



#### **ORGANISATION**

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## IN COLLABORATION WITH

AEI - ASE - EUREL - GDR ISIS (\*) - IEE - IEEE - Signal Processing & French Section-(\*) - SFO - SFP (\*) - SRBE - VDE - Fresnel Institute of Marseille - Ville de Marseille. (\*) TBC (To Be Confirmed)

#### **OBJECTIVE**

A critical element in the definition and evaluation of any **signal or image processing** method is the validity and the accuracy of the underlying **scene and sensor modeling** backgrounds, contrasts, fluctuations, speckle, propagation, disturbances and jammers, sensor calibration, etc...

The accuracy of this **physical description** and its **adequation to the signal processing** techniques becomes an essential challenge for sensor design, in the different fields of application - from medical to military, using radar, acoustic, optic, optronic and seismic sensors.

In this biennial workshop, detailed analyses of this **interaction between physics and signal or image processing** will be presented in ½ hour conferences, with applications in such different areas as radar and laser imaging and interferometry, PET or scintigraphic imaging, inverse scattering, 3D reconstruction, and polarimetric classification, models of random signals, atmospheric turbulence measurement, blind source separation, influence of multichannel models, robust subspace tracking, or

characterization of the mobile radio channel.

This forum of specialists coming from different application domains and countries, and sharing a common interest in adequation of physical models to signal processing techniques, is expected to generate ideas and innovations for further advances in the fast expanding techniques of signal and image processing.

#### **TOPICS**

- Signal and Image acquisition :
- Image formation and analysis,
- Tomography,
- SAR.
- Phase conjugation,
- New image acquisition systems (laser SAR, hyperspectral, polarization, etc..),
- · Acoustic and seismic signals acquisition,

## Modeling:

- · Description and modeling of physical phenomena: radiation, flow, propagation, motion, turbulence, scattering, etc.
- Scenes and interferences modeling,
- Physical and stochastic models for sensor processing,
- Noise and disturbances modeling and synthesis,
- Performance criteria.

## Processing algorithm :

- Clutter, reverberation, background elimination,
- Motion estimation, motion elimination
- Noise reduction
- Separation of sources
- Multidimensional processing (space-time, space-frequency, time-frequency, spacepolarization)
- Stereo and array processing of complex scenes
- Classification and identification
- Sensor fusion

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- P4.7 The Archimedes principle applied to separation of uniformly distributed sources
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- P4.9 Trilion project : source separation in H.F. digital communications
- A. BISIAUX, D. LEMUR & L. BERTEL (Laboratoire Radiocommunications, Université de Rennes I, France)
- P4.10 A model of random signals with long-range correlations
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## SESSION P5 - RADAR

- P5.1. Subspace based spectral analysis for VHF radar signals
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- P5.2 The application of multidimentional processing in airborne PD radar
- L. FUEN & W. XIUCHUN (Nanjing Research, Institute of Electronics Technology, China)
- P5.3. Determination of intrinsic characteristics for discrimination of radar targets in resonance region
- R. TORIBIO & J. SAILLARD (Ecole Polytechnique de l'Université de Nantes, France)
- P5.4. A new approach using pseudo Wigner-Ville distribution and hidden markov model to estimate pulse radar FM parameters
- C. de LUIGI & S. PARIS (MS/GESSY, ISITV, Université de Toulon et du Var, France)

## SESSION P6 - SAR

- P6.1. Speckle filtering and classification of ERS-1 SAR images Gamma distributed
- Y. SMARA & D. CHERIFI (Electronic Institute, Houari Boumedienne, University of Sciences & Technology, Algeria)
- P6.2 Two-dimensional profile reconstruction from multiple-frequency approach
- K. BELKEBIR (Institut Fresnel, Faculté de Saint-Jérôme, France)
- P6.3. Extraction of urban structures and DEM from airborne SAR images
- E. SIMONETTO & H. ORIO (ONERA DTIM/TI Chatillon, France)
- P6.4. Modelling SAR images with lpha -stable textures

-D

E.E. KURUOĞLU & J. ZERUBIA (INRIA, Sophia Antipolis, France)

