Measurements of General Relativity precessions in the field of the Earth with laser-ranged satellites and the LARASE program

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***Abstract*—The LAser Ranged Satellites Experiment (LARASE) represents a new experiment whose main goal is to provide accurate measurements for the gravitational interaction in the weak-field and slow-motion (WFSM) limit of Einstein's theory of general relativity by means of a very precise laser tracking of geodetic satellites orbiting around the Earth. Beside the quality of the tracking observations of the satellites orbit, guaranteed by the powerful Satellite Laser Ranging (SLR) technique of the International Laser Ranging Service (ILRS), also the quality of the dynamical models implemented in a software code plays a fundamental role in order to obtain precise and accurate measurements of relativistic physics. The models have to account for the perturbations provoked by both gravitational and non-gravitational forces in such a way to reduce as well as possible the difference between the *observed* range, from the tracking, and the *computed* one, from the models. In particular, LARASE aims to improve the dynamical models of the current best laser-ranged satellites in order to perform a precise and accurate orbit determination. This represents a first step towards new refined tests and measurements of GR in the field of the Earth. After a brief presentation of the main relativistic measurements which constitute the main goals of LARASE, the results obtained during last year will be discussed in terms of the improvements reached in the satellites orbit modelling and in their precise orbit determination.**

I. Introduction

After 100 years, Einstein's general relativity (GR) still represents the *standard model* of physics for the interpretation of the gravitational interaction [1]. Indeed, GR provides the best description of gravity both at the high and low energy scales and it is the pillar of modern cosmology to understand the universe that we observe. The successes of GR are very numerous, and this important anniversary was celebrated worthily with the first direct detection of gravitational waves [2]. This first detection opens the way to a new kind of astronomy, the gravitational astronomy, which will offer the opportunity to investigate compact sources with a different approach, further enriching astrophysics and cosmology [3], [4]. Moreover, the same theory of GR will be verified in its predictions under the regime of strong fields, or quasi-strong fields as outlined in [5], [6].

GR is a metric theory for the description of the gravitational interaction and is fully decribed by the metric tensor $g_{\mu\nu}$. However, other gravitational theories different from GR have been proposed [7]. Some of these theories are metric theories like GR, and they share with GR the same spacetime structure and the same equations of motion for test particles, but differ in the field equations form. Conversely, other theories provide more fundamental differences, such as violations of Einstein Equivalence Principle (EEP). These are non-metric theories of gravity. Moreover, the overall validity of GR is not only questioned by these alternative theories, but also from quantum theories of physics. Therefore, it is clear that it is very important to precisely test the consequences of GR, as well as those of competing theories, at all the accessible scales of distances and energies. In the following sections the activities performed within the LARASE research program are presented. In particular, the results concerning the improvements (with respect to our previous work [8]) reached for the satellites precise orbit determination (POD) will be discussed.

The rest of the paper is organized as follows. In Section II, the objectives of LARASE will be briefly summarized in term of the main relativistic effects to be measured in the near future. In Section III, the results in terms of the main activities performed aiming to improve the dynamical models for the satellites orbit are presented. Finally, in Section IV, our conclusions on the current status of LARASE activities are given.

II. LARASE GOALS

The research program and collaboration named LARASE [9] aims to provide an original contribution in testing and verifying the predictions of GR in its WFSM limit by means of the SLR technique [10]. One of the first goals of LARASE is to provide a reliable POD of a set of laser-ranged satellites belonging to the network of the ILRS [10].

Indeed, the test masses of LARASE will be some of the best laser-ranged satellites orbiting the Earth. These satellites are spherical in shape, fully passive, and with a generally low area/mass ratio in order to minimize the accelerations due to the non-gravitational perturbations (NGP). Among these satellites, the two LAGEOS and LARES will be the most important to consider because of the high accuracy of their orbit determination. The older LAGEOS (LAser GEOdynamic Satellite) was launched by NASA on May 4, 1976, LAGEOS II was jointly launched by NASA and ASI on October 22, 1992, finally LARES (LAser RElativity Satellite) was launched by ASI on February 13, 2012.

