Situated Conditionals - A Brief Introduction

(Extended Abstract)

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Abstract

We extend the expressivity of classical conditional reasoning by introducing *situation* as a new parameter. The enriched conditional logic generalises the defeasible conditional setting in the style of Kraus, Lehmann, and Magidor, and allows for a refined semantics that is able to distinguish, for example, between *expectations* and *counterfactuals*. We introduce the language for the enriched logic and define an appropriate semantic framework for it. We analyse which properties generally associated with conditional reasoning are still satisfied by the new semantic framework, provide a suitable representation result, and define an entailment relation based on Lehmann and Magidor's generally-accepted notion of Rational Closure.

Keywords

Conditional reasoning, non-monotonic reasoning, counterfactual reasoning, defeasible reasoning, belief revision

1. Introduction

Conditionals are at the heart of human everyday reasoning and play an important role in the logical formalisation of reasoning. Two very common interpretations, that are also strongly interconnected, are conditionals representing *expectations* ('If it is a bird, then presumably it flies'), and conditionals representing *counterfactuals* ('If Napoleon had won at Waterloo, all Europe would be speaking French'). The first example above assumes that the premises of conditionals are consistent with what is believed, while the second example assumes that those premises are inconsistent with an agent's beliefs. This poses a formal problem for the classical semantics of conditional reasoning, that we are going to explain in Example 1, but let us introduce some formal preliminaries first. A conference version of this work has been presented at AAAI-21 [1], while an extended technical report is available online [2].

2. Formal background

We assume a finite set of propositional *atoms* $\mathcal{P} = \{p, q, \ldots\}$, while the set of all propositional sentences is $\mathcal{L} = \{\alpha, \beta, \ldots\}$. The set of all valuations (worlds) is denoted $\mathcal{U} = \{u, v, \ldots\}$. Whenever it eases presentation, we represent valuations as sequences of atoms (e.g., p) and barred atoms (e.g., $\bar{\mathsf{p}}$), with the usual understanding. E.g., the valuation $\mathsf{b}\bar{\mathsf{f}}\mathsf{p}$ conveys the idea that b is true, f is false, and p is true. v *satisfies* α is indicated by $v \Vdash \alpha$, while $[\![\alpha]\!] \stackrel{\text{def}}{=} \{v \in \mathcal{U} \mid v \Vdash \alpha\}$ and for $X \subseteq \mathcal{L}$, $[\![X]\!] \stackrel{\text{def}}{=} \bigcap_{\alpha \in X} [\![\alpha]\!]$. $X \models \alpha$ denotes classical propositional entailment. Given a set of valuations V, $\mathsf{fml}(V)$ indicates a formula characterising the set V.

A *defeasible conditional* $\mid\!\sim$ is a binary relation on $\mathcal{L}$. A suitable semantics for rational conditionals is provided by *ranked interpretations*.

Definition 1. *A* ***ranked interpretation*** $\mathscr{R}$ *is a function from* $\mathcal{U}$ *to* $\mathbb{N} \cup \{\infty\}$*, satisfying the following* ***convexity property****: for every* $i \in \mathbb{N}$*, if* $\mathscr{R}(u) = i$*, then, for every* j $0 \leq j < i$*, there is a* $u' \in \mathcal{U}$ *for which* $\mathscr{R}(u') = j$*.*

Figure 1 gives an example of two ranked interpretations. For a given ranked interpretation $\mathscr{R}$ and valuation v, we denote with $\mathscr{R}(v)$ the *rank of* v. The number $\mathscr{R}(v)$ indicates the degree of *atypicality* of v. So the valuations judged most typical are those with rank 0, while those with an infinite rank are judged so atypical as to be implausible. We can therefore partition the set $\mathcal{U}$ w.r.t. $\mathscr{R}$ into the set of *plausible* valuations $\mathcal{U}^f_{\mathscr{R}} \stackrel{\text{def}}{=} \{u \in \mathcal{U} \mid \mathscr{R}(u) \in \mathbb{N}\}$, and *implausible* valuations $\mathcal{U}^\infty_{\mathscr{R}} \stackrel{\text{def}}{=} \mathcal{U} \setminus \mathcal{U}^f_{\mathscr{R}}$.

