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Abstract

Visual reconstruction problems tend to be mathematically ill-posed. They can be reformulated as well-posed variational problems using regularization theory. A generalization of the standard regularization method to visual reconstruction with discontinuities leads to variational problems which include the discontinuity contours in their unknowns. The minimization of the corresponding functionals is a difficult problem. This paper suggests the use of the Γ-convergence theory to approximate the functional to be minimized by elliptic functionals, which are more tractable. A Γ-convergence theorem which is of relevance to vision applications is discussed, and the results of a computer experiment with synthetic images are shown.

Key words: early vision, discontinuity detection, variational convergence.

Introduction

Visual reconstruction leads to inverse mathematical problems which are generally ill-posed. Regularization provides a method to make such problems well-posed. Poggio[12] has argued that many early vision problems may be unified mathematically by using the Tikhonov regularization theory. The basic idea of regularization methods is to restrict the class of admissible solutions of an ill-posed problem by imposing additional constraints. In the standard Tikhonov theory[15], a problem is made well-posed by restricting the solutions to a space of smooth functions. Through regularization, an ill-posed problem may be reformulated as a problem of calculus of variations. Finite element methods can then be used to compute a numerical solution[17].

The validity of the smoothness constraint in early vision is based on the assumption that the coherence of matter tends to give rise to smoothly varying characteristics in a visual scene. However, spatially localized physical transitions, such as sharp changes in surface geometry, surface color, or illumination, lead to discontinuities in the scene characteristics. The smoothness constraint and, consequently, the standard Tikhonov regularization theory, are clearly inadequate in the presence of visual discontinuities. Furthermore, such discontinuities are the most significant locations in any image, since they often indicate the boundaries of objects. Hence, their detection is an important task in early computational vision. It is thus necessary to extend the standard regularization methods in order to take discontinuities into account.

Both deterministic[4] and stochastic[9] methods have been proposed for visual reconstruction with discontinuities. In a recent paper, Blake[5] has argued that deterministic methods are more efficient and powerful than stochastic ones. The present paper deals with the deterministic approach to visual reconstruction. A variational method for the segmentation of images has been proposed by Mumford and Shah[11], and also discussed by Blake and Zisserman[4]. This variational method can be generalized for application to other vision problems. The generalized formulation can be considered as an extension of the regularization approach to ill-posed problems involving discontinuities.

Standard regularization methods require the minimization of an elliptic functional whose variational properties are well-known. Variational problems for regularization with discontinuities involve both an analogous elliptic functional and a measure of the length of the discontinuity contours. The discontinuity curves are themselves among the unknowns of the problem and this makes the minimization of the functional difficult. The problem of finding effective algorithms for computing the solutions of variational problems of this type is widely discussed. One of the methods which have been proposed is the graduated nonconvexity algorithm of Blake and Zisserman [4]. This efficient method can be developed only in the discrete setting. This paper suggests the use of variational convergence to approach the problems of calculus of variations arising in early vision. The theory of variational convergence is designed to approximate a variational problem by a sequence of different problems. A particular concept of variational convergence for functionals, the Γ-convergence introduced by De Giorgi [6], is suitable for problems of calculus of variations with free discontinuities. The approach presented in this paper employs a theorem of Γ-convergence, proved by Ambrosio and Tortorelli [2], which shows how a functional depending on discontinuities can be approximated by a sequence of elliptic functionals which are more tractable. The Γ-convergence makes it possible to go to the limit in the corresponding minimization problems: the minimizers of the functionals of the sequence converge, in an appropriate metric, to the minimizers of the original functional. As an example, the theorem of Γ-convergence is applied to the problem of the computation of stereo disparity, and the results of a computer experiment with synthetic images are shown.

Variational problems arising in early vision

In the standard Tikhonov regularization theory, the class of admissible solutions of an ill-posed problem is restricted to a Sobolev space of smooth functions [15]. Regularization with discontinuities requires a more general space of functions. A class of functions that contains discontinuous functions and is suitable for early vision problems is the space of functions of bounded variation, denoted by BV. The mathematical reason is that many theorems regarding variational problems with free discontinuities can be stated in BV spaces. A function of two variables u(x,y) is called a function of bounded variation in the domain Ω if u(x,y) is summable and there exists a constant K such that for any h_1 and h_2 the following inequality holds:

$$\iint_{\Omega_h} |u(x+h_1, y+h_2) - u(x,y)| \, dxdy \leqslant K \|h\|, \tag{1}$$

where $\|h\|$ denotes the norm of the vector h with components (h_1,h_2), and Ω_h is the subset of points of Ω whose distance from the boundary of Ω is greater than the norm of h. The functions of bounded variation form a Banach space. A description of the properties of BV functions can be found in Vol'pert and Hudjaev[16].

