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The role of the power law distributions in describing social complex systems. Implications in the field of knowledge performance assessment

PhD candidate: Stefania Giuffrida

Tutor and coordinatore:
Prof. Agostino La Bella

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A Francesco e Simone

"It does not matter how frequently something succeeds if failure is too costly to bear"

Nassim Nicholas Taleb – Fooled by randomness

Smaller and smaller
M.C. Escher (1956)

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INTRODUCTION

For centuries, scientists tried to connect all natural phenomena to the same, simple laws of physics by a reductionist approach, which explains the behaviour of all the systems by the properties of the first components of the matter (atoms, molecules, etc.). However, world around us is anything but simple: the biological structures, the dynamics of the planet (volcanoes, oceans, earthquakes, etc.), such as the political, social-economics phenomena are characterized by an intrinsic variability, which cannot always by easily described by the basic law of physics. The term “complexity”, indeed, has been coined to define a set of observed properties of natural, biological, social and economic entities elsewhere indescribable and unpredictable.

Although a unique definition of complexity doesn’t exist, in literature the “complex systems” are always described as composed by many sub-parts, they are far from equilibrium, non-linear interactions dominate and chaotic behaviour breakdowns the symmetry of existing structures. A critical point exists where some kind of self-organization emerges apparently without any central control. To this transitional situation, between order and disorder, has been often refereed as the “edge of chaos”, for some authors the necessary condition for life to occur (Waldrop, 1993).

Per Bak (1996) observed that real world is generated by instability and due to a succession of small or large catastrophes, as described by the self-organized criticality theory. Using the words of Nassim Nicholas Taleb, it is “a *small number of Black Swans”* that “*explain almost everything in our world, from the success of ideas and religions, to the dynamics of historical events, to elements of our own personal lives*” (Taleb, 2007). But what is a “black swan” and how it is connected with the complex theory? A black swan is a rare, unpredictable event, potentially with extreme impact: these three features have as a common statistical representation a power law distribution. The long, fat tails of this kind of statistics, in fact, can explain many empirical evidences, from the frequency of large disastrous earthquakes in nature to the periodical financial market crashes in economy. Whenever not linear interactions prevail and causes and effects are not proportional each other, the complex interactions among the component of a system are well described by a power function rather than by a Gaussian distribution, typical, instead, of linear, random, at equilibrium events.

The initial goal of this research is to verify the possibility to model one or more aspects of the "life" of a complex organization in terms of a power law distribution, in order to propose new approaches for the personnel and/or the processes management.

The idea originates from the study of the literature, where is abundantly treated the fascinating capability of the power functions to describe phenomena apparently very diverse in a large range of research fields. Power laws, in fact, have been found, since the seventies, as signature of physical, chemical, natural, but also social and organizational phenomena. Since Ilya Prigogine's work, Nobel graduated for his studies on irreversible processes, it has been clear that a new era of science has been starting, when concepts like "emergence", "scalability", "self-organization", "criticality", "chaos" became very popular. With the creation of the Santa Fe institute, New Mexico, then, the research on complexity started to involve systematically not only physics and computation, but also the social systems.

Nevertheless, an important limit of the literature for the transferability of complexity concepts to the social sciences is the large use of metaphors. A social system (i.e. an organization or an economic entities), indeed, is generally composed by many agents, whose relations can be affected by emotions, moods, aptitudes that can generate unpredictable effects, such as in a metastable situation "on the edge of chaos". For this reason, many authors transferred some complexity concepts into their studies on social-economic systems, sometimes with very disputable parallelisms to natural and biological world.

Starting from this point and following the evolution of social research in the framework of the complex theory, our first aim is modeling a managerial research problem through a rigorous mathematical model. For this purpose, in the first part of this thesis, we introduce the reader to the complex theory, since its original researches in the fields of hard and natural sciences (Chapter 1), in order to describe the fundamental basis and the common features of the complex systems that, during the last decades of research, have been characterizing always more also the social, organizational and managerial sciences (Chapter 2). The self-similarity of the complex structures, due to the scale invariance property of the power laws, is one of the most outstanding example. The scalability has been proved in a variety of phenomena, like nodes of internet, number of academic citations, prices fluctuations, size of cities, etc.

The scientist who more disclosed in an impressive and, in some way, artistic manner this aspect of complexity of nature was the Polish Lithuanian mathematician Benoit Mandelbrot, the father of the fractal geometry, who showed how small parts of many natural objects resemble the whole (from the cauliflowers to the trees, from the coasts of Britain to the galaxies, etc.). He had the merit to have applied mathematical concepts, otherwise purely theoretical, to economy and finance, demonstrating how Gaussian terms like mean and variance become meaningless in a scalable financial world, where rare events are not so rare, due to the fat tails of the power law distributions.

Other two concepts intrinsically linked to the development of studies on complexity are the self-organization and the emergence. In a system, a property is emergent if it cannot be explained from sum of interactions of the lower level components of the system itself. In a living system, for example, from the microscopic complex interactions between non-living molecules, at the macroscopic level, “emerges” a living cell. Arising from the model of the self-organized criticality, the emergent properties are the possible outputs of a system being on that edge between order and disorder, where unpredictable behaviour can occur. In natural science, many studies dealt with these issues and they have been applied to understand empirical evidence of local random interactions between group of agents, sometimes leading to the emergence of organized global behavior, such as in flocks of birds, ant colonies or schools of fishes, as described by the so called “swarm theory”. In social science, the main applications of these models regard human collective behaviour, from communication to marketing science.

After the discussion about the characteristics of complex systems, we try to face the difficulty in modeling social-economic complex systems, respect to physical or chemical phenomena, reviewing in particular those literature, mainly regarding economic issues, which adopted a more rigorous statistical and mathematical approach. In Chapter 3, in fact, we focuses on the growth dynamics model applied to economic entities like firms, countries, universities (Stanley et al., 1996; Amaral et al., 1997; Axtell, 2001; Lee et al., 1998; Gabaix, 1999; Plerou, 1999). Exploiting the scale free property of the power laws, these authors described a common growth mechanism for these apparently so different subjects, for which the fluctuations of the annual growth rate distributions always scale as a power law of the initial size. Actually, all these bodies share a complex structure (firms are composed by divisions, countries by regions or cities, universities by departments) that make them to evolve in an remarkably analogous way. As original contribution, we test the model also for Italian universities, even if limited to the published ISI papers. Our results confirm that also our national academies growth according the previous model.

The literature review and the in-depth analysis performed in the first is preparatory for the more explorative second part of our dissertation. The identified formal model is applicable to complex social organizations; it takes advantage of the scale invariance of a determined feature of these systems (in this case the probability distribution of the size of the entity); anyway, its fundamental weakness consists in the fact that it is substantially only descriptive of the complex phenomenon. Indeed, the considered growth model doesn't provide any tool for the management or the improvement of an enterprise, a country or a university, but it just detects a transversal property among the entities and try to find and explanation for that.

On the other hand, as in the model is shown how self-similar probability distributions can collapse on a single curve by a scaling process, this represents an intrinsic potential for the normalization of data.

In Chapter 4, we demonstrate that the scale-free property of power laws can be used in an innovative manner, as a scaling relationship to normalize data, in order to be independent of the size of the studied organization.

Since one of the field in which the dimensional bias represents a long-standing problem is knowledge and research quality assessment of the university, we decide to verify if a normalization process, based on the power law scale free property, could be applied to our case study, the most recent Italian Evaluation of Research Quality (ERQ), concerning 2004-2010 period of activity of the Italian university system (ANVUR Final Report, 2013).

Previous studies have investigated the correlation between university size and knowledge productivity, mainly to understand if it was possible to define a critical mass at which research groups can attain increased research capabilities and greater attraction of funds, or as part of the debate over the most suitable methods for normalization of data in research assessments (Adams and Thomson, 2011; Morgan, 2001; Kenna and Berche, 2011a; Kenna and Berche, 2011b; Abramo et al., 2012). Nevertheless, this literature often fails to fully consider the intrinsic complexity of the university structure and the related impacts on the measurement of research productivity.

Taking a new approach, respect to the studies on the complex behavior of social systems like firms, countries and universities, in this work we exploit the scale-free property of power laws for the normalization of data from differently sized universities, referring to productivity assessment. Our contribution aims at demonstrating that the fluctuations observed in the distribution of an indicator of university productivity of knowledge are correlated to university size in terms of a power law. This fact introduces a dimensional bias that can partially falsify the results of periodical university assessment exercises, favoring one university over another, sometimes with unfair consequences, particularly where a share of public funding is allocated on the basis of the research assessment.

The case considers 769 average scores obtained in the 14 research and teaching areas recognized in the Italian system, by 93 public and private universities in the EQR.

We show that: i) the distribution of the average scores for the university disciplinary areas, grouped for size, follows an exponential decay; ii) the width of the distribution scales as a power law of size, with the scaling exponent $\beta \approx 0.13$; iii) it is possible to apply the methodology to create a scale-independent rank of university performance, beginning from the measured data.

The proposed framework is intended to be generic, to be applied in any knowledge productivity or quality assessment at any time that the size of the monitored organizations becomes relevant and must be appropriately weighed to validate the evaluation process. In the case study included in this thesis we apply the described method to reduce the dimensional bias emerging in the knowledge productivity and quality assessments of research organizations, but the same steps can be also followed in the case of knowledge assessment in other sectors, such as in the entrepreneurial system, where dimensional bias can be relevant. However further validation of the approach must be done on a case by case basis.

CHAPTER 1: THE COMPLEX SYSTEMS

1.1. Definition and characteristics of complex systems

Complexity science includes a collection of theories, models, concepts and metaphors, which in the last seventy years have characterized branches of the knowledge very different each other: from physics to economy; from chemistry and biology to social sciences. In every research field, the effort was devoted to understanding why many phenomena appear complex. In physics, classical Newtonian mechanics reduces all complex systems to its basic components (atoms or particles) and describes the properties of the whole as the result of the deterministic and reversible evolution in time of its constituents. The classical Newton law, $f = ma$ (elegantly expressing that, when a force acts on an object, this accelerates proportionally to that force) is deterministic because a certain initial state produces a predictable final output. At the same time, it is reversible, that means that, by the knowledge of the starting conditions, it is always possible to predict the state of the system in the present, past and future. Newton law describes gravitational motion of planets and galaxies, such as the behavior of the electromagnetic waves in the Maxwell's equations and remains valid even in the Einstein's theory of relativity (coherent with the classical mechanics for objects moving at high velocities). Similarly, within the quantum mechanics, the Schrödinger equation determinates a reversible and deterministic change of the quantum state of a system (represented by a wave function), which can be compared to a reversible motion of a particle along a particular trajectory[1]. This equation evolves such as a corpuscular equation, due to the corpuscular-wave dualism of the particles. As well as a simple algebraic equation, also the Schrödinger equation admits as solution not a number, but a function (ψ), which has inside all the necessary information to know the state of the physical system in the past, present and future.

[1] The Schrödinger equation $i\hbar\, \partial\psi/\partial t = \widehat{H}\psi$ where $\hbar = h/2\pi$ with h Planck constant that is about $6.62 \cdot 10^{-34}$ Js and i is the imaginary unit so that $i^2 = -1$, says how the wave function ψ of the system (that is its quantum state) is modified by the application of the operator H called Hamiltonian operator, describing the total energy of the systems.

The argument that explains why the deterministic nature of the Schrödinger equation is not in contrast with the probabilistic nature of the quantic phenomena is that the Schrödinger equation doesn't predict where i.e. an electron will be at any time, but rather says exactly (deterministically) which is the probability to find the same electron in any point of the space and at any time.

Moreover, any property described by the wave function ψ has an indefinite value (the effective value is determined by the square module of ψ), until we do not make a measure, in some way altering the system and introducing an element of irreversibility, as expressed by the Heisenberg uncertainty principle[2].

However, generally in nature, the interaction among many different components of a system can be difficult to model, if we exclude limit cases i.e. ordered systems like crystals or sets of not interacting gaseous atoms at the equilibrium (in this case individual atoms move at different velocities in different directions but, on average, all atoms behave the same way and can be represented by classical rules).

The world around us is, actually, very different by the "classical" models. The biological forms of life, the interacting cells in the brain, the financial exchanges in the global economy, the behavior of human masses are all examples of complex phenomena, apparently not responding to the basic law of physics.

Due to the difficulty to directly connect complex phenomena to physical mechanisms working at a deeper level, researchers often referred to these phenomena as "emergent". Observed phenomena of self-organization create order out of the disorder and are responsible for the most of patterns and structures we find in the natural world.

Therefore, in contrast with the deterministic predictable systems, modeled by the classical physics, there are a lot of real world situations, where detailed long-term prediction are not possible.

In such complex scenario, also events considered statistically rare can give rise to potentially catastrophic consequences, when some components of a systems stimulate many others by a cascade effect. In geophysics, for instance, this aspect has been carefully considered in studying the earthquakes, as we better explain in the paragraph on the self-organized criticality (§ 2.2)

[2] It is the measuring process, according Bohr and Rosenfeld (Rosenfeld, 1965), to introduce an irreversibility element in the quantum systems. When we perform a measure in a quantum system, the wave function will collapse on a point because the measured property assumes, in that instant of time, only a particular value among those possible before the measuring (an eigenvalue of the wave function). The process to reducing the wave function on a particular eigenvalue is not reversible.

In literature, a lot of complex systems have been described (see Johnson, 2009 for an simple introduction to the most common daily life complex phenomena), all sharing the following general characteristics:

- there is a large number of elements interacting each other in the system (e.g. molecules in a biochemical system; investors and traders in the stock market; users in Internet; insect colonies or flocks in the animal world, etc.);
- the observed parameters are dynamic, far from the equilibrium and exhibit scaling properties (Stanley et al., 1996; Lee et al., 1998; Mandelbrot, 2001);
- some kind of self-organization emerges, these emergent phenomena can be not explained by the properties of the single component and they happen without any central control, as theorized by the swarm theory (Helbing, 1991);
- the interaction are typically non-linear, so effects are not proportional to the causes of the processes (it is said “small cause, large effect”), potentially generating also chaotic behaviors.

This last feature is particular important for its potentially disastrous consequences. When the interactions among components of a complex system are not linear, an amplification or a dampening of small initial fluctuations can happen, respectively if effects are larger or smaller than the causes. In the former case we talk about “positive feedbacks” among the components of the multi-agent system, so that also small initial perturbations can be transmitted through the system, occasionally inducing cascading effects (see the “butterfly effect” in the following paragraph). It is the case, for example, of the global epidemics, the shutdown of the power grids or the unexpected traffic jam (Helbing, 2009).

1.2. Irreversibility, non-linearity and chaos in the dynamics of complex systems

The challenge of the complexity theory is to prevent catastrophic events potentially generated by the chaotic behavior of some systems (i.e. cancer in biology, thunderbird in meteorology, market crash in economy, etc.). Often concepts like “complexity”, “non-linearity” and “chaos” are confused or used as synonyms. There are, instead, substantial differences among them.

As we said, complex systems are in general described by non-linear equations and the associated phenomena are mainly irreversible. The study of non-equilibrium dynamic systems became quite popular since the eighties, especially thanks to the work of the Belgian physical chemist Ilya Prigogine, Nobel Prize in Chemistry in 1977 for his contribution to the research on the irreversible phenomena and the non–equilibrium thermodynamics. He stated that the nature shows both irreversible and reversible processes, but the former are the rule, the latter the exception (Prigogine, 1997a).

The irreversible behavior of a thermodynamic system cannot be described by the Newton's laws, that are reversible and deterministic. Irreversibility means that the "arrow of time" exists and it is not possible to predict the state of a dynamic system at any instant of time, future or past, only by the knowledge of its initial conditions (Prigogine, 1997b).

Indeed, according to the second law of thermodynamics there exists a function S (entropy), which increases monotonically so that

$$dS = (dS_e + dS_i) \geq 0 \qquad (1)$$

until it reaches its maximum at equilibrium ($dS = 0$), being dS_e and dS_i respectively the entropy flow from the surroundings and the entropy production from irreversible processes inside the system.

Only irreversible processes contribute to the entropy production. For isolated systems $dS_e = 0$ and the (1) becomes

$$dS_i \geq 0 \qquad (2)$$

The equation (2) expresses the property for which, in an isolated system, S can only increase in time, associating irreversible processes to the arrow of time. In other words, the equilibrium state represents an "attractor", towards which all the non-equilibrium states evolve (Prigogine, 1978).

The apparent contrast between Newtonian motion laws (deterministic and reversible) and the thermodynamics (describing irreversible processes moving towards the attractor state of equilibrium) has been solved by the statistical mechanics.

Following in the Maxwell footsteps, who formulated the equations for the electromagnetism, Boltzmann applied the "probability" concept to the physics of gas. Let's consider the macroscopic state of a system of N particles with energy E confined by a permeable partition in two equal parts of a box.

To calculate the potential distribution of particles N_1 and N_2 in the two parts of the box, at the microscopic level, the dynamics of the single particle can be represented by the classical motion law. At the macroscopic level, the system evolves toward the most probable state, that is the thermodynamic equilibrium. This is reached when the fraction of particles N_1 and N_2 in the two halves of the container become and remain essentially equal, with N large enough (and exactly equal when $N \rightarrow \infty$). In fact, even if there are always particles flowing from one to the other side of the partition, statistically, their number will be in average the same $N_1 = N_2$.