P6.5. - Note on the SAR speckle simulation

F. DAOUT (Université Paris X, Lab. GEA, France), F. SCHMITT & C. DREVET (Ecole Navale, IRENav., France), L. PASTORE (ONERA, Department DEMR Palaiseau, France)

P6.6. - 3D GPR imaging of shallow buried objects in the ground

J.C. BUREAU & P. MILLOT (ONERA DEMR CERT, France), C. PICHOT, E. LE BRUSQ, E. GUILLANTON & J.Y. DAUVIGNAC (LEAT, University Nice-Sophia Antipolis/CRNS, France)

## REGISTRATION

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# MODELLING IMAGES WITH ALPHA-STABLE TEXTURES

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#### **Abstract**

In this paper we present an alternative to Gaussian textures for modelling images. In particular we introduce linear textures generated with alpha-stable innovations. The advantages alpha-stable textures present over Gaussian textures are two-folds: they can model both textures exhibiting rough-impulsive characteristic and those that exhibit unsymmetrical (skewed) characteristics. We also introduce new parameter estimation techniques for modelling and we demonstrate the success of the techniques on synthetic data. The alpha-stable textures are expected to work well especially for SAR images of urban scenes.

#### Keywords

Non-Gaussian models, alpha-stable distribution, linear alpha-stable fields.

#### 1.Introduction

Linear processes with Gaussian innovations have been used widely in modelling image textures. One of the most important applications of texture modelling has been the segmentation of images. Although Gaussian texture models have been successful in some applications, their performance have been disappointing in cases where the image contains impulsive and/or skewed features. Such examples are not rare in the case of SAR images [1]. The Gaussian model is very conservative in that, impulsive events occur with exponentially diminishing probability and skewness is simply not supported by the Gaussian density function, which is symmetric. In this paper, we present linear alpha-stable textures (fields with alphastable innovations) which can model both impulsive and skewed image characteristics and introduce techniques for estimating the process parameters (both process coefficients and density parameters) from observations. Finally, we present simulation results on estimating the parameters of synthetic alpha-stable textures.

## 2.Alpha-Stable Distributions

The alpha-stable distribution family is described most conveniently by its characteristic function, which is the Fourier transform of the probability density function (pdf) as:

$$\varphi(t) = \begin{cases} \exp\left\{j\mu t - \gamma |t|^{\alpha} \left[1 + j\beta \operatorname{sgn}(t) \tan\left(\frac{\alpha\pi}{2}\right)\right]\right\}, & \text{if } \alpha \neq 1\\ \exp\left\{j\mu t - \gamma |t|^{\alpha} \left[1 + j\beta \operatorname{sgn}(t) \frac{2}{\pi} \log|t|\right]\right\}, & \text{if } \alpha = 1 \end{cases}$$

where alpha  $\alpha \in (0,2]$  is the characteristic exponent which sets the impulsiveness of the distribution,  $\beta \in [-1,1]$  is the symmetry parameter which sets the skewness,  $\gamma > 0$  is the dispersion analogous to the variance and  $\mu \in (-\infty,\infty)$  is the location parameter. When  $\beta = 0$ , the distribution is symmetric around  $\mu$  and the distribution is called a symmetric  $\alpha$ -stable (S $\alpha$ S) distribution. When moreover  $\mu = 0$ , the distribution is centralised at the origin. The three important special cases of the  $\alpha$ -stable distribution are the Gaussian distribution ( $\alpha = 2$ ), the Cauchy distribution ( $\alpha = 1$ ,  $\beta = 0$ ) and the Pearson distribution ( $\alpha = 0.5$ ,  $\beta = 1$ ).

The  $\alpha$ -stable distribution has received great interest over the last few years due to its numerous attractive properties which can be summarised as: 1) it is a generalisation of the Gaussian distribution and shares many important properties with it, such as stability ( $\alpha$ -stable distributions are closed under linear operations); 2) it has a strong theoretical justification provided by the *generalised central limit theorem* [2]; 3) it provides a very flexible model: it can model not only varying degrees of impulsiveness but also skewness.