In order to obtain a refined POD, beside the quality of the tracking data of the satellites orbit — which is provided by the ILRS — reliable dynamical models have to be included in the software used for the orbit determination of the satellites. Therefore, as stated above, a major goal of LARASE is to improve the dynamical models of the LAGEOS and LARES satellites with a special attention to the NPG. With regard to the POD execution, we will take advantage of the software GEODYN II of NASA/GSFC [11], [12].

With regard to the relativistic measurements to be carried out, LARASE will mainly focus on the GR precessions related with the Earth's gravitoelectric and gravitomagnetic fields and with space curvature. The former field, analogous to the electric field due to electric charges of Coulomb's law, is produced by the Earth's mass, while the latter, analogous to the magnetic field due to electric currents, is produced by the Earth's current-of-mass, i.e., by the Earth's angular momentum.

These relativistic effects are: i) Einstein (or Schwarzschild) [1] precession, ii) Lense-Thirring [13] precession (see also [14]), iii) de Sitter [15] or geodetic precession. The first precession arises from the gravitoelectric field, the second is due to the gravitomagnetic field, while the latter is in part due to the gravitoelectric field and in part due to space curvature. These precessions are responsible of long-term and secular effects on two of the three Euler angles that define the orbit orientation in space, namely the argument of pericenter, ω, which is subject to Einstein and Lense-Thirring precessions, and the right ascension of the ascending node, Ω, which is subject to Lense-Thirring and de Sitter precessions.

With regard to the geodetic precession, it is the effect due to the motion of the Earth-Moon system in the background field of the Sun which matter for a satellite in orbit about the Earth, while the (direct) effect due to the Earth and the motion of the satellite around the Earth is negligible.

Concerning the secular effects produced on the orbit we have the following expressions:

$$\dot{\omega}^{\rm Schw} = \frac{3(GM_\oplus)^{3/2}}{c^2 a^{5/2}(1-e^2)}, \quad (1)$$

for the gravitoelectric precession of the argument of pericenter and

$$\dot{\omega}^{\rm LT} = \frac{-6GJ_\oplus}{c^2 a^3 (1-e^2)^{3/2}} \cos i, \quad (2)$$

for its gravitomagnetic part,

$$\dot{\Omega}^{\rm LT} = \frac{2GJ_\oplus}{c^2 a^3 (1-e^2)^{3/2}}, \quad (3)$$

for the gravitomagnetic precession of the satellite node and finally

$$\dot{\Omega}^{\rm dS} = \frac{3}{2}\frac{GM_\odot}{c^2 R^3_{\oplus\odot}} \left| \mathbf{R}_{\oplus\odot} \times (\mathbf{V}_\oplus - \mathbf{V}_\odot) \right| \cos\epsilon. \quad (4)$$

for the precession of the satellite node due to the de Sitter effect. This last precession is the same for all Earth's satellites.

In the above equations, G and c are, respectively, the gravitational constant and the speed of light, $M_\oplus$ and $J_\oplus$ represent the mass and angular momentum of the Earth, finally, a, e and i, are, respectively, the orbit semi-major axis, eccentricity and inclination. In the case of the geodetic precession, the expression is valid in the solar barycentric reference frame where $\mathbf{V}_\oplus$ and $\mathbf{V}_\odot$ represent the speed of the Earth and of the Sun, the vector $\mathbf{R}_{\oplus\odot}$ represents their separation, $M_\odot$ represents the mass of the Sun and finally ϵ represents the obliquity of the ecliptic plane with respect to the Earth's equator.

In the field of the Earth, the best measurements for the Lense-Thirring effect and for the advance of the argument of pericenter are those described respectively in [16] and in [17][1]. Finally, because a way to test the predictions of Einstein's GR with respect to those of other metric theories is through the measurements of the so-called parameterized post-Newtonian (PPN) parameters [18]–[21], a few of them (namely γ, β, α_1 and α_2) are the subject of the investigations and measurements of LARASE.

