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Multi-dimensional multi-option opinion dynamics leads to the emergence of clusters in social networks

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ABSTRACT

In real-world social networks, opinions evolve within a multidimensional space of multiple topics being concurrently discussed, and a multi-option decision-making process, rather than a simple binary choice, takes place. Our work introduces a multi-dimensional multi-option opinion dynamics model capturing the complexity of opinion evolution in social networks. The model exploits the coupling of inner opinion and outward action, emphasizing how similar actions strengthen interactions between agents. Unlike existing research, in which consensus, clustering or polarization result from specific network structures, we find that different attitude patterns towards neighbours lead to the spontaneous emergence of such macroscopic phenomena, which are therefore *independent* of network structural features. We provide analytical conditions for the transitions to these behaviours, confirming them via simulations on different networks. Thus, our model allows one to explain the emergence of collective phenomena observed in real-world situations, thereby providing insights in areas such as opinion guidance and multi-agent decision-making.

1. Introduction

In social settings, people may hold different attitudes or preferences on a wide range of topics. Such opinions not only significantly influence the behaviour of individuals, but they also shape their patterns of mutual interaction. As interpersonal communication, information dissemination, and interplay between information and individuals occur, opinions themselves evolve continuously. Understanding this evolution is crucial to analyse the formation of global trends and the rise of social movements, and to develop effective control strategies and policies [1]. Note that, as the final effects of such processes are mesoscopic or macroscopic, studying how shifts in individual opinions affect collective actions is more significant than focussing merely on observing the microscopic dynamics of the single agents. As a result, models of opinion dynamics have long been studied within quantitative sociology and complex system science. Many such models are 1-dimensional, and they simplify the preferences considered into a binary form, so that individuals are assumed to hold one between two mutually exclusive opinions, as in the classic voter model [2]. Thus, in this framework, a person could be considered for example as subscribing to either left-wing or right-wing politics, or as exclusively liking either classical music or pop music. While such simplifications may result in more treatable mathematical expressions, which, in turn, may produce more general results [3], they also come at the cost of a large loss in realism. Thus, two general types of modifications of opinion-formation models have been considered, driven by two corresponding considerations.

The first one is the recognition that, even within a single given generic area, opinions and beliefs are usually multidimensional. For instance, in politics, one can hold separate opinions about preferred social and economic policies, with varying degrees of correlation between them. This resulted in the development of models, measures and

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mechanisms of opinion dynamics in more than 1 dimension, such as the Axelrod model [4], the multidimensional bounded confidence [5], and works accounting for cognitive balance while studying the formation of multidimensional consensus [6,7].

The second realization is that the number of possible alternatives within any single topic is rarely, if ever, just 2. This exposed the need of multi-option models, in which the individuals may express different level of support for different choices relating to the same topic. For example, regarding fiscal matters, one may have different preferences for proportional, progressive and regressive tax schemes. Similarly, one could express varying levels of personal affinity towards ideologies across the political spectrum, including far-left, left, moderate left, centre, moderate right, right and far-right parties. Alternatively, given some specific consumer good, an individual may have different opinions about the possible product features and their combinations. This understanding led to the development of models in which the opinion space can be discrete or continuous [8–12].

These two extra levels of realism have also been combined, producing models and theories that are both multidimensional and multi-option [13–17]. At the same time, it became clear that, while the space of each person's opinion on any given subject is naturally continuous, in situations where one must eventually express a specific preference, the space of the final choices is most often discrete. Returning to the previous example of the political spectrum, even in a 1-dimensional model, one may have a distribution of preferences whose maximum is between the (arbitrarily chosen) points corresponding to "centre" and "moderate right". In this case, when they have to express their opinion by voting, they will have to operate some kind of inner compromise, and they will most likely end up choosing one of the two parties that are closest to their ideal preferred choice. After this idea of a distinction between opinions and final actions was first explored [18], a number of static and adaptive models were introduced [19–25]. However, in contrast to the extensive research existing on 1-dimensional opinion dynamics, the attention paid to multidimensional multi-option models with adaptive relationship between opinions and actions has been limited.

In this article, we propose a multidimensional multi-option networked opinion-dynamics model with coupling between opinions and actions. This coupling, and the rules for the evolution of the system, are chosen to be as simple as reasonably possible, while at the same time reflecting and incorporating the main mechanisms of real-world social interactions. Using analytical arguments and numerical simulations, we show that the model features steady states corresponding to a general consensus and to the polarization of opinions, and we study the transition between them. Also, we demonstrate that the behaviour of the system is independent of the specific network structure, in contrast to existing models, in which the final outcome can be predicted by tuning the network parameters.