Let $\mathscr{R}$ be a ranked interpretation and let $\alpha \in \mathcal{L}$. Then $[\![\alpha]\!]^f_{\mathscr{R}} \stackrel{\text{def}}{=} \mathcal{U}^f_{\mathscr{R}} \cap [\![\alpha]\!]$, and $\min [\![\alpha]\!]^f_{\mathscr{R}} \stackrel{\text{def}}{=} \{u \in [\![\alpha]\!]^f_{\mathscr{R}} \mid \mathscr{R}(u) \leq \mathscr{R}(v) \text{ for all } v \in [\![\alpha]\!]^f_{\mathscr{R}}\}$. A defeasible conditional $\alpha \mid\!\sim \beta$ can be given an intuitive semantics in terms of ranked

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interpretations as follows: $\alpha \mid\!\sim \beta$ is *satisfied in* $\mathscr{R}$ (denoted $\mathscr{R} \Vdash \alpha \mid\!\sim \beta$) if $\min[\![\alpha]\!]^f_{\mathscr{R}} \subseteq [\![\beta]\!]$, with $\mathscr{R}$ referred to as a *ranked model* of $\alpha \mid\!\sim \beta$. It is easily verified that $\mathscr{R} \Vdash \neg\alpha \mid\!\sim \bot$ iff $\mathcal{U}^f_{\mathscr{R}} \subseteq [\![\alpha]\!]$. Hence we frequently abbreviate $\neg\alpha \mid\!\sim \bot$ as α.

3. Situated conditionals

Back to our problem, let us present an extended version of the (admittedly over-used) penguin example.

Example 1. *Suppose we know that birds usually fly (*$\mathsf{b} \mid\!\sim \mathsf{f}$*), that penguins are birds (*$\mathsf{p} \to \mathsf{b}$*) that usually do not fly (*$\mathsf{p} \mid\!\sim \neg\mathsf{f}$*). Also, we know that dodos were birds (*$\mathsf{d} \to \mathsf{b}$*) that usually did not fly (*$\mathsf{d} \mid\!\sim \neg\mathsf{f}$*), and that dodos do not exist anymore. Using the standard ranked semantics (Definition 1) we have two ways of modelling this information.*

The first option is to formalise what an agent believes by referring to valuations with rank 0 *in a ranked interpretation. That is, the agent believes* α *is true iff* $\top \mid\!\sim \alpha$ *holds. In such a case,* $\top \mid\!\sim \neg\mathsf{d}$ *means that the agent believes that dodos do not exist. A model for this conditional knowledge base is shown in Figure 1 (left). The main limitation of this representation is that all exceptional entities have the same status as dodos, since they cannot be satisfied at rank* 0*. Hence we have* $\top \mid\!\sim \neg\mathsf{p}$*, just as we have* $\top \mid\!\sim \neg\mathsf{d}$*, and we are not able to distinguish between the status of the dodos (they do not exist anymore) and the status of the penguins (they are simply exceptional birds).*

The second option is to represent what an agent believes in terms of all valuations with finite ranks. That is, an agent believes α *to hold iff* $\neg\alpha \mid\!\sim \bot$ *holds. If dodos do not exist, we add the statement* $\mathsf{d} \mid\!\sim \bot$*. A model for this case is depicted in Figure 1 (right). Here we can distinguish between what is considered false (dodos exist) and what is exceptional (penguins), but we are unable to reason coherently about counterfactuals, since from* $\mathsf{d} \mid\!\sim \bot$ *we can conclude anything about dodos.*

∞	$\mathcal{U} \setminus ([\![0]\!] \cup [\![1]\!] \cup [\![2]\!])$
2	$\mathsf{p}\overline{\mathsf{d}}\mathsf{bf}$, $\overline{\mathsf{p}}\mathsf{dbf}$, pdbf
1	$\overline{\mathsf{p}}\overline{\mathsf{d}}\mathsf{b}\overline{\mathsf{f}}$, $\mathsf{p}\overline{\mathsf{d}}\mathsf{b}\overline{\mathsf{f}}$, $\overline{\mathsf{p}}\mathsf{db}\overline{\mathsf{f}}$, $\mathsf{pdb}\overline{\mathsf{f}}$
0	$\overline{\mathsf{p}}\overline{\mathsf{d}}\mathsf{bf}$, $\overline{\mathsf{p}}\overline{\mathsf{d}}\overline{\mathsf{b}}\mathsf{f}$, $\overline{\mathsf{p}}\overline{\mathsf{d}}\overline{\mathsf{b}}\overline{\mathsf{f}}$