Mumford and Shah[11] proposed a variational formulation for the problem of image segmentation, which is referred to as the weak membrane model by Blake and Zisserman[4]. Their approach can be generalized for application to other visual reconstruction problems. A variational method for the regularization of ill-posed vision problems involving discontinuities looks for a BV function u(x,y) which minimizes the following functional:

$$E(u) = \iint_{\Omega} \Phi(u) \, dxdy + \lambda \iint_{\Omega} \|\nabla u\|^2 dxdy + \alpha \, \mathscr{L}(S_u), \tag{2}$$

where S_u is the set of discontinuity points of u(x,y), $\mathscr{L}(S_u)$ denotes the total length of S_u which is considered as the union of curves, constants λ and α are positive weights, and $\Phi(u)$ is a non-negative function. If $\Phi(u) = (u-f)^2$, with function f(x,y) representing the data, the corresponding functional is called the weak membrane functional by Blake and Zisserman. The first two terms in the functional have a meaning similar to the functionals employed in standard regularization. The first term, which depends on the problem, measures the discrepancy between the solution u(x,y) and the input data. The second term obliges the solution to be smooth outside the discontinuity set S_u. By minimizing the total length of the curves along which the solution is discontinuous, the third term prevents the formation of an incoherent discontinuity set. The contour length has been chosen because it is the simplest reasonable measure of the set of jump points. A functional which also includes a measure of contour curvature might be defined in order to obtain smooth discontinuity curves. However, the minimization of such a functional is likely to be a computationally infeasible problem. Furthermore, Blake and Zisserman[4] argued that the functional which only involves the contour length performs quite well in discontinuity detection. It should be noted that the gradient of a BV function, which appears in the functional, has a different meaning from the one used in standard mathematical analysis. It can be shown[1] that the derivative (in the distributional sense) of a BV function can be decomposed into a summable part, called the approximate differential, and a singular part. Only the approximate differential appears in the second integral in the expression of E(u), and this makes the integral meaningful. A more detailed discussion can be found in Vol'pert and Hudjaev[16]. The weights λ and α are the parameters of the problem. It is not clear how they should be chosen. Blake and Zisserman discussed the meaning of these parameters for discontinuity detection with the weak membrane. They found that the weight λ is a scale parameter, while the square root of the ratio $2\alpha/\lambda^{1/2}$ is a contrast threshold which determines step detection[4].

The problem of minimizing functional E(u) can be considered as an extension of the classical regularization method in the presence of discontinuities. It has not been proved if this problem is well-posed, but it can be conjectured that this is true. Ambrosio[1] proved that the weak membrane functional has minimizers in a suitable subclass of the functions of bounded variation, denoted by SBV (special functions of bounded variation). The SBV functions have the property that the singular part of the distributional derivative is concentrated along the discontinuity set (like a δ distribution). Unfortunately, the SBV space also includes functions with highly irregular discontinuity sets S_u. Since visual scenes are generally expected to give rise to piecewise regular discontinuity contours, the proof of the existence of solutions in a class of functions whose discontinuity sets are the union of regular curves would be of relevance to computer vision applications. At present, the theoretical analysis of properties, such as well-posedness, of variational problems of this type is still an open question.

The computation of the discontinuity contours makes the minimization of the functional E(u) a difficult problem, both theoretically and practically, because the set S_u of the discontinuity curves is an unknown of the problem. For instance, it is quite difficult to compute a numerical solution of the Euler-Lagrange equations found by Mumford and Shah. The concept of Γ-convergence is a mathematical tool suitable for variational problems with free discontinuity sets. In particular, the theorem of Γ-convergence proved by Ambrosio and Tortorelli[2], allowing the approximation of functional E(u) by elliptic functionals, makes the problem numerically more tractable.