The so called "Boltzmann order principle" is expressed through the competition between energy E and temperature T in the Boltzmann factor $e^{-\frac{E}{kT}}$, where k is the Boltzmann universal constant, that is a competition between energetic and entropic factors. While, in fact, at the thermodynamic equilibrium, entropy is maximum for an isolated system, free energy $F = E - TS$ is minimum. Ordered structures (i.e. the crystals) appear by lowering of temperature, becoming small the entropy contribution to the free energy. At high temperatures, instead, the entropy is dominant and the molecular disorder produces the liquid and gaseous states (Prigogine and Stengers, 1979).

Although the equilibrium thermodynamics well describes a lot of physical-chemical phenomena, it does not explain many ordered structures present in nature, many of which existing far from the thermodynamic equilibrium. It has been observed that the thermodynamic evolution towards the equilibrium conflicts with the theory of Darwinian evolution for biological systems, that is a statistical selection of rare events (Bak, 1996). Indeed, irreversibility in thermodynamics expresses a statistical way of evolution to the most probable state, where the disorder is maximum and rare configurations statistically disappear. In biology, in contrast, irreversible processes happens with an increase of organization and with the emergence of ordered complex structures. Prigogine wondered if it was possible to conciliate these two apparently opposite aspects of evolution.

In 1977, he says: "*To obtain a thermodynamic theory for this type of structure*" (the living system) "*we have to show that non equilibrium may be a source of order. Irreversible processes may lead to a new type of dynamic states of matter which I have called dissipative structures. […] These structures manifest a coherent super-molecular character which leads to new quite spectacular manifestations*" (Prigogine 1977).

Indeed, it is possible to distinguish three big fields of thermodynamics: the "equilibrium thermodynamics", where the entropy production, the rate of irreversible processes (flows) and the involved forces are all zero; the "near-equilibrium thermodynamics", where thermodynamic forces are weak and the flows are linear functions of the same forces; the "non-equilibrium thermodynamics", where flows are non-linear functions of forces.

Far from equilibrium, systems are generally open and can import energy from the environment, increasing their "negentropy" (the negative entropy) and producing self-organized ordered structures (those that Prigogine called "dissipative structures") where imported energy is then again dissipated (with a new increase of entropy). This type of phenomena are well known in fluid hydrodynamics, where, at high speed, the liquid flow can create turbulences that appear chaotic at a macroscopic level, but that are highly organized at a microscopic scale, so that the transition from a horizontal fluid layer to a turbulence represents a self-organization process. In the Bénard[3] problem, a horizontal fluid layer is heated from below so that the temperature gradient opposes the effects of the gravitational forces. For small fluctuations of the temperature (states closed to equilibrium) only a conductive bottom-up transfer of heat happens. But, for critical values of the gradient, a spontaneous internal convective motion is established, which generates very regular organized patterns like hexagonal cells (Prigogine and Nicolis, 1971). Rayleigh–Bénard convection is a typical example of how microscopic perturbations of the initial conditions produce a non-deterministic macroscopic effect. This inability to predict long-range conditions and sensitivity to initial-conditions are characteristics of chaotic or complex systems.

Almost at the same time of Prigogine, at the Santa Fe Institute in New Mexico (see Waldrop, 1993), several scientists dealt with non-linearity and chaos theory, using cybernetics and computer simulation, but their approach to the chaotic systems was, in general, deterministic as it followed the classical Newtonian equation of trajectories.

In fact, if small errors are made in measuring, at time t, the variables describing two apparently identical states of a dynamic system described by non-linear equations, then even large differences can be observed later in time.

[3] Henri Claude Bénard (1874–1939) was a French physicist who studied convection phenomena in liquids. He gave his name to the Rayleigh–Bénard convection, occurring in a plane horizontal layer of fluid heated from below, in which the fluid develops a regular pattern of convection cells known as Bénard cells. If the gradient of temperature increases , the structures become more complex and the turbulent flow becomes chaotic. Convective Bénard cells tend to approximate regular hexagonal prisms or square prisms or spirals.

This can generate an apparently stochastic behavior, even if only deterministic equations have been used to describe the system: this is the so called "deterministic chaos" (Schuster and Just, 2005; Anderson, 1999). Random fluctuations can affect such a system amplifying non–linear interactions (positive feedbacks) among its components, since the system reaches a critical point, called "bifurcation point".

If the evolutionary trend of a system is described by a diagram of bifurcations with many branches, the choice of a branch, at a bifurcation point, depends on small local fluctuations that constantly occur. From a bifurcation point, small fluctuations can induce the system towards different configurations, some stationary and stable, others where chaos prevails and any symmetry of existing structures is broken. So, at any bifurcation threshold, the system continues absorbing energy and different configurations can emerge: stationary states (we say there is a stable attractor); oscillatory states (limit cycle attractor), chaotic states (strange attractor). Non periodic strange attractors, in fact, characterize systems with chaotic behavior. Mathematically, given a dynamic system $u(t)$ in an Euclidean space $\Re^N$ where $u(0)$ is the state of the systems at t=0, we say the system has an attractor if exists a subset A of $\Re^N$ so that, for each $u(0)$ and for large values of t, $u(t)$ is close to A (Mandelbrot, 1983).

Strange attractors are those points around which the system continuously and chaotically orbits, without any clear reason. In general, in a non-linear system, small perturbations to its initial conditions can produce unpredictable and amplified systemic[4]

effects. This is sometimes referred to as "butterfly effect" because two non-linear systems, even starting by the same initial conditions, can evolve differently in the course of the time. The name of the effect derived from the studies of Edward Lorenz[5],

[4] There are a lot of examples of natural phenomena where non-linear interactions produce systemic effects. For example the glycolysis, the set of metabolic reactions that convert glucose in ATP molecules (adenosine triphosphate), the energy source for the life beings, is generated by a time oscillation in the concentrations of the intermediates, which is a typical non-linear, far from equilibrium phenomenon of amplification of a chemical signal. In physics, the laser is an emission of coherent light generated through a process of optical amplification of non-linear interactions among atoms, put inside an optical cavity and excited by means of an outside light source or an electrical field. This supplied energy is absorbed by atoms and transformed in emitted light. If the applied pump power reaches a particular threshold, than the power of the recirculating light can rise exponentially and all the atoms start to emit a coherent, uniformly polarized and often monochromatic light (the laser beam).

[5] Edward Norton Lorenz (1917-2008) was an American mathematician and meteorologist, who developed a simplified mathematical model for the atmospheric convection, where a system of three ordinary differential equations (the Lorenz equations) admits both stable and chaotic solutions, depending on the value assumed by the equation's parameters. In the first case we talk about fixed point attractors; in the former, we say that a strange (or Lorenz) attractor exists.

who described how his weather model was influenced by small perturbations (Lorenz, 1963), so that a very small change in the initial conditions can generate significantly different outcomes.

At the 139th meeting of the American Association for the Advancement of Science in 1972, in fact, he title his speech *Does the flap of a butterfly's wings in Brazil set off a tornado in Texas?*. The same concepts are also at the base of the "self-organized criticality" theory, we will expose in the next chapter.

Non-linearity may create order, bringing it out of the complexity of elementary processes and allowing new coherence to emerge. Fluctuations with an external or internal origin, can so generate new structures.

CHAPTER 2: WHY THE POWER LAWS BETTER DESCRIBE NATURAL PHENOMENA

2.1. From a Gaussian to a Paretian description of phenomena

In the previous chapter we discussed how in a complex system the interactions among the components are typically non-linear. This characteristic is the main reason for which also deterministic systems may show, in time, unpredictable behavior, sometimes even with catastrophic consequences.

Furthermore, the interactions among the system elements often change the statistical model that best fits the general behavior of the global system. Complex phenomena exhibit Paretian rather than Gaussian distribution, mainly due to the correlation among the events. Normal distributions typically describe the dynamics of systems at equilibrium, where any linear applied solicitation generates effects proportional to the causes, with a global behavior of the system due to the superposition of the single components' contribution. In a non-linear system, where the parts (atoms, molecules, organisms, processes, people, companies, nods of a network, etc.) interact, influencing each other with positive feedbacks effects, than the complexity increases and a power law distributions emerge.

As we have seen in classical physics and chemistry (Chapter 1), also in economics the linearity of phenomena is generally considered as an acceptable assumption, but it often fails in describing real situations, for instance why the effects of a bankruptcy of a firm propagate through its network of creditor–debtor (Hong et al. 2007) or why some disastrous rare events like the market crashes are more frequent than expected (Sornette et al, 1995). For example, the Black-Scholes' model, extensively used in the financial sector, assumes: prices distributed on a bell curve; a constant volatility during the life of the options; prices not experimenting jumps; no taxes and commissions to be paid. These conventions imply a normal distribution of events, with no possibility to predict events like that of 19 October 1987 (the Black Monday world crash). In fact, the Black Monday would be at 35 sigma from the mean of the distribution, which is equivalent to attribute to such an event a probability of zero (Hudson and Mandelbrot, 2004).

Indeed in nature, as in the most of social phenomena, interconnected events are more common than independent ones. As already said, the statistical distribution governing interrelated events is not normal, but rather it has heavy tails. As a consequence, while for a normal distribution we can well define mean and variance, in a power law Pareto distribution the statistical moments depend on the exponent of the power law. In fact, if we express the Pareto probability distribution as a function:

$$p(x) = Cx^{-\alpha} \qquad \text{with} \quad x \geq x_{min}$$

where the normalization constant is derived from the condition

$$\int_{x_{min}}^{\infty} Cx^{-\alpha} dx = 1$$

than the first moment (the mean or average) is

$$< x > = \int_{x_{min}}^{\infty} xp(x)dx = \frac{C}{2-\alpha} \left[x^{2-\alpha}\right]_{x_{min}}^{\infty}$$

which is finite only for $\alpha > 2$.

The generic momentum k, instead, is

$$< x^k > = \int_{x_{min}}^{\infty} x^k p(x)dx = \frac{\alpha - 1}{\alpha - 1 - k} x_{min}$$

which is finite only for $\alpha > k + 1$.

Consequently, the variance of the distribution ($k = 2$) will be finite only for exponents $\alpha > 3$. This conditional definition of mean and variance do not permit to precisely characterize the distribution, nor in terms of a stable average value, neither in terms of confidence intervals for statistical significance of data (Andriani and McKelvey, 2005).

So, in comparison with the vanish tails of the Gaussian distribution, the fat and more slowly decaying tails of a Pareto distribution are the reason of the increased probability for rare events to happen (Fig. 2.1). In a log-log plot the distribution $p(x)$ is a straight line, because

$$log[p(x)] = log(Cx^{-\alpha}) = -\alpha log\, x + log\, C$$

As example, we cite the statistical probability of a financial crisis: according to the Gaussian point of view of Buchanan (2004), a market loss of 10% in a day happens every 500 years; according to the power law distribution described by Mandelbrot (Hudson and Mandelbrot, 2004), it occurs one every 5 years.

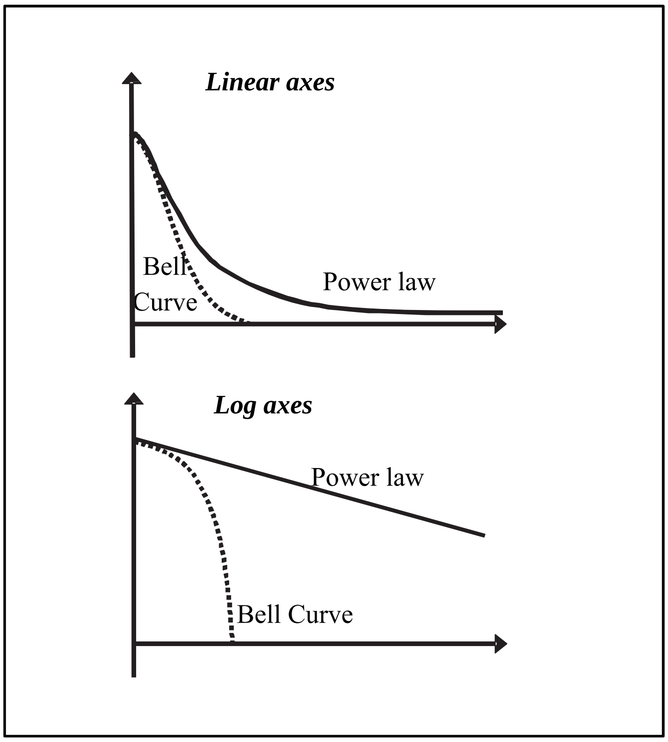

Fig. 2.1: comparison between a Gaussian and a Pareto probability distribution in linear and logarithmic scale (*Source: Andriani and McKelvey, 2011*)

Researchers have been studying the strong descriptive capability of the class of power functions for decades, not only in physical, chemical and biological systems (where so called theory of the complex systems first originated), but also in economic and social phenomena. Examples are Vilfredo Pareto's observations on the distribution of wealth (Pareto, 1897); Zipf' studies of the frequency of words (Zipf, 1949) and Mandelbrot's research on price fluctuations of cotton market (Hudson and Mandelbrot, 2004).

In the following paragraphs we will review the most relevant properties of the complex systems described by power law functions or distributions (i.e. self-organization, scalability and self-similarity, preferential attachment and small world, emergence, etc.) within the theory frameworks in which they have been studied.

2.2. The power laws in the self-organized criticality

Often, in nature, fluctuations characterize parameters which describe a lot of phenomena (i.e. the turbulence in liquids, the neuronal activity, the movement of the earth crust, etc.). These instabilities can generate the emergence of new states not tuned by any external agent (such as a magnetic field, a thermodynamic variable, etc.), occasionally showing an unpredictable behavior (see the deterministic chaos in Chapter

1). In these cases is not possible to predict the final configurations of the system, but what is more problematic is that, in some circumstances, the results of a phenomenon are important.

Let think to complex phenomena like the earthquakes, the stock market collapse or the cancer proliferation: statistical properties of these events, such as the average number of earthquakes per year or the frequency of world financial crisis, do not help in avoiding a devastating “big-one” or in losing suddenly our financial savings.

For all these reasons, great popularity gained in the last thirty years the study of the so called “self-organized criticality”, so defined by Per Bak (Bak, 1990). He carried out searches on the capability of some dynamic systems to experiment critical states, during which a self-organized configuration can occasionally evolve through an “avalanche” effect, due to minimal modifications of the external conditions. A sand pile is the classical example. When grains of sand are trickled onto the top of a pile, the slope of the heap growths and continuous, always bigger, avalanches fall downwards, until the angle of repose is reached (the self-organized stationary state), so that the addition of any single grain can generate both small or big avalanches (Fig. 2.2).

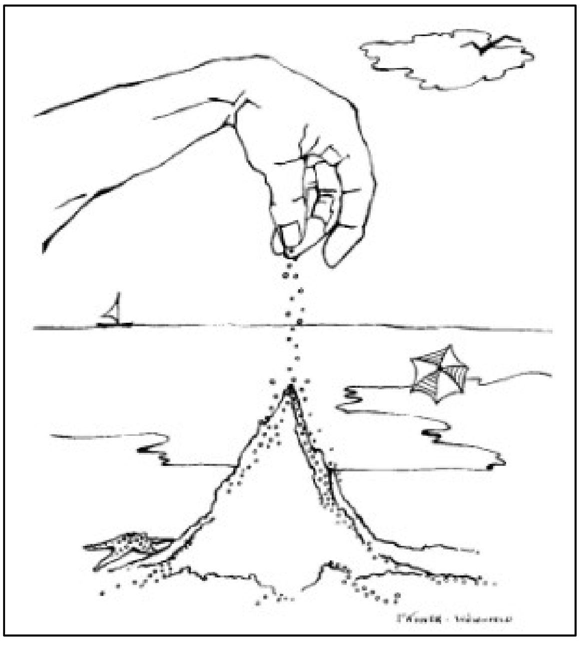

Fig. 2.2: a sand pile in the Bak’s model (*Source: Bak, 1996*)

In the same way, earthquakes generated by the sudden movement of the tectonic plates inside the planet (considered as a fluctuation of the stationary critical of the earth’s crust) propagates seismic waves up to the earth’s surface, such as an avalanche of big or small intensity in a sand pile. The Gutenberg-Richter law:

$$N(E) \approx E^{-b} \tag{3}$$

gives the number of earthquakes that release a certain energy E, being b a constant value between 1,2 and 1.5 (Gutenberg and Richter, 1956). Computer simulations revealed the chaotic behavior of the earthquakes: small changes in the initial conditions, in fact, can give rise to a large amount of effects, so that it is not possible, without knowing all the details about the system (that is unrealistic), to make predictions about its evolution. The Gutenberg-Richter law has the typical mathematical expression of many complex phenomena. It is a power law and gives us information about the frequency of earthquakes of low or high intensity (respectively with small or large value of E). Due to the fat tails of the power law distributions, it will be much more likely to have many low energy earthquakes than to experiment a "big one".