2. Results

2.1. The model

We consider a network of N agents with adjacency matrix $\mathbf{A}$, so that $A_{i,j} = 1$ if there is a link between agent i and agent j, and $A_{i,j} = 0$ otherwise. The agents have opinions on R topics. To avoid confusion, in the following we use Greek indices to refer to specific topics, and Latin ones to refer to individual agents and to possible choices within topics. Given a topic α, we indicate the number of its available options with M_α. Note that, in general, each topic may offer a different number of choices, so that, given two topics α and β, M_α and M_β are not necessarily equal. Also, we call M the total number of options across topics, so that by definition $M = \sum_{\alpha=1}^{R} M_\alpha$.

The opinion of each agent about any given option of a topic is allowed to take a continuum of values, with negative ones indicating opposition and positive ones indicating support, and with the magnitude being proportional to the strength of the opinion held. Then, the state of the system at any given time can be represented as an $M \times N$ matrix $\mathbf{X}$, so that the element $X_{i,j}$ is the opinion of agent j on option i. Note that this also means that the ith column of $\mathbf{X}$ can be thought of as an M-dimensional vector $\mathbf{x_i}$ that encodes the full opinion state of the agent i. To indicate the opinion state of agent i restricted to topic α, we use the notation $\mathbf{x_i^{(\alpha)}}$. Also, no constraints are imposed on the elements of $\mathbf{X}$, allowing for the representation of conflicted agents, who have a positive opinion about more than one option of some topic. The action state of agent i on topic α is described by an M_α-dimensional vector $\mathbf{y_i^{(\alpha)}}$. Given the fact that, upon acting, agents operate a choice in favour of one option, necessarily excluding all others, all the components of each action-state vector are 0, except the one that corresponds to the choice operated, which is 1.

We further assume that if two agents are connected in the network, then they always exert a mutual influence, whose strength has a minimum value of 1 and can increase depending on the degree of similarity between them. However, we also consider that, in principle, one cannot know with certainty the full extent of someone else's set of opinions. Instead, for all practical purposes, each individual infers the beliefs of other people by observing their behaviour. Thus, in our model, we make the strength of the influence between two connected nodes depend on the similarity of their actions, rather than that of their opinions. Specifically, to the minimum strength due to the mere existence of a link between the two agents, we add a term that is equal to the average number of topics for which they operate the same choice. This yields a weighted adjacency matrix $\mathbf{W}$, such that the element $W_{i,j}$ is the strength of the influence that agents i and j have on each other, and which can be expressed as

$$W_{i,j} = A_{i,j} \left(1 + \frac{1}{R} \sum_{\alpha=1}^{R} \sum_{l=1}^{M_\alpha} \delta_{y_{i\,l}^{(\alpha)}, y_{j\,l}^{(\alpha)}} \right), \tag{1}$$

where $\delta_{x,y}$ is Kronecker's symbol, defined to be 1 if $x = y$ and 0 otherwise. Note that, while the underlying structure of the network, given by $\mathbf{A}$, is fixed, the weights of the links between agents, given by $\mathbf{W}$, change in time.

To model the evolution of the opinions of the agents, we start from a number of considerations.

- First, we assume that if an agent were completely isolated, then the strength of their opinions would slowly decrease, eventually reaching a state in which the agent is neutral about all options for any given topic, having an opinion whose magnitude is close to 0 for all of them.
- Next, we account for the fact that, in realistic situations, all options of a topic are correlated. For example, a person who holds left-wing political ideas will be more likely to positively regard moderate-left ideologies than right-wing ones. Thus, for each topic α, we introduce an opposition matrix $\boldsymbol{Y}^{(\alpha)}$, such that the element $Y_{i,j}^{(\alpha)}$ is a measure of ideological distance between options i and j of topic α.
- Concerning the interactions between agents, whose strengths are given by Eq. (1), we want to be able to model both positive and negative attitudes.
- Additionally, we want correlations and external influences to be bounded. For this, we map their compound effects via a saturation function $S(z)$, which we choose to be continuous, odd, and to have derivative 1 at 0. Also, given a finite domain $[-Z, Z]$, we impose that $S(z) = S(-Z)$ for all $z < -Z$ and $S(z) = S(Z)$ for all $z > Z$.
- Finally, we want to keep the mean of the opinion distribution of each agent constant on each individual topic.

