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Statistical Analysis of Triggered Seismicity in the Kresna Region of SW Bulgaria (1904) and the Umbria-Marche Region of Central Italy (1997)

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Abstract—A version of the restricted trigger model is used to analyse the temporal behaviour of some aftershock sequences. The conditional intensity function of the model is similar to that of the Epidemic Type Aftershock-Sequence (ETAS) model with the restriction that only the aftershocks of magnitude bigger than or equal to some threshold M_{tr} can trigger secondary events. For this reason we have named the model Restricted Epidemic Type Aftershock-Sequence (RETAS) model. Varying the triggering threshold we examine the variants of the RETAS model which range from the Modified Omori Formula (MOF) to the ETAS model, including such models as limit cases. In this way we have a quite large set of models in which to seek the model that fits best an aftershock sequence bringing out the specific features of the seismotectonic region struck by the crisis. We have applied the RETAS model to the analysis of two aftershock sequences: The first is formed by the events which followed the strong earthquake of $M = 7.8$ which occurred in Kresna, SW Bulgaria, in 1904. The second includes three main shocks and a large swarm of minor shocks following the quake of 26 September 1997 in the Umbria-Marche region, central Italy. The MOF provides the best fit to the sequence in Kresna; that leads to the thought that just the stress field changes due to the very strong main shock generate the whole sequence. On the contrary, the complex behaviour of the seismic sequence in Umbria-Marche appears when we make the threshold magnitude vary. Setting the cut-off magnitude $M_0 = 2.9$ the best fit is provided by the ETAS model, while if we raise the threshold magnitude $M_0 = 3.6$ and set $M_{tr} = 5.0$, the RETAS model turns out to be the best model. In fact, observing the time distribution of this reduced data set, it appears more evident that especially the strong secondary events are followed by a cluster of aftershocks.

Key words: Epidemic-type models, modified Omori law, trigger model, thinning simulation, triggering magnitude.

1. Introduction

The clustering feature in earthquake occurrence has focused the researchers' attention for a long time. Sequences of events have been classified in three main types according to their distribution in time: (i) A main shock followed by a number of aftershocks of decreasing frequency; (ii) a slow build-up of seismicity (foreshocks)

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leading to a type (i) sequence; and (iii) a gradual increase and decay of seismicity in time without a distinct main shock (MOGI, 1963). Type (iii) sequences, known as swarms, occur in areas with more complex tectonic structure.

Many approaches have been proposed in the literature to model the gradual decay of the aftershocks triggered by a strong earthquake in type (i) sequences. The most widely used model is the so-called Omori Law (OMORI, 1894), which UTSU (1961) transformed into the modified Omori formula (MOF). It assumes that all the events in an aftershock sequence are triggered by the stress field change due to the main shock, are conditionally independent and follow a nonstationary Poisson process. Some sequences show more complex behaviour, such as the presence of two twin main shocks or of several strong aftershocks (primary events), each followed by a subcluster of secondary events. This requires enrichment of the MOF with additional terms. Following such an approach OGATA (1988) introduced into the MOF the idea of self-similarity by extending the capacity of generating secondary events to every aftershock of the sequence. This led to formulation of the Epidemic Type Aftershock-Sequence (ETAS) model. Besides the triggered seismicity the ETAS model may also describe the background activity, represented by a constant term μ added to the conditional intensity function. Between these two limit cases, the MOF model with only one parent-event and the ETAS model in which every event shares in the generation of the subsequent ones, there is a range of different versions of trigger models. The distinctive difference is in the nature of the primary events: In the original trigger model (VERE-JONES and DAVIES, 1966; VERE-JONES, 1970) they are mutually independent, whereas in the restricted trigger model (OGATA, 1988) they are 'children' of the preceding primary events. The number of such primary events and their location in the sequence can be estimated by an extraordinarily complicated combinatorial computation, or fixed on the basis of physical considerations. An empirical relationship in seismology, known as BATH's law (1965, 1973), states that the difference between the main shock magnitude and the magnitude of the strongest aftershock is constant, on average 1.2 for Richter and 1.4 for Utsu (OGATA, 2001). By extending this principle to the subsequences generated by primary events as well, we derive that the difference between the weakest primary event and the weakest event in the aftershock sequence must be at least 1.2. This suggests choosing the primary events among those of magnitude larger than a suitable threshold M_{tr}.

The aim of our work is to examine the class of trigger models obtained by varying M_{tr} between the cut-off M_0 and the maximum magnitude $M_{\max}$. At first we have simulated two data sets according to the ETAS and RETAS model respectively, we have implemented Fortran subroutines and computed the maximum likelihood estimates of their parameters and then we have checked that the true model was identified using the Akaike criterion (Section 3). In Section 4 we present the results of the analysis of two aftershock sequences showing a different behaviour: The first was generated by the strong earthquake of $M = 7.8$ which occurred on April 4, 1904 in the Kresna region, SW Bulgaria, while the second is the seismic swarm which started on September 26, 1997 in the Umbria-Marche region, central Italy. Our aim is to seek out the model that best fits the data according to the Akaike criterion and hence to identify which physical interpretation can be given to each different behaviour, among the following: The aftershock sequence was triggered by the only main shock, by every event or by just some selected events.

2. Trigger Models

According to the MOF, the decaying frequency of aftershocks per unit time is given by the inverse power law (UTSU, 1961)

$$n(t) = \frac{K}{(t+c)^p}, \tag{1}$$

where t is the time elapsed from the occurrence of the main shock, K is a parameter related to the magnitude of the main shock and to the cut-off magnitude M_0, p is a coefficient of attenuation and c is a constant.