Γ-convergence for functionals

A new concept of convergence for sequences of functionals has appeared in mathematical analysis in recent years. This concept is specially designed to approach the limit in the sequences of the corresponding variational problems and is called variational convergence [3]. This notion of convergence can be used to approximate one variational problem by another, which may have quite different computational properties. When applied to minimization problems, the convergence of the sequence of functionals $F^n(x)$ to the limit functional $F(x)$, in a variational sense, requires that minimizers of $F^n(x)$ converge (in a suitable metric) to minimizers of $F(x)$ as n tends to infinity. Furthermore, the minimal values of the $F^n(x)$ must converge to the minimal value of $F(x)$. Hence, a variational convergence is a weak notion of convergence for sequences of functionals which makes it possible to go to the limit in the corresponding minimization problems.

A particular theory of variational convergence, the Γ-convergence introduced by De Giorgi,[6] has an interesting computational application to problems of calculus of variations with free discontinuity sets. A sequence of functionals $F^n(x)$ defined on a metric space X is said to be Γ-convergent to the functional $F(x)$ if the following two conditions hold for all $x_0 \in X$:

(i) for every sequence x_n converging to x_0 (in the metric of the space X) one has

$$\liminf_{n \to \infty} F^n(x_n) \geqslant F(x_0); \qquad (3)$$

(ii) there exists a sequence x_n converging to x_0 such that

$$\limsup_{n \to \infty} F^n(x_n) \leqslant F(x_0). \tag{4}$$

The limit functional F(x) is called the Γ-limit of the sequence $F^n(x)$. The Γ-limit when it exists is unique. The meaning of the inequality (3) is the following: if the limit of the numerical sequence $F^n(x_n)$ exists, it is greater than or equal to $F(x_0)$; otherwise the limit of every convergent subsequence $F^{n_k}(x_{n_k})$ is greater than or equal to $F(x_0)$. The inequality (4) likewise means that all limit values of the numerical sequence $F^n(x_n)$ are less than or equal to $F(x_0)$. It should be noted that Γ-convergence is a different concept from pointwise convergence. This can be understood when examining the two conditions for Γ-convergence: condition (i) is a stronger assumption than the pointwise convergence, since one asks that inequality (3) holds for any convergent sequence x_n, and not only for $x_n = x_0$; condition (ii) is a weaker requirement than the pointwise convergence in which inequality (4) must hold with $x_n = x_0$. Hence, in general, Γ-convergence is not implied by and does not imply pointwise convergence. They are two independent concepts, comparable in the sense that if F(x) and $F_1(x)$ are, respectively, the Γ-limit and the pointwise limit of sequence $F^n(x)$, then $F(x) \leqslant F_1(x)$ for every $x \in X$.

The fundamental variational property of Γ-convergence can now be formulated: let $F^n(x)$ be a sequence of functionals defined on the metric space X which is Γ-convergent to the limit functional F(x) as n tends to infinity. Assume also that the functionals $F^n(x)$ have minimizers in X. It can then be shown[3] that if a sequence x_n^* of minimizers of $F^n(x)$ converges, then the limit is a minimizer of F(x), and $F^n(x_n^*)$ converge to the minimal value of F(x). Hence, the Γ-convergence is a notion of variational convergence. If n is large enough, the problem of minimizing F(x) can then be replaced by the problem of minimizing $F^n(x)$.

There is another property of Γ-convergence which is of relevance to the calculus of variations. If $F^n(x)$ Γ-converges to F(x), it can then be proved[3] that $F^n(x) + G(x)$ Γ-converges to F(x) + G(x) for every continuous functional G(x). This property means that Γ-convergence is stable under continuous perturbations. This stability feature plays an important role in the application of the theorem of Ambrosio and Tortorelli to the problem of the minimization of the functional E(u).

A theorem of Γ-convergence

The concept of Γ-convergence is designed to approximate one variational problem by a sequence of problems which may perhaps involve different mathematical variables. In the first years of development of Γ-convergence, Modica and Mortola[10] showed that Γ-limits of elliptic functionals may be a completely different functional containing, in their case, the measure of a surface. Ambrosio and Tortorelli extended the result of Modica and Mortola to show also that the weak membrane functional is the Γ-limit of a sequence of elliptic functionals.