∞	$\mathcal{U} \setminus ([\![0]\!] \cup [\![1]\!] \cup [\![2]\!])$
2	$\mathsf{p}\overline{\mathsf{d}}\mathsf{bf}$
1	$\overline{\mathsf{p}}\overline{\mathsf{d}}\mathsf{b}\overline{\mathsf{f}}$, $\mathsf{p}\overline{\mathsf{d}}\mathsf{b}\overline{\mathsf{f}}$,
0	$\overline{\mathsf{p}}\overline{\mathsf{d}}\mathsf{bf}$, $\overline{\mathsf{p}}\overline{\mathsf{d}}\overline{\mathsf{b}}\mathsf{f}$, $\overline{\mathsf{p}}\overline{\mathsf{d}}\overline{\mathsf{b}}\overline{\mathsf{f}}$

Figure 1: Left: a ranked interpretation of the KB in Example 1 satisfying $\top \mid\!\sim \neg\mathsf{d}$. Right: a ranked interpretation of the KB expanded with $\mathsf{d} \mid\!\sim \bot$.

We introduce a logic of *situated conditionals* to overcome this problem. The central insight is that adding an explicit notion of context to standard conditionals allows for a refined semantics of this enriched language in which the problems described in Example 1 can be dealt with adequately. It also allows us to reason coherently with counterfactual conditionals such as 'Had Mauritius not been colonised, the dodo would not fly'. Moreover, it is possible to reason coherently with situated conditionals without needing to know whether their premises are plausible or counterfactual. In the case of penguins and dodos, for example, it allows us to state that penguins usually do not fly assuming to be in a situation in which penguins existing, and that dodos usually do not fly, assuming dodos exist, while being unaware of whether or not penguins and dodos actually exist. At the same time, it remains possible to make statements about what necessarily holds, regardless of any plausible or counterfactual premise.

A *situated conditional* (SC) is a statement $\alpha \mid\!\sim_\gamma \beta$, with $\alpha, \beta, \gamma \in \mathcal{L}$, which is read as 'given the situation γ, β holds on condition that α holds'.

To provide a suitable semantics for SCs we define *epistemic interpretations*, a refined version of the ranked interpretations. We distinguish between two classes of valuations: plausible valuations with a *finite rank*, and implausible valuations with an *infinite rank*. Within implausible valuations we further distinguish between those that would be considered as *possible*, and those that would be *impossible*. This is formalised by assigning to each valuation u a tuple of the form $\langle f, i\rangle$ where $i \in \mathbb{N}$, or $\langle \infty, i\rangle$ where $i \in \mathbb{N} \cup \{\infty\}$. The f in $\langle f, i\rangle$ is intended to indicate that u has a *finite rank*, while the ∞ in $\langle \infty, i\rangle$ is intended to indicated that u has an *infinite rank*, where finite ranks are viewed as more typical than infinite ranks. Implausible valuations that are considered possible have an infinite rank $\langle \infty, i\rangle$ where $i \in \mathbb{N}$, while those considered impossible have the infinite rank $\langle \infty, \infty\rangle$, where $\langle \infty, \infty\rangle$ is taken to be less typical than any of the other infinite ranks.

Formally, let $R \stackrel{\text{def}}{=} \{\langle f, i\rangle \mid i \in \mathbb{N}\} \cup \{\langle \infty, i\rangle \mid i \in \mathbb{N} \cup \{\infty\}\}$. We define the total ordering $\preceq$ over R as follows: $\langle x_1, y_1\rangle \preceq \langle x_2, y_2\rangle$ if and only if $x_1 = x_2$ and $y_1 \leq y_2$, or $x_1 = f$ and $x_2 = \infty$, where $i < \infty$ for all $i \in \mathbb{N}\}$. We need to extend the notion of convexity of ranked interpretations to epistemic interpretations: let e be a function from $\mathcal{U}$ to R. e is said to be *convex* (w.r.t.$\preceq$) if and only the following holds: i) If $e(u) = \langle f, i\rangle$, then, for all j s.t. $0 \leq j < i$, there is a $u_j \in \mathcal{U}$ s.t. $e(u_j) = \langle f, j\rangle$; and ii) if $e(u) = \langle \infty, i\rangle$ for $i \in \mathbb{N}$, then, for all j s.t. $0 \leq j < i$, there is a $u_j \in \mathcal{U}$ s.t. $e(u_j) = \langle \infty, j\rangle$.