Self-organized criticality has been also found in completely different sectors such as the economy. With the divergent interests of suppliers and customers, the irrationality of people and the political changes affecting a globalized market, the world economy act in a continuous critical edge, where small fluctuations of one or more parameters can generate a financial bubble or a market crash. Chains of firm bankruptcies are also described in this framework (Fujiwara, 2004). The competition among companies, oriented to make products of better quality or sold at the best price, pushes profits toward a critical value where some companies inevitably die. This reduces the competitive pressure among the remaining enterprises, which can increase the profits again. As a consequence, new competitors will enter the market and the system moves again toward a new critical point (Aleksiejuk and Hołyst, 2001).

2.3. The self-similarity of the power laws: the fractal mathematics

In 1961, Benoit Mandelbrot[6], invited for a talk to Harvard University, wondered why his diagrams on income distribution through the society were so similar to those describing cotton market's price fluctuations on a century base, which his host, Professor Houthakker, was at that time working on. He noticed that price variations followed a power law distribution, such as the frequency of words in the Zipf' studies or the wealth distribution in Pareto's researches. The most exiting evidence, anyway, was

[6] Benoit Mandelbrot (1924-2010) was a Polish mathematician who invented the term "fractal" (as it derived from the Latin word "fractus", broken) to describe the "roughness" in the shapes of mountains and coastlines, the structures of plants, blood vessels and lungs; the clustering of galaxies, etc. He had the merit to merge the works of other mathematicians in order to turn them into an integrated powerful mathematic tool able to explaining non-smooth, "rough" objects and phenomena in the real world (Mandelbrot, 1983). His studies dealt also with other fields such as information technology, economics, and fluid dynamics.

the fact that the fluctuations of prices on daily, monthly or yearly rate fitted the same way (Hudson and Mandelbrot, 2004). Actually, the reason of this behavior was not structural, as it didn't depend on the type of data, neither on the time, but it is a specific property of the power law functions, called self-similarity or scalability. In fact, multiplying the argument of a power law

$$f(x) \sim x^{-\alpha}$$

for a constant k, the function itself is proportionally scaled as in (4)

$$f(kx) = (kx)^{-\alpha} = k^{-\alpha} f(x) \qquad (4)$$

This property has, in some cases, important implications. Let's come back to the earthquakes: although there is no way to predict an earthquake, the devastating and extremely improbable events are those which must be taken particularly into account, as they represent a risk for the people. Nevertheless, since that of Gutenberg-Richter is a scale-free law (3), it gives one the great opportunity to investigate the behavior of earthquakes of large magnitude by studying the much more frequent low intensity earthquakes, for which there is a larger availability of data.

Generally, as we have seen in the § 2.1, the power laws and their probability densities describe phenomena where variables at stake are not independent, typically the sign of an elevated degree of complexity of the system. The power laws, thus, are good clues of a not linear and scale-free dynamic. Mandelbrot, with his studies on cotton market, was one of the first scientists to consider the potential effects of the scale invariance in the economic research. First consequence of the self-similarity at different scales is that, to study the general properties of a system, it is not possible to use a reductionist approach, as the classical physicists used to do. It means that it is not possible to derive the characteristics of a complex systems as a whole, by observing the behavior of its basic components. In nature many objects and phenomena appear self-similar changing the scale: the trees, the lightning in a thunder, the lumps, some vegetables and the distribution of galaxies and clusters in the universe (Pietronero, 1987) are just few examples (Fig. 2.3).

Although so common, these structures are difficult to be mathematically represented, as it is not possible to use the traditional analytical tools. In fact, typically, a tangent can be drawn everywhere along the perimeter of a curve. A tangent is a straight line drawn to a curve in such a manner that it touches the curve at only one point.

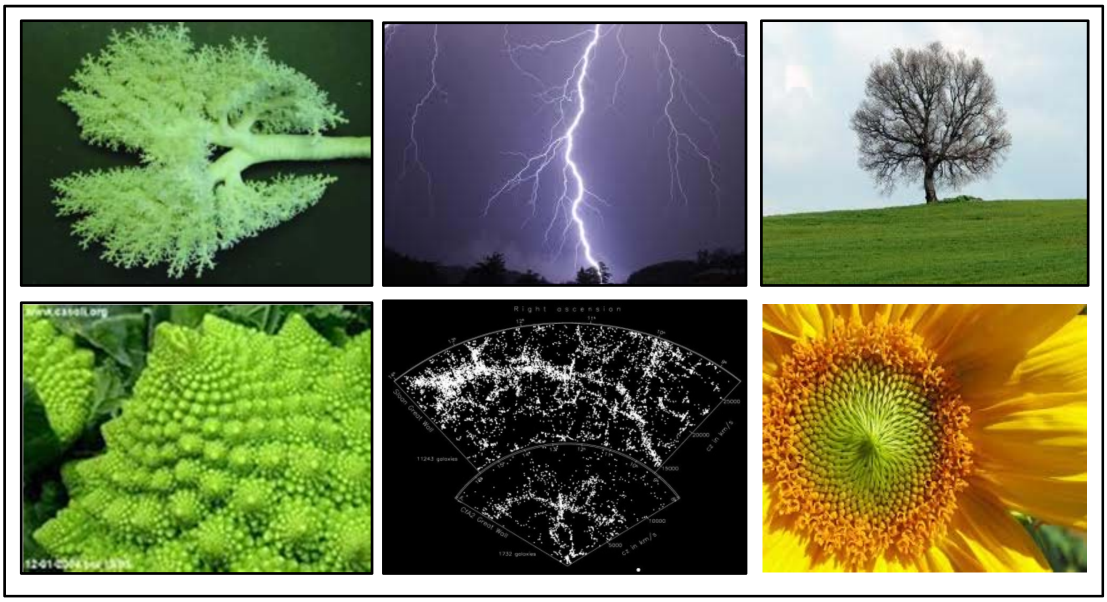

Fig. 2.3: examples of fractal structures present in nature

In scale invariant objects, each small portion of the *ensamble* has the same complexity of the entire structure, so no tangent line can be drawn anywhere along i.e. a Kock snowflake (see Fig. 2.7) in order to describe its perimeter (Pietronero, 2011).

To introduce the fractal theory, Mandelbrot used the geography, wondering how long was the coast of Britain. The answer he gave is that it depends upon how far you look at it. Measuring with a ruler, in fact, one can approximate the length of a generic curve by adding the sum of the straight lines which connect some points upon the curve. The lesser is the number of straight lines used to approximate the length of the curve, the more imprecise will be the measure. The shorter will be the straight lines the more the sum of their length will approach the curve's true length. However, a fractal curve cannot be measured in this way, as its complexity is maintained changing the scale. Whereas approximations of a smooth curve get closer and closer to a single value, as measurement precision increases, the measured length of a fractal, instead, always diverges to infinity. A coastline is an example of fractal figure: plotting the length of the ruler versus the measured length of the coastline on a log-log plot it gives a straight line (Fig. 2.4), the slope of which is the fractal dimension of the coastline, a number between 1 and 2 (Weisstein, 2015).

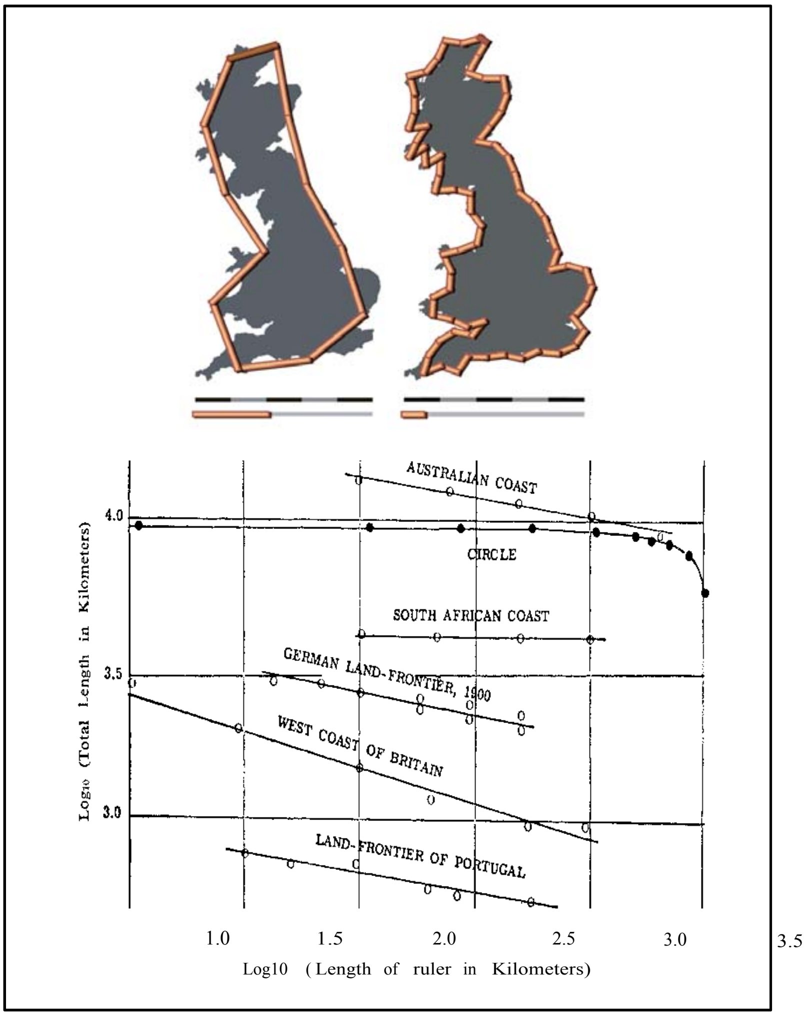

Fig. 2.4: how the measure of the coasts' length depends on the length and the number of the rulers used for its estimation, Lewis Fry Richardson first noted that the relation between length estimate and length of ruler is linear on a log-log plot. (*Source: Mandelbrot 1967*)

2.3.1. The fractal dimension

Mandelbrot's idea, starting from the observation of the coastline, was that the real complex objects around us can be represented by successive approximation, such as in the case of the coastline. To deduct a general rule, we can start dividing into sections regular figures. Let's start splitting a straight line first in two segments, then in three and in four and so on, obtaining each time N equal parts, self-similar to the entire line, so that

$$N = k^D$$

where k is the divisional factor and D is a number that, in this case, coincides with the Euclidean dimension ($D = 1$ for a line).

Then we can consider a square: dividing each side in two, three, four, etc. equal parts, we obtain a number of small squares, self-similar to the initial one, which are again $N = k^D$ but, this time, whit $D = 2$ (Euclidean dimension of a flat plane). Finally, we can repeat the exercise with a cube, having the same results with $D = 3$ (Fig. 2.5).

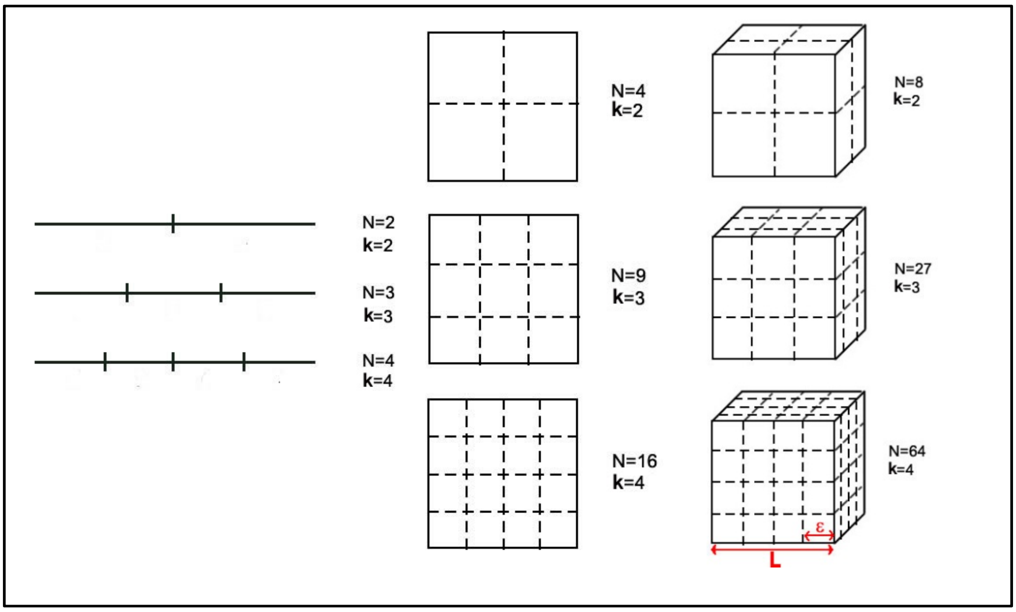

Fig. 2.5: partition of regular figures

The same method can be used also to section less regular figures. In general, the number of equal parts in which we divide a linear segment is

$$k = \frac{L}{\varepsilon}$$

where L is the total length of the segment and ε is the length of the generated sections. Similarly to the regular figures, the N equal parts produced in the process will be:

$$N = k^D = (L/\varepsilon)^D = c\,(1/\varepsilon)^D = c\,\varepsilon^{-D} \qquad (5)$$

where c is a constant that depends on the unit of measure of the length L.
The equation (5) is a particular power function, as in logarithmic scale it defines the *Hausdorff - Besicovitch* dimension

$$D = \frac{logN}{logk}$$

Mandelbrot called D “fractal dimension”, as it represents the level of roughness of the geometric shape. He introduced this notion to describe the way in which an object fills in the space, using the Kolmogorov’s concept of “capacity of a geometrical figure” (Kolmogorov, 1958). According to Mandelbrot (Mandelbrot, 1983), in fact, a fractal set is, by definition, an ensemble for which the *Hausdorff - Besicovitch* dimension strictly exceeds the topological dimension ($DHB > DT$). The understanding of this concept is quite simple. The topological dimension is always an integer natural number lower (or equal) than 3; indeed, it can be considered just like the Euclidean dimension, so that a point has $DT = 0$, a line has $DT = 1$, a flat surface has $DT = 2$ and a solid figure in the space has $DT = 3$. For regular geometric shapes, like lines, surfaces or cubes, the fractal dimension coincides with the topological dimension, because, for any divisional factor k, the ratio between the log of resulting parts and the log of k is always 1, 2 or 3 respectively (see again Fig. 2.5).

For complex and jagged shapes, the fractal dimension is a fractional value, generally different from the Euclidean dimension. In terms of capacity to fill in the space, in fact, in a 3-dimensional Euclidean space, a cube fill in all the space at its disposal, with nor holes neither hollows. Its fractal dimension is 3, just like its Euclidean dimension. But let think to a sponge in a box (that represent the 3-dimensional space in which it is). Not all the space at its disposal is filed in, because there are a lot of empty zones. So, it is easy to imagine that the sponge's fractal dimension is more than the dimension of a plane ($DT = 2$), but lesser than the dimension of a cube ($DT = 3$): indeed, it is be a fractional number between 2 and 3.

The Sierpinski gasket is another example of irregular geometrical figure. Given an equilateral triangle, the median points of each side can be considered the vertexes of a new inscribed overturned triangle (Fig. 2.6). If this is extracted, three new smaller triangles are generated, similar to the first one. The process can be iteratively repeated, each time dividing each side of a triangle in two equal parts (k=2) and giving rise to N=3 new triangles. For this reason, the Sierpinski gasket's fractal dimension is

$$D = \frac{log3}{log2} \approx 1{,}58$$

The topological dimension of the Sierpinski gasket is 1 because, iterating the triangle extraction at infinitum, it remains only a tangle of straight lines. So it can verified that the the Hausdorff - Besicovitch (fractal) dimension exceeds its topological dimension, as in the Mandelbrot definition of a fractal.

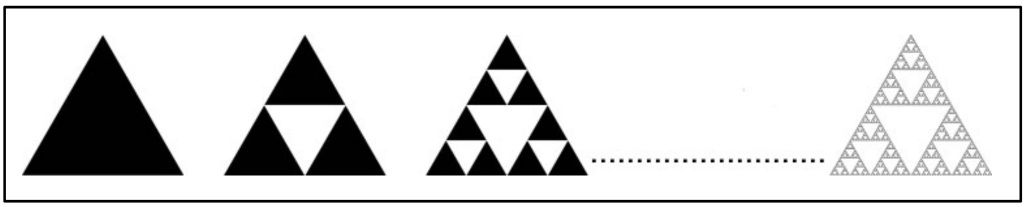

Fig.2.6: the Sierpinski gasket (*Source: Schroeder, 1991*)

The topological dimension of the Koch curve (Koch snowflake) is equal to 1 for the same reason, while its fractal dimension is

$$D = \frac{log4}{log3} \approx 1{,}26$$

as, in each iteration, the side of a triangle is divided in three equal parts (k=3) and the middle segment is replaced by two segments having the same length to generate an equilateral triangle (Fig. 2.7), so that 4 identical segments are generated on each side (N=4).

Usually the fractal dimension is a fractional, irrational number, but it can be also an integer. It is the case of a particle, that moves with a Brownian motion on a flat surface. Its trace is considered a fractal, with fractal dimension equal to 2 as, at infinitum, the particle touch all the points of the surface, such as the Peano curve.

