}
 281 estimations, while Fig. 10 shows the paths comparison. 287

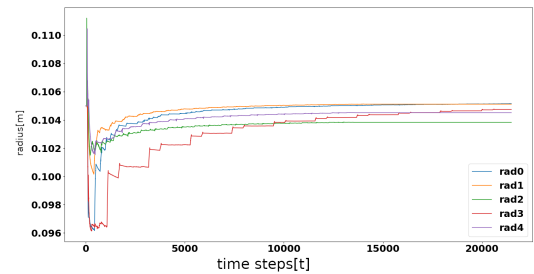


Figure 7: radius trend, for a single instance of the parallel estimation method. Standard deviation of estimated radius: 0.00033m

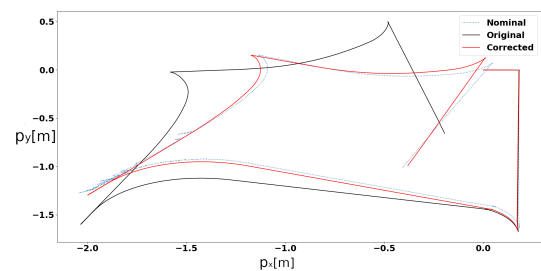


Figure 8: Nominal, original and corrected paths, for a single instance of the parallel estimation case. Value of $\sigma_c = 0.57$, as computed in equation 21

282
 283 **C. TEST CASE B - DIFFERENTIAL DRIVE MOBILE PLATFORM**

284 The test is performed on a Turtlebot3 Waffle commercial
 285 mobile platform, endowed with 2 active wheels (100Hz en-
 286 coders) and 2 ball casters.

For this specific robot, equation 2 takes this form:

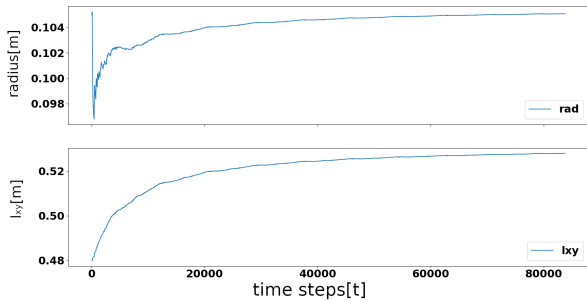


Figure 9: radius and l_{xy} trends , for a single instance of series estimation method

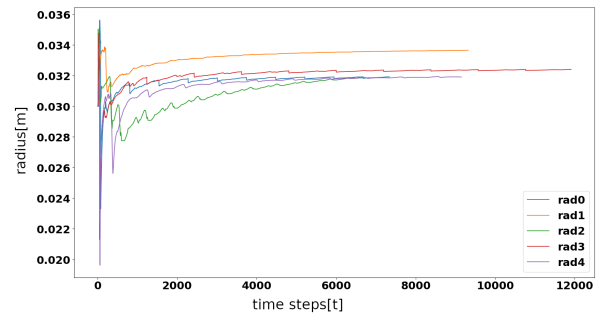


Figure 11: radius trend, for a single instance of the parallel method. Standard deviation of estimated radius: 0.00059m

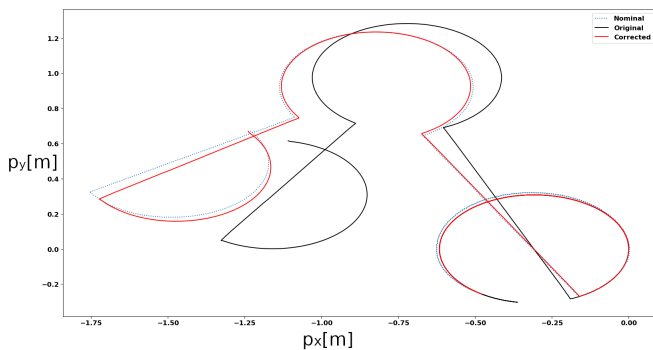


Figure 10: Nominal, original and corrected path, for a single instance of the series estimation case. Value of $\sigma_c = 0.48$, as computed in equation 21

odometry and the corrected one, using the same concept as in 21, the results are shown in 1. The overall resulted accuracy is higher in the omniwheel robot, due to a better manufacture and devices quality .

Table 1: Docking performance improvement for both mobile robots

$\bar{\sigma}_c$ (eq 21)	Parallel	Series
Omniwheel	0.76	0.82
Turtlebot3	0.37	0.51

Table 2: Omniwheel robot estimation results, on 20 trials

	Parallel	Series
$\bar{r}[m]$	0.101	0.104
$Std r[m]$	0.0031	0.00035
$\bar{l}_{xy}[m]$	0.495	0.525
$Std l_{xy}[m]$	0.0086	0.0017
$\bar{\sigma}_c$ (ref eq 21)	0.26	0.428

$$\begin{bmatrix} v_x^t \\ v_y^t \\ v_z^t \\ \omega_1^t \\ \omega_2^t \\ \omega_3^t \end{bmatrix} = r \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -\frac{1}{l_d} & \frac{1}{l_d} \end{bmatrix} \begin{bmatrix} \dot{\theta}_0^t \\ \dot{\theta}_1^t \end{bmatrix} \quad (22)$$

309

288 where l_d is the distance between left wheel and right wheel,
289 refer to [47] for a complete survey on mobile robot kinemat-
290 ics.

291 An example of the radius value trend during the parallel
292 estimation in this case is shown in Fig. 11.

293 D. RESULTS

294 The results are presented in Table 2 for the omniwheel robot,
295 and in Table 3 for the Turtlebot3 waffle, where Std refers
296 to the standard deviation of the estimated parameter, and
297 $\bar{\sigma}_c, \bar{r}$ and \bar{l} are the mean values, over the 50 trials. While
298 the overall odometry error decreased in both cases, it is
299 noticeable that the series estimation brought better results in
300 both the experiments. The estimation in the series case, is
301 more precise, due to higher $\bar{\sigma}_c$, and more reliable, due to a
302 smaller $Std r$ and $Std l$. In order to assess the docking
303 capability improvement, we also compare the final position
304 distance between the nominal path, the original computed

311 V. CONCLUSIONS AND FURTHER DEVELOPMENT

312 In this work, a novel method to improve the IMR docking
313 quality and calibrate the kinematic parameters of a mobile
314 platform is presented. An Unscented Kalman Filter is used
315 to estimate the parameters of the dynamic model of the mobile
platform, tracked using an external camera, and wheels en-
coders. The method is tested on two different IMRs, gathered
results show a positive outcome. In order to improve the
method, future works will focus on filtering the data, to raise
the value of information-rich data, and filter out the data
that is deemed not important. This approach can be pursued
using the Fisher information theory. Furthermore, the method
will be tested in a 3D scenario, including problematics re-
lated to 3D outdoor navigation and mobile platforms with
suspensions. Other improvements can be done to lessen the
computation time, in order to make the algorithm more
suitable for online use.

Table 3: Turtlebot3 estimation results, on 20 trials

	Parallel	Series
$\bar{r}[m]$	0.027	0.031
$Std\ r[m]$	0.0064	0.004
$\bar{l}_d[m]$	0.332	0.340
$Std\ l_d[m]$	0.021	0.012
$\bar{\sigma}_c$ (ref eq 21)	0.122	0.27

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