 $\alpha$ -stable distributions have found many applications such as in modelling atmospheric noise, noise on telephone lines [3] and financial time series [4]. Unfortunately, up to now the applications of  $\alpha$ -stable distributions in image processing have been very limited other than a few isolated works [5]. Moreover, work in both signal and image processing have been limited to only symmetric  $\alpha$ -stable (SaS) distributions, ignoring skewed distributions in all other than a couple of works on parameter estimation [6] while some physical phenomena clearly exhibit skewed characteristics.

In this paper, we defend the view that the  $\alpha$ -stable processes provide a very flexible framework for modelling textures in images. Contrary to previous work in signal and image processing, here we consider general  $\alpha$ -stable distributions including the skewed case.

## 3.Linear \alpha-Stable Textures

In this paper, we consider only deterministic textures; random fields such as  $\alpha$ -stable Markov random fields will be addressed in a follow-up paper. The models we consider are: 2-D AR processes with  $\alpha$ -stable innovations which can be described with:

$$X(n,m) = \sum_{i,j \in N} a_{i,j} X(n-i,m-j) + W(n,m)$$
 (2)

where N denotes the neighbour index set and the innovations W(n,m) are distributed with (possibly skewed)  $\alpha$ -stable distributions.

Unlike the Gaussian fields,  $\alpha$ -stable fields can provide textures which have skewed and impulsive features and therefore have an inhomogeneous look. To picture this significant difference in character we present examples of synthetic Gaussian and  $\alpha$ -stable textures in Figures 1,2,3 and 4.

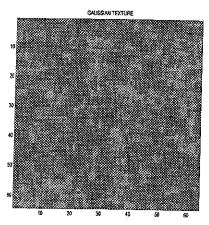


Figure 1: Gaussian texture.

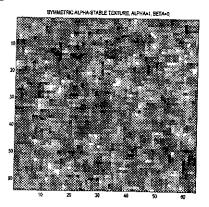


Figure 2: Symmetric  $\alpha$ -stable texture,  $\alpha=1$ ,  $\beta=0$ .

## 3.1.Estimation of $\alpha$ -Stable Field Coefficients

There are various techniques suggested for the estimation of linear 1-D symmetric  $\alpha$ -stable processes. In particular, Kanter and Steiger suggest classical least squares estimation which is surprisingly consistent despite the lack of second order moments and

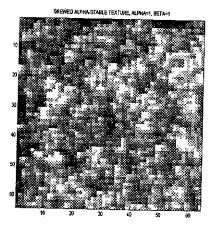


Figure 3: Skewed  $\alpha$ -stable texture,  $\alpha=1$ ,  $\beta=1$ .

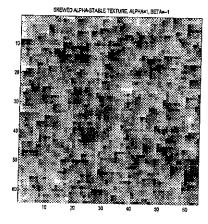


Figure 4: Skewed  $\alpha$ -stable texture,  $\alpha=1$ ,  $\beta=-1$ 

generalised Yule-Walker estimation [7]. Gross and Steiger later suggested least absolute deviations (LAD) estimation and demonstrated its consistency [8]. However, unfortunately the techniques required for LAD estimation are computationally very expensive. Nikias and Shao provide a performance comparison of these techniques by simulations [2]. Finally, Kuruoglu et al. suggest least lp-norm estimation [9] and demonstrate the higher rate of convergence when compared to least squares as predicted by Davis et al. [10].

The simulation studies provided in these papers consider only the symmetric case. For the case of 2-D skewed  $\alpha$ -stable fields we present a new estimation technique. The technique can be simply summarised as first symmetrising the field without changing the linear field coefficients and then to use extensions of the techniques developed earlier for 1-D SaS processes on the symmetrised field.

## 3.1.1.Symmetrising and Centralising Transforms

In this section, we suggest a symmetrising and centralising transform which converts the skewed AR sequence into a symmetric one. The transformation is

motivated with the following properties of the  $\alpha$ -stable distribution.