With these choices, we can formalize the opinion dynamics for each topic as follows. First, we gather the internal correlations and the external influences in a vector $\mathbf{H_i^{(\alpha)}}$; then, we include the internal relaxation dynamics, obtaining a vector $\boldsymbol{\Phi}_{\mathbf{i}}^{(\alpha)}$; finally, we correct the expression for the evolution of each $\mathbf{x}_{\mathbf{i}}^{(\alpha)}$ by subtracting the average change resulting from the internal and the external dynamics. In formulae:

$$\begin{aligned} \mathbf{H}_{\mathbf{i}}^{(\alpha)} &= \beta \left[-\boldsymbol{\Upsilon}^{(\alpha)} \mathbf{x}_{\mathbf{i}}^{(\alpha)} \pm \left(\mathbf{I} - \boldsymbol{\Upsilon}^{(\alpha)}\right) \sum_{j=1}^{N} W_{i,j} \mathbf{x}_{\mathbf{j}}^{(\alpha)} \right], \\ \Phi_{i\,l}^{(\alpha)} &= -x_{i\,l}^{(\alpha)} + S(H_{i\,l}^{(\alpha)}), \\ \Delta x_{i\,l}^{(\alpha)} &= \Phi_{i\,l}^{(\alpha)} - \frac{1}{M_\alpha} \sum_{k=1}^{M_\alpha} \Phi_{i\,k}^{(\alpha)}. \end{aligned} \tag{2}$$

In the expressions above, β is a positive parameter that reflects the overall strength of correlations and influences, $\mathbf{I}$ is the identity matrix and the sign within the equation for $\mathbf{H}_{\mathbf{i}}^{(\alpha)}$ determines the type of dynamics. Specifically, choosing the positive sign results in a dynamics in which neighbouring agents behave in an accommodating or cooperative way with each other, moving towards the middle point of their respective opinions. The opposite choice of a negative sign causes the agents, on the balance, to shift their own opinion distribution towards giving more weight to options that are in ideological opposition to those of their neighbours.

Next, we define a rule for each agent to choose one of the possible options for each topic at each time step. Our rule resembles the classic public goods game, in which participants pay a generically different cost and share the resulting benefits equally. In our case, we consider that the cost an agent has to pay in order to choose a given option is a maximum of 1 and is exponentially inversely proportional to the strength of their opinion about that option. So, given a topic α and an option l, the cost for agent i to choose it is

$$C_i^{(\alpha)}(l) = 1 - \frac{e^{x_{i\,l}^{(\alpha)}}}{\sum_{k=1}^{M_\alpha} e^{x_{i\,k}^{(\alpha)}}}. \tag{3}$$

However, rather than representing a monetary cost, in our model this quantity is more akin to an emotional price, similar to an effort one has to put in order to go against one's own convictions. Then, for choosing an option, each agent will perceive a shared reward that depends on the cost invested by those amongst their neighbours who have chosen the same option. This simulates the effect of peer pressure, and, accounting for the agent's own effort, it results in a contribution that can be written as $\frac{1}{d_i+1}\left(C_i^{(\alpha)}(l) + \sum_{j=1}^{N} A_{i,j} C_j^{(\alpha)}(l) y_{j\,l}^{(\alpha)}\right)$, where d_i is the degree of node i, i.e., the number of neighbours of agent i. To perform the actual choice, the agents try to maximize their total payoff, which we express as the sum of three terms, namely the contribution just described, the remaining emotional strength, and a third term representing a bonus that the agent receives that is proportional to the strength of their opinion about the option considered. Since the maximum possible investment is 1, the second term is $1 - C_i^{(\alpha)}(l)$. Thus, the total payoff is

$$P_i^{(\alpha)}(l) = \frac{1}{d_i+1}\left(C_i^{(\alpha)}(l) + \sum_{j=1}^{N} A_{i,j} C_j^{(\alpha)}(l) y_{j\,l}^{(\alpha)}\right) + \left(1 - C_i^{(\alpha)}(l)\right) + x_{i\,l}^{(\alpha)}. \tag{4}$$

Then, at each time step, each agent tries to maximize the expression above for each topic, assuming that all the neighbours will repeat the same choice they operated in the last round. So, for agent i and topic α, the action state vector at time $t+1$ is

$$y_{i\,l}^{(\alpha)} = \begin{cases} 1 & \text{if } l = l^* \\ 0 & \text{otherwise,} \end{cases} \tag{5}$$

where $l^* = \operatorname{argmax}_l P_i^{(\alpha)}(l)$ and, in the expression for $P_i^{(\alpha)}$, the elements of the action vectors $\mathbf{y}_{\mathbf{j}}^{(\alpha)}$ are taken at time t.