A sequence of functionals which approximate the functional E(u) in the sense of Γ-convergence is the following:

$$E^n(u,z) = \iint_\Omega \Phi(u)dxdy + \lambda \iint_\Omega (1-z^2)^{2n} \|\nabla u\|^2 dxdy$$
$$+\alpha \iint_\Omega \left[(1-z^2)^{2n} \|\nabla z\|^2 + n^2 z^2/4\right] dxdy. \tag{5}$$

If n is large enough, the problem of minimizing the functional E(u) with respect to the variable u can then be replaced by the quite different problem of minimizing the functional $E^n(w)$ with respect to the vector variable w=(u,z). This is an instance of approximation, via Γ-convergence, of one variational problem by another, with different variables. The expression of the approximating functionals $E^n(w)$ now contains a first part which still measures the discrepancy between the solution and the data, and a second part which will be denoted by $\hat{E}^n(w)$. Ambrosio and Tortorelli proved that the sequence of functionals $\hat{E}^n(w)$ Γ-converges to the sum of the last two terms in the expression of the functional E(u), as n tends to infinity. The proof of the theorem is highly complex and is given in Ambrosio and Tortorelli[2]. The stability property of

Γ-convergence can now be used to apply the theorem of Ambrosio and Tortorelli to the whole functional E(u). From this property the sequence $E^n(w)$ Γ-converges to E(u) if the functional which measures the discrepancy between the solution and the data is continuous. This functional is continuous if, for instance, the function $\Phi(u)$ is piecewise continuous, a condition generally satisfied in vision problems.

The auxiliary variable z(x,y) is related to the set S_u and has a value ranging between 0 and 1. This function plays the role of control variable on the gradient of u. The function z_n^* which minimizes $E^n(w)$ is close to 1 in a neighbourhood of the set of jumps S_u, whereas far from this neighbourhood it is much lower. The neighbourhood shrinks as n tends to infinity. When n is large, the function z_n^* is then concentrated along a small neighbourhood of S_u, thus the gradient of u_n^* is permitted to become arbitrarily large along S_u, generating a jump in the solution. Asymptotically, the minimizers z_n^* converge to a continuity control function as intended by Terzopoulos[14], whose value is 1 along the set of jumps S_u, and 0 everywhere else. This explains why the Γ-limit E(u) does not depend on the variable z. The limit functional E(u) is defined on the SBV space, while the approximating functionals $E^n(w)$ are defined on the subclass of the summable functions which are regular in the points where $z(x,y) \neq 1$. Both the Γ-limit and the functionals of the sequence are thus defined on a subclass of the L^1 space of the summable functions, and, consequently, the convergence of the minimizers u_n^* of $E^n(w)$ to the minimizers of E(u) takes place with respect to the L^1 norm[2]. With respect to this norm, the minimizers z_n^* converge to 0.

The presence of the term measuring the length of the discontinuity contours makes the minimization of the functional E(u) difficult. In particular, the design of a discrete approximation scheme using finite elements is not straightforward. Mumford and Shah showed that the minimization of the functional with respect to the discontinuity set S_u leads to a differential equation for the discontinuity curves. This equation is coupled to a classical Neumann problem with boundary conditions defined on the unknown set S_u [11]. To compute a numerical solution of the system of coupled equations is a very difficult problem. Using the theorem of Γ-convergence the unknown discontinuity contours are represented by the control function z(x,y). The minimization with respect to the set S_u is now replaced by the minimization with respect to an ordinary function, which can be performed by classical calculus of variations. The numerical resolution of the resulting system of Euler-Lagrange equations is simpler, even if the equations are still nonlinear. Furthermore, the approximating variational problems now contain the terms $\hat{E}^n(u,z)$ which are functionals of elliptic type. As is known, elliptic functionals admit a simple finite element discrete approximation. Once discretized this way, the functional can be directly minimized using an optimization technique. Hence, the Γ-convergence method introduces considerable simplifications, as it permits us to resort to classical calculus of variations and standard finite element methods. However, the minimization problem is still nonconvex, and may thus have multiple local minima. A global optimization algorithm is then needed to overcome nonconvexity. Blake and Zisserman[4] proposed the graduated nonconvexity algorithm to avoid the local minima in a discrete formulation of the weak membrane problem. Their algorithm is an efficient tool in the discrete setting. Finally, the exponential dependence of the approximating functionals on the index n suggests that even small values of n can give good approximations of the solution.

