# Sensitivity Analysis of Microwave Non Destructive Diagnostic Procedures Based on the Use of Capacitive Sensors

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## Abstract

This paper deals with the use of capacitive sensors in Non Destructive Techniques (NDT) at microwave frequencies, with the aim of analyzing the measuring conditions which allow to improve the detection sensitivity. Particular reference was made to the case where the capacitive sensor is used in connection with a Vectorial Network Analyzer (VNA).
Detection sensitivity was first theoretically studied with reference to the following measured variables: input impedance, its real and imaginary part, absolute value and phase of the complex reflection coefficient. The influence of the measuring reference plane for such variable was discussed and the choice of its position was proved to be of great importance in order to maximize the measuring sensitivity.
The experimental results confirmed the criteria derived from the theorical analysis to improve the measuring procedure.

**Categories and Subject Descriptors**: I.2.9 [**Computing Methodologies**]: Robotics – *Sensors*; J.2 [**Computer Application**]: Physical Sciences and Engineering – *Electronics*; J.7 [**Computer Application**]: Computer in Other System – *Industrial Control – Process Control*

## 1 – Introduction

Microwave non destructive techiques ( NDT ) have a long history and the interest for these techniques has been greatly increased in recent time also in connection to the fact that ever expanding materials technology, by which lighter and stroger electrically insulating composites are replacing metals in many applications, demands inspection techniques alternative to the other NDT methods.
Microwave signals penetrate inside dielectric media, the depth of penetration essentially depending on the permittivity of the medium and the used operating frequency. Measurements can be conducted by contact or non-contact procedures, by reflection or transmission techniques. Microwave NDT techiques allow to detect dimensional variations of the tested body and are sensitive to the presence of subsurface dielectrically inhomogeneous regions, which can be related to defects of many types.

These techniques are of great interest in different industrial applications due to the fact that complicated post signal processing are, generally, not required, and due to the capability of providing real-time informations. An extensive bibliography relative to the use of microwaves in non destructive evaluation is reported in [1].
Excluding particular applications, such ground probing radars or heating systems, the required power is in the few milliwatts range, so that no environmental hazard is present.
The fundamental part of any microwave non destructive technique is the device by which microwave signal is applied to the body under test; such a device is commonly called sensor. In the technical literature sensors of far different types have been proposed, generally derived from rectangular or cilindrical waveguide transmission systems, from coaxial systems and from microstrip systems. A complete review relative to microwave sensors design and applications is reported in [2].
It is very important that the type of the sensor is suitably chosen with reference to the physical and geometrical characteristics of the body to be examined and with reference to the specific application. In addition, the design of the diagnostic procedure is of fundamental importance to achieve the best results from the specific sensor type.
In this paper the case is considered where a capacitive sensor is used. Particularly, the sensor is simply assumed in the form of an open-ended coaxial line, faced to the surface of the body under test and connected to the measuring system via a lossless transmission line. The aim of the paper is the analysis of the detection sensitivity with reference to the commonly measured parameters, the reflection coefficient ? and the input impedance Zi, for example, in order to optimize and the detection capability.

## 2 – Analysis of the measuring procedures

The basic schema of a microwave measuring system for the Non Destructive Testing (NDT) of materials, based on the use of capacitice sensors, is shown in Fig. 1.

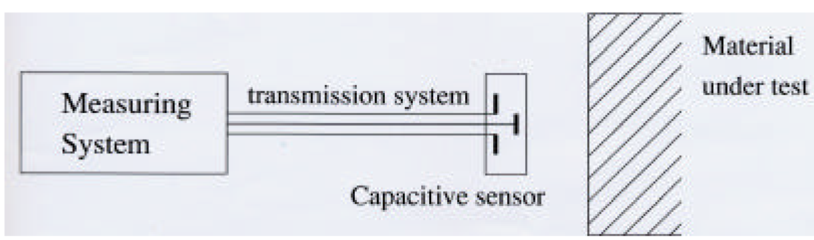



Fig. 1 – NDT system based on the use of a capacitive sensor

In Fig. 2 the equivalent circuit is reported which models the interaction of the material with the measuring system itself.
The presence of inhomogeneous regions inside the material causes variations in capacitive reactance Xc: the focal point is the design the of a suitable procedure which can maximally improve the measure sensitivity, that is the variation of the measured variable with respect to the variation of Xc.