2.2. Bifurcations of opinion states

We are interested in studying how the opinions of the agents develop in our model, exploring the conditions under which consensus or polarization appear as steady states of the dynamics. To do so, we first prove that the model is well-posed, by showing that the opinion states, represented by the matrix $\mathbf{X}$, are bounded.

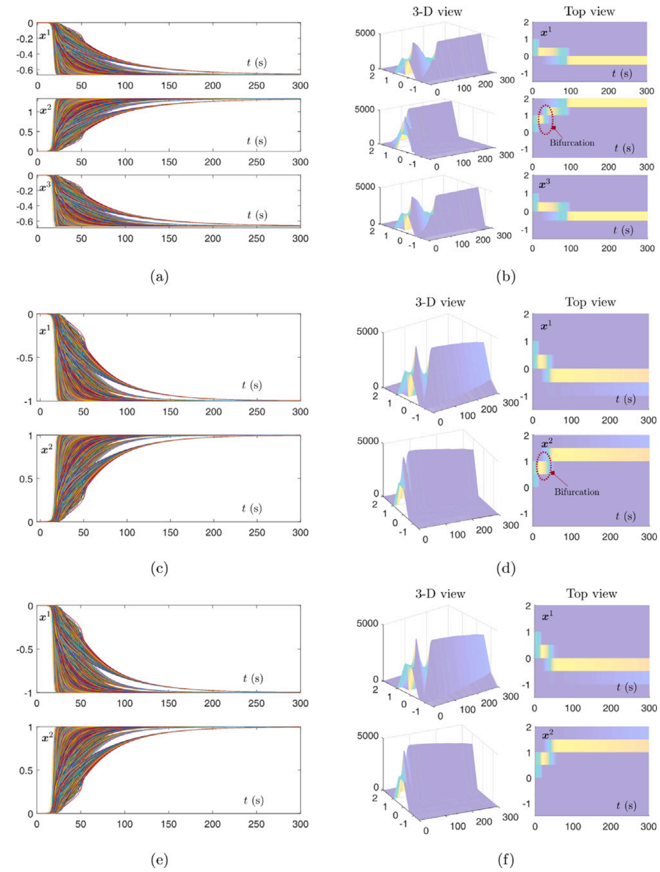

Fig. 1. Evolution of consensus on scale-free networks for cooperative dynamics. At steady state, the opinions of all agents on the three topics, shown in panels (a), (c) and (e), reach a consensus, supporting option 2 for all topics, and opposing option 1 and option 3 (when available). The distributions of the opinions, shown in panels (b), (d) and (f), illustrate more clearly how, after β crosses the threshold β_2, shifts start occurring, eventually leading the opinion distributions towards their final state.

Theorem 1. *Let S be a continuous, odd function S with a symmetric finite domain $[-Z, Z]$, such that $S(0) = 0$, $S'(0) = 1$, $S(z) = S(-Z)$ for all $z < -Z$ and $S(z) = S(Z)$ for all $z > Z$. Then, with the dynamics described in the previous section, all the elements $X_{i,j}$ of the state matrix $\mathbf{X}$ are bounded at all times $t > 0$.*

Next, we characterize the presence of bifurcation points of the dynamics:

Theorem 2. *If all the topics have a single option, the dynamics described in the previous section always results in a consensus state. If, instead, the topics have multiple options, two different general regimes are found. Specifically:*

- *When the agents evolve in a competition regime, the neutral state, in which all the agents are neutral about all options, is stable when $\beta < \beta_1$, where $\beta_1 = \frac{1}{1-2\min_{\lambda_i}\{\Re(\lambda_i)\}}$ and λ_i are the eigenvalues of the weighted adjacency matrix $\mathbf{W}$. The point at β_1 is a bifurcation point, so that, as β increases and crosses it, different branches of polarization emerge in the system.*
- *When the agents evolve in a cooperation regime, the neutral state is stable when $\beta < \beta_2$, where $\beta_2 = \frac{1}{1+2\max_{\lambda_i}\{\Re(\lambda_i)\}}$. The point at β_2 is a bifurcation point, so that, as β increases and crosses it, different branches of consensus emerge in the system.*