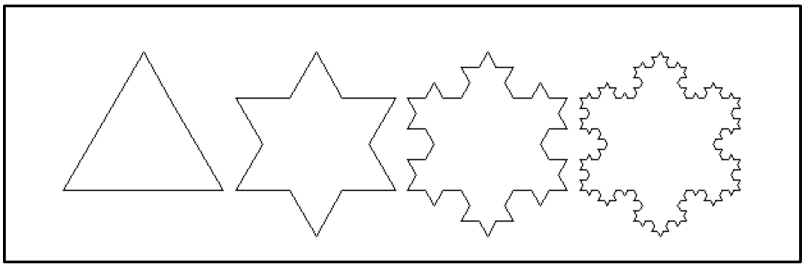

Fig. 2.7: the Koch snowflake (*Source: Schroeder, 1991*)

2.3.2. The fractal evidence of the allometric growth

In the previous paragraph we show how the self-similarity of many complex natural objects and phenomena derives from the definition of their fractal dimension, expressed by a power law (equation 5). After all, "fractality" is not a mathematic game. Fractal evidence was found in a lot of research fields. In nature, for example, a power law has been found to describe the scaling behavior of several biological parameters. The equation:

$$Y = Y_0 M^b$$

where Y_0 is a normalization constant, M is the body mass and Y is a variable (such as the metabolic rate, the blood circulation time, the life span, etc.), is known as "allometric equation". It has a fascinating property: the scale exponent b, whatever the parameter Y represents, takes on a value multiple of ¼ for a lot of biological phenomena connected with the metabolic system. For instance, the diameter of the tree trunks is proportional to $M^{3/8}$, the blood circulation time and the life span to $M^{1/4}$, the heart rate to $M^{-1/4}$ (West et al., 1997; West et al., 1999), the metabolic rate to $M^{3/4}$ (Schmidt-Nielsen, 1984). The allometric equation explains a lot of natural evidences, such as the reason for which, in larger organisms, the heart beats slower; why, for unit mass, they consume less energy (because $M^{3/4}/M = M^{-1/4}$); why smaller organisms die before.

These facts make it evident that, at least for mammals, nature has replied, at different scales, the same biological mechanism and functionalities for different animals, from mice to elephants.

But how to explain the self-similarity of the allometric law? West et al. (1999) observed that, by evolution, living beings have been selected responding to efficiency principles. It implies a balance between the maximization of metabolic performance (capability to produce energy necessary to the survival) and the minimization of distances to be covered to transport materials internally to the organisms (e.g. blood, nourishment, etc.). While the first property is directly connected to the surface area, which exchanges resources with the external environment (so it should change with the mass to the power $2/3$), the latter, instead, depends on the volume of the organism (we would expect it changes proportionally to the mass). The fractional exponent of the allometric growth reflects, therefore, the dynamic constrains inside the organisms, that are the internal fractal-like distribution networks, such as the vascular system, common to different organisms at any scale and emerging to optimize energy transport.

The parallelism between allometric growth and fractal structural development seems to be confirmed also by the studies on the shape and evolution of urban areas. Bettencourt (Bettencourt et al., 2007), on the base of the biological model for metabolic networks, have demonstrated that all the cities of a country, in the same year, scale according to an allometric power law such as:

$$Y = Y_0 N(t)^{\beta} \qquad (6)$$

where $N(t)$ is the population, at time t, and the exponent β assumes around the same value for different classes of structural or social indicators (the variable Y in equation 6). For example, they have shown that physical infrastructures like roads or electrical supply networks growth more slowly than the population of a city (β is 0,83 and 0,87 respectively); vice versa, social quantities, such as processes driven by innovation and wealth creation (patents, GDPs, bank deposits, etc.) increase faster than the urban population, being β always larger than 1 (Batty, 2008; Bettencourt et al., 2007). Fractal nature of urban agglomerates is far-back known (Batty and Longley, 1994), but it is still to demonstrate if the self-similar urbanization has a responsibility or, on the contrary, it depends on the social and economic dynamics of the cities. Anyway, theories exists that assume that cities exist due to a trade-off between benefits (i.e. income and resources) and costs (infrastructures and physical links such as streets) of agglomeration, where the first prevail given that cities continue to grow ever larger (Batty, 2013).

2.4. The power laws within the study of the complex networks

One of the most applied methodology for the investigation of complex systems is the complex network analysis. As, by definition, a complex system is composed by many different interacting constituents, a lot of scientists simulated these interactions by the edges of an intricate network, where the nodes are the single agents.

In table 2.1 we show some examples of complex networks described in literature with the correspondent application domains.

Authors	Described network	Application domain
Jeong, et al., 2000	Metabolic network	Biology-
Steyvers and Tenenbaum, 2005	Semantic network	Linguistics
Reijneveld et al., 2005	Neuronal brain physiology	Medicine
Onody and de Castro, 2004	Brazilian soccer players'	Sport
Sznajd-Weron and Sznajd, 2000	Opinion formation	Sociology
Abe and Suzuki, 2006	Earthquakes networks	Seismology
Albert et al., 1999	Internet diameter	ICT
Watts and Strogatz, 1998	Power grid of the Western USA	Energy
Fass et al., 1996	Bacon number, actors's minimal working distance from Kevin Bacon	Cinema
Saito et al., 2007	Interfirm relationships	Economy

Table 2.1: examples of complex networks reported in scientific literature

Traditionally, in the graph theory, networks with a complex topology are described by the Erdős–Rényi (ER)'s probabilistic model. It supposes that N nodes are randomly connected each other with probability p (Erdős and Rényi, 1959).

A random network, therefore, is rather homogeneous, due to the Poisson distribution of the nodal degree, that is the probability that a node has k edges (fig. 2.7).

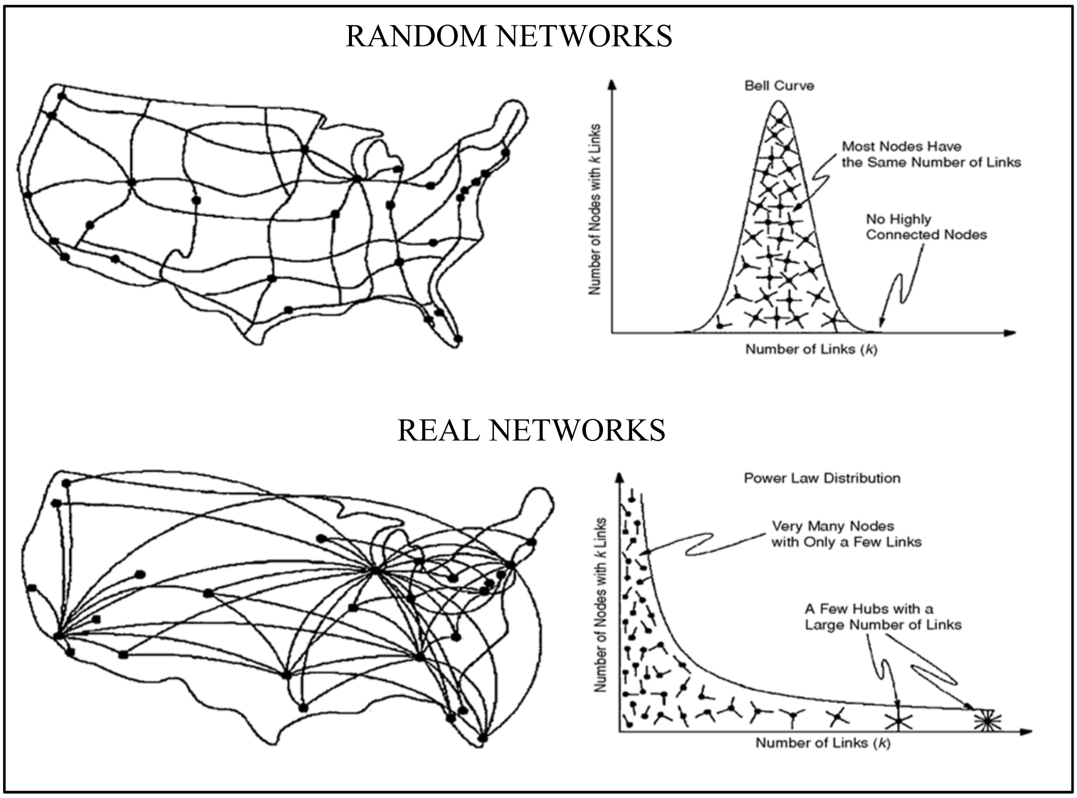

Fig. 2.7: random vs real networks (*Source: Barabási, 2007*)

Nevertheless, real networks show structural properties very different from the random graphs described by the ER model. Above all, in the real networks, the probability that a vertex is connected with k other nodes follows the power law

$$P(k) \sim k^{-\gamma} \tag{7}$$

As a consequence, a real complex network behaves as a scale-free system, where only a few nodes are highly connected (for this reason they are called “hubs”), while the most nodes have only a few connections with the others. Indeed, the fat tailed power law (7) admits that highly connected nodes (large k) have chance to occur; on the contrary, in the ER model, the vertex with large connectivity are practically absent, as the probability to find highly connected nodes decreases exponentially with k (Barabási and Albert, 1999).

In table 2.2 we show briefly the main differences between random and real networks (Braha and Bar-Yam, 2007). As observed by Barabási and Albert (1999) random networks model assume a fixed number of nodes, whereas the real networks are often characterized by a continuous addition of new vertices (i.e. new web pages in the world wide web, new electric stations in a power grid, etc.).

	Random networks	Real networks
Degree distribution	$p(k) = \frac{e^{-\lambda}\lambda^k}{k!}$ with $\lambda = N\frac{(N-1)!}{k!\,(N-1-k)!}p^k(1-p)^{N-1-k}$	$P(k) \sim k^{-\gamma}$
Average degree	$< k > \cong pN$	$< k > = \frac{\sum_{i=1}^{N} k_i}{N}$
Average distance between two nodes (path length)	$l \sim \frac{\ln(N)}{\ln < k >}$	$l = \frac{1}{N(N-1)}\sum_{i\neq j} d_{ij}$ with dij number of edges along the shortest path between the nodes i and j
Clustering coefficient	$C = p \cong \frac{< k >}{N}$	$C = \frac{1}{N}\sum_{i=1}^{N} c_i$ with $c_i = \frac{2n_i}{k_i(k_i-1)}$ ki = neighbors of vertex i n_i = edges among the k_i nodes

Table 2.2: comparison between parameters of random and real complex networks

Furthermore, while in the ER model the connections between nodes happen randomly and uniformly, real networks exhibit a *preferential attachment*. In fact, observations demonstrated that the new attaching nodes prefer to connect to nodes with a higher degree. The most evident example is the WWW, where new web pages preferably links to the most popular web sites i.e. like Google, Facebook or Twitter, but it has been also demonstrated the same behavior also for scientific publications, where there is the tendency for authors to cite other papers already very cited.

The airline map is another example of real networks, where the concept of hub, a highly connected node, is represented by the few airports that have connections with a lot of travel destinations in the same country or in the world.

The inhomogeneity of the real scale-free networks make them extremely robust against random failures or random removal of nodes. The resilience of the real networks, in fact, is due to the power low distribution of nodal degree: since there are few hubs and, instead, a large amount of weakly connected nodes, a random removal of nodes much more likely will affect the less-connected nodes, with no significant malfunctioning of the total network (Barabási, 2007). On the other hand, real networks are, for the same reason, very vulnerable to targeted attacks, with potential disastrous consequences for the network security (i.e. hacker attack targeting hubs can “switch off” strategic sections of a network).

Another property of the real networks is the *small-world* phenomenon (Watts and Strogatz, 1998), often referred to as "six degrees of separation". Although the real networks are more clustered than the random ones, they show a small path length similar to the random ER model networks, due to the long-range connections generated both in the real and in the random process. The American sociologist Stanley Milgram (Travers and Milgram, 1969) empirically demonstrated that any person in the world can reach any other individual through a short chain of social ties, which average length is approximately six (each person can be connected to any other by a knowledge chain made only of five intermediates). Many sociological researches claimed that, in a social network, members are grouped in small clusters, where each node is connected to friends and acquaintances, but it has only a few links to the rest of the world. According to Barabási, also the WWW is a small world, but with a grade of separation of about 19 (Barabási, 2007).

2.5. The "emergence" phenomenon in the swarm theory

Recently, scientists are orienting their efforts not for predicting the behavior of complex systems, which, as previously observed, can be unfeasible, but rather for taking advantage of their emergent properties such as the self-organization. Systems with many components interacting in a non-linear way, do not necessarily behave chaotically. Often, a spontaneous coordination emerges, such as in the case of the colonies of ants and termites, the flocks of birds and the swarms of bees. This is the reason for which the study of the collective intelligences (also called "swarm theory") is becoming an important branch of the complex theory within the social sciences (Helbing and Lämmer, 2008).

Swarm intelligence is based on local and decentralized interactions among the various components of the group, acting without a central control, neither any knowledge of the global pattern, where a systemic behavior emerges, which is efficient for the group and often very robust against the external attack (i.e. a fish schools able to baffle their predators).

Applications of these researches can be found in fields apparently very different. In the anthropology, i.e. they have been applied to model the collective dynamics of walking people, which self-organize in ordered lanes within a crowd. Walking patterns emerge from local interactions among people, aiming at adapting the group to the reduced availability of space (Mussaïd et al, 2010).

Other applications of the swarm theory regard the opinion formation and the so called “bandwagon effect” in politics or in consumers’ preferences. Individuals are inclined to do something because other people are doing it, regardless of their own beliefs. For example, individuals may vote for a politician because he or she is most popular and the voters want to be considered as part of the majority. In the web, this is the strategy used by common social networks for marketing scopes; a great number of “I like” under a Facebook tag is both the indicator of a collective preference and a way to catch the attention of other people, which can becomes a service to be sold to companies.

CHAPTER 3: THE EVIDENCE OF THE POWER LAWS IN SOCIAL AND ECONOMIC SCIENCES

3.1. Power laws in the organization sciences

We have already introduced, in the previous Chapter, some examples of how power law functions well describe not only natural or physical-chemical phenomena, but also economic and social events: from Mandelbrot's studies on price fluctuations of cotton market, to the Pareto's investigations on the wealth distribution, until to the firm bankruptcies and some complex networks such as the ways of opinion formation and the inter-firm relationships.

However, there are many other social and organization processes described in literature by a Pareto, scale-free function. Andriani and McKelvey (2009) have tried to classify, according to their causes, some power law distributions relevant for organization sciences presented in literature by many authors, correlating them with theories such as the allometric growth, the self-organized criticality (SOC) and the preferential attachment.

Actually, from Prigogine's studies on, power laws have been correlated to complex systems, out of equilibrium, at a critical phase (the "bifurcation" point), where non-linear interactions take place, with the potential emergence of new properties or behaviors. So the question is: how, when and why an organization show similar conditions?

First of all, we want to underline that, even if there is a great variety of researches on social and economic issues that refer to the complexity theory (Allen, 2001; Gardini et al., 2008; Schneider and Somers, 2006; Zang, 2009; Plate, 2010 are only a few examples), several of them are based on empirical and metaphoric considerations, mainly those regarding people behavior. Only some of them, mostly those concerning measurable economic parameters, instead, are treated with scientific accuracy through the use of mathematical models.

Gilstrap (2005), for example, deals with the properties of complex organizations and their dynamics by the metaphoric parallel between the mathematical concept of "attractor" (see § 1.2) and some social or economic parameters. He describes a monopolistic market as the metaphor of a fixed point attractor (a final state that a dynamical system evolves towards); the financial budget cycles of an organization as the metaphor of a periodic attractor (a set of states, so that the evolution of a system results in moving cyclically through each state); finally, the human interactions, such as a shared vision, the team processes and the information flows, as the metaphors of a strange attractor (when the system evolution through the set of possible states is chaotic).

Schneider and Somers (2006) associate the self-similarity of the complex adaptive systems to the organizational identity, where people's perception of belongingness to a social human aggregate reflects on the organizational processes.

Farazmand (2003) interprets modern organizations as open systems where dissipative structures (importing energy by the environment) have self-organizing capabilities, evolving towards new organization entities. The chaotic changes, the non-equilibrium of the economic context and the environmental feedbacks are read as producing dynamism in the organization, so that task of the leader is fostering chaotic changes in order to "shake-up" stable systems for their revitalization.

A certain level of difficulty in modeling social-economic complex systems certainly exists. As observed by Helbing (2010), the study of systems composed by people is drastically different from the study of a physical or chemical phenomenon, characterized by atoms and molecules. Humans are influenced by ungovernable parameters like morale, emotions, individual preferences, so that, some "emergent" properties are hardly ascribable to any cause or evolution of the environment.

In the following paragraphs, we review those organization and economic researchers which, in our opinion, are the most interesting investigations where the power law distributions have been detected. They are, indeed, mathematically well described, so that they are the base on which we have developed our model for the knowledge performance assessment we will present in the last Chapter of this work.