The frequency $n(t)$ (1) can be considered as the conditional intensity function of a point process, i.e.,

$$n(t) \approx \lambda(t)$$

where $\lambda(t)$ signifies

$$Pr\{\text{an event occurs in } (t, t+dt)|\mathcal{H}_t\} = \lambda(t|\mathcal{H}_t)\ dt + o(dt).$$

Here $\mathcal{H}_t$ denotes the history of the process, which for the MOF is just the occurrence time of the main shock. Some aftershock sequences show a secondary cluster in correspondence with the largest aftershock; in this case a further term has to be added to the MOF and the conditional intensity function becomes:

$$\lambda(t) = \frac{K_0}{(t+c_0)^{p_0}} + I_{\{t>T_1\}} \frac{K_1}{(t-T_1+c_1)^{p_1}},$$

where I is the indicator function, K_0, K_1, c_0, c_1, p_0 and p_1 are parameters and T_1 is the time elapsed between the primary and the secondary main shock (largest aftershock). To extend the possibility of generating 'descendants' to all events in the sequence, OGATA (1988) proposed the so-called epidemic type aftershock-sequence (ETAS) model with conditional intensity function

$$\lambda(t|\mathcal{H}_t) = \mu + \sum_{t_i<t} \frac{K_i}{(t-t_i+c)^p}, \tag{2}$$

where μ is the rate of background activity, the history $\mathcal{H}_t$ consists of the times t_i and magnitudes M_i, $i = 1, 2, \ldots$, of all the events which occurred before t and the sum is taken over these events. Clearly, in this case $i = 1$ indicates the main shock. OGATA gave the following explicit formulation to the term K_i:

$$K_i = K_0 \; e^{\alpha(M_i - M_0)}, \tag{3}$$

where K_0 is a constant and α measures the effect of magnitude on the production of 'descendants'. The choice of the exponential function form (3) is based on the linear relationship between the logarithm of aftershock areas and the magnitude of the main shocks due to UTSU (1971). The variety of the trigger models spreads between the MOF (1) and the ETAS model (2); they are defined by the conditional intensity function:

$$\lambda(t|\mathcal{H}_t) = \mu \; + \sum_{\{m \in \mathcal{P}; t_m < t\}} \frac{K_m}{(t - t_m + c)^p}, \tag{4}$$

being $\mathcal{P} = \{m = i_k, k = 1, \ldots, L\}$ the subset of primary events triggering offspring and μ, c, p, $K_1, \ldots, K_m$ the parameters to be estimated. A discriminant element among the models of this class is the subset $\mathcal{P}$, which can comprise events randomly distributed (VERE-JONES and DAVIES, 1966), or identified before estimation (OGATA, 1988) and possibly, in their turn, triggered by events belonging to the same subset $\mathcal{P}$, which occurred previously (OGATA, 2001).

In this work we have examined the model in which, as in OGATA (2001), the primary events are those with magnitude larger than or equal to a threshold M_{tr}, however, contrary to OGATA (2001), the parameters K_m obey the restriction (3); hence the conditional intensity function of our model becomes

$$\lambda(t|\mathcal{H}_t) = \mu \; + \sum_{\substack{t_i < t \\ M_i \geq M_{tr}}} \frac{K_0 \; e^{\alpha(M_i - M_0)}}{(t - t_i + c)^p}. \tag{5}$$

Moreover, we note that in this model, contrary to the original trigger model, each primary event belongs to the offspring of the preceding primary events. Because of the points in common with both the ETAS and the restricted trigger model we have named this model Restricted Epidemic Type Aftershock-Sequence (RETAS) model. The idea of a gap between the magnitude M_{tr} of the triggering event and that of the largest event generated is borrowed by Bath law, however we have not fixed the size of this gap; even better, by varying M_{tr} we have examined all the models between the MOF and the ETAS model on the basis of the Akaike criterion given by:

$$AIC = (-2) \; \max_{\theta} \log L(\theta; 0, T) + 2k, \tag{6}$$

where k is the number of parameters of the model and $\log L$ is the logarithm of the likelihood function, given by

$$\log L(\theta; 0, T) = \sum_{i=1}^{N} \log \lambda_\theta(t_i|\mathcal{H}_{t_i}) - \int_0^T \lambda_\theta(s|\mathcal{H}_s)\ ds. \tag{7}$$

In the above expression, N is the number of earthquakes of magnitude larger than or equal to M_0 which occur at times $t_i, i = 1, \ldots, N$, in the interval under study $[0, T]$. The minimum value of AIC identifies the best model. Summarising, the different models that we analyse in the next Sections can all be defined by the conditional intensity function (5) in which the set of triggering events changes as indicated in Table 1. We point out that in the MOF μ is generally set equal to 0 and $K_0\ e^{\alpha(M_1 - M_0)} = K$ for the identifiability of the model.

Likelihood maximization in the MOF and in the ETAS model has been performed by the package SASeis of the IASPEI Software Library (UTSU and OGATA, 1997). Moreover, we have implemented a program in Fortran 95 which exploits subroutines of the IMSL library to maximize the likelihood of the RETAS model following a quasi-Newton method.

After having chosen the best model among those proposed, we have evaluated its goodness of fit through the residual analysis. The integral

$$\Lambda(t) = \int_0^t \lambda(s|\mathcal{H}_s)\ ds \tag{8}$$

of the nonnegative conditional intensity function produces a 1-1 transformation of the time from t to $\tau = \Lambda(t)$ so that the occurrence dates $t_1, t_2, \ldots, t_N$ are transformed into $\tau_1, \tau_2, \ldots, \tau_N$. It is known that the $\tau_i, i = 1, \ldots, N$, are distributed as the standard stationary Poisson process if $\lambda(\cdot)$ is the intensity function of the process actually generating the data. Using the MLE conditional intensity function $\lambda_{\hat{\theta}}(t|\mathcal{H}_t)$, the corresponding $\hat{\tau_1}, \ldots, \hat{\tau_N}$, called residual process (OGATA, 1988), provide a measure of the deviation of the data from the hypothesized model. In particular the difference between the observed and expected cumulative number of events at time τ, $0 < \tau < \tau_0$, is a random variable distributed according to the normal $N(0, \tau(1 - \tau/\tau_0))$ (OGATA, 1992).