Property 1: Let 
$$X_1 \approx S_{\alpha}(\beta_1, \gamma_1, \mu_1)$$
 and  $X_2 \approx S_{\alpha}(\beta_2, \gamma_2, \mu_2)$ 

be independent stable random variables. Then,

$$X_1 + X_2 \approx S_{\alpha}(\beta, \gamma, \mu)$$

where

$$\beta = \frac{\beta_1 \gamma_1^\alpha + \beta_2 \gamma_2^\alpha}{\gamma_1^\alpha + \gamma_2^\alpha} \,, \quad \gamma = \left( \gamma_1^\alpha + \gamma_2^\alpha \right)^{1/\alpha} \,, \quad \mu = \mu_1 + \mu_2$$

Property 2: Let  $X \approx S_{\alpha}(\beta, \gamma, \mu)$  and  $c \in \Re$ . Then,

$$cX \approx S_{\alpha} \left( \operatorname{sgn}(c) \beta, |c| \gamma, c\mu \right), \quad \text{if } \alpha \neq 1$$

$$cX \approx S_1 \left( \operatorname{sgn}(c)\beta, |c|\gamma, c\mu - \frac{2}{\pi}c(\ln|c|)\gamma\beta \right), \text{ if } \alpha = 1$$

Consider segmenting the AR process into two, from the middle, and subtracting one segment from the other, sample by sample. That is:

$$X(n) - X\left(n + \frac{L}{2}\right) = \sum_{i=1}^{N} a_i \left(X(n-i) - X\left(n - i + \frac{L}{2}\right)\right) + W(n) - W\left(n + \frac{L}{2}\right)n = 1, 2, ..., L$$

Calling Y(n) = X(n) - X(n+L/2) and U(n) = W(n) - W(n+L/2), it is easy to see that the resulting sequence is an AR sequence with the same coefficients and driven by a SaS process (U(n)) distributed with the same  $\alpha$ ,  $\beta$ =0, and scale parameter  $2\gamma$ . To estimate the AR coefficients of this process, one can simply use the 2-D extensions of any of the previously suggested techniques for 1-D SaS AR processes.

#### 3.1.2.Zeroth Order Term

An alternative solution to the problem is motivated by Davis *et al.*'s work on the convergence properties of M-estimators for linear processors in the domain of attraction of a stable law [10]. Here we cite their Theorem 5.1:

Theorem: Let X(t) be an AR(p) process given by  $X(t) = \phi_0 + \phi_1 X(t-1) + ... + \phi_L X(t-N) + Z(t)$ 

with  $E[Z]^p < \infty$  for some  $\eta > 0$ . If the function

 $m(x) = E|Z - z|^p$  has a unique minimum at  $z = \tilde{z}$ , then  $\hat{\Phi} \to (\Phi_0 + \tilde{z}, \Phi_1, ..., \Phi_N)$  a.s. where  $\hat{\Phi}$ 

minimizes the  $l_p$  norm estimation error:

$$\sum_{t=1}^{L} |X(t) - \hat{\phi}_0 - \hat{\phi}_1 X(t-1) - \dots - \hat{\phi}_L X(t-N)|^p$$

Motivated by this theorem we suggest that one can simply introduce an artificial zeroth order term in the

model, obtain the parameters using one of the techniques for symmetric processes and then simply discard the artificial term. The remaining AR coefficients are the actual estimates we are looking for. The only drawback seems to be the increased computational cost due to the additional term, which converges more slowly than the other coefficients [11].

#### 3.2.Order Determination

Bhansali provided a consistent estimation technique in [12] for the determination of the order of 1-D AR processes with innovations in the domain of attraction of a symmetric stable law. His technique is based on a generalisation of Akaike's FPE criterion.

Given observations  $X_1, X_2, ..., X_L$ , the generalised FPE criterion of Akaike is:

$$FPE_{\eta}(k) = \sigma_k^2 \left( 1 + \frac{\eta k}{L} \right)$$
 where  $\eta > 0$  is an arbitrary

$$\sigma_k^2 = \frac{1}{L} \sum_{t=1}^L (X(t) + \hat{a}_{k_1} X(t-1) + ... \hat{a}_{kk} X(t-k))^2$$

Then, the order is estimated as:

$$N = \inf_{k} \text{ FPE}_{\alpha}(k), \quad k = 0,1,...N_{\text{max}}$$

Once one symmetries the AR process as described in section 3.1.1, Bhansali's method can be used to obtain the order of the process.