### *i) Procedure based on the measure of the input impedance Zi*

In the case where the measuring apparatus is connected to the capacitive sensor by means of a lossless transmission line, the input impedance Zi can be expressed as

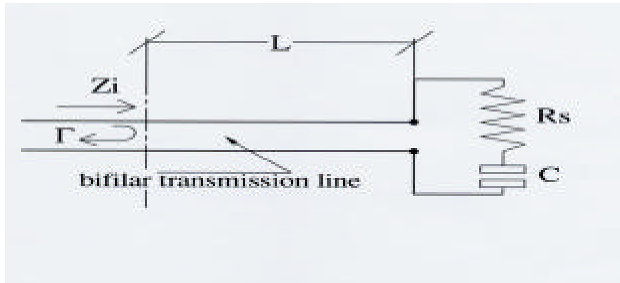



Fig. 2 – The equivalent circuit which models the interaction between the material and the capacitive sensor

$$Zi \; ? \; Zo\frac{(Rs \; ? \; jXc).\cos x \; ? \; jZo.sinx}{Zo.\cos x \; ? \; j(Rs \; ? \; jXc).sinx} \qquad (1)$$

where: Zo = characteristic impedance of the transmission line;
x = 2?L/ ? , with ? wavelength on the transmission line and L distance of the reference plane from the input section of the sensor.
By using Eq. (1), the sensitivity of the normalized input impedance Zi/Zo with respect to the normalized capacitive reactance Xc/Zo results:

$$S(x) = \frac{d(Zi/Zo)}{d(Xc/Zo)} \; ? \; ? \; j\frac{1 \; ? \; \tan^2 x}{?1 \; ? \; j(Rs/Zo \; ? \; jXc/Zo).\tan x?} \qquad (2)$$

Let’s now examine the following cases.

*ia) Material with very low loss factor ( Rs ?? Xc).*
In this case we have:

$$S(x) \; ? \; ? \; j\frac{1 \; ? \; \tan^2 x}{(1 \; ? \; Xc/Zo.\tan x)^2} \qquad (3)$$

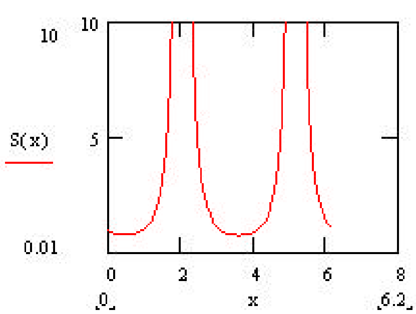



Fig. 3 – Sensitivity of Zi/Zo with respect to Xc/Zo for low loss factor materials( Rs ?? Xc, Xc/Zo = 0.5) vs. x = 2 ?L/ ?

A typical graph of function |S(x)| vs. x is shown in Fig. 3. We can see that the sensitivity can be greatly improved if the reference plane distance L for the measure of Zi is chosen suitably near to the values determined by the equation

$$\tan(x) \; ? \; ?1/(Xc/Zo) \qquad (4)$$

Fig. 3 is relative to the case Xc/Zo = 0.5.

*ib) Materials with high loss factor (Rs ? ? Xc)*
In this case Eq. (2) becomes

$$S(x) \; ? \; ? \; j\frac{1 ? \tan^2 x}{(1 ? \; j(Rs/Zo).\tan x)^2} \qquad (5)$$

For example, in Fig. 4 the typical shape of |S| vs. x is shown in the particular case where Rs/Zo = 6.