Table 1

Synthesis of the elements characterizing the three models: MOF, RETAS and ETAS

Model	Triggering magnitude	Parents
MOF	$M_{tr} = M_1$	main shock
RETAS	$M_0 \le M_{tr} \le M_1$	some events
ETAS	$M_{tr} = M_0$	all the events

3. Model Simulation

To test the ability of model selection of the Akaike criterion we have first simulated some data sets by following the thinning method (OGATA, 1999). Let $F(t|t_1,\ldots,t_n)$ be the conditional distribution of an event occurring at time t, given the history of the point process expressed through the sequence of the events which occurred in $t_1,t_2,\ldots,t_n$. Supposing that f is the density function of F, the relationship between the distribution F and the conditional intensity function λ characterizing the point process is given by

$$\lambda(t \mid \mathcal{H}_{t^-}) = \frac{f(t \mid t_1,\ldots,t_n)}{1 - F(t \mid t_1,\ldots,t_n)}.$$

Solving this equation we have:

$$F(t \mid t_1,\ldots,t_n) = 1 - \exp\left\{-\int_{t_n}^{t} \lambda(s \mid \mathcal{H}_{s^-})\, ds\right\}.$$

Now, drawn a value u_{n+1} from the uniform distribution on $(0,1)$, the time t_{n+1} of the next event can be simulated by solving numerically the equation:

$$F(t \mid t_1,\ldots,t_n) = u_{n+1}. \tag{9}$$

By substituting in (9) λ with the expressions (2) and (5) we secured two samples from the ETAS and RETAS models, respectively. The parameters common to both the models are: $\mu = 0.0238$, $c = 0.00234$, $\alpha = 0.474$, while for ETAS we have set $K_0 = 0.0365$, $p = 1.25$, $M_0 = 2.9$, and for RETAS $K_0 = 0.297$, $p = 0.872$, $M_0 = 3.5$, and $M_{tr} = 4.5$. As for the size of each event, we have assumed that magnitude and occurrence time are independent and the magnitude follows an exponential distribution with parameter $\beta = b \log_e 10$, where the b parameter of the Gutenberg-Richter formula is 0.889.

The sample generated according to the ETAS model includes $N = 1000$ events; through the maximum likelihood estimation method we have fitted the RETAS model to this data set for increasing values of the triggering magnitude starting from $M_{tr} = M_0$. In Figure 1a we have reported the cumulative number and the magnitude of the data set, while Figure 1b shows the *AIC* values obtained by (6) varying M_{tr}. As can be seen, the best-fitting model corresponds to $M_{tr} = M_0$, that is the case in which the RETAS model degenerates into the ETAS model. Hence, the *AIC* criterion has identified correctly the simulated model.

The events generated according to the RETAS model were $N = 300$, with $M_0 = 3.5$ and $M_{tr} = 4.5$. Figure 2a shows the cumulative number and the magnitude of those events with respect to the occurrence time, while Figure 2b shows the *AIC* values obtained by varying the value of M_{tr} from M_0 to the maximum observed

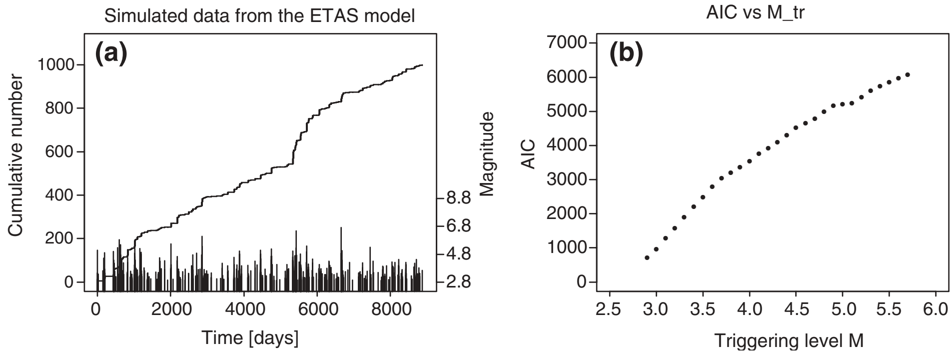

Figure 1
Data set simulated from the ETAS model; a) cumulative number of the simulated events and their magnitude versus the ordinary time (days); b) *AIC* values from the application of the RETAS model to the simulated data with different triggering magnitudes M_{tr}; the smallest *AIC* value corresponds to $M_{tr} = M_0$, which means that the algorithm correctly identifies the model from which the data were simulated.

magnitude. The best model detected by the smallest *AIC* value is again the right RETAS model with $M_{tr} = 4.5$.

One could think that the results obtained from the first data set depend on the low cut-off magnitude and that, by raising the value of M_0, the RETAS model could turn out as the best model. To check this we have reduced the first data set by neglecting the events of magnitude smaller than $M_0 = 3.6$; the resulting subset is represented in Figure 3a. We have repeated the ML estimation procedure and the

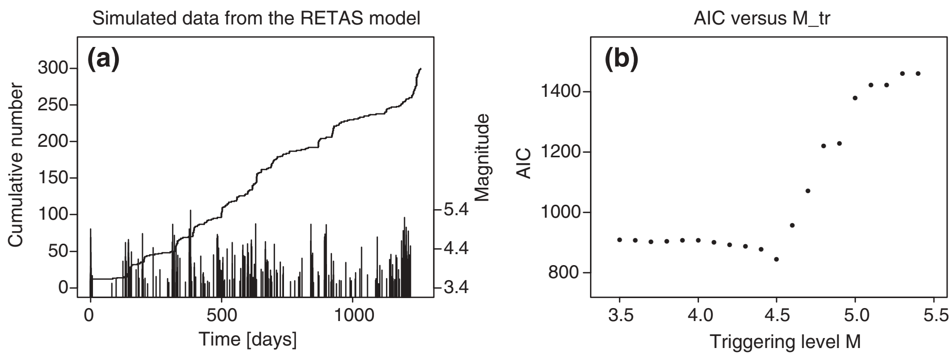

Figure 2
Data simulated from the RETAS model for $M_0 = 3.5$ and $M_{tr} = 4.5$; a) cumulative number of the simulated events and their magnitude versus the ordinary time (days); b) *AIC* values from the application of the RETAS model to the simulated data with different triggering magnitude levels; the smallest *AIC* value corresponds to $M_{tr} = 4.5$, which means that the algorithm has correctly identified the model from which the data were simulated.