3.2. Growth of companies

For a long time, the dominant model for describing the growth dynamics of firms was the Gibrat model (Gibrat, 1931). Its basic assumptions are that the dynamic of growth of the firm size can be represented by a random walk of a variable S (the size), so that the dimensional variation between two period of time t and $t + \Delta t$ can be described by a random multiplicative process where:

$$S_{t+\Delta t} - S_t = \varepsilon_t \phi(S_t)$$

where, at different instant of time t, $\varepsilon_1, \varepsilon_2, \dots \varepsilon_t$ represent a series of small random independent, identically distributed, proportionate changes and $\phi(S_t)$ is a function of S at step t. In particular, in the Gibrat law, $\phi(S_t) = S_t$ that means that, at any step, the variation of the variable is a random proportion of the previous value of the same variable.

$$S_{t+\Delta t} - S_t = \varepsilon_t S_t \tag{8}$$

For example, at time $t = 2$ the (8) will be:

$$S_2 = S_1(\varepsilon_1 + 1) = S_0(\varepsilon_0 + 1)(\varepsilon_1 + 1)$$

so that, at the last step, at time T, we can write:

$$S_T = S_0 \prod_{t=0}^{T-1} (\varepsilon_t + 1)$$

The log of both the members will be:

$$log S_T = log S_0 + \sum_{t=0}^{T-1} log\,(\varepsilon_t + 1) \tag{9}$$

As the central limit theorem states that the distribution of the sum (or average) of a large number of independent, identically distributed variables will be approximately normal, regardless of the underlying distribution, this means that the distribution of equation (9) will be asymptotically normal, so S_T will be log-normally distributed.
Furthermore, from (8) follows that:

$$\sum_{t=1}^{n} \frac{S_{t+\Delta t} - S_t}{S_t} = \sum_{t=1}^{n} \varepsilon_t$$

and if $t >> \Delta t$ than

$$\sum_{t=1}^{n} \frac{S_{t+\Delta t} - S_t}{S_t} \sim \int_0^n \frac{dS}{S} = log\, S_n - log\, S_0 = \sum_{t=1}^{n} \varepsilon_t \tag{10}$$

From (10) derives the main consequence of the Gibrat's model: as the increase of the logarithm of firm size is equal to the sum of the random variables ε_t, than the growth rate of an enterprise is a random variable independent by its initial dimension. This means, using the Mansfield's words (1962), that "*a firm with sales of $100 million is as likely to double in size during a given period as a firm with sales of $100 thousand*".

Although the Gibrat's law was used in many models, literature reports a lot of empirical cases in which it has to be rejected (Relander, 2011). Mansfield (1962) tested the Gibrat's law showing that its validity depends on the different test specifications. First, he verified the law fails to hold in the case in which all the firms are taken into account, including those that leave the industry within the study period. The principal reason of such an evidence is that the probability that a firm will die cannot be considered independent of its size. Second, he showed the law fails also excluding those firms that leave the industry in the period. Indeed, as small and slow growing firms are more likely to exit, this will result in a nonrandom sample. Finally, he demonstrated that Gibrat's law applies, instead, for larger firms. Formally this means that firm growth rate is independent of size (so the Gibrat's law is valid) only for those firms that have surpassed the level of minimum efficient scale, as in the hypothesis of Simon and Bonini (1958). Furthermore, Mansfield revealed an inverse relationship between the variance of the growth rate and the initial size of the firms, regardless the firm's chance of death. This means that the smaller companies tend to have higher and more variable growth rates than the larger ones. Hall in 1987 investigates the dynamics of approximately 1800 firms growth, in terms of employees, in the U.S. manufacturing sector from 1972 until 1983. He confirmed the negative correlation between firm size and growth rate, already observed by Mansfield, and he found that Gibrat's law was rejected for the smaller firms while it was accepted for the larger ones, even considering other measures of size (i.e. the employees).

Some authors have analyzed the size dimension of firms and their mechanism of growth, observing entrepreneurial data of a large amount of firms in several years. In particular, Axtell (2001) verified that firms with workers between 1 and 10^6 are Zipf-distributed (with an exponent about 1.06), considering the number of employees as measure of size, as showed in Fig. 3.1.

Stanley et al. (1996) and Amaral et al. (1997), instead, demonstrated that the conditional (that means considered for groups of homogeneous companies) distribution p of the logarithm of the growth rate (in terms of sales) of US manufacturing firms is not Gaussian, as expected from the Gibrat approach, but rather exponential (with a tent-shaped form):

$$p(r|S_0) = \frac{1}{\sqrt{2}} \frac{1}{\sigma(S_0)} exp(-\frac{|r-\bar{r}(S_0)|}{\sigma(S_0)}) \tag{11}$$

where $r = log(S_1/S_0)$ and S_1 and S_0 are the sales at two consecutive years (Fig. 3.2).

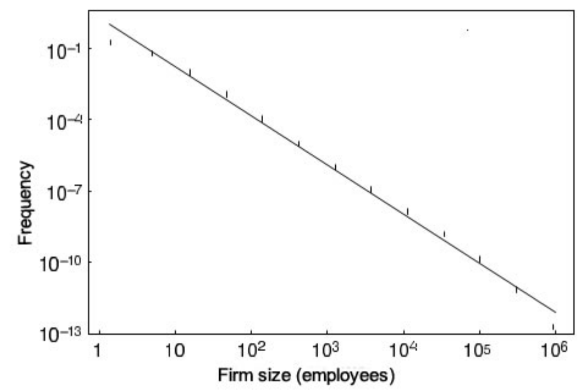

Fig. 3.1: power law evidence in the size distribution of US firms by employees (*Source: Axtell, 2001*)

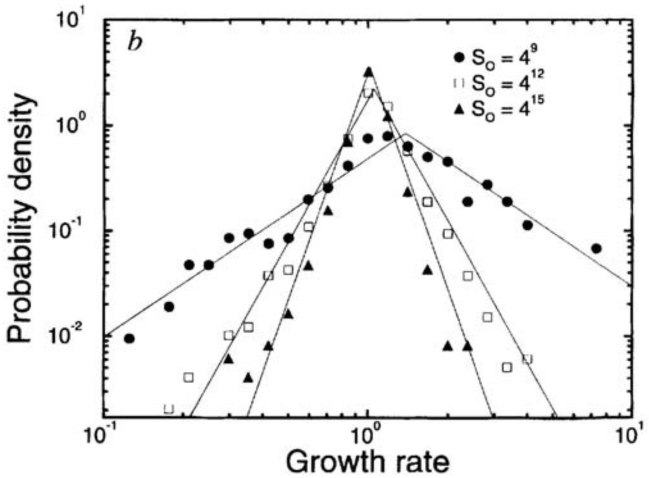

Fig. 3.2: probability density of the growth rate for all the publicly traded US manufacturing firms from 1990 to 1991 by Compustat database. Solid lines are fits to eq. 3.5 (*Source Stanley et al. 1996*)

Moreover, the authors showed that the fluctuations of the growth rates (the width of the probability density distribution) scale with firm size as the power law:

$$\sigma \propto S_0^{-\beta} \tag{12}$$

both in terms of sales and employees, with β=0,15 and β=0,16 respectively (Fig. 3.3).

This observed behavior means that largest companies have smaller fluctuations in their growth rate, so they don't experiment big change in a period of time. Results seem universal across markets (same for different SIC codes) and it is a confirmation of the Mansfield research, who had already found an inverse relationship between the variance of the growth rate and the initial size of the firms.

The fact that the distribution of growth rates has the same tent shape, even if with a width that varies inversely with a power of firm size, for all the different groups of firm's size, suggests an idea of universality. In fact, the results obtained by Stanley, Amaral and coauthors regard companies in different manufacturing sectors and the slope is quite the same if firm size is measured by the number of employees (rather than by the total annual sales).

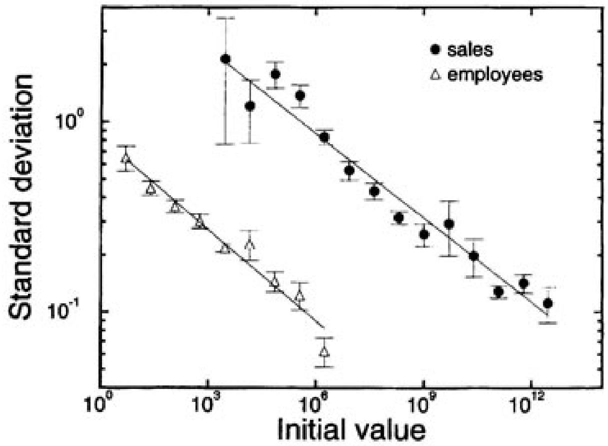

Fig. 3.3: standard deviation of the growth rate of companies in terms of sales and employees as function of the initial values. Solid lines are fits with slopes β=0,15 and β=0,16 respectively (*Source Stanley et al. 1996*)

The self-similarity of the probability density, instead, has been quantitatively proved by plotting the "scaled" probability density as a function of the opportunely "scaled" growth rate. In Fig. 3.4 the scaled probability density

$$p_{scal} = \sqrt{2}\,\sigma(S_0)p(r|S_0) \tag{13}$$

is plotted as function of the scaled growth rate

$$r_{scal} = \sqrt{2}\,\frac{r-\bar{r}\,(S_0)}{\sigma(S_0)} \tag{14}$$

As expected, with this scaling procedure, all the distribution collapse upon the same curve. A potential explanation of such a behavior could be the analogy between the statistical mechanics of physical systems, composed by many interacting components, and the hierarchical composition of companies, made up of many interrelated subunits (divisions).

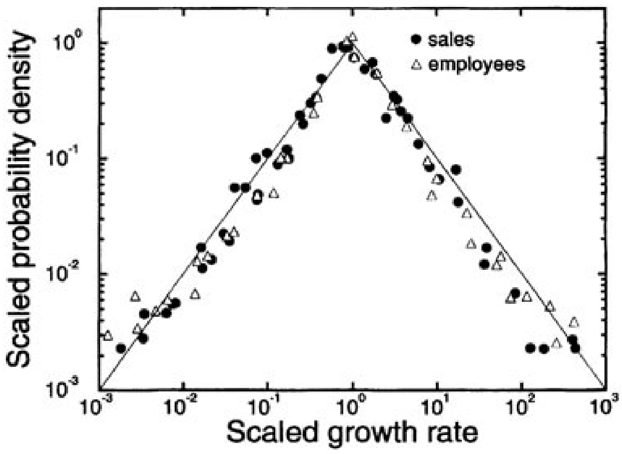

Fig. 3.4: scaled probability density as a function of the scaled growth rate (*Source Stanley et al. 1996*)

3.3. Growth of GDP

Lee et al. (1998) found that an identical tent-shaped distribution was valid for the growing and shrinking of the economies of countries. They considered the gross domestic product (GDP) of 152 countries in the period 1950–1992. While data on different GDPs are consistent with a lognormal distribution, the distribution of the logarithm of the annual growth rate $r_1 = log[GDP(t+1)/GDP(t)]$ for countries with about the same GDP has, in a given range, an exponential form as in equation (11) (Fig. 3.5a):

$$\rho(r_1|GDP) = \frac{1}{\sqrt{2}}\frac{1}{\sigma(GDP)}exp(-\frac{|r_1-\overline{r_1}(GDP)|}{\sigma(GDP)})$$

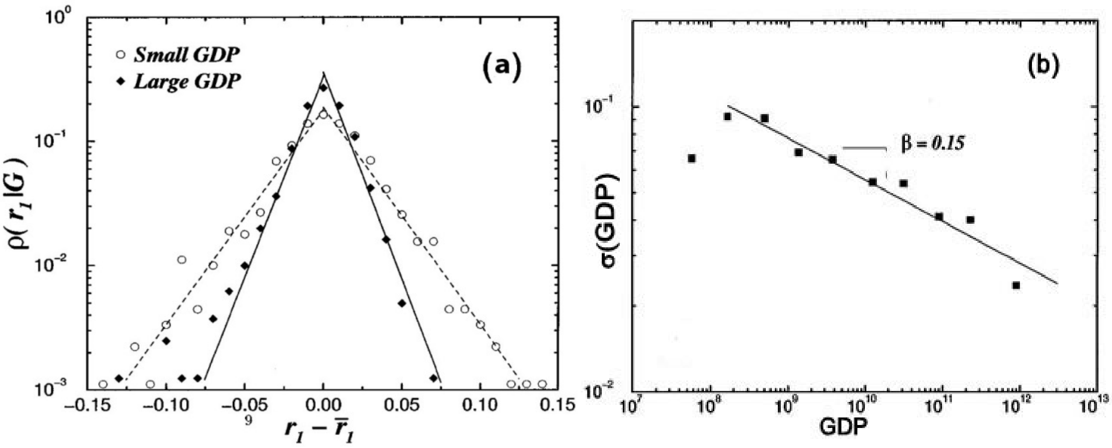

Fig. 3.5*a*: probability density function of average annual growth rate r_1 of GDP for the period 1950–1992 with an exponential fit. *b*: standard deviation of the distribution of annual growth rates as a function of GDP, with a power law fit (*Source Lee et al. 1998*)

Like in the case of firms, also for countries the width of the distributions scale as a power law of the initial GDP (Fig. 3.5b):

$$\sigma \propto GDP^{-\beta}$$

with β≈0,15.

Again, the self-similarity of the probability density is tested by plotting its scaled values

$$\rho_{scal} = \sigma(GDP)\rho(r_1|GDP)$$

versus the scaled annual growth rate $r_{1scal} = [r_1 - \overline{r_1}\,(GDP)]/\sigma(GDP)$, obtaining that all data collapse onto a single curve (Fig. 3.5 c).

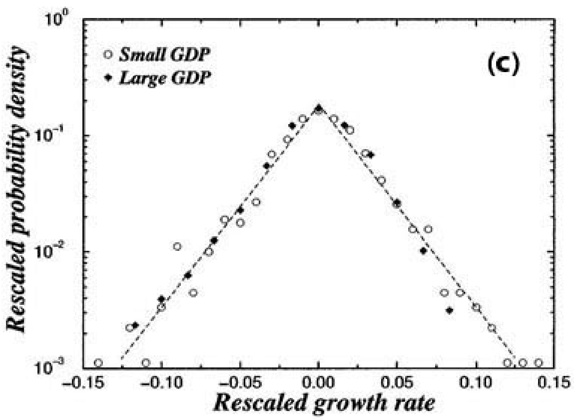

Fig. 3.5*c*: Scaled probability density as a function of the scaled growth rate of countries (*Source Lee et al. 1998*)

The observed behavior is quantitatively the same for firms and countries' economy, which suggests the hypothesis that a common mechanism might characterize the growth dynamics of economic organizations with complex internal structure. The authors, as in the case of firms, explained the growth dynamics of countries with a model for which the subunits composing the global country economy have characteristic size increasing with the size of the GDP and leading, for the standard deviation, to a an exponent β smaller than ½.

3.4. Growth of universities

For demonstrating the universality of the growth dynamics of complex organization, Plerou et al. (1999) analyzed the growth rate of hundreds American, English and Canadian universities between 17 years, in terms of R&D expenditures, number of papers published each year and number of patents granted. While the sizes result log-normally distributed, the probability distribution of the logarithm of the growth rate has the same functional tent-shaped form than in the studies on firms and countries (in Fig. 3.6a directly the scaled distributions), with a standard deviation that has a power law fitting with exponent β≈0,25 (Fig. 3.6b).

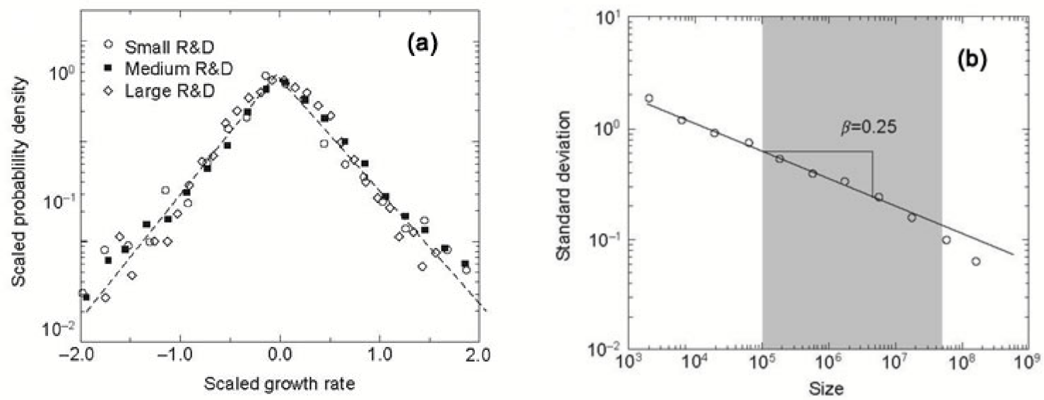

Fig.3.6*a*: scaled growth rate distribution of R&D expenditure and its standard deviation The data, by a scale factor, collapse in a single curve and have the same shape also for papers number and for patents granted. *b*: standard deviation of the annual growth rates distribution as a function of the university size (*Source: Plerou, 1999*).

The authors explain the analogy between university, firms and countries observing that research activity has a cost, which take university departments to act on the market such as a business firm or a governmental agency, offering their research capabilities to the "best offering" financial source. Moreover, the combination of peer review and government direction can be compared with the market forces (consumer evaluation and product regulation) acting on a company or a country.

