A locally anisotropic fluid-structure interaction remeshing strategy for thin structures with application to a hinged rigid leaflet

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ABSTRACT

We apply an "immersed" finite element approach discussed in [1] for the 2D steady incompressible Stokes equations to a 2D fluid-structure interaction problem in a tube with a thin (i.e., codimension 1) hinged rigid leaflet sets in motion by an incompressible unsteady flow. In order to change the minimal possible fraction of elements we remesh only the triangles that are cut by the structure. Furthermore, with this approach essential constraints between the fluid and the solid may be strongly enforced in the finite element spaces and it is fairly easy to allow the fluid stress to be discontinuous across the structure.

An obvious consequence of the proposed remeshing strategy is the presence of anisotropic triangles, for which it is known that the standard finite element method may be used (see, e.g., [2]). However, for mixed elements most of the inf-sup stability proofs require isotropic meshes and only few results are available on distorted triangles. In [1], the inf-sup stability of the Hood-Taylor element with the present method has been discussed. It was shown that instabilities may occur but also that adding a bubble in the velocity space stabilizes the element.

In the present work we apply and extend the results presented in [1] to a 2D fluid-structure problem with a codimension 1 structure immersed in the fluid domain. As a consequence, it may be convenient to use mixed elements with a discontinuous pressure. Indeed, with such elements the discontinuity in the pressure field across the structure is straightforwardly enforced. We discuss the use of the P2/P0 and the P2-bubble/P1-discontinuous elements with the proposed remeshing algorithm. However, we also consider the elements in [1] with a discontinuous pressure is across the structure.

Numerical tests show that the two elements with a discontinuous pressure suffer inf-sup stability issues in a more severe way than the Hood-Taylor element. The P2/P0 and P2-bubble/P1-discontinuous elements show spurious modes along the structure, and thus not only in corners as in the case of the Hood-Taylor element. On the contrary, the Hood-Taylor element with an additional bubble do not show any spurious modes in the performed tests. The deleterious effects of inf-sup instabilities are also observed in the conditioning of the linear system.

Furthermore, in [3] a similar fluid-structure interaction algorithm to the one presented here is employed using the P2-bubble/P1-discontinuous element. However, a smoothing strategy is used such that all triangles are isotropic. Our results show in particular the necessity of such a smoothing with that element.

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