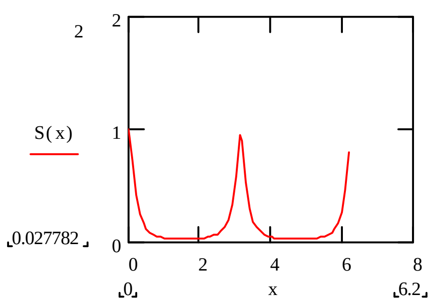
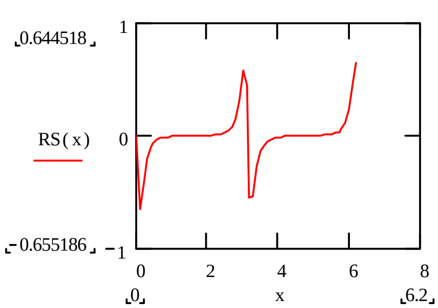


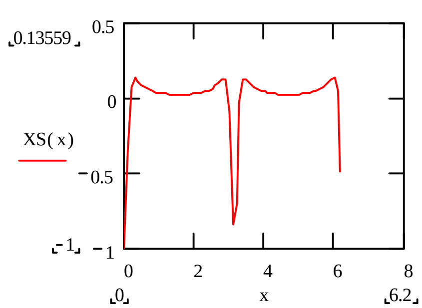

Fig. 4 – Sensitivity of Zi/Zo with respect to Xc/Zo for high loss factor materials ( Rs? ? Xc, Rs/Zo = 6) vs. x = 2 ?L/ ?

Moreover, for Rs/Zo = 6, in Fig. 5 and 6 the shape of the real and imaginary part of the sensitivity S(x), RS(x) and XS(x), respectively, are shown.
From Figs. 4, 5, 6 we can see the convenience of measuring the impedance Zi, or its real or its imaginary part, directly at the input section of the capacitive transducer ( L = 0) or at distances L = k ?/2.

Fig. 5 – Sensitivity of Ri/Zo with respect to Xc/Zo vs. x = 2 ?L/ ? ( Rs? ? Xc, Rs/Zo = 6)

Fig. 6 – Sensitivity of Xi/Zo with respect to Xc/Zo vs. x = 2 ?L/ ? ( Rs? ? Xc, Rs/Zo = 6)



*ic) The general case*

With reference to the general case, where the assumptions Rs ?? Xc or Rs ?? Xc are not alloweed, in Fig. 7 the typical shape of |S(x)| obtained from Eq . (2) is shown; the particular case Xc/Zo = 0.5, Rs/Zo = 0.2 was considered.

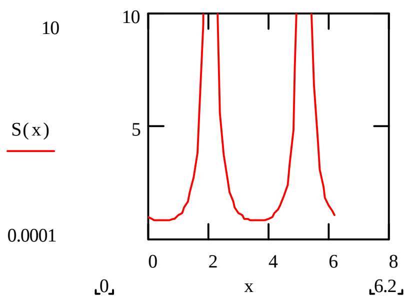



Fig. 7 - Sensitivity of Zi/Zo with respect to Xc/Zo for the general case vs. x = 2 ?L/ ?
( Rs/Zo = 0.2)

Also for this case considerations similar to those relative to the case *ia)* can be made.

***ii) Procedure based on the measure of the Reflection Coefficient*** ?

The Reflection Coefficient ? at distance L from the capacitive sensor can be expressed as

$$? \; ? \; \frac{(Rs/Zo \; ? \; Xc/Zo.\tan x \; ? \; 1) \; ? \; j(\tan x \; ? \; Rs/Zo\tan x \; ? \; Xc/Zo)}{(Rs/Zo \; ? \; Xc/Zo\tan x \; ? \; 1) \; ? \; j(\tan x \; ? \; Rs/Zo\tan x \; ? \; Xc/Zo)} \qquad (6)$$

After some calculations the following expressions for the module and the phase of ? , respectively, can be obtained

$$|?| \; ? \; \sqrt{\frac{((Xc/Zo)^2 \; ? \; (Rs/Zo)^2 \; ? \; 1 \; ? \; 2Rs/Zo)}{((Xc/Zo)^2 \; ? \; (Rs/Zo)^2 \; ? \; 1 \; ? \; 2Rs/Zo)}} \qquad (7)$$

$$? \; = \; \tan^{?1}\frac{(1 \; ? \; Rs/Zo)\tan x \; ? \; Xc/Zo}{Rs/Zo \; ? \; Xc/Zo\tan x \; ? \; 1} \; ? \; \tan^{?1}\frac{(1 \; ? \; Rs/Zo)\tan x \; ? \; Xc/Zo}{Rs/Zo \; ? \; Xc/Zo\tan x \; ? \; 1} \qquad (8)$$

Obviously, due to the assumption of a lossless transmission line, | ?| results indipendent from x.