Definition 2. *An* ***epistemic interpretation*** E *is a total function from* $\mathcal{U}$ *to* R *that is convex.*

We let $\mathcal{U}^f_E \stackrel{\text{def}}{=} \{u \in \mathcal{U} \mid E(u) = \langle f, i\rangle \text{ for some } i \in \mathbb{N}\}$ and $\mathcal{U}^\infty_E \stackrel{\text{def}}{=} \{u \in \mathcal{U} \mid E(u) = \langle \infty, i\rangle \text{ for some } i \in \mathbb{N}\}$. We let $\min[\![\alpha]\!]_E \stackrel{\text{def}}{=} \{u \in [\![\alpha]\!] \mid E(u) \preceq E(v) \text{ for all } v \in [\![\alpha]\!]\}$, $\min[\![\alpha]\!]^f_E \stackrel{\text{def}}{=} \{u \in [\![\alpha]\!] \cap \mathcal{U}^f_E \mid E(u) \preceq E(v) \text{ for all } v \in [\![\alpha]\!] \cap \mathcal{U}^f_E\}$, and $\min[\![\alpha]\!]^\infty_E \stackrel{\text{def}}{=} \{u \in [\![\alpha]\!] \cap \mathcal{U}^\infty_E \mid E(u) \preceq E(v) \text{ for all } v \in [\![\alpha]\!] \cap \mathcal{U}^\infty_E\}$. We can now provide a semantic definition of situated conditionals in terms of epistemic interpretations.

$\langle\infty,\infty\rangle$	$[\![\mathsf{p}\wedge\neg\mathsf{b}]\!]\cup[\![\mathsf{d}\wedge\neg\mathsf{b}]\!]$
$\langle\infty,1\rangle$	$\overline{\mathsf{p}}\mathsf{dbf}$, pdbf
$\langle\infty,0\rangle$	$\overline{\mathsf{p}}\mathsf{db}\overline{\mathsf{f}}$, $\mathsf{pdb}\overline{\mathsf{f}}$
$\langle f,2\rangle$	$\mathsf{p}\overline{\mathsf{d}}\mathsf{bf}$
$\langle f,1\rangle$	$\overline{\mathsf{p}}\overline{\mathsf{d}}\mathsf{b}\overline{\mathsf{f}}$, $\mathsf{p}\overline{\mathsf{d}}\mathsf{b}\overline{\mathsf{f}}$
$\langle f,0\rangle$	$\overline{\mathsf{p}}\overline{\mathsf{d}}\mathsf{bf}$, $\overline{\mathsf{p}}\overline{\mathsf{d}}\overline{\mathsf{b}}\mathsf{f}$, $\overline{\mathsf{p}}\overline{\mathsf{d}}\overline{\mathsf{b}}\overline{\mathsf{f}}$

Figure 2: Model of the statements in Example 2.

Definition 3. *$E \Vdash \alpha \mid\!\sim_\gamma \beta$ (abbreviated as $\alpha \mid\!\sim^E_\gamma \beta$) if*

$$\begin{cases} \min[\![\alpha\wedge\gamma]\!]^f_E \subseteq [\![\beta]\!] & \text{if } [\![\gamma]\!]\cap\mathcal{U}^f_E \neq \emptyset; \\ \min[\![\alpha\wedge\gamma]\!]^\infty_E \subseteq [\![\beta]\!] & \text{otherwise.} \end{cases}$$

Intuitively, this definition evaluates $\alpha \mid\!\sim_\gamma \beta$ as follows. If the situation γ is compatible with the plausible part of E (the valuations in $\mathcal{U}^f_E$) then $\alpha \mid\!\sim_\gamma \beta$ holds if the most typical plausible models of $\alpha\wedge\gamma$ are also models of β. On the other hand if the situation γ is not compatible with the plausible part of E (that is, all models of γ have an infinite rank) then $\alpha \mid\!\sim_\gamma \beta$ holds if the most typical implausible (but possible) models of $\alpha\wedge\gamma$ are also models of β. SCs and epistemic interpretations allow to model more correctly the conditionals in Example 1.

Example 2. *Consider the following rephrasing of the statements in Example 1. 'Birds usually fly' becomes $\mathsf{b} \mid\!\sim_\top \mathsf{f}$. Defeasible information about penguins and dodos are modelled using $\mathsf{p} \mid\!\sim_{\mathsf{p}} \neg\mathsf{f}$ and $\mathsf{d} \mid\!\sim_{\mathsf{d}} \neg\mathsf{f}$. Given that dodos don't exist anymore, the statement $\mathsf{d} \mid\!\sim_\top \bot$ leaves open the existence of dodos in the infinite rank, which allows for coherent reasoning under the assumption that dodos exist (the context d). Moreover, information such as dodos and penguins necessarily being birds can be modelled by the conditionals $\mathsf{p}\wedge\neg\mathsf{b} \mid\!\sim_{\mathsf{p}\wedge\neg\mathsf{b}} \bot$ and $\mathsf{d}\wedge\neg\mathsf{b} \mid\!\sim_{\mathsf{d}\wedge\neg\mathsf{b}} \bot$, relegating the valuations in $[\![\mathsf{p}\wedge\neg\mathsf{b}]\!]\cup[\![\mathsf{d}\wedge\neg\mathsf{b}]\!]$ to the rank $\langle\infty,\infty\rangle$. Figure 2 shows a model of these statements.*