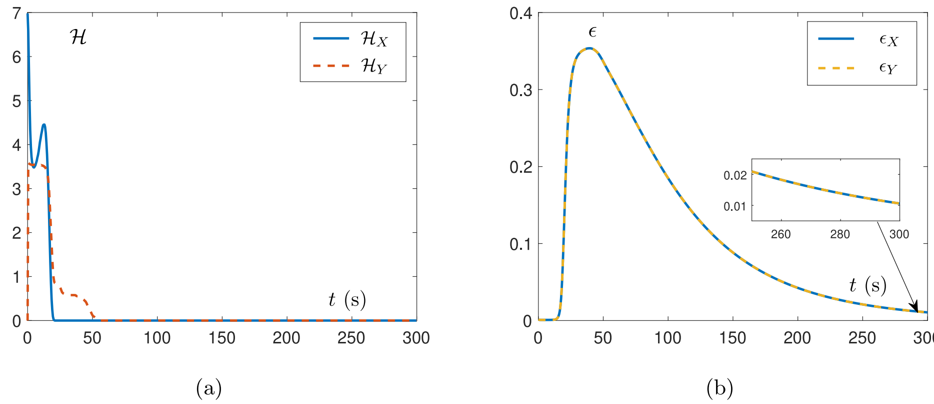

Fig. 2. Diversity and consensus for cooperative dynamics on scale-free networks evolve towards a uniform state. (a) The Shannon information entropy of the set of opinion clusters, $\mathcal{H}_X$, and that of the set of action clusters, $\mathcal{H}_Y$, converge to 0 as the system evolve, indicating the formation of a single cluster in both cases. (b) The consensus errors of opinion clusters, ϵ_X, and of action clusters, ϵ_Y, are indistinguishable from each other, and converge to a value smaller than 0.05, indicating that the uniformity of opinion drives the uniformity of action, and that the system evolves towards a single generalized consensus state.

Detailed proofs of both theorems are given in the Supplementary Material.

To verify these theoretical predictions, we carried out numerical simulations on scale-free networks with $N = 5000$ agents interacting over $R = 3$ topics. Topic 1 had 3 options, whereas topics 2 and 3 had 2 options each, for a total of $M = 7$ options. The initial opinion state was extracted from a Gaussian distribution with 0 mean and a variance of 5×10^{-4}, yielding a neutral state with some small stochastic noise. The simulations were run up to a final time $T = 300$, using tanh as the saturation function. Additionally, when studying the cooperative dynamics, we let β change with time t according to $\beta = \beta_2 + 0.1\frac{t}{T} - 0.05$, whereas when studying the competition dynamics, we used $\beta = \beta_1 + 0.1\frac{t}{T} - 0.05$.

The results, shown in Fig. 1 for the cooperation dynamics, confirm the analytical predictions. The states of the opinions of all the agents for the three topics and their respective choices eventually converge towards consensus, as shown in panels (a), (c) and (e). Also, at intermediate times, corresponding to intermediate values of β, the distributions of opinions for each individual option, shown in panels (b), (d) and (f), undergo continuous transitions, shifting from one mean value to another, after β has crossed the bifurcation point identified in Theorem 2.

To provide further quantitative insights into the dynamics observed, we grouped the agents into different types of clusters, and studied their evolution in time. Specifically, we define an opinion cluster to be the set of all agents who share the same qualitative opinion states. In other words, a cluster consists of all the agents who support or oppose the same options for all topics, regardless of the strengths of their preferences. Similarly, we define an action cluster to be the set of all agents who choose the same options for all topics. Then, we measured the evolution of the Shannon information entropy $\mathcal{H}$ and of the consensus errors ϵ for the set of clusters of both kinds. We recall here that, given a partition of the network into a set of N_c clusters with sizes $S_1, S_2, \dots, S_{N_c}$, the Shannon information entropy, measured in bits, is given by

$$\mathcal{H} = -\sum_{i=1}^{N_c} \frac{S_i}{N} \log_2 \frac{S_i}{N} . \tag{6}$$

Also, we define the consensus error of a cluster C as

$$\epsilon = \max_{i,j \in C} \left\| \mathbf{x_i} - \mathbf{x_j} \right\| . \tag{7}$$

Note that in the definition above, C may be equivalently an opinion cluster or an action cluster. The analysis shows that the cluster diversity as measured by the Shannon information entropy vanishes relatively quickly as the system evolves, both for opinion-based clusters