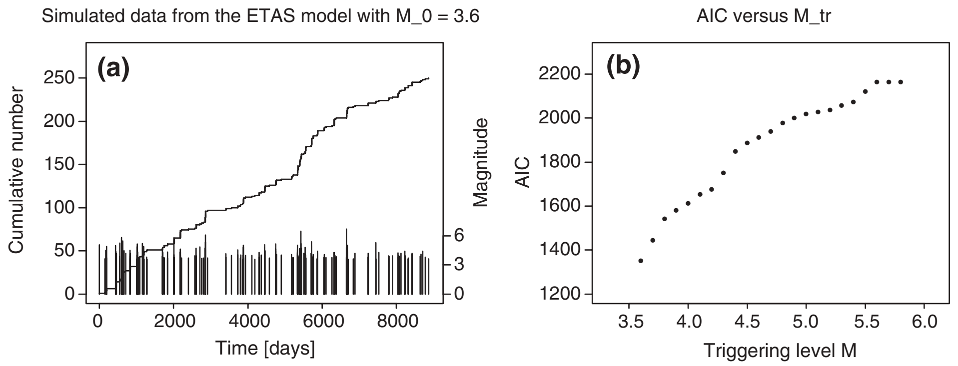

Figure 3
Subset of the data simulated from the ETAS model with $M_0 = 3.6$; a) cumulative number of the simulated events and their magnitude versus the ordinary time (days); b) *AIC* values from the application of the RETAS model to the simulated data for different triggering magnitude levels; the smallest *AIC* value is for $M_{tr} = M_0$, that is the algorithm has again identified the model used in the simulation.

evaluation of the *AIC* criterion varying M_{tr}; again the algorithm correctly identifies the ETAS model.

4. Data Analysis

Two aftershock sequences have been analysed by fitting to the data the class of the RETAS models obtained by making the triggering magnitude M_{tr} vary in the range of the magnitude so that also the MOF and the ETAS models are considered as limit cases.

The 1904 aftershock sequence in the Kresna region, SW Bulgaria—One of the two data sets analysed is from the Kresna seismic region in SW Bulgaria. On April 4, 1904 a very strong earthquake of $M = 7.8$ struck the region and triggered an aftershock sequence which continued for a long period. This event is considered as one of the strongest for the Balkan area in the twentieth century (Ranguelov *et al.*, 2001). We have analysed a catalogue for the Kresna region compiled by Dineva *et al.* (1999). It contains 472 events of magnitude $M \geq 3.0$ which occurred in the period 52 A.C. - 1993; actually, only seven of the earthquakes reported in that catalogue date back to 1800. All magnitudes are expressed as M_S magnitudes.

The catalogue completeness has been examined in the framework of the change-point problem (Rotondi, 1999; Rotondi and Garavaglia, 2002) and the results obtained indicate that the catalogue can be considered complete since 1890 for events of magnitude $M \geq 4.0$.

The area under study is bounded by the points of coordinates (40.8N, 22.0E), (40.8N, 24.4E), (42.4N, 22.0E), (42.4N, 24.4E). We assume that it contains all the aftershocks of the 1904 main shock of $M_S = 7.8$.

We have first applied a version of the space-time window method (CHRISTOSKOV and LAZAROV, 1981), calibrated on the Bulgarian peninsula, in order to select the data forming the aftershock sequence. Sixty earthquakes of magnitude $M \geq 4.0$ have been recognized as aftershocks of the 1904 strong event in a time period of 1230 days. This set has been used as input for the maximum likelihood estimation procedure of the RETAS model parameters.

Figure 4 shows the *AIC* values obtained by fitting the class of the RETAS models to the temporal distribution of the aftershocks. The best model, associated with the smallest *AIC* value, corresponds to $M_{tr} = 7.8$; in this case only the main shock generates 'descendants' and the model coincides with the MOF. The ML estimates of the parameters of this model are reported in the top row of Table 2. The differences between the *AIC* values of the other RETAS models are very small; we think this is due to both the very strong dependence of the aftershock frequency on the magnitude of the triggering event, represented by the high value of the α parameter, and the small value of the K parameter. As an example, in the second top row of Table 2 we report the results concerning the RETAS model with $M_{tr} = M_0 = 4.0$, i.e., when it coincides with the ETAS model. Because of the very small $K = 1.E - 10$ and of the large $\alpha = 6.28$, besides the main shock, only the two aftershocks of $M \geq 6.0$

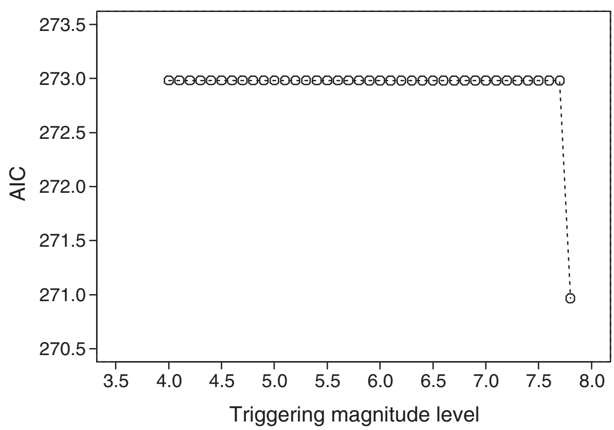

Figure 4
Kresna region: *AIC* value of the RETAS model for different M_{tr} triggering magnitudes applied to the aftershock sequence of the 1904 strong earthquake. The minimum *AIC* value indicates that the best model is the RETAS model with $M_{tr} = 7.8$, equivalent to the MOF where only the main shock can trigger descendants.