Companies	Contries	Universities
β≈0,20	β≈0,15	β≈0,25

Table 3.1: comparison between the different values for the exponent β

In Table 3.1 are reported the different values of the exponent β obtained by fitting with a power law the standard deviation for firms, countries and universities' growth distribution. The differences are explained in Amaral et al. (1998) and Plerou et al. (1999) by a simulation model, where each organization (university, firm or country) is made up of many components (i.e. departments, divisions, cities, respectively), assumed as growing independently one from each other through an independent random multiplicative process with variance W^2. In this model the exponent β is inversely proportional to the width D of the distribution of the minimum sizes of the component units.

$$\beta = \frac{W}{2(W+D)} \qquad (15)$$

So, while companies and countries can have units, with dimension that can be very different case by case (with a consequent larger distribution of the minimum sizes), usually there is not such a big difference between universities (that have consequently a narrower distribution and a larger value as from 15).

3.5. An experimental confirmation of the growth model for Italian universities

On the base of the Plerou's research, we analyzed the growth rate of Italian public universities between 1997 and 2013 in terms of published papers, according to the database Web of Science (WoS) of the Institute for Scientific Information (ISI). The average yearly number of papers published by scientists affiliated to Italian universities in ISI journals is not log-normally distributed, such as for American, English and Canadian universities, but rather it is a power law of the size (Fig. 3.7). Even Liang et al. (2006), analyzing the growth dynamic of Chinese universities, found a similar results when the papers are considered as a measure of size. A possible reason of this difference respect to the Plerou's results can be the much smaller numbers of papers than in the US universities. The power law is generated when a lower bound for size is taken into account. The minimum number of papers (1 in our case, excluded by the Fig. 3.7) acts effectively like a lower bound. This bound is obviously less important for US universities with their huge numbers of papers.

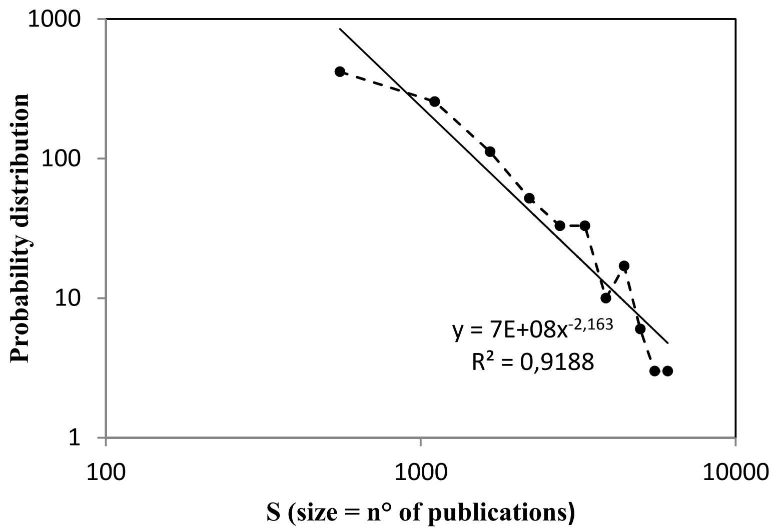

Fig.3.7: distribution of the average yearly number of papers published by Italian public universities in ISI WoS between 1997 and 2013

The distribution of the logarithm of the growth rate, instead, has the same tent-shaped form already shown by Plerou for US universities (Fig. 3.8).

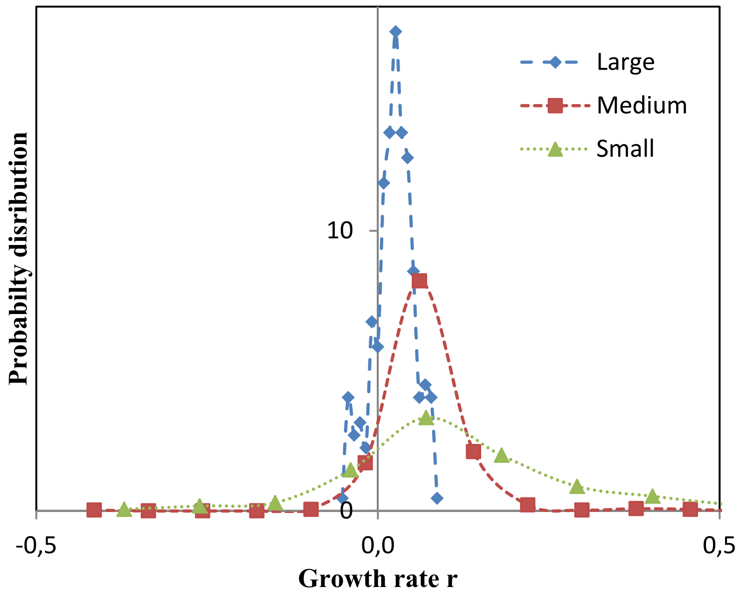

Fig. 3.8: probability distribution of the annual growth rate for three different dimensional classes of universities (S<200; 201<S<1000; 1001<S<4300 where S is the average number of publication in the period).

The results confirm that, also for Italian universities, the width of the distribution increases while decreasing the size (fig. 3.8). To confirm this evidence, we plot the conditional standard deviation of the distribution as a function of size. Fig. 3.9 shows that standard deviation follows the law:

$$\sigma = aS_0^{-\beta}$$

with a = 0,55 and β≈0,25.

The value of β is equal to that of Plerou's paper, which confirm the similar growth model for the university system.

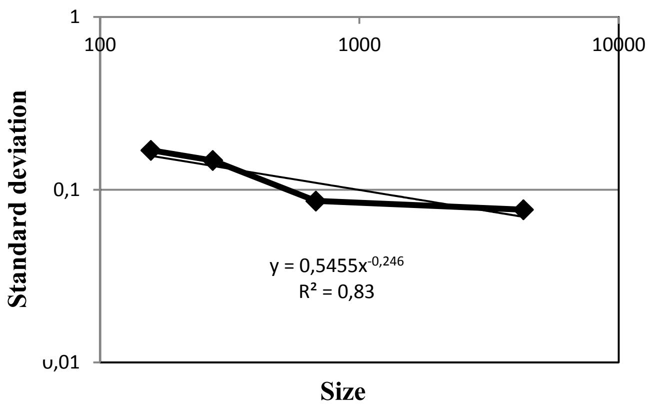

Fig. 3.9: standard deviation of the annual growth rate as a function of the size of the universities. The solid line is least-square fit to the data in logarithmic scale with slope β≈0,25.

We plot, then, the scaled conditional probability distribution $p_{scal}(r|S)$ (as in equation 13) versus the calculated scaled growth rate (as in equation 14) for the three groups of size. $\bar{r}$ and σ are, respectively, the average growth rate calculated for each group of universities (large, medium and small) and the standard deviation previously calculated. Fig. 3.10 shows as the three conditional curves collapse upon a same curve.

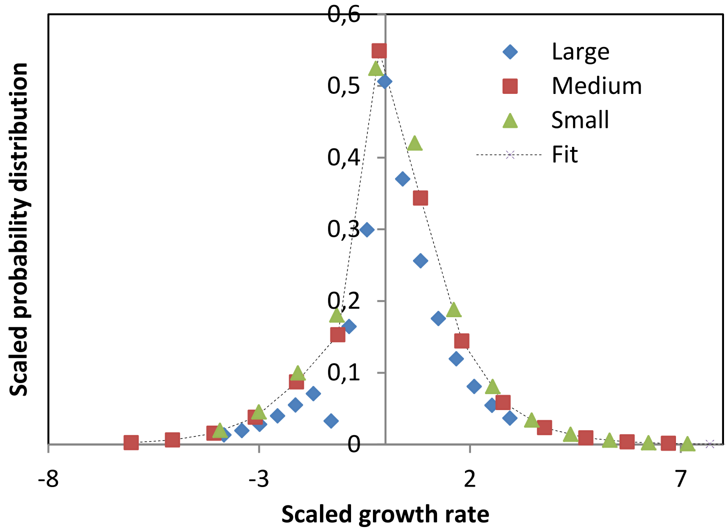

Fig. 3.10: scaled probability distribution as function of the calculated value of r_{scal}

CHAPTER 4: A CASE STUDY ON THE ASSESSMENT OF RESEARCH PRODUCTIVITY AND QUALITY

4.1. How and why measuring the university research performance

The debate concerning the development of a performance measurement system for the university sphere often centers around the difficulty of defining the proper method and most suitable performance indicators for consideration of academic institutions (Moed and Halevi, 2014). The choices of indicators and weightings are of great importance in all the research assessments, since this reflects the value judgments and priorities of the decision-maker. The metrics must be fit-for-purpose, appropriate and verifiable (European Commission, 2010).

Literature reports examples of the methodologies applied in different countries. Katharaki and Katharakis (2010), for example, use the data envelopment analysis (DEA) for the Greek case, as was previously already done by Avkiran (2001) for Australian universities, while Shin (2009) classifies the performance of Korean universities by means of Hierarchical Cluster Analysis. Phusavat et al. (2011), instead, take a statistical approach to identify the most important criteria for university classification in Thailand. Finally, Wu et al. (2012) compare the official rankings of the 12 private universities of Taiwan by a hybrid multiple-criteria decision making (MCDM) model, which weights the performance evaluation indices by the analytic hierarchy process (AHP).

Other issues of discussion are the appropriate degree of combination between peer review and bibliometric approach, such as through mixes of quantitative and qualitative indices and the level of error that can be tolerated in the process (Torres-Salinas et al., 2011).

Governments capabilities in measuring the performance of academic institutions have policy implications, which become particularly relevant given the decreasing resources dedicated to the research sector. Many governments have started to allocate annual funds on the basis of rankings that research institutions obtain in national assessment. One of the potential misrepresentation of this approach regards the affiliation of top scientists: Abramo et al. (2011a) reveal that in national systems where top-performing researchers are dispersed among many universities, the high quality of their production could be insufficient to influence the ranking of their own institutions, and the top scientists could be so penalized by the overall low performance of their home university and the resultant lower funding that the institution then receives.

Van Dalen and Klamer (2005) considered the use of citations in evaluating scientific outputs sometimes dysfunctional, when citation statistics become the sole measuring method in evaluating and rewarding investments in science. The high percentage of uncited scientific research, indeed, could bring to the worrisome conclusion that the money spent on that research has been a waste, with potential ruinous consequences both for internal competition in the academies and in the policy makers' incentive design.

4.2. The problem of "size" in the knowledge assessment

The available methods for evaluating the knowledge production of universities are often considered as unsuitable, because the measurement criteria utilized are sometimes not completely in accordance with the characteristics of different university types (Chen et al., 2010) or because there is a lack of a comprehensive performance evaluation metric (Wu, 2012).

The size of the assessed units is considered an important issue to be taken into account. The paradigm that "the higher the size of research units means higher scientific productivity" is frequently invoked to support policies of concentrating resources in larger institutes, under which small institutions tend either to join to obtain a critical mass or to disappear completely. Recent studies have investigated the relationship between the size of research units and the quality of their performance, questioning whether there is a critical mass for research, in terms of group size (Adams and Thomson, 2011). On the base of an analysis of the UK's 2008 Research Assessment Exercise (RAE), they indicate quality is indicated as a driver of scale and not vice versa, as often small and medium-sized units perform as well as or better than larger units, demonstrating that there is no significant correlation between dimension and productivity. Similarly, Morgan (2001) considers the large dimension of departments in some UK universities as a consequence, rather than a direct cause of their research excellence, due to the greater ability of the excellent universities to attract internal and external funding. On the other hand, if a research unit is of larger size it should be much more able to use productive resources and so increase productivity. Kenna and Berche (2011a) demonstrated that research quality is indeed correlated with group size: in fact, while there seems to be a size below which a group is exposed to extinction, there is also a further threshold at which the correlation between research quality and group size begins to decline.

In order to compare data among different universities or countries, normalization of outputs is often carried out. Nevertheless, normalization in some cases can generate erroneous judgments. For example, for the 2008 RAE, Kenna and Berche (2011b) criticize the normalization method, based on averaging of all the research groups, since the different disciplines may be characterized by different levels of research production. Abramo et al. (2012) instead standardize the research products according to their subject category, using scaling factors to render the distributions of the citations of articles comparable. Billaut et al. (2010) also severely criticize the normalization procedures of the "Academic ranking of World Universities", published annually by the Jiao Tong University Institute of Higher Education, since every year the authors change the normalization of the criteria, but without changing the weights of the diverse criteria in order to cope with their new normalization, with the consequence that their relative "importance" in the evaluation is rendered meaningless.

In the research policy sector, Katz (2006b) shows that common measures of R&D intensity and national wealth scale with the sizes of European countries. He demonstrates that where a performance indicator is derived from any ratio of two measures that exhibit scaling properties and scale correlation between them, then it could be that this indicator will not be properly normalized for size. However, the existence of this scaling correlation can actually help to construct of a "scale-independent indicator".

A scale-independent indicator is an indicator derived from a power law distribution or correlation. In fact, indicators that have been derived from a power law can be normalized by a scaling relationship in order to make them comparable over a wide range of sizes.

In the previous chapters we have already explained in detail how concepts like dimension, scale of representation and interaction among data are all intrinsically interconnected when a phenomenon is described by a power law. In this Chapter , we will exploit the scale-free property of power laws to propose an innovative and effective mechanism of normalization of research productivity indicators, in order to overcome the bias introduced by the size of the universities.

4.3. Solving dimensional bias in the assessment of research productivity and quality of innovation systems

An innovation system consists of all those public or private actors, institutions and organizational units that contribute directly or indirectly to the development or production of innovation. Many authors argued that the innovation systems can be considered “complex phenomena” (Schneider and Somers, 2006, Ravasz and Barabasi, 2003; Katz and Cothey, 2006a), as they are dynamic and reveal a correlation between some variables of the system (so also exhibit scaling properties), expressed by a power law (Stanley et al., 1996; Lee et al., 1998; Mandelbrot, 2001).

Like enterprises, universities can certainly be considered as innovation systems, since their primary mission is new knowledge creation. Moreover, in a university, research department personnel work together, exchange information and influence each other, thus generating large amount of not linear interactions and the emergence of complex phenomena.

As described in the previous Chapter, many studies on the growth dynamics of companies, countries and universities have exploited the scale-free property of power laws to demonstrate that firms, countries and universities shared a common mechanism of growth. Nevertheless, the various authors tend not to investigate the potential application of this kind of normalization, particularly for innovation systems. In the following paragraphs we will demonstrate how these findings can be innovatively used to normalize, by a scaling relationship, the distributions of outcomes of any knowledge productivity assessment, thus overcoming whatever dimensional biases are present in the data. In particular, we test the method on the most recent national research productivity and quality assessment of the entire Italian research system for the period of observation 2004-2010.

4.3.1. The assessment of Italian universities

The assessment of knowledge productivity and quality of public and private universities in Italy is quite young in practice and suffered of a lack of continuity. At times there has been a tendency for evaluation to generate collateral effects such as opportunistic behaviors by those to be evaluated, potentially in attempts to conform to the directions set by the assessment process (Rebora and Turri, 2010; Abramo et al., 2011; Turri, 2014). Over the past 18 years, there have been continuous reorganizations of national research management policies in an attempt to address critical issues in the efficiency and effectiveness of the Italian public research and higher education systems, going hand in hand with attempts to identify successful models of evaluation (Silvani and Sirilli, 2001).

The most recent knowledge productivity assessment exercise performed was the Italian Evaluation of Research Quality (ERQ), concerning the 2004-2010 period of research activity (ANVUR Final Report, 2013). The exercise involved the entire Italian research system, including the 14 different academic research and teaching areas defined under Italian Ministerial Decree of 29 July 2011, as present in the 93 public and private Italian universities. In the final report, a score was assigned to all the research products (articles in journals, books, chapters on books, patents, software, databases, etc.) submitted by researchers working in all the university research fields (a potential of 1,302 scores for the 14 official research areas in the 93 universities). The evaluation of the products was carried out partly by a bibliometric analysis of impact factors of the publishing journals and the citations to the publications and partly by an independent peer review of experts. The final scores for the research products were attributed considering the following criteria, with different weights:

- relevance in terms of knowledge advancement and impact;
- originality and innovation in the related sector;
- internationalization, in terms of collaborations with groups of other nations and appreciation by the scientific community.

The final score for each area was then calculated as the sum of products considered excellent, good, average or poor, minus penalty scores for failure to present sufficient products, for unassessable products and those showing apparent plagiarism.

The ERQ is an enormous and valuable source of recent and free information on the knowledge production of the Italian universities. Moreover, ERQ shows an empirical correlation between dimension of the assessed disciplinary areas (estimated in terms of number of evaluated knowledge workers) and the correspondent average scores obtained in the evaluation. In fact, the smallest university disciplinary areas are observed as present at both the top and at the bottom of the rankings, while the biggest ones are generally placed at the average. This result could be influenced by the practice of averaging: given the procedure of averaging by the number of knowledge workers, any excellent or poor performances have more opportunity for detection in a small institution than in a larger one. Moreover, when we increased the number of researchers evaluated (i.e. the larger university disciplinary areas), their average score naturally tends towards the average score of the global assessment, which is an another dimensional bias to be taken into account when considering such data sets.