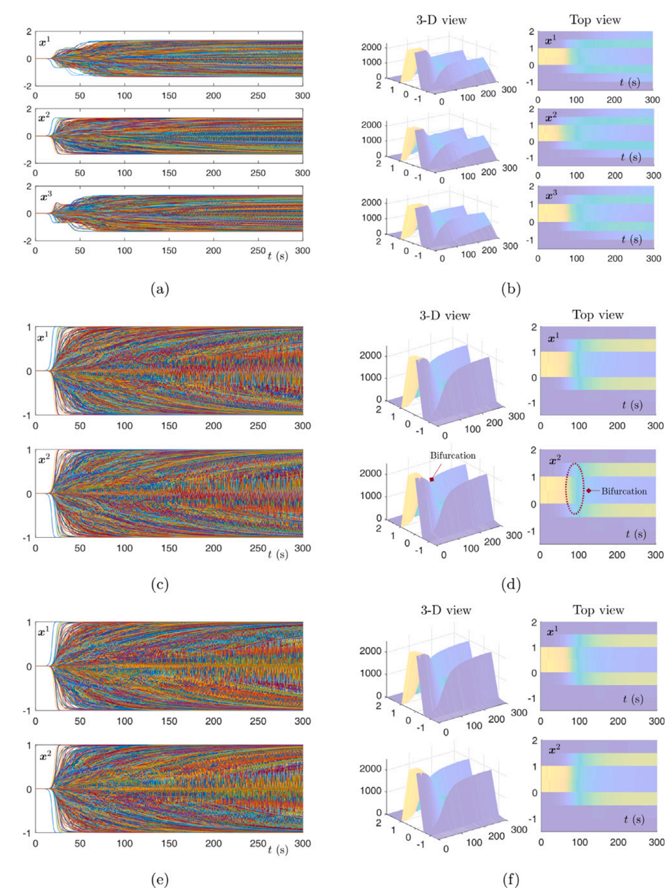

Fig. 3. Evolution of polarization on scale-free networks for competitive dynamics. At steady state, the opinions of the agents on the three topics, shown in panels (a), (c) and (e), are spread across a range, from strong opposition to strong support. The distributions of the opinions, shown in panels (b), (d) and (f), illustrate more clearly that, after β crosses the threshold β_1, the opinions start shifting, eventually reaching a bimodal distribution, with peaks in correspondence to values of opposite signs.

and for action-based ones, as illustrated in Fig. 2(a). This suggests that the consensus of opinions emerging from the network evolution promotes homogeneity of action, a consideration that is confirmed by the evolution of the consensus errors, shown in Fig. 2(b). In fact, the consensus errors for both types of clusters are indistinguishable, and they eventually reach values smaller than 0.05, allowing us to conclude

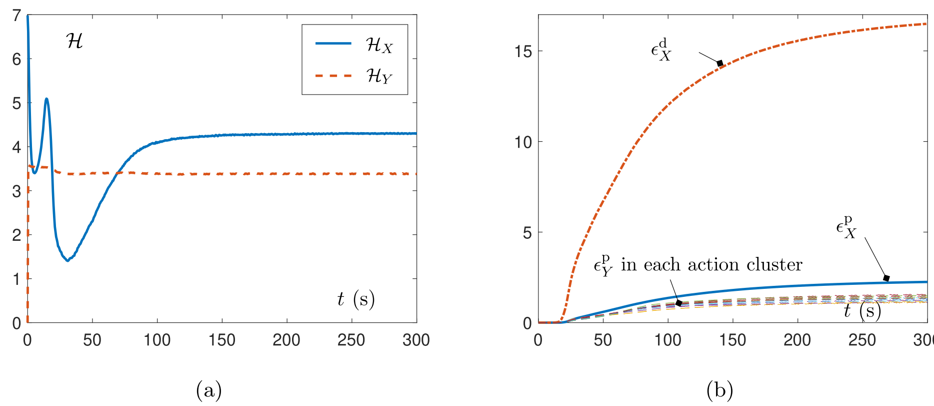

Fig. 4. Diversity and consensus for competitive dynamics on scale-free networks evolve towards a polarized state. (a) The Shannon information entropy of the set of action clusters, $\mathcal{H}_Y$, converges to its final state much faster than that of the set of opinion clusters, $\mathcal{H}_X$, consistently with the definition of action dynamics and opinion dynamics, and with the underlying inspiring real-world concepts. (b) The polarization indices for opinion clusters, ϵ^{p}_X, and for action clusters, ϵ^{p}_Y, converge to non-zero values; similarly, the disagreement index computed over the whole network, ϵ^{d}_X, reaches very large values, indicating a general strong conflict of opinions between neighbours.

that all agents form a single cluster with uniformity of opinion and of action.