## 3.3. Estimation of distribution parameters

A number of techniques for the estimation of the distribution parameters from i.i.d. samples of a skewed α-stable random variable were introduced in [6]. However, we do not possess the innovations sequence and the distributions parameters need to be estimated using the information provided by observations of the AR process. To be able to use the techniques based on fractional moments for estimating distribution parameters directly from the AR process observations, one needs to establish that the AR process is ergodic. It is a well known result by Maruyama that MA processes with innovations from an infinitely divisible distribution are ergodic [13]. Since an AR process which has no roots on the unit circle can be converted to an MA process with infinite order, the ergodicity of AR processes with α-stable innovations, which are subsets of indivisible distributions, is readily established.

Due to the linearity of the AR process, from the stability property, the characteristic exponent,  $\alpha$  of the samples from AR process is equal to the characteristic exponent of the innovations process. Hence, one can estimate the characteristic exponent directly from AR

process samples. It can be shown from Properties 1 and 2 that  $\beta_{observatio~ns} = \beta_{innovation~s}$  and

$$\gamma_{observations} = \gamma_{innovations} \left( \sum_{i} |c_{i}|^{\alpha} \right)^{1/\alpha}$$

where  $c_i$ 's are the coefficients of the infinite MA process obtained from the AR process by long division. From these equalities, the rest of the parameters are readily obtained, having obtained the parameters of the innovations sequence using the fractional moments based techniques given in [6].

## 5. Simulations

Consider estimating the simple  $\alpha$ -stable AR field with coefficients  $a_{12} = 0.5$ ,  $a_{21}=0.5$ ,  $a_{22}=-0.2$ (neighbourhood cells: upper left corner),  $\alpha=1.1$ ,  $\beta$ =0.5,1 and  $\gamma$ =1. We have used the estimation strategy we have suggested in section 3.1.1: we first symmetrised the field and then applied least lp-norm estimation which we calculated via the 2D version of IRLS [9]. The results are given in Table 1 and Figures 5,6,7. For reasons of comparison we also provide results obtained using least squares (LS) estimation assuming a symmetric field. The difference in performance is obvious. Symmetrising the field and then applying least squares significantly improves the performance in this case over direct least square (a stable field is obtained) however more samples are need when compared to least lp-norm estimation and this fact is reflected in the increased variance of the estimates.

	$a_{12}$	$a_{21}$	$a_{22}$
LS	0.55	0.54	-0.15
LPS	0.50	0.50	-0.20

**Table 1:**  $\beta$ =0.5, LS: Least Squares, no symmetrisation, LPS: Least Squares with Symmetrisation.

	$a_{12}$	$a_{21}$	$a_{22}$
LS	0.46	0.57	-0.03
LPS	0.50	0.51	-0.21

**Table 2:**  $\beta$ =1.0, LS: Least Squares, no symmetrisation, LPS: Least Squares with Symmetrisation.

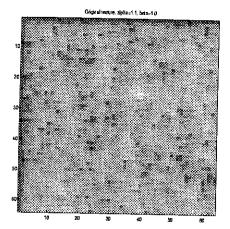


Figure 5: Original texture,  $\alpha=1.1,\beta=1.0,\gamma=1.$ 

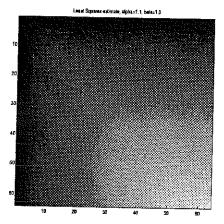


Figure 6: texture generated using parameters estimated with Gaussian texture assumption.

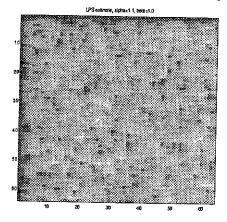


Figure 7: texture generated using parameters estimated with least lp-norm estimation with symmetrising transform.

Currently, we are working on the application of these techniques in the segmentation of synthetic aperture radar images of urban areas which show impulsive and skewed characteristics.

## Acknowledgements

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