From Eq. (7) we have

$$d(|?|^2)/d(Xc/Zo) = Sgm = \frac{8(Xc/Zo)(Rs/Zo)}{[(Xc/Zo)^2 \; ? \; (1 \; ? \; Rs/Zo)^2]} \qquad (9)$$

A simple analysis of Eq. (9) shows that sensitivity Sgm has its maximum value for Xc/Zo = (1+Rs/Zo)/ ?3; the maximum value is equal to (3 ?3/2(Rs/Zo))/(1+Rs/Zo)$^{3}$. For example, Fig. 8 shows the graph of Eq.(9) vs. xc= Xc/Zo for the case Rs/Zo = 6.

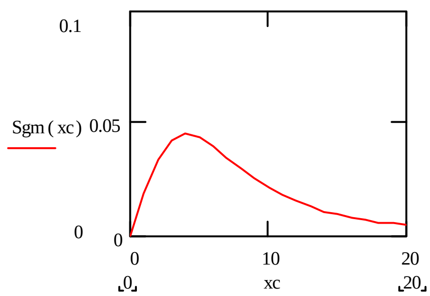



Fig. 8 – Sensitivity of |?|$^{2}$ with respect to Xc/Zo vs. Xc/Zo for the case Rs/Zo = 6

From Eq. (8) we have: d ? /d(Xc/Zo) = Sgf = (10)

$$\frac{R_1(1 ? \tan^2 x)}{(R_1 ? Xc/Zo \tan x)^2 ? (R_1 \tan x ? Xc/Zo)^2} ? \frac{R_2(1 ? \tan^2 x)}{(R_2 ? Xc/Zo \tan x)^2 ? (R_2 \tan x ? Xc/Zo)^2}$$

with $R_1$ = 1-Rs/Zo and $R_2$ = 1+Rs/Zo.
It can be proved that also Sgf is indipe ndent from x; its final expression becomes:

$$\text{Sgf} = \frac{R_1[R_2^2 ? (Xc/Zo)^2] ? R_2[R_1^2 ? (Xc/Zo)^2]}{[R_1^2 ? (Xc/Zo)^2][R_2^2 ? (Xc/Zo)^2]} \quad (11)$$

In Fig. 9 the graph of Sgf, measured in degree, vs. xc = Xc/Zo is reported for the already considered case Rs/Zo = 6.
Figs. 8 and 9 show that measuring procedures based on the module or phase variations of Reflection Coefficient generally offer very low sensitivity.

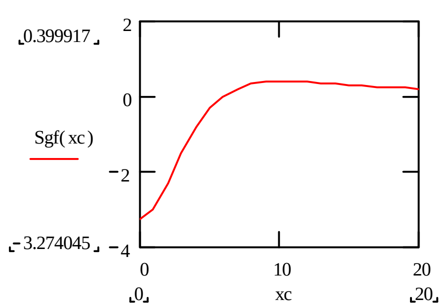



Fig. 9 - Sensitivity of ? with respect to Xc/Zo vs. Xc/Zo for the case Rs/Zo = 6

**3 – Experimental results**

A set up was implemented to experimentaly test the sensitivity analysis results reported in the previous paragraph. The measuring system is based on the use of the HP 8753B Vectorial Network Analyzer (V. N. A.) which allows to measure the input im pedance and the complex reflection coefficient at any reference plane located between the output plane of the V. N. A. and the input plane of the capacitive sensor. The used sensor was the open-ended coaxial cable capacitive sensor supplied with the kit HP 85070 Dielectric Probe. A schema of the experimental set -up is shown in Fig. 10.