For the systems evolving under competitive dynamics, we observe similar findings, but of an opposite character. There, the spontaneous formation of clusters with diverging opinions occurs after crossing the bifurcation point β_1. Then, for each option, a subset of agents emerges who support it, while the rest oppose it, leading to the polarization illustrated in Fig. 3(a). The competitive dynamics is such that for each agent it promotes options that are ideologically distant from those mostly supported by their neighbours. In this sense, the network edges act as channels over which conflicting opinions of agents meet, resulting in a higher degree of disagreement and polarization with respect to the cooperative dynamics. To better quantify this effect, we introduced and studied the polarization index ϵ^{p} and the disagreement index ϵ^{d}, drawing inspiration from similar indices used to study Fredkin–Johnsen dynamics [26]. Given a cluster C, these indices measure the extent to which opinions deviate from the cluster average $\bar{\mathbf{x}}$ and the difference of opinions between neighbours, respectively, and they are defined as

$$\epsilon^{\mathrm{p}} = \sum_{i\in C} \left\| \mathbf{x_i} - \bar{\mathbf{x}} \right\| \tag{8}$$

and

$$\epsilon^{\mathrm{d}} = \sum_{i,j\in C} A_{i,j} \left\| \mathbf{x_i} - \mathbf{x_j} \right\| . \tag{9}$$

As in the case of the consensus error, also in the two equations above, the cluster may be an opinion cluster or an action cluster. The analysis of these two quantities confirms our considerations, revealing a non-negligible spread of opinions within clusters, and, especially, a very large difference of opinion state between neighbours (Fig. 4). Additionally, the Shannon information entropy for both types of clusters shows that, while the system rapidly converges to its final distribution of actions, the opinion clusters keep changing, until they converge at approximately $t = 150$. This is consistent with our definition of the dynamics of opinions and actions, and with the fact that one's own opinions can keep changing, and sometimes even significantly so, but still result in the same choice of action.

Simulations carried out on Watts–Strogatz networks, and discussed in the Supplementary Material, show the same behaviour observed on scale-free networks for both cooperative and competitive dynamics. Summarizing, these results indicate that opinions do not remain in a neutral state for a long time, but rather they evolve either towards consensus or towards polarization, depending on the balance of cooperative influences and competitive ones.

3. Discussion

The results presented here demonstrate the diversity of steady states that emerge in a dynamical social network when one accounts for multiple correlated topics of discussion, as well as for the interplay between the personal opinion dynamics of each agent and the outward actions that they take on each topic, which are chosen from a set of multiple possible options. These ingredients allow us to reproduce the key features of real-world opinion formation, leading to final states including consensus, polarization and clustering. This is due at least partly to the fact that, regardless of the positive or negative feeling an agent has towards their neighbour's opinion, similarity of action leads to a strengthening of the communication channel between them. Also, importantly, an agent's actions need not always correspond to the option for which they hold the most positive opinion, as external and internal influences may affect their final decision. This is reminiscent of mechanisms and dynamics that are regularly observed in real-world situations, such as peer pressure, cognitive dissonance, or tactical voting. As a result, agents may tend to align their opinions and actions with those of their neighbours, effectively reaching a cooperative state, or maintain and promote differences, favouring the clash between different opinions, leading eventually to competitive state. These findings are independent from the type of network, in a substantial difference with prior models, which instead exhibit different final states for different network structures. Also, they imply that neutral states are not stable in realistic situations, and a collective behaviour of consensus or disagreement will spontaneously emerge from them. From a theoretical point of view, we established the rigorous conditions to observe transitions between these different macroscopic states, showing the presence of a bifurcation point of the dynamics, whose type depends on the relative importance of positive and negative external influences. Finally, due to its ability to reproduce real-world patterns of behaviour and its amenability to analytical treatment, we believe that our model will find use in studies of specific social systems, such as evolving political landscapes, where the relatedness of topics and the possible necessity of compromise choices play a fundamental role.

CRediT authorship contribution statement

Yimeng Qi: Conceptualization, Investigation, Methodology, Writing – original draft, Writing – review & editing. **Songlin Zhuang:** Conceptualization, Funding acquisition, Investigation, Writing – original draft, Writing – review & editing. **Xinghu Yu:** Funding acquisition, Investigation, Writing – original draft, Writing – review & editing. **Zhihong Zhao:** Investigation, Writing – original draft, Writing – review & editing. **Weichao Sun:** Methodology, Writing – original draft, Writing – review & editing. **Zhan Li:** Methodology, Writing – original draft, Writing – review & editing. **Jianbin Qiu:** Methodology, Writing – original draft, Writing – review & editing. **Yang Shi:** Investigation, Writing – original draft, Writing – review & editing. **Fangzhou Liu:** Conceptualization, Funding acquisition, Methodology, Writing – original draft, Writing – review & editing. **Charo I. del Genio:** Conceptualization, Writing – original draft, Writing – review & editing, Funding acquisition. **Stefano Boccaletti:** Conceptualization, Funding acquisition, Writing – original draft, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.chaos.2024.115983.