4.3.2. Scaling correlation of the universities' productivity performance index

In order to overcome the impact of university size on assessment of knowledge production, we will first show how the distribution of average assessment scores for Italian universities is size correlated. We then use an approach similar to that applied in previous studies of the growth dynamics of enterprises (Stanley et al., 1996; Amaral et al., 1997; Axtell 2001), countries (Lee et al., 1998) and universities (Plerou, 1999) to normalize the data and create a new scale-free ranking.

To verify the presence of a dimensional bias in the ERQ assessment, we build an indicator V (16), generated by dividing the log of the total score obtained in the EQR by each university disciplinary area for the total number of knowledge workers in the that area, in keeping with a method often applied in university ranking systems (e.g. The Times Higher Education World University Rankings 2013-2014):

$$\mathrm{V} = \mathrm{Log(Average\ score)} = \mathrm{Log}_{10}\frac{\mathrm{v_{ij}}}{\mathrm{N_{ij}}} = \mathrm{Log}_{10}\frac{\text{Score for area j of the university i}}{\text{Number of researchers in the area}} \quad (16)$$

The number of workers active in each area is determined on the base of records maintained in the CINECA database[7]. We calculate the log of the average score to permit a direct comparison with the growth rate model described in Chapter 3.

The choice to use the score of the disciplinary areas, rather than the results for the university as a whole, is due to two main reasons: first of all, to obtain a quantity of data comparable to that of previous works on the subject (Stanley et al., 1996; Amaral et al., 1997; Lee 1998; Plerou et al., 1999); secondly, because there is evidence that research performance depends on disciplinary specialization, and thus a university could readily obtain a top position (high score) in one discipline, while performing poorly (low scores) in others (López-Illescas et al., 2011; Kivinen et al., 2013). This can happen also if the top-performing scientists are not concentrated in single institutions or disciplinary

[7] CINECA is an interuniversity consortium that supports the Italian Ministry of University and Research (MIUR) in the coordination and the monitoring of all processes and activities concerning Italian universities.

area, but they rather dispersed among the areas of all the universities, because their performance could then be insufficient to increase any one university's general performance (Abramo et al., 2011a). From this point forward, we consider the number of researchers working in a particular university disciplinary area as the size of that area.

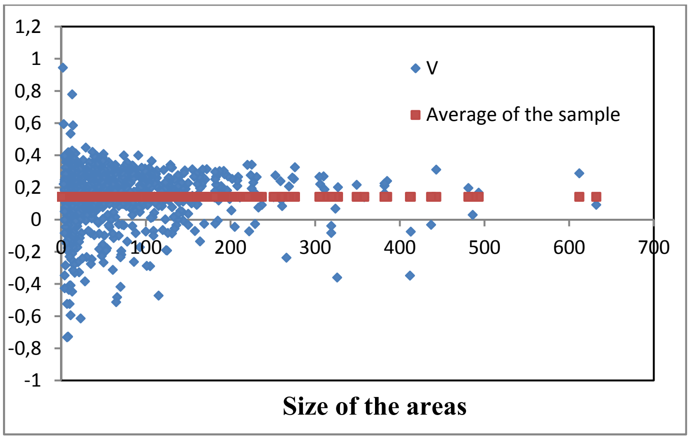

Figure 4.1: index V (16) calculated for the 14 academic research and teaching areas of all the evaluated universities (the size of each area is measured as the number of knowledge workers)

In Figure 4.1, each point represents the log of the ratio between the score attained by a university disciplinary area and the number of its knowledge workers (16). It is evident that, for the smallest sized areas, V has fluctuations with respect to the average of the sample that are larger than the fluctuations for the biggest ones. To test for any correlation between the data, we divide the university areas into three groups according to size. We considered 769 average scores, as reported in table 4.1.

The available data are effectively less than the 1,302 potential area average scores because: i) not all disciplinary areas are represented in all the universities; ii) for some of them the score is missing; iii) we consider only areas with at least one researcher and with an average score other than zero (to avoid undefined value for the $\log_{10}$).

Size of the area	Number of average scores
Large (> 80 researchers)	250
Medium (10 ÷ 79 researchers)	402
Small (<10 researchers)	117

Table 4.1: number of average scores considered in each size group

The conditional probability distribution of V is not Gaussian, but rather has an exponential tent-shaped form (Fig. 4.2), depending on the bin of size N considered, in keeping with equation (17):

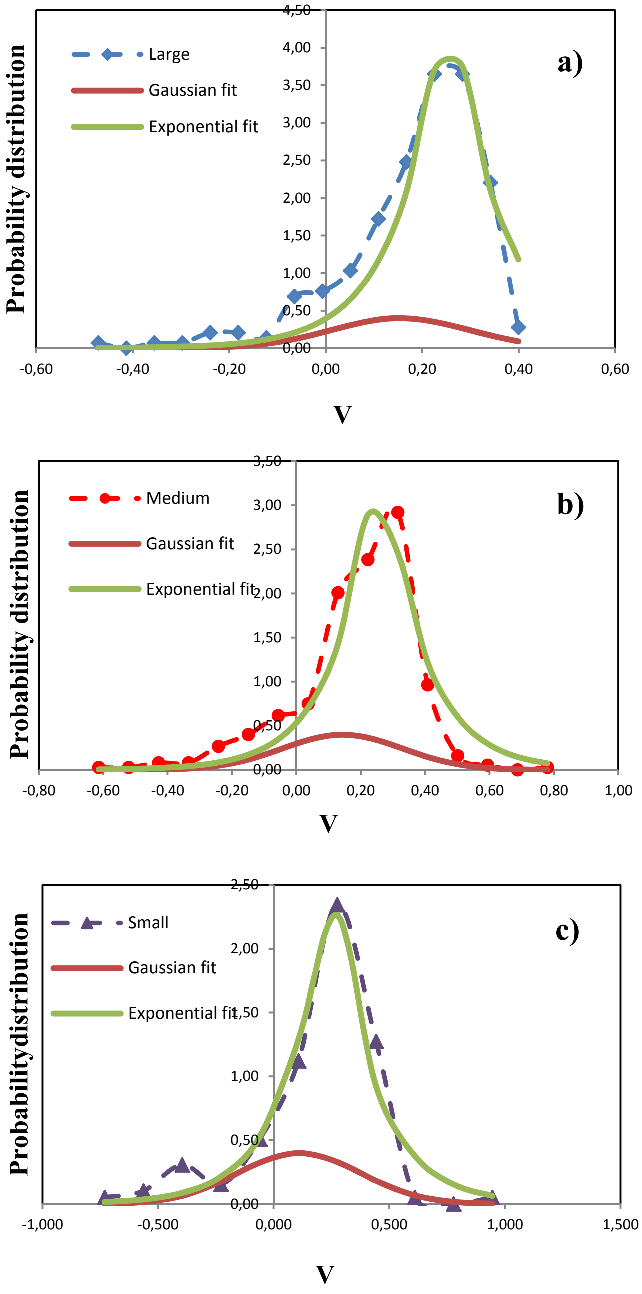

Figure 4.2: conditional probability distribution of the indicator V for the 14 academic research and teaching areas of 93 public and private universities. *Notes*: Figure 4.2a, b and c represent respectively large, medium and small university areas. Green solid lines are fits to equation (17) using mean ($\overline{v}$) and standard deviation (σ) of the conditional data set; red solid lines are Gaussian fits obtained by calculating the normal distribution of the variable z=(V-$\overline{V}$)/σ.

$$p(V|N) = \frac{1}{\sqrt{2}}\frac{1}{\sigma(N)}exp(-\frac{|V-C\overline{V}|}{\sigma(N)}) \qquad (17)$$

where C is a constant equal to 1.65, 1.8 and 2.3 (respectively) for large, medium and small university disciplinary areas.

By plotting in the same graph the conditional probability distributions for large, medium and small university disciplinary areas (Fig. 4.3) we can detect an analogy with the conditional distribution of the growth rate of enterprises, in terms of how the width of distribution depends on the classes of size (Fig. 3.2), as well as shown in the case of studies on countries and universities (Stanley et al., 1996, Lee et al., 1998 and Plerou et al. 1999).

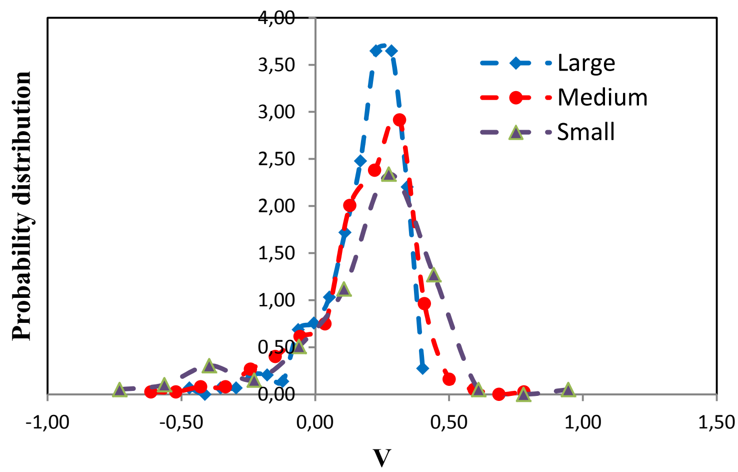

Figure 4.3: comparison of the conditional probability distributions of large, medium and small university disciplinary areas: it is evident that the width of the distributions increases with the decrease of the size of the areas, as in the growth rate model of firms, countries and universities

Figure 4.3 demonstrates that the width of the distributions (standard deviation σ) increases when size N of the disciplinary areas decreases. Thus, we find that $\sigma(N)$ is well approximated by a power law over roughly three order of disciplinary area magnitude (from about 10 to 10^3 researchers), as expressed in (18):

$$\sigma(N) = aN^{-\beta} \qquad (18)$$

with $a \approx 0{,}36$ and $\beta = 0{,}13 \pm 0{,}08$ (Fig. 4.4).

Continuing the analogy with the growth rate model, we now plot the scaled conditional probability distribution $p_{scal}(V|N)$ (19) versus the calculated scaled average score (20) for the three groupings of university areas by size.

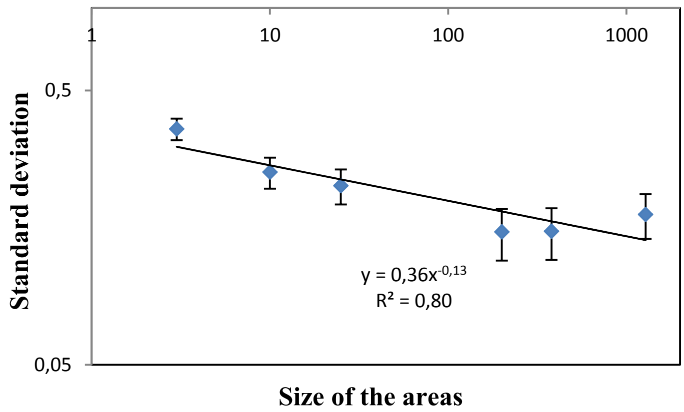

Figure 4.4: standard deviation of the indicator V as a function of the size of the areas: the solid line is least-square fit to the data in logarithmic scale with slope $\beta = 0{,}13 \pm 0{,}08$. The figure also shows standard the error bars for the standard deviation.

$\bar{V}$ and σ are respectively the average value calculated for the group of university disciplinary areas (large, medium and small) and the standard deviation calculated in (18):

$$V_{scal} = \sqrt{2} * \frac{(V - C\bar{V})}{\sigma(N)} \tag{19}$$

$$p_{scal}(V|N) = \frac{1}{\sqrt{2}} \frac{1}{\sigma(N)} exp(-|V_{scal}|) \tag{20}$$

In confirmation that the scaling procedure introduced under the growth rate model can also be applied to our current application, Fig. 4.5 shows that the conditional curves in fact collapse on a single curve, when the equation (4.5) is calculated for each of the three groups of disciplinary areas with the correspondent values of $\bar{V}$ and σ.

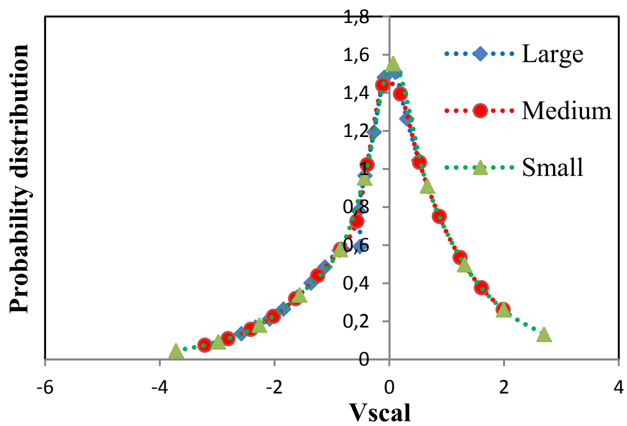

Figure 4.5: scaled probability distribution (20) as a function of the calculated value of V_{scal} (19)

By comparing Fig. 4.5 with Fig. 3.4, Fig. 3.5c and Fig. 3.6a, a common normalization procedure can be derived. Our observations have confirmed that the fluctuations in the probability distribution of the indicator V are correlated to university size by a power law (18), in the same way that the growth rate distribution of firms, countries and universities are correlated with the dimension of the related entities. Therefore the findings of the previous studies on the growth rates of firms, countries and universities can be applied in an innovative manner to exploit the scaling relationship between the probability distributions of the index V in Italian universities, in order to normalize the data and thus deal with the dimensional bias in assessments of knowledge productivity and research quality, as observed by the Italian EQR.

Opportunely scaling the index V, we can attribute to all the probability distributions related to the different size groups of the university areas the same level of fluctuations, which are a potential source of a distortion in the assessment of results.

4.3.3. A scale-independent rank of performance

In order to construct a scale-adjusted ranking of performance, we calculated V_{scal} for all our initial data, applying equation (19). We use the values of constant C as obtained in the equation (17) and the value for σ(N) as calculated according to equation (18). In Fig. 4.6 we show the original values of the index V of the 14 academic research and teaching areas of all the universities evaluated (Fig. 4.6a) and the corresponding values obtained after the application of the method for scale adjustment of V (Fig. 4.6b).

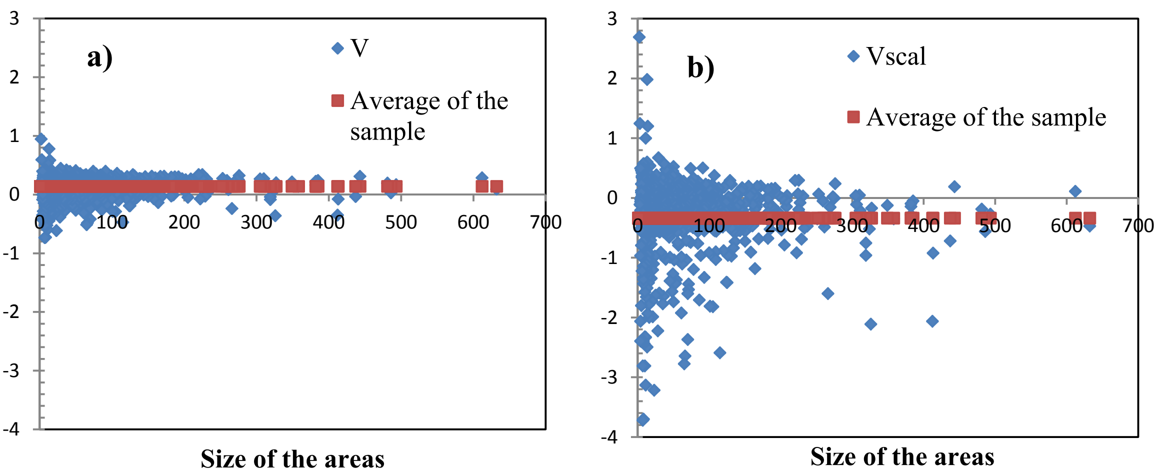

Figure 4.6a: original index V for the 14 academic research and teaching areas of all the evaluated universities. b) Scaled value of V according to the growth rate model.

The two set of data appear self-similar at different scale, also termed in mathematical terminology as showing "fractal" behavior. This outcome is a consequence of the equation (16) and (19), as V and V_{scal} are correlated through a power function expressed by the standard deviation. Furthermore, the proposed method magnifies the relative distances among the rankings of the different areas, especially in the case of the small and the medium sized areas, but maintains the position of the outliers, which express of the best and worst performing areas.

In order to detect the impact of the scaling method on the rankings and its relationship with the size of the areas, we plot the variation of the index V versus the dimension of the areas.