In the following section some of the results that we have recently obtained on the improvements of the models for some of the main perturbations acting on the two LAGEOS satellites and on LARES will be described.

[1]These papers also include an estimate of the systematics error sources. In particular, in [17] the quadrupole coefficient of the Earth's gravitational field has been estimated together with the satellite state-vector in the case of various models for the Earth's field expansion in spherical harmonics.

III. Results

We focus on some of the main results reached during last year of the LARASE activities concerning the models improvements with respect to those described in [8]. For a full view and description of the goals of LARASE we refer to [9].

A. Internal structure

As we described briefly in [8], a characteristic of the approach of LARASE in order to improve the dynamical models of the two LAGEOS satellites as well as of LARES, in particular with regard to the models related with the NGP, is to reconstruct information about the structure, the materials used, and the moments of inertia of the satellites. We did this activity starting from the original drawings of the LAGEOS and LARES satellites, and we built a 3D-CAD model of the satellites structure which is useful for finite element-based analysis[2]. This activity is now concluded in the case of the two LAGEOS and fully described in [22]. Indeed, a main goal of this work was to have an independent estimate of their moments of inertia — that were not measured on the flight model of the two satellites — and to have a refined model of the satellites able to provide a reference for the development of a new thermal model in order to account properly of the quite complex perturbation produced by the thermal thrust effects. In Table I the results obtained for the moments of inertia of the two LAGEOS satellites are shown.

TABLE I. Mass and moments of inertia of LAGEOS and LAGEOS II (flight arrangements). The masses are the one measured. The moments of inertia are those computed in [22] with normalized densities.

Satellite	Moments of inertia (kg m^2)		
	I_{xx}	I_{yy}	I_{zz}
LAGEOS	11.42 ± 0.03	10.96 ± 0.03	10.96 ± 0.03
LAGEOS II	11.45 ± 0.03	11.00 ± 0.03	11.00 ± 0.03

A section view of the two satellites from our CAD model is shown in Fig. 1, where the main parts of the structure are visible: i) two hemispheres of aluminum containing the CCRs, ii) the brass core that contribute to increase the mass of the satellite, iii) the copper beryllium shaft that fastens the different parts of the satellites.

It is interesting to compare our Fig. 1 with Fig. 1 of [23]. The two figures differ for the internal dimensions and for the material of the shaft: copper beryllium vs brass. Indeed, in [22] we also closed some contradictory information provided in the historical literature of LAGEOS on some of these parameters.

B. Spin dynamics

Among the plethora of the NGP, the thermal forces (as the Earth-Yarkovsky and Yarkovsky-Schach thermal effects) depend on the knowledge of the satellite spin vector orientation and rate. In this regard we have focused our attention on the spin dynamics. Indeed, as we highlighted in [8] the rotational dynamics of passive satellites like the two LAGEOS has been deeply investigated in the past by many authors (see [24]–[30]). Our new model includes the gravitational torque due to the oblateness of the satellite, the magnetic torque due to the eddy currents induced in the rotating satellite, see [24] and [26] for details, and the torques due to the reflection asymmetry and the non coincidence between the center of mass and the geometrical center of the satellite, see [27], [30]–[32]. However, with respect to the previously quoted papers we have gone a step forward. Indeed, besides solving equations that have been averaged over the orbital period and the day for the various torques involved, as in case of the solution shown in [8] in the case of LAGEOS II, we have also solved the full set of Euler equations in the general case, valid for any rotational period of the satellite. In Figure 2, in the case of the older LAGEOS satellite, the comparison between our spin model — in the general case — with the available observations is shown.

[2] We took advantage of SOLIDWORKS® 3D-CAD software and its capability to evaluate 3D model solid mass properties.