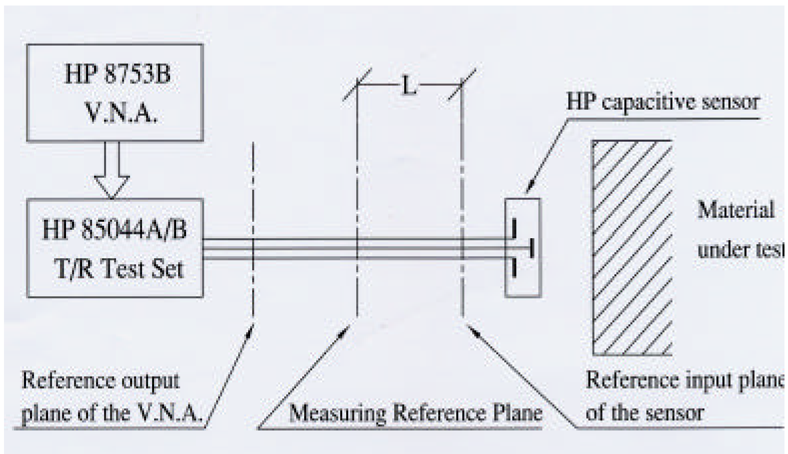



Fig. 10 – The experimental set -up used to test the results of the sensitivity analysis

At the frequency of 2.45 GHz, the input impeda nce Zi and the reflection coefficient ? = |? |exp(-j? ) were measured vs. different values of L between the measuring reference plane and the input plane of the capacitive sensor. For this purpose the Function “Reference Plane Extension” available in the V . N. A. was used. The value L = 0 was preliminary setted by operating the V. N. A. in the swept frequency mode, with central frequency 2.45 GHz, and by varing the plane extension until a maximally flat diagram for the phase ? was obtained. Successively, f or different values of L the measure of Zi and ? was made in the condition where the capacitive sensor was in free air, and in the condition where it was placed upon a one centimeter thick Plexiglass plate. The results are reported in the following Table I .

**Table I**

| **l [cm]** | **Zia [? ]** | **Zid [? ]** | **\|dZi\| [? ]** | **dRi [? ]** | **DXi [? ]** | **d\|? \|** | **d? [°]** |
|---|---|---|---|---|---|---|---|
| 0 | 293.6-j19.37 | 290.86-j30.6 | 11.6 | -2.74 | -11,3 | 0,0004 | 0.796 |
| 0.75 | 48.87-j99.68 | 47.6-j98.2 | 1.95 | -1.27 | 1.48 | 0.00055 | 0.809 |
| 1.25 | 21.89-j60.49 | 21.6-j59.78 | 0.767 | -0.29 | 0.71 | 0.0004 | 0.706 |
| 1.75 | 13.37-j37.12 | 13.27-j36.67 | 0.461 | -0.1 | 0.45 | 0.00036 | 0.692 |
| 2.25 | 9.98-j20.69 | 9.94-j20.32 | 0.372 | -0.04 | 0.37 | 0.00033 | 0.748 |
| 2.75 | 8.65-j7.23 | 8.66-j6.91 | 0.32 | 0.01 | 0.32 | 0.00074 | 0.738 |
| 3.25 | 8.57+j5.29 | 8.60+j5.59 | 0.301 | 0.03 | 0.3 | 0.00053 | 0.702 |
| 3.75 | 9.68+j18.49 | 9.73+j18.83 | 0.344 | 0.05 | 0.34 | 0.000134 | 0.709 |
| 4.25 | 12.64+j34.29 | 12.80+j34.77 | 0.506 | 0.16 | 0.48 | 0.000768 | 0.794 |
| 4.75 | 19.98+j56.19 | 20.34+j56.91 | 0.805 | 0.36 | 0.72 | 0.000563 | 0.799 |
| 5.25 | 42+j92 | 43+j93.14 | 1.516 | 1 | 1.14 | 0.000141 | 0.715 |
| 5.75 | 142.34+j142.8 | 147.16+j142.84 | 4.82 | 4.82 | 0.04 | 0.000321 | 0.667 |
| 6.12 | 292.84-j22.5 | 290.56-j32.5 | 10.257 | 2.28 | 10 | 0.0004 | 0.73 |
| 6.25 | 245.14-j108.44 | 238.34-j113.22 | 8.312 | -6.8 | -4.78 | 0.000603 | 0.687 |