Data availability

Raw simulation data (Original data) (github)

References

[1] Hassani H, Razavi-Far R, Saif M, Chiclana F, Krejcar O, Herrera-Viedma E. Classical dynamic consensus and opinion dynamics models: A survey of recent trends and methodologies. Inf Fusion 2022;88:22–40.

[2] Holley RA, Liggett TM. Ergodic theorems for weakly interacting infinite systems and the voter model. Ann Probab 1975;3:643–63.

[3] Jin C, Yin C, Jin X, Min Y, Li Y, Chen N, et al. Group-based rewiring rules of binary opinion competition dynamics. Sci Rep 2018;8:14423.

[4] Axelrod R. The dissemination of culture: A model with local convergence and global polarization. J Conflict Resolut 1997;41:203–26.

[5] Huet S, Deffuant G. Openness leads to opinion stability and narrowness to volatility. Adv Complex Syst 2010;13:405–23.

[6] Schweighofer S, Garcia D, Schweitzer F. An agent-based model of multidimensional opinion dynamics and opinion alignment. Chaos 2020;30:093139.

[7] Banisch S, Olbrich E. An argument communication model of polarization and ideological alignment. J ASSS – J Artif Soc S 2021;24:1.

[8] DeGroot MH. Reaching a consensus. J Amer Statist Assoc 1974;69:118–21.

[9] Friedkin NE, Johnsen EC. J Math Sociol 1990;15:193–206.

[10] Deffuant G, Neau D, Amblard F, Weisbuch G. Mixing beliefs among interacting agents. Adv Complex Syst 2000;3:87–98.

[11] Hegselmann R, Krause U. Opinion dynamics and bounded confidence: models, analysis and simulation. J ASSS – J Artif Soc S 2002;5:2.

[12] Liu J, Huang S, Aden NM, Johnson NF, Song C. Emergence of polarization in coevolving networks. Phys Rev Lett 2023;130:037401.

[13] Battiston F, Nicosia V, Latora V, San Miguel M. Layered social influence promotes multiculturality in the axelrod model. Sci Rep 2017;7:1809.

[14] Ye M, Liu J, Anderson BDO, Yu C, Basar T. Evolution of social power in social networks with dynamic topology. IEEE Trans Autom Control 2018;63:3793–808.

[15] Franci A, Golubitsky M, Bizyaeva A, Leonard NE. https://arxiv.org/abs/1909.05765 (2019).

[16] Baumann F, Lorenz-Spreen P, Sokolov IM, Starnini M. Emergence of polarized ideological opinions in multidimensional topic spaces. Phys Rev X 2021;11:011012.

[17] Franci A, Bizyaeva A, Park S, Leonard NE. Analysis and control of agreement and disagreement opinion cascades. Swarm Intell 2021;15:47–82.

[18] Martins AC. Continuous opinions and discrete actions in opinion dynamics problems. Internat J Modern Phys C 2008;19:617–24.

[19] Fan K, Pedrycz W. Opinion evolution influenced by informed agents. Phys A 2016;462:431–41.

[20] Leshem A, Scaglione A. The impact of random actions on opinion dynamics. IEEE Trans Signal Inf Process Netw 2018;4:576–84.

[21] Gaisbauer F, Olbrich E, Banisch S. Dynamics of opinion expression. Phys Rev E 2020;102:042303.

[22] Zino L, Ye M, Cao M. A two-layer model for coevolving opinion dynamics and collective decision-making in complex social systems. Chaos 2020;30:083107.

[23] Zhan M, Kou G, Dong Y, Chiclana F, Herrera-Viedma E. Bounded confidence evolution of opinions and actions in social networks. IEEE Trans Cybern 2021;52:7017–28.

[24] Aghbolagh HD, Ye M, Zino L, Chen Z, Cao M. Coevolutionary dynamics of actions and opinions in social networks. IEEE Trans Automat Control 2023;68:7708–23.

[25] Zhan M, Su X, Zha Q, Xu X, Chen X. Opinion and action interactive evolution based on social network leadership and opinion estimation of action. IEEE Trans Syst Man Cy. – S 2024;54:1037–48.

[26] Chen X, Lijffijt J, De Bie T. Quantifying and minimizing risk of conflict in social networks. In: Proc. ACM SIGKDD int. conf. knowl. disc. data min. 2018, p. 1197.