An application to stereo vision

The Γ-convergence theorem has been experimented on the problem of computing stereo disparity. This is a typical ill-posed problem of visual reconstruction. A regularization approach to the problem of computing stereo disparity has been presented in previous papers by the author[7,8]. In this problem, the function $\Phi(u)$ takes the form

$$\Phi(u) = \Big[L(x,y)-R(x+u(x,y),y)\Big]^2, \tag{6}$$

where u(x,y) is the disparity function, and R(x,y) and L(x,y) are the right and left image intensities, or a simple function of them (such as a Laplacian). Vertical disparity is assumed to be negligible. Once disparity has been computed, depth may easily be recovered by triangulation. The disparity discontinuities are generally due to the occluding boundaries between different surfaces.

The computer simulations here presented were executed with synthetic stereo pairs of images corresponding to simple patterns. Figure 1 shows the two images of a stereo pair representing three patches of flat surfaces. The stereogram shown in Figure 2 represents a curved surface portrayed against a plane background. The brightness pattern of all the surfaces is a linear combination of spatially orthogonal sinusoids. The spatial frequency of the sinusoids is chosen to give a reasonably strong brightness gradient such as that usually required for binocular stereo matching. The stereo disparity in the images is discontinuous along the occluding boundaries between the different surfaces. The stereo pairs are obtained by simulating a simple perspective projection of the objects yielding a horizontal disparity inversely proportional to depth, and no vertical disparity (see also March[7]). The data are not noisy.

The limit functional was replaced by $E^n(u,z)$ with a fixed value of n. The approximating functional was discretized by means of standard rectangular finite elements[17], and then minimized using a conjugate gradient method. Even if the functional is nonconvex, the computer simulations showed that the descent method yields a satisfactory solution, at least for the simple images used here. In more general situations, appropriate global optimization methods will be needed. The neighbourhood of the discontinuity set in which the minimizer z_n^* is near to 1 shrinks as n tends to infinity. This suggests that the discretization step of the finite element method must decrease in an appropriate manner as n increases. However, because of the above mentioned exponential dependence of $E^n(u,z)$ on the index n, small values of n can be used. The images used in the present simulations were discretized using a rectangular grid of 64x64 nodes. For this discretization, the computer experiments showed that even a small value such as $n=2$ gives a satisfactory reconstruction of the surfaces in proximity to the disparity discontinuities. In order to improve the solution, the finite elements where the control function z exceeds a suitable threshold were removed, thereby allowing the surfaces to fracture freely (see also Terzopoulos[13]). The values of disparity obtained were finally converted into depth values.

The conjugate gradient algorithm was started with an initial estimate of the disparity function u equal to a constant value, and setting the control function z equal to a small constant, near to 0, everywhere. The optimal choice of the parameters of the functional is a difficult problem, and will not be examined here. In the computer simulations, the values $\lambda = 2$, $\alpha = 0.01$, and $n = 2$ were chosen on the basis of the results of a number of experiments. The value 0.7 was used for the threshold on the function z. The simulations showed that the function z converges to values very close to 1. However, a not too high threshold yields a similar solution while requiring a smaller number of iterations of the conjugate gradient algorithm. It should be noted that the choice of the parameters is the same for both the stereograms. Figures 3 and 4 show the surfaces reconstructed from the respective stereograms. The simulations show that the control function z plays an essential role in the reconstruction of the surfaces along the occluding boundaries. In the points where z converges to 1, the disparity gradient is allowed to increase arbitrarily, deactivating the smoothness constraint along the discontinuity set and then yielding the correctly reconstructed depth discontinuities.

Conclusion

Visual reconstruction gives rise to problems of calculus of variations with free discontinuity contours. This paper has suggested the use of the theory of Γ-convergence to approach this type of variational problem. The concept of Γ-convergence allows the approximation of one variational problem by another which may appear quite different. In particular, the Γ-convergent approximation of functionals depending on discontinuities by elliptic functionals makes the computation of the minimizers easier. The representation of the discontinuity contours by a control function allows the use of the finite element method, which is very convenient for numerical computation. The functionals of the approximating sequence used in the present paper are complicated. However, different sequences of functionals may converge to the same Γ-limit. Ambrosio and Tortorelli (personal communication) have recently found another sequence of elliptic functionals which converges to the measure of the discontinuity set. The new functionals are simpler and, in particular, more "quadratic", even if still nonconvex. An accurate reconstruction of the discontinuities is possible if index n is sufficiently large, and the discretization step is consequently small. The convergence of the control function z to the discontinuity set as n increases, when a finite element discretization is used, is a feature of the method that requires a deeper understanding from a numerical point of view. This aspect will be further examined in a forthcoming paper.