Table 2

Kresna region: ML estimates of the parameters and AIC value for the MOF and the ETAS model, provided by our Fortran 95 program (top two rows) and by the IASPEI Software Library (bottom two rows); in bold the minimum AIC value denoting the best model

Model	K	α	c	p	AIC
RETAS → MOF	2.0003	0.127	0.00202	0.816	**270.9668**
RETAS → ETAS	1E-10	6.28	0.00202	0.816	272.9815
MOF (OGATA)	3.269		0.00202	0.816	270.9668
ETAS (OGATA)	3.8E-9	5.342	0.00203	0.816	273.0234

significantly contribute to the generation of 'descendants' through the term K_i (3). In this case, the AIC value is not sensitive to the M_{tr} variations.

We have set μ equal to 0 because we think that the background activity is null in the period considered and that the stress field change due to the main shock controls the aftershock process by itself. The bottom two rows in Table 2 give the results related to the MOF and the ETAS models provided by the SASeis (Statistical Analysis of Seismicity) programs by UTSU and OGATA, included in the IASPEI Software Library (UTSU and OGATA, 1997). We used them to validate our program. The results produced by the two software packages match perfectly in the case of the MOF and show very small differences in the ETAS model, probably due to computational issues.

Substituting the ML estimates of the best model parameters (first row in Table 2) in (5) with $M_{tr} = 7.8$, we have calculated the expected cumulative number of events up to time t, for $t \in [0, T], T = 1230$ days. The corresponding curve is represented in Figure 5a by a solid line; the open circles indicate the real cumulative number of events. In Figure 5b we have plotted the same expected and real cumulative numbers

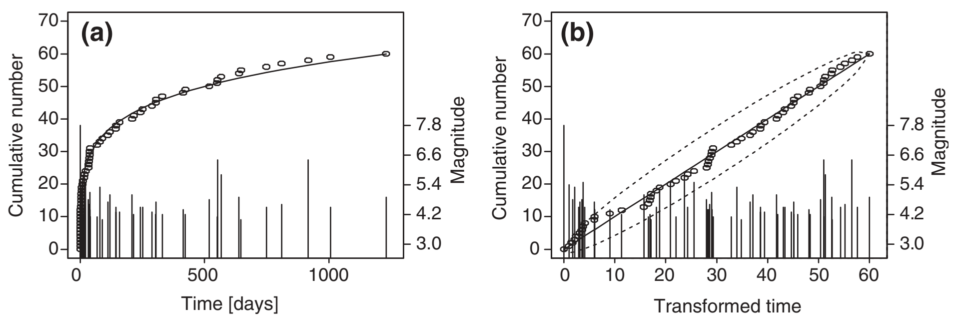

Figure 5

Diagrams of the cumulative number and magnitude of the 1904 aftershocks in Kresna region, SW Bulgaria; observed (circles) and estimated (continuous line) cumulative number of events versus (a) the ordinary and (b) transformed time. The plug-in estimate corresponds to the RETAS = MOF model ($M_{tr} = 7.8$). The dotted lines in (b) are the twofold standard deviations of the residual point process.

versus the transformed time; dotted curves represent the two-fold standard deviations of the residual point process, a measure of the deviation of the model from the data. We note that at the beginning of the aftershock sequence the observations exceed the expected number of events. We think this could be explained by the series of foreshocks preceding the main shock, some of which were so strong as to influence the start of the aftershock process. The model fits well the rest of the aftershock sequence.

In conclusion, the analysis performed on the relaxation process after the 1904 strong earthquake in the Kresna region reveals that we have a typical aftershock sequence in which the process is controlled by the stress-field change caused by the main shock. The best model is the RETAS model with $M_{tr} = 7.8$, that is the MOF, where only the main shock triggers all the aftershocks.

The 1997 aftershock sequence in Umbria-Marche region, central Italy—A seismic sequence struck the Umbria-Marche region during September-October 1997. It was characterized by the occurrence of six earthquakes of magnitude larger than 5 in a period of 20 days. A few hours after the first two strong shocks of duration magnitude $M = 5.6$ and $M = 5.8$ on September 26, a dense seismological network was installed in the epicentral area by the Istituto Nazionale di Geofisica (ING), the Camerino University (CU) and the Géosciences Azur (GA) groups (AMATO *et al.*, 1998); this allowed to follow in detail the spatio-temporal distribution of the aftershock sequence. Records related to more than 2000 events were collected in the CD-Rom *Waveforms, arrival times and locations of the 1997 Umbria-Marche aftershocks sequence*. The data set covers the period from September 26, 1997, when the sequence started, up to November 3, 1997. The obtained catalogue can be considered complete above magnitude 2.9 and above this cut-off it contains 508 events. Studies on different features of this sequence reveal that it is an atypical compound aftershock sequence for a magnitude 6 event, if we refer to the largest event. The rupture process is unlikely to be explained only by an elastic fracture mechanism, but should also be related to temporal changes in stress in the highly faulted upper crust of the region (DESCHAMPS *et al.*, 2000).

For faults maps of the Umbria-Marche region with the aftershocks epicenters for the two cut-off magnitudes, $M_0 = 2.9$ and $M_0 = 3.6$, we refer to DESCHAMPS *et al.* (2000); that study suggests that the strongest events delineate three subzones and are spatially close to the rupture zones of the main shocks. Consequently, we expect that our analysis points out that those events are the triggering events.