We have identified relevant *situated* rationality postulates, that represent desirable properties for SCs:

(Ref) $\alpha \mid\!\sim_\gamma \alpha$

(LLE) $$\frac{\models \alpha\leftrightarrow\beta,\ \alpha \mid\!\sim_\gamma \delta}{\beta \mid\!\sim_\gamma \delta}$$

(And) $$\frac{\alpha \mid\!\sim_\gamma \beta,\ \alpha \mid\!\sim_\gamma \delta}{\alpha \mid\!\sim_\gamma \beta\wedge\delta}$$

(Or) $$\frac{\alpha \mid\!\sim_\gamma \delta,\ \beta \mid\!\sim_\gamma \delta}{\alpha\vee\beta \mid\!\sim_\gamma \delta}$$

(RW) $$\frac{\alpha \mid\!\sim_\gamma \beta,\ \models \beta\rightarrow\delta}{\alpha \mid\!\sim_\gamma \delta}$$

(RM) $$\frac{\alpha \mid\!\sim_\gamma \beta,\ \alpha \not\mid\!\sim_\gamma \neg\delta}{\alpha\wedge\delta \mid\!\sim_\gamma \beta}$$

(Inc) $$\frac{\alpha \mid\!\sim_\gamma \beta}{\alpha\wedge\gamma \mid\!\sim_\top \beta}$$

(Vac) $$\frac{\top \not\mid\!\sim_\top \neg\gamma,\ \alpha\wedge\gamma \mid\!\sim_\top \beta}{\alpha \mid\!\sim_\gamma \beta}$$

(Ext) $$\frac{\gamma\equiv\delta}{\alpha \mid\!\sim_\gamma \beta \text{ iff } \alpha \mid\!\sim_\delta \beta}$$

(SupExp) $$\frac{\alpha \mid\!\sim_{\gamma\wedge\delta} \beta}{\alpha\wedge\gamma \mid\!\sim_\delta \beta}$$

(SubExp) $$\frac{\delta \mid\!\sim_\top \bot,\ \alpha\wedge\gamma \mid\!\sim_\delta \beta}{\alpha \mid\!\sim_{\gamma\wedge\delta} \beta}$$

These properties are inspired by both the KLM characterisation of conditional reasoning [3, 4] and the AGM approach to belief revision [5]. A situated conditional relation that is closed under all these properties is a *Full Situated Conditional* (FSC). A representation theorem connects the class of FSC's to the class of epistemic interpretations.

Theorem 1. *Every epistemic interpretation generates an FSC. Every FSC can be generated by an epistemic interpretation.*

Beyond investigating the properties characterising the class of epistemic interpretations, we have also modeled a first form of non-monotonic entailment relation, *minimal closure*, that is based on the classical *rational closure* defined for ranked models [4].

For a detailed explanation of the properties characterising FSC's, the proof of the representation theorem, and a presentation of the minimal closure, we refer the reader to the technical report [2].

4. Concluding remarks

The main contributions of this work can be summarised as follows: (*i*) the motivation for and the provision of a simple situation-based form of conditional which is general enough to be used in several application domains (e.g., planning [2, Example 5.1]); (*ii*) an intuitive semantics which is based on a semantic construction that has proven useful in the area of belief change and that is more general and also more fine-grained than the standard preferential semantics; (*iii*) an investigation of the properties that situated conditionals satisfy and of their appropriateness for knowledge representation and reasoning, in particular when reasoning about information that is incompatible with background knowledge, and (*iv*) the definition of a form of entailment for contextual conditional knowledge bases based on the widely-accepted notion of rational closure, which is reducible to classical propositional reasoning.

Next steps are the extension of this approach to other logics. Description Logics, for which rational closure has already been reformulated [6, 7, 8], are the first candidates. We also plan to investigate refinements of RC such as lexicographic closure [9] and their variants [10, 11, 12].

A conference version of this work was presented at AAAI-21 [1], and, while an extended version of the paper is under review at the moment, a technical report can be found online [2].

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