The distance L reported in Table I is the equivalent free space distance between the reference measure plane and the capacitive sensor input plane. The considered range start from L equal zero (reference plane just put at the input plane of the sensor) up to about the half free space wavelength at the frequency of 2.45 GHz ( ? = 12.244 cm). Zia and Zid are the the input impedance values measured when the sensor is in free air and when it is placed upon the dielectric slab, respectively; |dZi| = |Zid -Zia|; dRi = Real?Zid-Zia?; dXi = Imag?Zid-Zia?; d|?| and d? are the differences in the module and in the argument of the reflection coefficient ?, respectively. On the basis of table I, in Fig. 11 the graph of |dZi| vs. L is shown. Fig. 12 and 13 show the correspondent graphs of dRi and dXi.

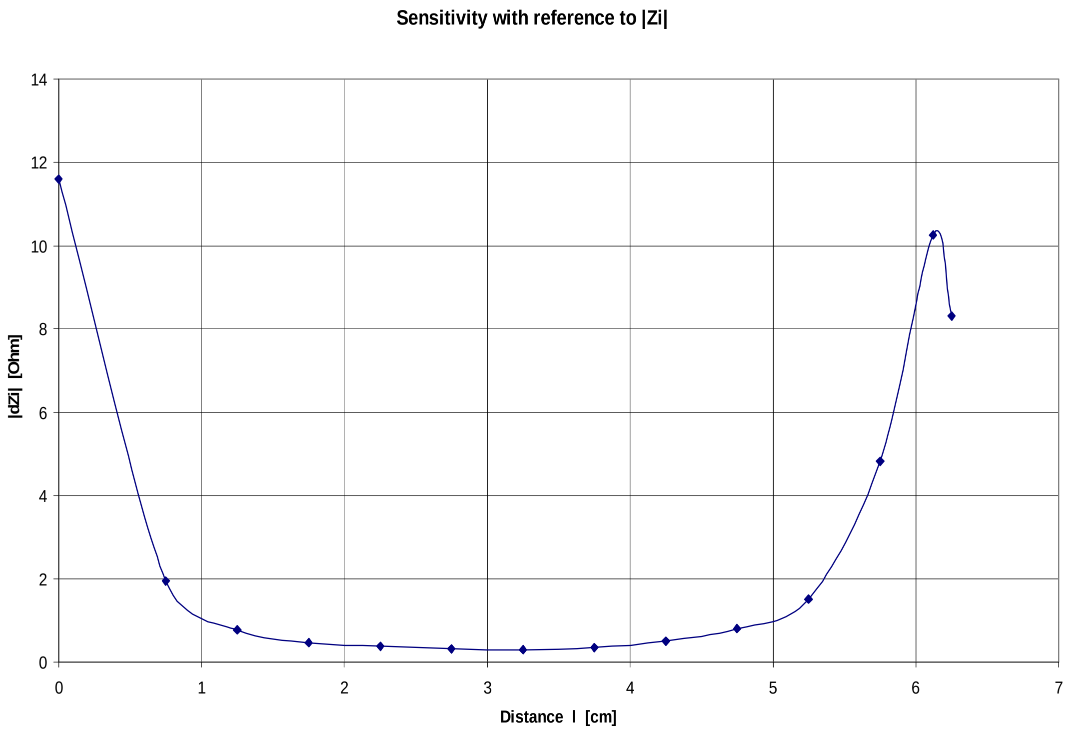



Fig. 11 – The measured values of the variation in the input impedance Zi vs. the equivalent free space distance L between the reference plane and the capacitive sensor input plane