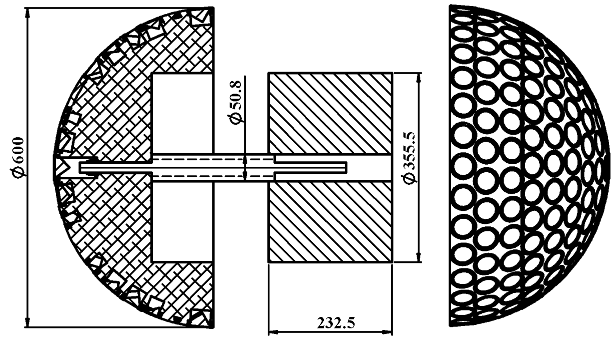
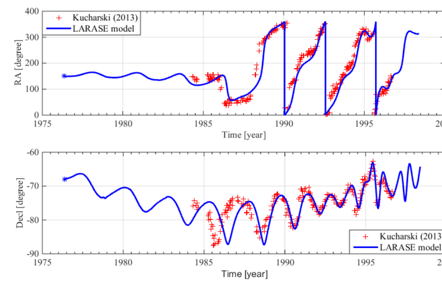
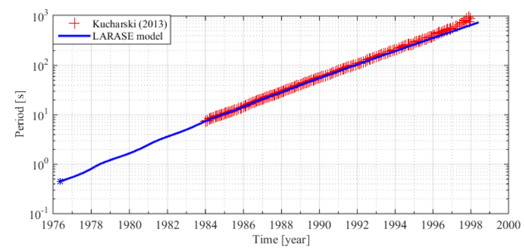


Fig. 1. From [22]. The LAGEOS satellites assembly. The dimensions are in mm. The two aluminum hemispheres are shown with the section of the cavities containing the CCRs together with the internal brass cylinder and Cu-Be shaft.



(a) LAGEOS right ascension (top) and declination (bottom)



(b) LAGEOS rotational period

Fig. 2. LAGEOS spin orientation (a) and period (b) in the J2000 reference frame. The units are degrees for the two spherical equatorial coordinates (α, δ) and seconds in the case of the rotational period P. The results for the spin evolution, as we obtained from our model (blue line), are compared with the observations in the literature as provided by [33].

Of course, the knowledge of the moments of inertia of a satellite [22] plays a significant role in the case of the gravitational torque. Very important is also a reliable model of the magnetic torque, which also plays a central role, especially in the difficulties to be solved in the passage from averaged equations to non-averaged equations.

In conclusion, we are now able to model the spin evolution of the two LAGEOS and LARES satellites with two different models: a model valid in the rapid spin approximation and a more general model based on the solution of the Euler equations. A paper is under preparation with the details of the results and the comparison between the two models for the three satellites.

C. Neutral drag

As highlighted in previous papers [8], [9], the impact of neutral drag on LARES orbit is much stronger with respect to that on the two LAGEOS because of its much lower orbit (1450 km height vs. 5900 km). We therefore reviewed the drag effects on the orbit of these satellites and we started a number of different activities as: i) the comparison of the predictions of the different atmospheric models at the altitudes of the satellites, ii) the estimate of the perturbing accelerations acting on the satellites, iii) the estimate of the disturbing effects on their orbit, and iv) the estimates of their drag coefficient C_D. We focus here on the last point, i.e., on the decay of the semi-major axis of LARES. In particular, in developing such activity, we jointly used the software GEODYN II with a modified verion of SATRAP (SATellite Reentry Analysis Program). SATRAP is able to load several different models for the Earth's atmosphere together with the appropriate geomagnetic and solar activities indices, see [34], [35] for details. Therefore, with SATRAP we have been able to investigate directly the impact of the neutral drag on the satellites orbit using the current best available models for the atmospheres main constituents.

With GEODYN II we first performed a POD of LARES over a time span of about 3.7 years. From this analysis we have been able to measure an orbital decay in the residuals of the satellite semi-major axis of about 1 m/yr, that corresponds to a transversal mean acceleration of about $-1.444 \cdot 10^{-11}$ m/s^2, see Figure 3. In this POD neither the neutral drag nor the thermal effects have been included in the dynamical models of GEODYN II.