We have fitted the class of RETAS models to the Umbria-Marche aftershock sequence; the *AIC* values obtained for the different triggering magnitudes are plotted in Figure 6. That figure shows that the best model (smallest *AIC* value) is the RETAS model with $M_{tr} = M_0 = 2.9$, that is the ETAS model. Table 3 contains the ML estimates of the parameters with $\mu = 0$. The α parameter generally varies in the range 0.4–3.0 and, according to OGATA (1999, p. 487), its value is smaller

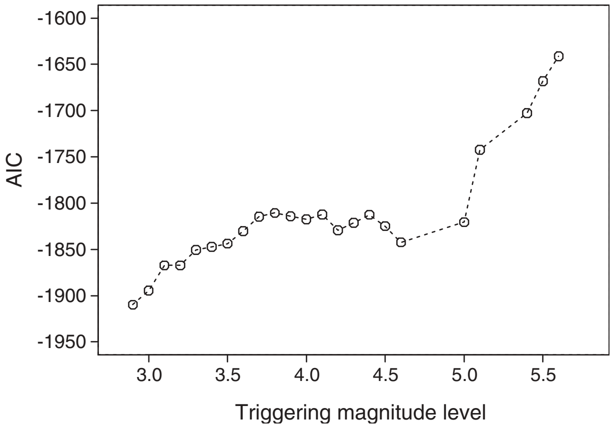

Figure 6
Umbria-Marche region: *AIC* value of the RETAS model for different M_{tr} triggering magnitudes applied to the aftershock sequence of the 1997 earthquake. The minimum *AIC* value indicates that the best model is the RETAS model with $M_{tr} = 2.9$, equivalent to the ETAS model where all the shocks can trigger descendants.

Table 3

Umbria-Marche region: ML estimates of the parameters and AIC value for the ETAS model fitted to the 1997 aftershock sequence with $M_0 = 2.9$

Model	K	α	c	p	*AIC*
RETAS → ETAS	0.11	0.89	0.103	1.55	−1909.85

in case of swarm-type activity than in ordinary mainshock and aftershock activities. In our case $\alpha = 0.89$ classifies the sequence as swarm; indeed, the occurrence of 6 events of magnitude between 5 and 6 in a 19-days period does not correspond to the common observations of typical main shock-aftershock sequences.

Assigning to each parameter its ML estimate, reported in Table 3, we obtain the plug-in estimate of the conditional intensity function (2); its integral (8) in $(0, t)$ provides the expected cumulative number of aftershocks in t time from the main shock. Figure 7 compares the observed (solid line) and estimated (dashed line) cumulative number of events versus the (a) ordinary and (b) transformed time. As can be seen, the strongest events generate secondary sequences identified by jumps in the curve. Visually the model seems to favorably fit the data (see Fig. 7a), however the residual analysis reveals that the number of the observed events exceeds that of the expected ones in nearly the whole time interval under study, even though the cumulative number curve remains mostly within the two-fold standard deviations (Fig. 7b).

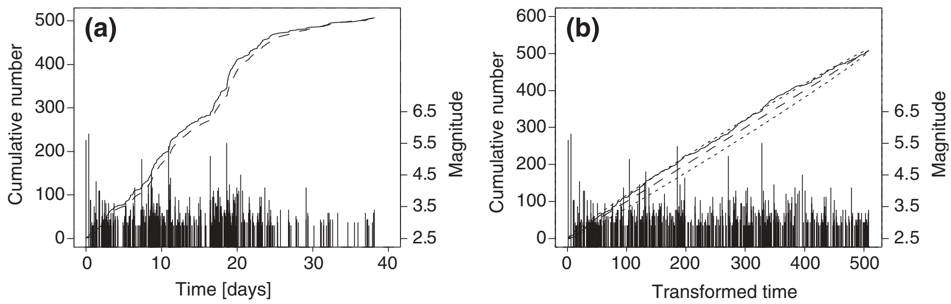

Figure 7
Diagrams of the cumulative number and magnitude of the 1997 aftershocks in Umbria-Marche region, central Italy; observed (solid line) and estimated (dashed line) cumulative number of events versus (a) the ordinary and (b) transformed time. The plug-in estimate corresponds to the RETAS = ETAS model ($M_{tr} = 2.9$). The dotted lines in (b) are the two-fold standard deviations of the residual point process.

Up to this point the results obtained agree on the following preliminary conclusions: a) the Umbria-Marche aftershock sequence is far from the usual main shock-aftershock sequence; b) a number of strong events has occurred; each of them triggering secondary aftershocks; c) the MOF is completely inappropriate in this case.

Let us compare the sets of *AIC* values obtained by analysing the first simulated data set and the Umbria-Marche sequence (Fig. 1b and Fig. 6, respectively), that is cases in which ETAS is the best model. We identify some differences, the main one being the presence of a local minimum in Figure 6 in correspondence of the triggering magnitude $M_{tr} = 4.6$. This observation led us to the idea of applying RETAS models with $M_{tr} \geq 4.6$ to the subset of the Umbria-Marche aftershock sequence with cut-off magnitude $M_0 = 3.6$, so that the difference between M_{tr} and the new M_0 is at least 1. Our purpose was to analyse the process at a higher energy level since stronger earthquakes usually correlate better to the seismotectonic structures and to the physical processes of a region. We have considered $N = 76$ aftershocks of magnitude bigger than or equal to 3.6. By fitting the RETAS model to these data and varying the triggering magnitude, we have obtained that the minimum *AIC* value is achieved for $M_{tr} = 5.0$; this means that, by raising the threshold magnitude, secondary clusters appear in the so reduced Umbria-Marche sequence, generated by the six strongest events. The *AIC* values versus the triggering magnitudes are plotted in Figure 8.

The ML estimates of the best RETAS model parameters are reported in Table 4; again the μ background rate is set equal to zero. Also in this case the small value of α parameter, $\hat{\alpha} = 0.078$, agrees with the classification of the sequence as swarm. Integrating the plug-in estimate of the conditional intensity function (5) we have calculated the expected cumulative number of events.

