

HCEQ2025-154**Productivity as an indicator of the cascade property of aftershocks**Molchan George² **Peresan Antonella**^{1,*} Varini Elisa³

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The widely used Epidemic Type Seismicity Model (ETAS) describes the clustering of seismic events as an epidemic type process (A-property), assuming that the productivity F1 distribution of the number of direct aftershocks is Poissonian (B-property). Both properties are non-obvious and require justification.

The real data favor the geometric distribution F1 (e.g. Shebalin et al., 2018, Dokl. Akad. Nauk, 481, 963–966). The F2 distribution of the number of cluster events with main shock m_0 and magnitudes greater than $m_0 - \Delta$ is also often attributed to geometric type. However, the coincidence of distribution types F1 and F2 turns out to be in contradiction with the cascade A-property (Molchan et al., 2022, GJI, 231,1545-1557), and the geometric type F1 is in contradiction with the Poissonian B-property. This study, which is analyzing and resolving the mentioned contradictions, develops in the following three steps.

Step 1 - Generalization of the ETAS model, designed to use any F1 distribution type, and selection of a very broad special class of F1 distributions, including both Poisson and geometric distributions (Molchan and Varini, 2024, GJI, 236,1440-1454).

Step 2 - Consider, instead of the F2 distribution, its more natural analog F2a. It refers to cluster events with magnitude greater than $m_a - \Delta_a$, where m_a is the mode in the theoretical distribution of the strongest aftershock with main shock m_0 . Under the conditions of Bath's law, namely $m_a = m_0 - 1.2$, both distributions coincide if $\Delta_a = \Delta - 1.2$. We find the limiting distribution F2a at $m_0 \gg 1$. It is noteworthy that in the subcritical regime the distribution type of this limit coincides with that of F1, and the limiting distribution itself depends only on the relative threshold Δ_a . In practice, this asymptotic result applies to the magnitude range where we expect self-similarity in seismicity, namely for the largest events globally.

Step 3 - Comparison of the limit F2a distributions corresponding to the geometric F1 distribution, with different Δ_a and with b-value $b=1$, with their real counterparts obtained from the ANSS global catalog for large events with magnitude $m_0 > 6$ (Molchan and Peresan, 2024 GJI, 239,314-328).

In the absence of any fitting, we obtained surprisingly good agreement between the empirical and theoretically predicted F2a distributions within the limits of the main values (0-0.95). The tails of the empirical F2a distributions can be ignored because they are unstable, obviously retain a pre-limit character, and are therefore of little information; atypical clusters, possibly of non-cascading nature, fall there. Thus, the cascade A-property of the ETAS model and the consistency of the choice of the geometric distribution F1 in its generalization are confirmed. This justifies the use of such a model to simulate seismicity.

Keywords: Probability distributions; ETAS model; Earthquake interaction, forecasting and prediction