Then, with SATRAP, the neutral drag perturbation has been computed over the same time span accounting for the measured decay and considering the real evolution of the solar and geomagnetic activities for several atmospheric models. In particular, assuming as reference for the unmodeled transversal acceleration due to the neutral atmosphere the above value, the drag coefficient estimated by SATRAP is comparable to the average value estimated by GEODYN II in a least square fit of the tracking data. We obtained $C_D \lesssim 4$. This means that the current best models developed for the atmosphere behavior are able to account for the observed decay, within their intrinsic errors (around 15%) and range of applicability.

Finally, it is worth of mention that after modelling the neutral drag perturbation in GEODYN II, a residual and very small decay is still present in the integrated residuals of LARES semi-major axis. Therefore, a further analysis is needed in order to extract from the observed decay a possible smaller contribution related with other unmodeled effects, as the thermal ones, acting on the satellite. In this context it will be necessary to fix the contribution of the signature of the drag and of the thermal effects in the residuals of the other orbital elements of LARES.

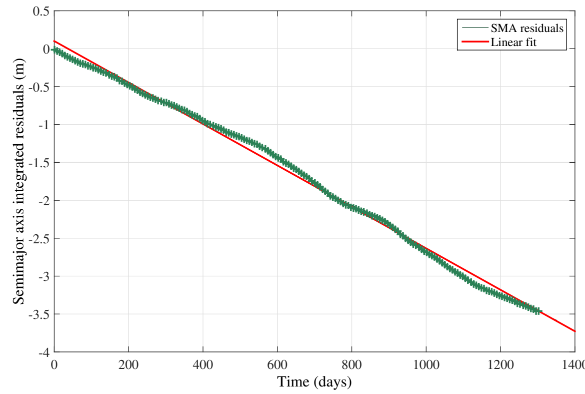


Fig. 3. Decay of LARES semi-major axis residuals (green) as obtained by GEODYN II over a time span of about 3.7 yr and its best fit with a straight line (red). The observed decay of LARES semi-major axis residuals is 0.9988 m/yr (i.e., about 2.7 mm/day!). Such decay corresponds to an average along-track deceleration of about $1.444 \cdot 10^{-11}$ m/s^2. Very interesting, after modeling the neutral drag perturbation in GEODYN II, we still observed a residual decay that corresponds to an average deceleration of about $2 \cdot 10^{-13}$ m/s^2. Probably, the thermal-thrust effects come to play a role at this level.

D. Tides

With regard to the gravitational perturbations, in our previous work [8] we concentrated upon solid tides. During last year we extended our analysis to ocean tides as well as to the POD improvements related with the use in GEODYN II of different models for the background gravitational field in order to identify the best model(s) to be used for the relativistic measurements. In this paper we focus on some of the results we obtained for the impact of ocean tides in the satellites node. Contrary to the case of solid tides which are (mainly) due to the elastic reaction of the whole Earth to the tide-generating potential, the reaction of the ocean leads to equipotential surfaces. In fact, the relative displacement between the ocean and the solid Earth is the origin of ocean tides. Although ocean tides account approximately for only 10% of the total response to the Moon and Sun tidal disturbing potential, they are characterized by larger uncertainties with respect to solid tides, because of the greater complexity of the involved phenomena, and consequently major difficulties in their modelling.

In Table II are shown the results (amplitude and period) we obtained for a few long-period ocean tides in the case of the right ascension of the ascending node Ω of the three satellites. We considered the GOT99.2 ocean model [36], that is the one currently implemented in our setup of GEODYN II, but other more recent models are under consideration for future analysis. As we can see, in the case of LARES, due to its lower height, the perturbation provoked by the tides has a larger impact on the orbit. In particular, the uncertainty in the amplitude of the K_1 tide, which, for each satellite, has the same period of that of the right ascension of the node, represents a relevant issue in this context because the ascending node of the two LAGEOS and of LARES represent the main observables for the relativistic measurements to be performed.