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FIGURE CAPTIONS

Fig. 1: Synthetic stereo pair of images representing flat surfaces with discontinuities.

Fig. 2: Synthetic stereo pair of images representing a curved surface in front of a plane. Disparity is discontinuous along the occluding boundary.

Fig. 3: Reconstructed depth map of the flat surfaces with their occluding boundaries.

Fig. 4: Reconstructed depth map of the curved surface with the discontinuity.

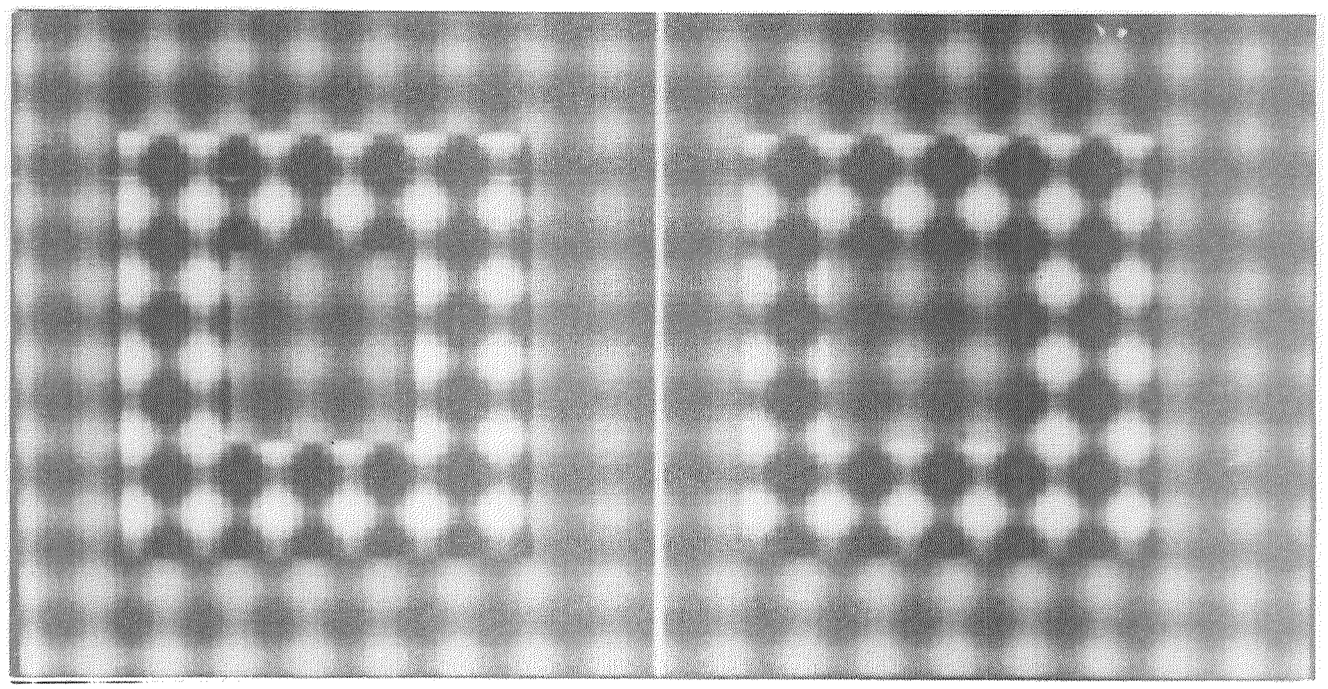

Figure 1

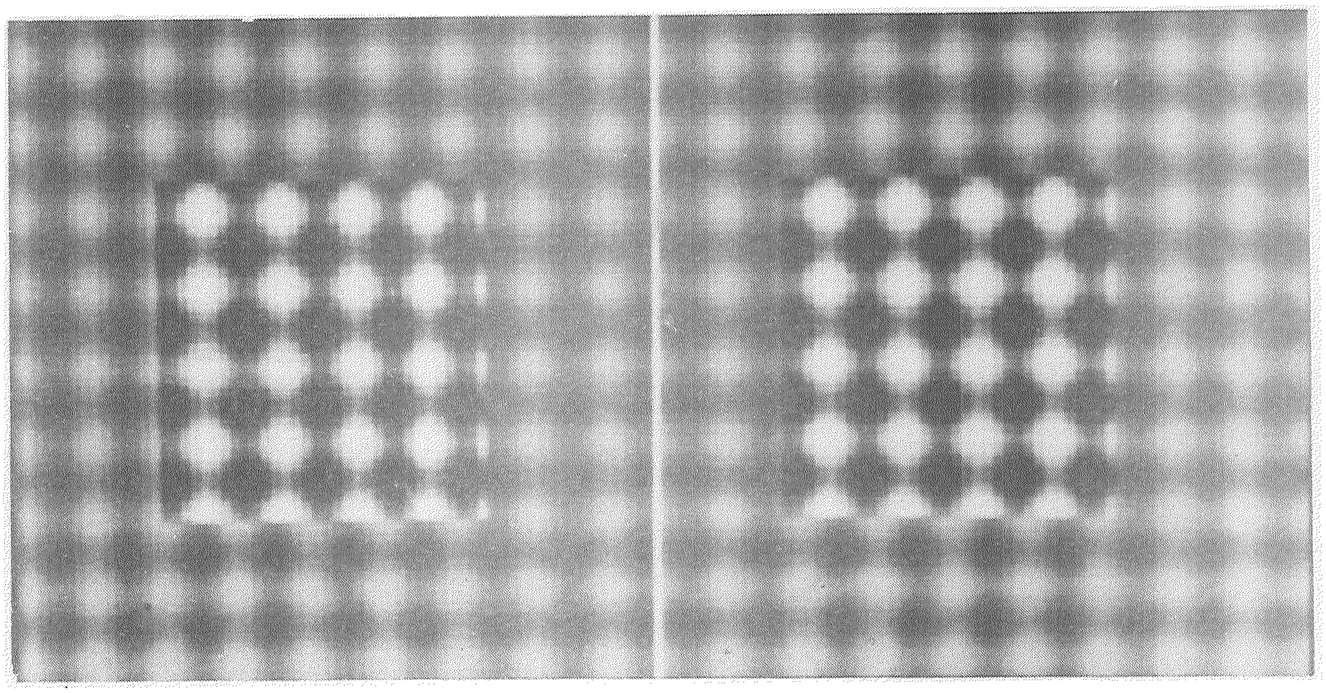

Figure 2

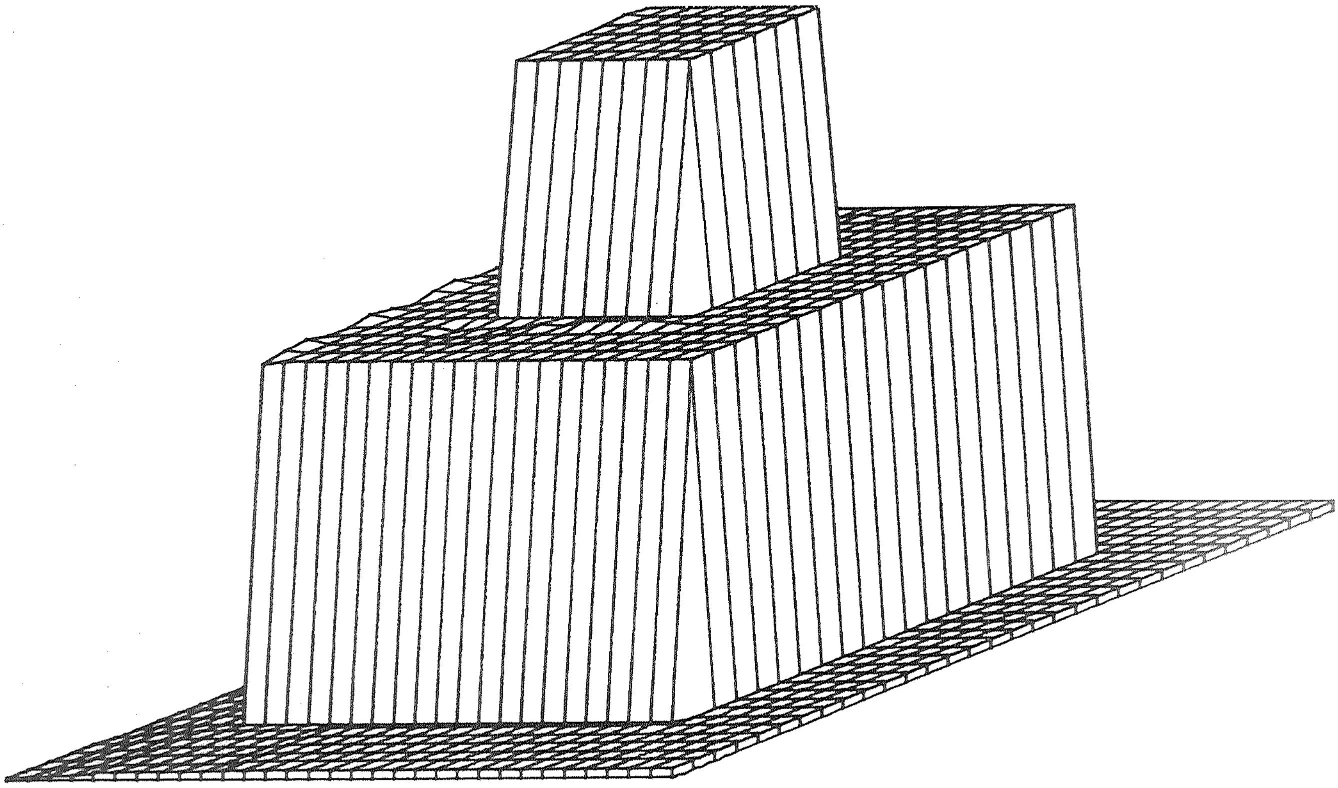

Figure 3

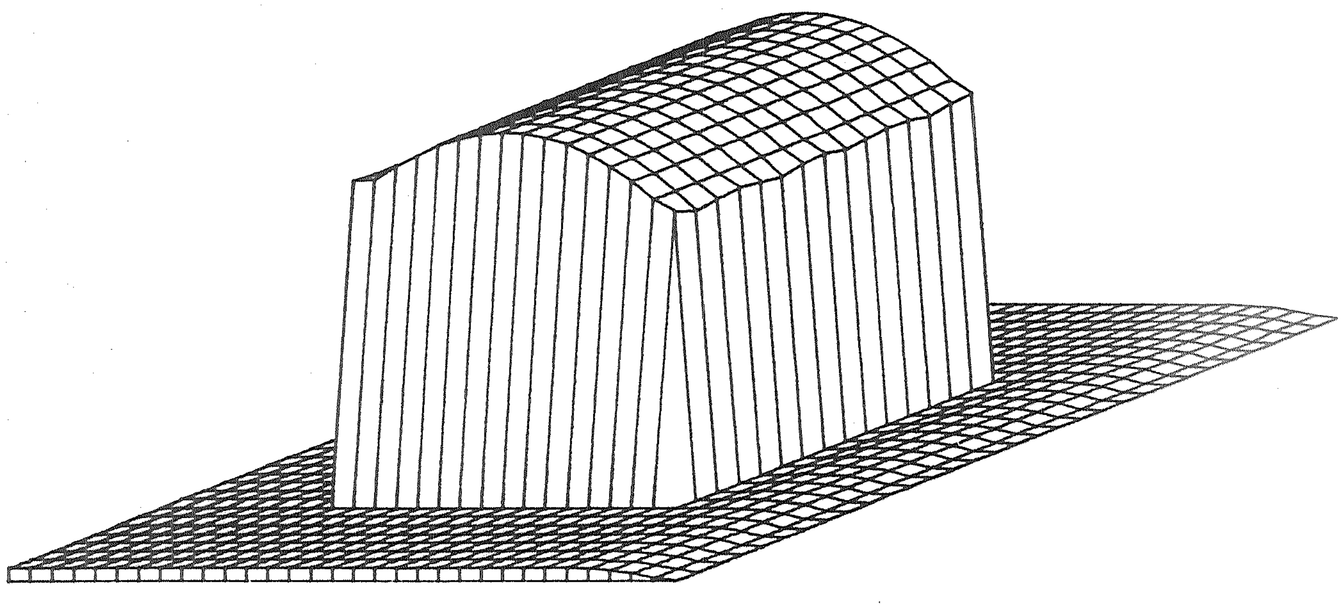

Figure 4