In Figure 9a the observed cumulative number of events is superimposed to the expected number curve; the bars indicate the aftershocks magnitude. It is easy to

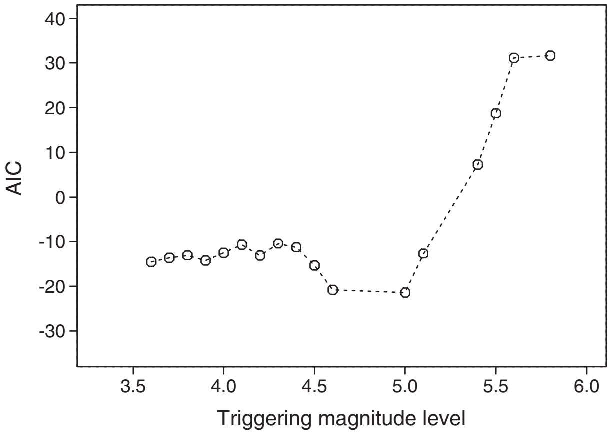

Figure 8
Umbria-Marche region: *AIC* value of the RETAS model for different M_{tr} triggering magnitudes applied to the aftershock sequence of the 1997 earthquake with cut-off magnitude $M_0 = 3.6$. The minimum *AIC* value indicates that the best model is the RETAS model with $M_{tr} = 5.0$, in which only the six shocks of magnitude larger than or equal to 5 can trigger descendants.

identify the subcluster generated by each of the events of $M \geq 5$. The same elements are drawn in Figure 9b versus the transformed time together with the two-fold standard deviations of the residual point process. Apart from a slight decrease of the activity between the 13th and the 16th day, the observed cumulative number of events is always within the error bounds.

In Table 5 we have summarised the main features of the seismic sequence pointed out by the stochastic modelling and those described in seismological studies appeared in the literature on the same case. We note that the two disciplines highlight the same characteristics. Indeed the seismological studies point out:

- a general increase of seismic activity due to stress changes induced by the first two strong shocks;
- part of the weakest events is caused by this general 'activation' of the zone;
- each strong event causes a spatially located increase of the static stress, leading to the occurrence of the next strong event in the sequence and of its aftershocks. Therefore the strongest events in the sequence seem to be more correlated in space and time to such subsequent changes in the static stress field.

Table 4

Umbria-Marche region: ML estimates of the parameters and AIC value for the RETAS model with $M_{tr} = 5.0$ *fitted to the 1997 aftershock sequence with* $M_0 = 3.6$

Model	K	α	c	p	AIC
RETAS	7.79	0.078	1.24	1.58	–21.58

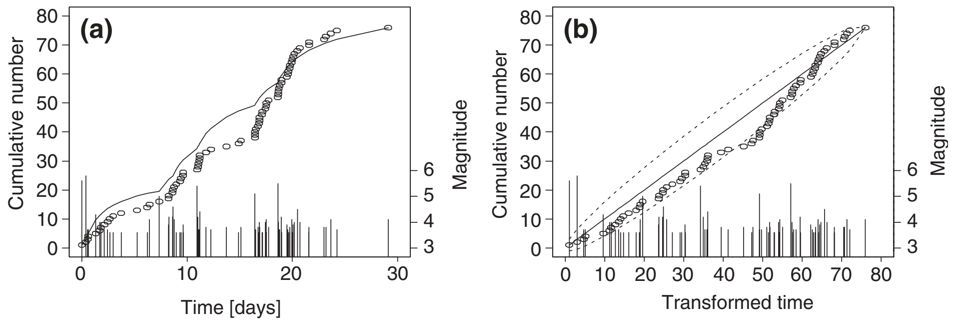

Figure 9
Diagrams of the cumulative number and magnitude of the 1997 aftershocks in Umbria-Marche region, central Italy, for cut-off magnitude $M_0 = 3.6$; observed (circles) and estimated (continuous line) cumulative number of events versus (a) the ordinary and (b) transformed time. The plug-in estimate corresponds to the RETAS model with $M_{tr} = 5.0$. The dotted lines in (b) are the twofold standard deviations of the residual point process.

Analogously, on the statistical side we have:

- the MOF fails to fit the data according to the 'swarm-type' nature of the aftershock sequence with six events of comparable strength;
- the ETAS model is the best model for all the events of $M \geq 2.9$, being able to capture, better than other models, both the aspects of the process, that is subsequent strong events with secondary aftershocks and a general increase of weaker seismicity, randomly distributed in space and time;
- the RETAS model provides the best description for the set of the events of $M \geq 3.6$, revealing that just the strongest events trigger aftershocks. In other

Table 5

Correspondence between results of stochastic analysis and of seismogical studies on the Umbria-Marche 1997 sequence

Stochastic analysis	Seismological studies
• MOF does not fit well the data	• no event is much stronger than the others (not a typical aftershock sequence)
• with $M_0 = 2.9$ the best model for the whole data set is the ETAS model (RETAS converging to ETAS with $M_{tr} = M_0 = 2.9$)	• each strong event triggers aftershocks — the sequence is a '*series of rupture episodes, each one followed by aftershocks, ...* }' (CATTANEO *et al.*, 2000)
• if we raise the cut-off magnitude to $M_0 = 3.6$, the best model for the reduced data set turns out to be the RETAS model with $M_{tr} = 5.0$, in which only the strongest earthquakes in the sequence can trigger secondary sequences	• epicenters delineate three subzones • in a short period ($\sim$ 20 days) six events of magnitude $5.0 \leq M \leq 6.0$ occur (DESCHAMPS *et al.*, 2000) • Coulomb stress changes correlate well with the position and the parameters of nearly all subsequent strong earthquakes in the sequence (COCCO *et al.*, 2000)

words, it shows the existence of a clearer correlation between strong events and seismicity of the areas where the static stress highly increases, owing to each of the largest events in the sequence.

The diagram in the Appendix summarises the steps carried out in the analysis.

5. *Conclusions*

In this article we have examined a version of the restricted trigger model in order to study the temporal distribution of aftershock sequences. We name this model Restricted Epidemic Type Aftershock-Sequence (RETAS) model since its conditional intensity function is similar to that of the ETAS model with the restriction that only the aftershocks of magnitude larger than some threshold M_{tr} can trigger secondary events. By assigning all the possible different values to the triggering threshold, we examine all the possible variants of the RETAS model placed between the limit cases: the MOF and the ETAS model.