E. Precise orbit determination

As highlighted in Section II, a POD for the orbit of the considered satellites is of fundamental importance in order to perform reliable and refined measurements in gravitational physics, we refer to [9], [16], [17], [37]–[43] for details, as well as in space geodesy applications (see [23], [44]–[49]).

Obviously, because of the intrinsic precision of the SLR data a comparable quality for the models used to describe each satellite orbit is needed.. These models can be roughly divided into three main categories:

- satellite dynamics;
- measurement procedure;
- reference frame transformations.

In this context, we are trying to follow as much as possible well established modelling conventions and resolutions, as those from the International Earth Rotation and Reference Systems Service (IERS) and from the International Astronomical Union (IAU). The IERS Conventions (2010) [50] constitute the general framework for reference systems related issues and measurement models. The IAU 2000 Resolutions [51] recommend the use of a well-defined relativistic framework in dealing with celestial mechanics in the Solar System. We stress that such conventions and resolutions, in turn, are not static and are usually updated to cope with the state of the art in observation and theory. At the end, all these activities will lead us to fix, on the basis of the i) tracking precision, ii) level of gravitational and non-gravitational perturbations to be modelled and iii) final precision of the POD, the amplitude of the unmodelled relativistic effect(s) to be measured with our least-squares fit and the consequent analysis of the orbital residuals of the satellites. The current LARASE project modelling setup is described in Table III.

Therefore, our preliminary analyses included a preparatory data reduction for the satellites orbit with a tailored setup for the models implemented in the software. Together with the satellites state-vector and selected station biases, the radiation coefficient and the corrections to polar motion and length of day have been estimated. Empirical accelerations have been also used when deemed necessary. The results of these preliminary analyses are shown in Figures 4 and 5. In Figure 4 it is shown the post-fit root-mean-square (RMS) of the satellites (mean) range residuals, while in Figure 5 the mean range residuals are shown.

As we can see from Figure 4, while LAGEOS and LAGEOS II orbits are recovered with a mean error roughly between 1.5 and 1 cm (or smaller), LARES orbit has a slightly higher error. This is due to a currently non-optimal modelling for the dynamics of LARES. The decreasing trend that we obtained for the RMS of the range residuals in the case of the two LAGEOS, which approaches the 5 mm (mean) value at the end of the time span, is also in quite good agreement with the results obtained from the data reduction of the orbit

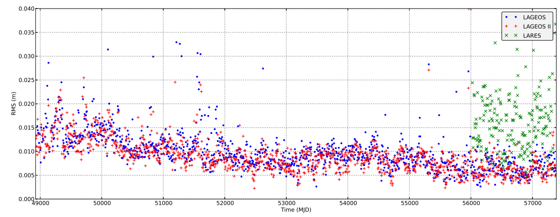


Fig. 4. Post-fit RMS of the range residuals for the POD of the three satellites when empirical accelerations have been estimated. The arc length is 7 days. LAGEOS (blue), LAGEOS II (red) and LARES (green), the units are in meters. The time is given in Modified Julian Date (MJD). In the case of the two LAGEOS, the starting epoch (MJD 48925) corresponds to October 24, 1992, while, in the case of LARES, the starting epoch (MJD 55975) corresponds to February 18, 2012. Notice the higher uncertainty associated with the LARES analysis, showing its currently non-optimal modelling.

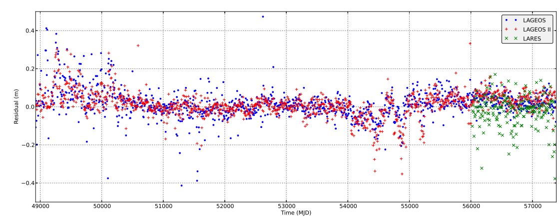


Fig. 5. Mean of the range residuals for the POD of the three satellites when empirical accelerations have been estimated. The arc length is 7 days. A mean close to zero corresponds to a good modelling of the satellite orbit.

of the two LAGEOS satellites performed by the main Analysis Centers of the ILRS network.