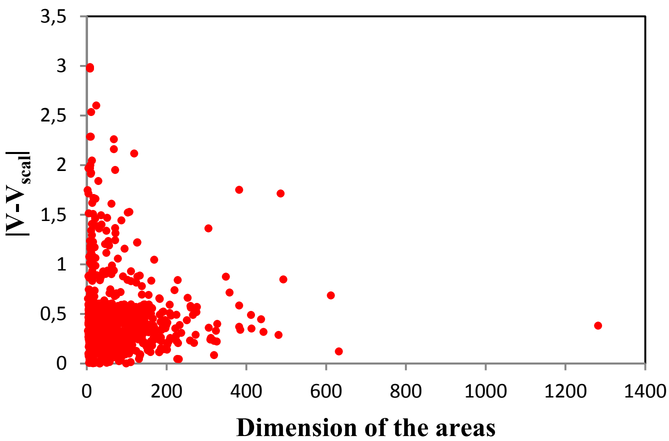

Figure 4.7: impact of application of the scaling relationship on index V versus the size of the disciplinary areas

Figure 4.7 shows that the main differences between V and $Vscal$ are for small and medium size areas. To quantify how much the application of the scaling relationship impacts the final rankings, we calculate the probability density of the difference between the rank position of each area according to the index V and that based on the scaled value of V (Δ_{rank}).

In Figure 4.8 we demonstrate that the medium-sized areas are those with the greatest tendency to lose positions (negative values of Δ_{rank}), while large and small universities areas experience increases of mainly moderate extend.

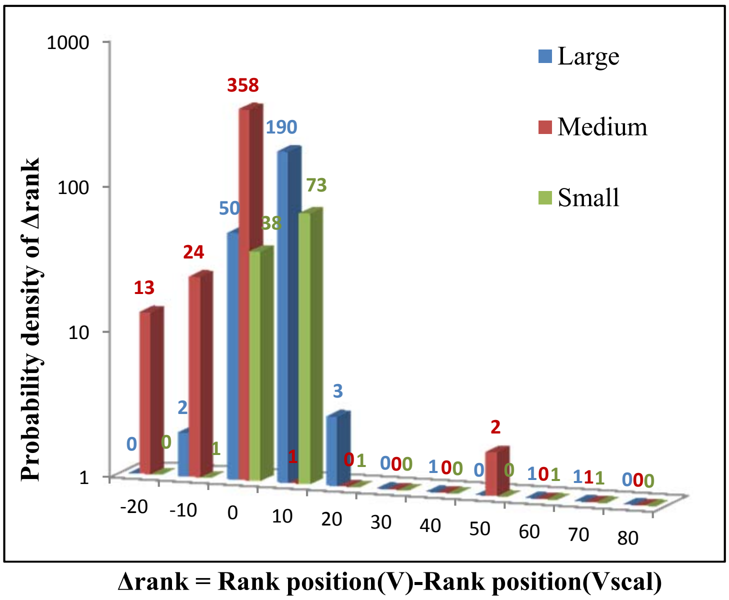

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Figure 4.8: probability density of the change of position in the ranking tables, after the application of the scaling relationship on the index V. Medium-sized university areas are the ones that lose the most positions. Large and small areas are instead only moderately affected by the scaling

In the scaling process, the most of changes for the rank position happens for a particular range of $|V - V_{scal}|$ between 0,2 and 0,6 (Fig. 4.9). These are the rank positions for which the differences between before and after the application of the scaling relationship are most significant.

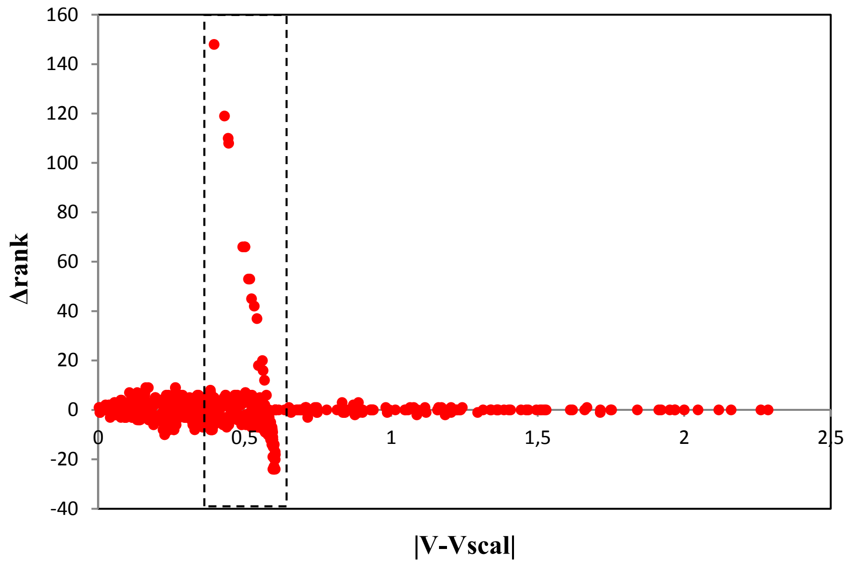

Figure 4.9: evidence that only a particular range of $|V - V_{scal}|$ generates significant changes in the rank position of the university areas.

Considering only the university areas included in this range, we next examine those with the maximum values for difference $|V - V_{scal}|$.

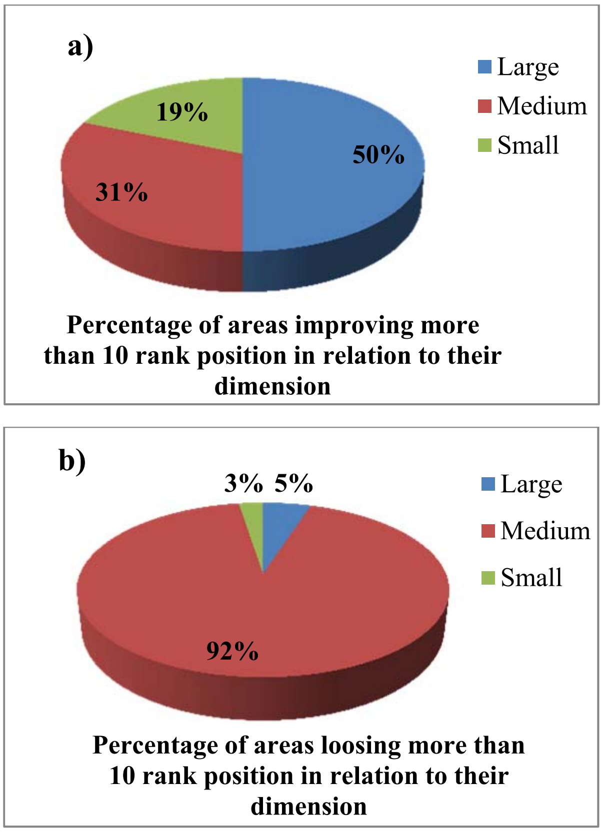

Figure 4.10: percentage of university areas improving or losing more than 10 rank positions

Notes: in a) the percentage of university areas that improve their rank position by more than 10 positions (total of 16 areas, gaining between 12 and 148 positions) in relation to their dimension, considering only those areas that fall in range of $| V - V_{scal}|$ (highlighted in Fig. 4.9). In b) the percentage of areas losing more than 10 rank positions (total of 40 areas, losing between 10 and 24 lost positions) in relation of their dimension, considering only the university areas falling in range of $| V - V_{scal}|$.

We find 16 areas with an improvement of more than 10 rank position (between 12 and 148 positions gained), of which 50% are large areas, 31% medium and 19% are small areas (Fig. 4.10a). On the other hand, the group also includes 40 university areas that lose more than 10 positions (between 10 and 24 positions), of which 5% are large, 92% are medium and 3% are small areas (Fig. 4.10b).

We can conclude that the scaling relationship has more impact on the medium-sized and large university areas: the first are the most penalized by the scaling procedure while the latter gain the most advantages.

Given the definition of the index V itself (16), in the case in which scores at the numerator are equal, than the small areas obtain a bigger V compared to large areas, which are more affected by the average process. However in the application of (16), it is the medium areas that are potentially those that change their original score the least, and for this reason are likely the most affected by the scaling process.

The proposed normalization method has three main advantages. Firstly, it magnifies the relative distances among the rankings of the areas, especially in the case of the smallest and the medium sized areas (fig. 4.6a and 4.6b), thus permitting the evaluators to better distinguish among the different performances of the university areas; on the other hand, the best and worst performer areas maintain their respective leader or tail-end positions. This aspect is particularly relevant for the fairer allocation of public funding on the basis of research assessment exercises. A second advantage consists in the capability of the method to balance an assessment that is evidently too skewed in favor of the smaller university areas. Indeed, top and worst research performers have less opportunity to emerge in large areas (where they are lost in the average), while they could be inopportunely exalted in small areas. The third, but not least, advantage of the proposed method is its transferability to other sectors. We would expect that dimensional bias could similarly affect also other innovation systems such as enterprises, research clusters, research institutions, etc. The proposed method can thus also tested on other national knowledge assessments and verified using different knowledge indicators, as long as a power law dependency is observed in the data.

4.4. How to apply the method to knowledge performance indicators

It is worth noting that the method presented can be easily applied to other knowledge performance indicators, other than V (16). The method is intended to be generically applicable to any assessments of knowledge productivity and quality, whenever the size of the monitored organizations becomes relevant in the assessment process. Fig. 4.11 represents the methodological steps to be followed in order to normalize any knowledge performance indicator affected by dimensional bias, according the proposed method. First of all, it is necessary verify whether the frequency distribution of the performance indicator under consideration is size sensitive (dividing the organization in classes by dimension).

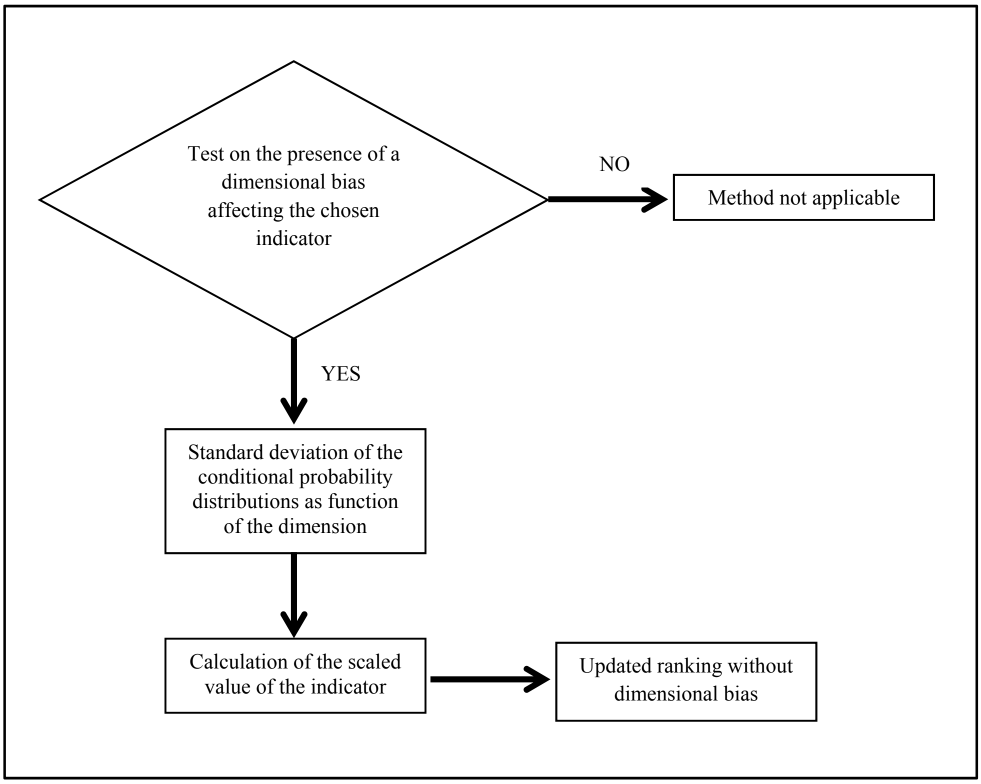

Figure 4.11: Schema for the application of the proposed normalization method to any knowledge performance indicator.

If it is the case, then, the standard deviation of the probability distributions must be calculated as function of the dimension (18), to verify that there is a power law of size. The scaled value of the indicator under consideration can then be calculated according to (19) and the conditional distributions plotted according to (20): the curves should now collapse on a single curve as in Fig. 4.5, demonstrating the success of the normalization procedure. The scaled value of the indicator of interest can then be used to modify any previously calculated rankings, eliminating the dimensional bias.

CONCLUSIONS

The capacity to measure knowledge productivity is a critical factor in countries' competitiveness, since the advancement of technological innovation and ultimately of national economic development depends on effective knowledge management (Chen et al., 2009). Although in the last two decades there has been some diversification in the locations of knowledge production, public institutions still play a central role in generating new knowledge and spillovers, especially towards the private sector. Research-focused universities remain the main source of tacit knowledge, human capital and research results that are potentially transferable to innovative companies (Audretsch et al., 2012).

In recent years the evaluation of knowledge productivity and research quality in the universities has become a key aspect of the management of national research system, with an increasing share of public funding being based on national assessment exercises. However the specificity of each country and the different characteristics of their academic systems make very complex to identify a general formula for application in the evaluation exercises of all nations. Furthermore, the methods used to assess the knowledge production of universities are sometimes affected by limits and biases, as extensively revealed in the scientific literature.

Understanding the relationships between research, innovation and economic development requires analysis of the practices of knowledge production and distribution. Theoretical models, such as the Triple Helix concept, reveal the complex dynamics that characterize a knowledge-based economy. Given such complexity, the university itself can be considered as a "system of innovation", consisting of a huge number of collaborations among researchers of different countries, various institutions and numerous competences (Etzkowitz and Leydesdorff, 2000; Leydesdorff, 2010).

Such a system can be characterized by complex dynamics of development, due to varying policy aims, technological evolution, social modifications and market forces. Nevertheless, especially for publicly-funded universities, a return contribution to society is always expected. This implies clear definitions of the impact of university research and the way such impact is to be measured (Martin, 2011).

In order to deal with the complex nature of university and the resulting difficulties in measuring its performance in the production of new knowledge, in this thesis we use concepts borrowed from the theory of the complex systems. Many studies reviewed in this work, indeed, suggested that power laws, Pareto distributions, fractals and scale-free theories concern not only natural, chemical and physical issues, but also social and organizational research. Several reasons are at the base of this consideration: the ICT diffusion, with the consequent always higher level of connectivity between individuals, as well as the market globalization and the facility of information exchange make people interdependent and exposed to recurrent feedbacks by the surroundings and to non-linear chaotic consequences of their actions. Standard Gaussian econometric approach often consider extreme events like irregularities to be ignored, while long tails Pareto distributions are actually more fitting for describing a lot of real situations. In such a case, normal statistics is not applicable any longer, as dynamics of interconnected events have mean and variance unstable, so useless to describe a lot of complex social phenomena. By consequence, researchers often fail in finding an appropriate tool for representing social and organizational problems, governed by humans behaviour and individual choices, and they frequently only rely on metaphors and parallelisms with the natural and physical world. We tried to face this difficulty, pinpointing those researches, which deal with economic and managerial complex problems by a more “robust” mathematical approach, in order to partially fill up the gap between description of social phenomena and provision of managerial/decision-makers solutions. Among the complexity theory concepts, one of the most easy to model is the scale free property, due to the formal representation of the power law functions. For this reason, we decide to exploit this feature for managerial aims, identifying a field of possible application.

In particular, we investigate the impact of the university size on the knowledge production, focusing our attention on the bias introduced in evaluation exercise outputs as a result of varying university size. Since we verify the existence of a correlation between a specific, *ad hoc* created performance indicator and the dimension of the assessed disciplinary areas, observed in the Italian case, we provide a method based on the scale-free property of the power laws, in order to overcome the dimensional bias in current methodologies of knowledge production assessment.

Previous research on the growth dynamics of other complex entities like enterprises, countries and universities has already use the scale-free property of power laws, but only to demonstrate that such organizations shared a common mechanism of growth, independent of the scale of the observed parameters.

In our work, we first verify the model applying it also to Italian university context, where we found a broadly correspondence with the literature results.

Then, as case study, we take advantage of the above mentioned growth rate model to provide a method able to reduce the dimensional bias in the knowledge production assessments of universities. In particular, we construct a scaled performance indicator that permits the normalization of the evaluation's "average score" distributions and makes different dimensioned university disciplinary areas comparable over a wide range of size.

As demonstrated by our case study for the Italian context, the method provides a solution for the dimensional bias that often occurs in such research knowledge assessments. The relevance of the method becomes more notable in view of the ever increasing share of public funding currently being allocated on the basis of periodic research assessments.

However, the proposed normalization process is intended to be generically applicable in any assessments of knowledge productivity or quality, wherever the size of the monitored organizations becomes relevant and dimensional bias has the potential to undermine the results of the evaluation. The described method could be applied to the research quality assessments of different countries in a worldwide context, although such applications must necessarily be accompanied by a deeper analysis of the various national evaluation systems, to consider the contingent specificities of the different cases. A comparison among many countries would allow verification of whether the dimensional issue has similar impact on different assessment systems, representing an issue to be addressed in future research and problem-solving.

Finally, the generality of the approach means that it can be gainfully applied to other innovation systems such as enterprises or other type of research centers, for the identification of dimensional bias in the assessment of other systems of knowledge creation.

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