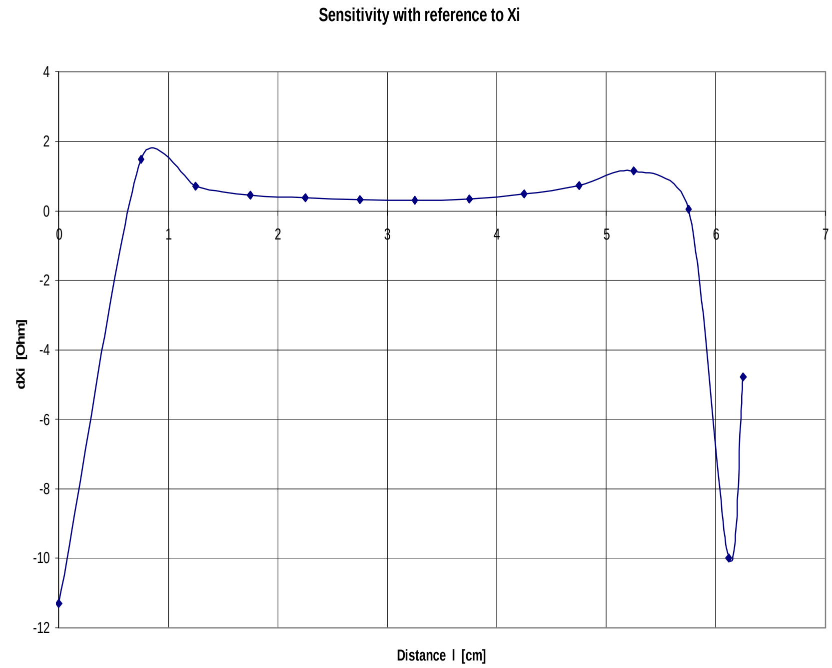



Fig. 12 – The measured values of the variation in t he imaginary part Xi of the input impedance Zi vs. the equivalent free space distance L between the reference plane and the capacitive sensor input plane

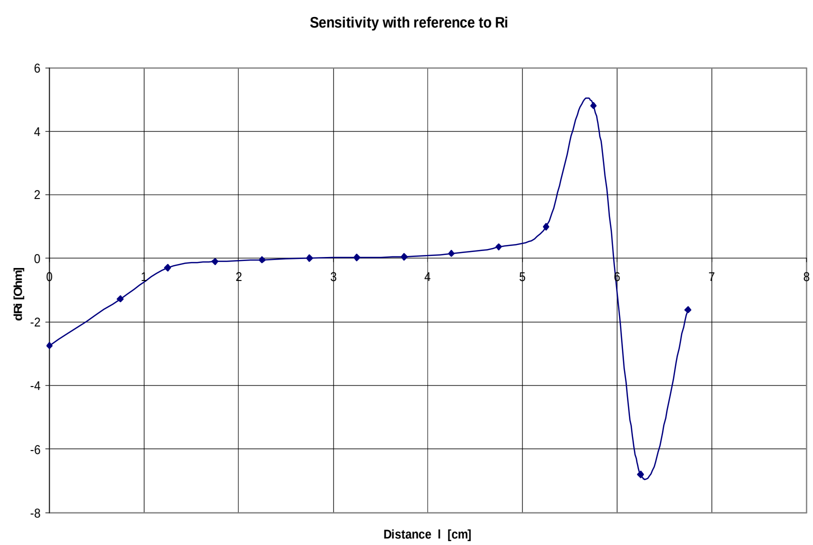



Fig. 13 – The measured values of the variation in  the real part Ri of the input impedance Zi vs. the equivalent free space distance L between the reference plane and the capacitive sensor input plane

## 4 – Discussion and conclusions

The experimental results reported in Figs . 11, 12, 13 are relative to conditions with Rs? ? Xc and with Rs/Zo ? 6 ( see first row of Table I). A comparison of such results with the results of the theoretical analysis reported in Figs. 4, 5, 6, relative to similar conditions, are quite satisfactory. Therefore we can conclude that the sensitivity which can be obtained in the use of a capacitive sensor for NDT applications can be greatly improved if the measured variable is relative to opportunely located reference planes.
It was also proved the conve nience that the measured variable is the magnitude of the input impedance or its real or imaginary part.
Both analytical computations as experimental results confirmed that, in the case where the assumed measured variable is the reflection coefficient, the sensitivity is indipendent from the location of the reference plane, and that its value is far lower than the value obtained in the previous case (see the last two columns of Table I).
Obviously, by assuming as measured variable the magnitude of the refle ction coefficient, the structure of the measuring system can be greatly simplified. Therefore such solution may be of interest whenever high sensitivity is not of paramount importance

## References

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