IV. Conclusions

We have described the state of the art of the LARASE research program in terms of the improvement currently reached in the satellites orbit modelling and we have recalled the main objectives of this collaboration in the field of fundamental physics measurements. The activities aiming to improve the modelling setup are very important not only because they allow to increase the final precision in the orbit fit of the considered satellites, i.e., in their POD, but especially because these activities allow to estimate an accurate error budget for the relativistic parameters to be measured for what regards their systematic error sources. A number of papers are in preparation with the details of the improvements reached in the modelling of: i) solid and ocean tides, ii) neutral drag, iii) spin evolution and iv) POD precision in various scenarios. The current level reached in the modelling of the orbit of the three satellites, although it must be further improved as part of the final objectives of LARASE (see [9] for details), is anyway enough to allow to start a new series of relativistic measurements also including the data analysis of LARES in addition to that of the two LAGEOS.

Acknowledgment

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TABLE II. PERTURBATIVE AMPLITUDES IN MAS PER PERIOD ON THE NODE Ω DUE TO OCEAN TIDES: $\ell = 2, p = 1, q = 0$. THE PERIODS ARE IN DAYS WHILE THE AMPLITUDES ARE IN MILLI ARC SECONDS. THE POSITIVE SIGN (+) OF THE PERIOD REFERS TO WESTWARD TIDAL WAVES, WHILE THE NEGATIVE SIGN ($-$) REFERS TO EASTWARD ONES.

Tide	LAGEOS		LAGEOSII		LARES	
Doodson number	Period [days]	Amplitude [mas]	Period [days]	Amplitude [mas]	Period [days]	Amplitude [mas]
065.455 (M_m)	27.55	-0.53	27.55	0.97	27.55	2.62
056.554 (S_a)	365.25	-21.47	365.25	39.30	365.25	106.40
075.555 (M_f)	13.66	-0.62	13.66	1.14	13.66	3.08
057.555 (S_{sa})	182.62	-6.52	182.62	11.94	182.62	32.32
165.555 (K_1)	1043.67	162.30	-569.21	-37.23	-233.53	-173.64
163.555 (P_1)	-221.36	-11.59	-138.26	-3.05	-102.48	-25.66
145.555 (O_1)	-13.84	-1.90	-13.34	-0.77	-12.91	-8.45
135.655 (Q_1)	-9.21	-0.29	-8.99	-0.12	-8.79	-1.31
275.555 (K_2)	521.835	-7.18	-284.61	-7.17	-116.77	-7.96
273.555 (S_2)	-280.95	14.84	-111.24	-10.75	-71.23	-18.63

TABLE III. CURRENT LARASE MODELLING SETUP FOR THE ANALYSIS OF GEODETIC SATELLITES SLR DATA. THE PERTURBATIONS ARE DIVIDED IN GRAVITATIONAL AND NONGRAVITATIONAL PERTURBATIONS AND IN REFERENCE FRAMES REALIZATIONS.

Model for	Model type	Reference
Geopotential (static)	EIGEN-GRACE02S	[52]
Geopotential (time-varying, tides)	Ray GOT99.2	[36]
Geopotential (time-varying, non tidal)	IERS Conventions (2010)	[50]
Third–body	JPL DE-403	[53]
Relativistic corrections	Parameterized post-Newtonian*	[51], [54]
Direct solar radiation pressure	Cannonball	[12]
Earth albedo	Knocke-Rubincam	[55]
Earth-Yarkovsky	Rubincam (1987-1990)	[56]–[58]
Neutral drag	NRLMSISE-2000	[59]
Spin	LARASE (2015)	To be published
Stations position	ITRF2008	[60]
Ocean loading	Schernek and GOT99.2 tides	[12], [36]
Earth Rotation Parameters	IERS EOP C04	[61]
Precession	IAU 2000	[62]
Nutation	IAU 2000	[63]

*We emphasize that selected parts of these post-Newtonian corrections have not been included in the modellization setup used for specific analyses of relativistic effects.

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