We have analysed two aftershock sequences: The former follows the earthquake of $M = 7.8$ which occurred on April 4, 1904 in the Kresna region, SW Bulgaria, the latter is the sequence with several events of moderate strength which started on September 26, 1997 in the Umbria-Marche region, central Italy. The best model for the Bulgarian sequence is the RETAS model with triggering threshold $M = 7.8$, which corresponds to the MOF; this means that the process is controlled by the stress-field change caused by the main shock. The Umbria-Marche sequence appears more complex. The RETAS model with $M_{tr} = M_0 = 2.9$ provides the best fitting to the global data set. In this case it coincides with the ETAS model and indicates that all the events generate secondary aftershocks. We have also analysed subsets of the data set varying the cut-off magnitude; it has turned out that for $M_0 = 3.6$ the best model is the RETAS model with triggering magnitude $M_{tr} = 5.0$, that is, just the six strongest shocks of this reduced data set trigger secondary activity.

We have compared the physical interpretations suggested by the results obtained through the stochastic modelling with the description of the seismic crisis provided by detailed seismological studies. It results that the process shows two main aspects: a) a general increase of the seismic activity, triggered by the first two strong shocks, that is also responsible for part of the weakest events, and b) a subsequent, spatially localised increase of static stress, leading to the occurrence of the next strong event in the sequence and of its aftershocks. The strongest events in the sequence seem to be more correlated in space and time to these subsequent changes in the static stress field.

The historical catalogue (Boschi *et al.*, 1997) reports several earthquakes of comparable magnitude in this area. According to Pino and Mazza (2000) the 1997 seismic sequence can be considered quite typical for this area, with events of moderate magnitude and fault length not exceeding 15 km. Also surface geology supports this conclusion, pointing out the main role played by the transverse structures in limiting the extension of the faults, thereby allowing repeated ruptures on the same fragments. It appears therefore that the RETAS model we have proposed can be applied for modelling future seismicity in the region, and in general for modelling compound seismic sequences in regions of complex tectonic structure with highly fractured earth crust. As for any model, we underline that the reliability of the conclusions depends strongly on the quality and quantity of the available data.

Appendix

Let $\{t_i, M_i\}_{i=1}^{N}$ be the aftershock sequence we have to analyse, where (t_1, M_1) indicate time and magnitude of the main shock. The algorithm we have applied is composed of two stages. The first consists in the following steps:

(i) order the set $\mathcal{D} = \{M_i\}_{i=1}^{N}$ increasingly and build the set of different magnitude levels $\{Mag_k\}_{k=1}^{K}$ obtained by removing from $\mathcal{D}$ the repeated values, so that $Mag_k < Mag_{k+1}, k = 1, \ldots, K-1, \forall k\ Mag_k \in \mathcal{D}$, and $\forall i\ \exists k : M_i = Mag_k$;
(ii) set $k = 0$;
(iii) $k \leftarrow k + 1$, $M_{tr} = Mag_k$;
(iv) given the conditional intensity function

$$\lambda(t|\mathcal{H}_t) = \mu + \sum_{\substack{t_i < t \\ M_i \geq M_{tr}}} \frac{K_0\ e^{\alpha(M_i - M_0)}}{(t - t_i + c)^p},$$

maximize $\log \lambda(\cdot)$ and evaluate AIC_k;
(v) if $k = 1$, then set $k_{best} = 1$ and $AIC_{best} = AIC_1$; otherwise, if $AIC_k < AIC_{best}$, then set $k_{best} = k$ and $AIC_{best} = AIC_k$;
(vi) if $k < K$ then go to (iii), otherwise stop.

The final output, expressed by the value k_{best}, identifies the best model: ETAS if $k_{best} = 1$, MOF if $k_{best} = K$ and RETAS with triggering magnitude $Mag_{k_{best}}$ if $1 < k_{best} < K$.

If the ETAS model eventually becomes the best model, it is interesting to examine whether particular correlations between the strongest aftershocks and the remaining seismicity emerge by raising the threshold M_0. In this second stage the analysis can be carried out through the following steps:

(i) plot the set of pairs $\{(Mag_k, AIC_k)\}_{k=1}^{K}$;
(ii) if this set shows a monotonic trend, then stop; otherwise find the index of the local minimum, let's say j, set $M_{tr} = Mag_j$ and $M_0 \approx M_{tr} - 1$, and go back to the first stage considering the subset formed by the events of size exceeding M_0.

As for the step (iv), the explicit expression of the log-likelihood is obtained by substituting (5) in (7). Then it turns out

$$\log L(\theta) = \sum_{j=1}^{N} \log \left[\mu + \sum_{\substack{t_i < t_j \\ M_i \geq M_{tr}}} \frac{K_0 \; e^{\alpha(M_i - M_0)}}{(t_j - t_i + c)^p} \right] - \mu T - \int_0^T \sum_{\substack{t_i < t \\ M_i \geq M_{tr}}} \frac{K_0 \; e^{\alpha(M_i - M_0)}}{(t - t_i + c)^p} \, dt.$$

The solution of the integral depends on the value of the parameter p; in particular, if $p = 1$, it is given by

$$\int_0^T \sum_{\substack{t_i < t \\ M_i \geq M_{tr}}} \frac{K_0 \; e^{\alpha(M_i - M_0)}}{(t - t_i + c)^p} \, dt = \sum_{\substack{i = 1 \\ M_i \geq M_{tr}}} K_0 \; e^{\alpha(M_i - M_0)} [\log(T - t_i + c) \; - \log(c)]$$

wheras, if $p \neq 1$, then it holds

$$\int_0^T \sum_{\substack{t_i < t \\ M_i \geq M_{tr}}} \frac{K_0 \; e^{\alpha(M_i - M_0)}}{(t - t_i + c)^p} \, dt = \sum_{\substack{i = 1 \\ M_i \geq M_{tr}}} \frac{K_0 \; e^{\alpha(M_i - M_0)}}{1 - p} [(T - t_i + c)^{1-p} - c^{1-p}] \; .$$

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